Dinner Organisation

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1 Modeling the Problem

1.1 Parameters

- $N = 10$: number of participants.
- $H = 5$: number of houses, each occupied by one couple.
- $K = 5$: number of courses.
- Each house can host at most 5 participants per course.
- Each couple must host exactly 2 courses.
- Every course is served in two houses simultaneously.
- Guest distribution: in each course, each hosting couple has exactly 3 guests.

1.2 Decision Variables

- Integer variable
$$\text{attend}_{p,k} \in \{0, 1, 2, 3, 4\}, \quad p = 0, \ldots, 9,\ k = 0, \ldots, 4,$$
indicating the house where participant p attends course k.
- Boolean variable
$$\text{serve}_{h,k} \in \{\text{False}, \text{True}\}, \quad h = 0, \ldots, 4,\ k = 0, \ldots, 4,$$
indicating whether house h hosts course k.
- Auxiliary integer variable
$$\text{meet}_{p,q} \in \{0, \ldots, 5\}, \quad 0 \leq p < q \leq 9,$$
counting the number of courses where participants p and q attend the same house:
$$\text{meet}_{p,q} = \sum_{k=0}^{4} [\text{attend}_{p,k} = \text{attend}_{q,k}].$$

1.3 Constraints

Course hosting.

$$\forall k: \quad \sum_{h=0}^{4} \text{If}(\text{serve}_{h,k}, 1, 0) = 2, \qquad \forall h: \quad \sum_{k=0}^{4} \text{If}(\text{serve}_{h,k}, 1, 0) = 2$$

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Couples attend their own house when hosting. For house h, let the couple be participants $2h$ and $2h+1$:

$$\forall h, k : \quad \text{serve}_{h,k} \Rightarrow (\text{attend}_{2h,k} = h \land \text{attend}_{2h+1,k} = h)$$

Non-hosting participants attend exactly one house per course.

$$\forall p, k : \quad \sum_{h=0}^{4} [\text{attend}_{p,k} = h] = 1$$

Each serving house has exactly 5 participants (couple + 3 guests).

$$\forall h, k : \quad \sum_{p=0}^{9} [\text{attend}_{p,k} = h] = \text{If}(\text{serve}_{h,k}, 5, 0)$$

Meeting counts. For each pair $p \neq q$:

$$\text{meet}_{p,q} = \sum_{k=0}^{4} [\text{attend}_{p,k} = \text{attend}_{q,k}]$$

Meeting frequency constraints.

- property 1: $\text{meet}_{p,q} \geq 1$
- property 2: $\text{meet}_{p,q} \leq 3$

Couples never meet outside their own house (property 3).

$$\forall h, k : \quad (\text{attend}_{2h,k} = \text{attend}_{2h+1,k}) \Rightarrow (\text{attend}_{2h,k} = h)$$

Distinct guests per house (property 4). For house h hosting courses k_1 and k_2, the guest sets must be disjoint:

$$\forall h, i \notin \{2h, 2h+1\} : \quad \sum_{t \in \{k_1, k_2\}} [\text{attend}_{i,t} = h] \leq 1$$

2 Solver Implementation using Z3

- Integer variables $\text{attend}_{p,k}$ encode participant attendance at houses during courses.
- Boolean variables $\text{serve}_{h,k}$ encode which houses host each course.
- Couples are fixed in their own house during hosting.
- Attendance constraints ensure each serving house has exactly 5 participants (couple + 3 guests).
- Each couple hosts exactly 2 courses, and each course is served in exactly 2 houses.
- Part (a):
  - Require each two people meet at least once.
  - Require couples never meet outside or guests are distinctive, but not both.
- Part (b):
  - Require each pair meets at most 3 times.
  - Couples never meet outside.
  - Guests are distinctive per house.

3 Results Part (a)

3.1 Schedule

Table 1: Dinner schedule for Part (a) - Scenario 1 (Property 1 + Property 3)

Course	Hosting Houses	Guest Distribution
0	1, 2	1: [0, 2, 3, 6, 9], 2: [1, 4, 5, 7, 8]
1	0, 1	0: [0, 1, 5, 7, 9], 1: [2, 3, 4, 6, 8]
2	0, 2	0: [0, 1, 2, 7, 8], 2: [3, 4, 5, 6, 9]
3	3, 4	3: [0, 2, 4, 6, 7], 4: [1, 3, 5, 8, 9]
4	3, 4	3: [1, 3, 4, 6, 7], 4: [0, 2, 5, 8, 9]

Table 2: Dinner schedule for Part (a) - Scenario 2 (Property 1 + Property 4)

Course	Hosting Houses	Guest Distribution
0	2, 3	2: [1, 4, 5, 8, 9], 3: [0, 2, 3, 6, 7]
1	0, 4	0: [0, 1, 2, 5, 7], 4: [3, 4, 6, 8, 9]
2	2, 4	2: [3, 4, 5, 6, 7], 4: [0, 1, 2, 8, 9]
3	1, 3	1: [0, 1, 2, 3, 4], 3: [5, 6, 7, 8, 9]
4	0, 1	0: [0, 1, 4, 6, 8], 1: [2, 3, 5, 7, 9]

Table 3: Dinner schedule for Part (a) - Scenario 3 (Property 1 + Property 3 + Property 4)

Course	Hosting Houses	Guest Distribution
No results		

3.2 Solver Performance and Optimization Results

Table 4: Optimization results summary - Scenario 1

Metric	Value
Feasible schedule found	Yes
Solver runtime (s)	0.0405
Solver status	sat

Table 5: Optimization results summary - Scenario 2

Metric	Value
Feasible schedule found	Yes
Solver runtime (s)	0.0189
Solver status	sat

Table 6: Optimization results summary - Scenario 3

Metric	Value
Feasible schedule found	No
Solver runtime (s)	4.7659
Solver status	unsat

- Scenario 1 confirms that a schedule satisfying Property 1 and Property 3 exists.
- Scenario 2 confirms that a schedule satisfying Property 1 and Property 4 exists.
- Scenario 3 confirms that satisfying Properties 1, 3, and 4 simultaneously is impossible (expected UNSAT).

4 Results Part (b)

4.1 Schedule

Table 7: Dinner schedule for Part (b) (template: fill with solver output)

Course	Hosting Houses	Guest Distribution
0	1, 4	1: [1, 2, 3, 4, 6], 4: [0, 5, 7, 8, 9]
1	1, 4	1: [0, 2, 3, 5, 7], 4: [1, 4, 6, 8, 9]
2	2, 3	2: [0, 2, 4, 5, 8], 3: [1, 3, 6, 7, 9]
3	0, 3	0: [0, 1, 3, 4, 9], 3: [2, 5, 6, 7, 8]
4	0, 2	0: [0, 1, 2, 7, 8], 2: [3, 4, 5, 6, 9]

4.2 Solver Performance and Optimization Results

Table 8: Optimization results summary (Part b)

Metric	Value
Feasible schedule found	Yes
Solver runtime (s)	0.0180
Solver status	sat

- The solver verified that property 2 can indeed be satisfied together with both 3 and 4.