Poster Printing using Z3

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1 Modeling the Problem

1.1 Parameters

- $N\_canvas = 3$: number of each canvas.
- $N\_poster = 12$: number of each canvas.
- $w = [5, 5, 4, 3, 7, 6, 5, 4, 6, 4, 6, 5]$: width of each poster.
- $h = [6, 6, 10, 11, 7, 10, 13, 10, 9, 15, 10, 10]$: height of each poster.
- $price = [10, 14, 13, 15, 10, 17, 21, 16, 16, 23, 19, 17]$: price of each poster.
- $W = [12, 12, 20]$: width of each canvas.
- $H = [12, 12, 20]$: height of each canvas.
- $cost = [30, 30, 90]$: cost of each canvas.
- $minimal\_profit = 60$: minimal profit of printing.

1.2 Decision Variables

To fit posters into canvases, we introduce the following variables:

- $z_{c,p} \in \mathbb{B}$ for $c = 1, ..., N\_canvas,\ p = 1, ..., N\_poster$: the value of $z_{c,p}$ will be true if and only if post[p] will be printed on canvas[c]
- $r_{c,p} \in \mathbb{B}$ for $c = 1, ..., N\_canvas,\ p = 1, ..., N\_poster$: the value of $r_{c,p}$ will be true if and only if post[p] will be turned 90°
- $u_c \in \mathbb{B}$ for $c = 1, ..., N\_canvas$: the value of u_c will be true if and only if canvas[c] will be used
- $x_{c,p},\ y_{c,p} \in \mathbb{N}$ for $c = 1, ..., N\_canvas,\ p = 1, ..., N\_poster$: the values of $x_{c,p}$ and $y_{c,p}$ indicate the bottom-left coordinate (x, y) of $poster[p]$ placed in $canvas[c]$
- $w\_eff\_i, h\_eff\_i, w\_eff\_j, h\_eff\_j \in \mathbb{N}$: the width and height of post[i] and post[j]
- $total\_profit \in \mathbb{N}$: the value of total profit after printing

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1.3 Constraints

For *post*[p], it cannot be printed more than once. This is expressed by the formula

$$\sum_i z_{p,i} \leq 1$$

Next we determine that whether *post*[p] fits into *canvas*[c]. This is expressed by the formula

$$\begin{aligned} z_{c,p} \Rightarrow \Big(& (\neg r_{c,p} \wedge x_{c,p} \geq 0 \wedge y_{c,p} \geq 0 \wedge x_{c,p} + w[p] \leq W[c] \wedge y_{c,p} + h[p] \leq H[c] \wedge w[p] \leq W[c] \wedge h[p] \leq H[c]) \\ & \vee (r_{c,p} \wedge x_{c,p} \geq 0 \wedge y_{c,p} \geq 0 \wedge x_{c,p} + w[p] \leq W[c] \wedge y_{c,p} + h[p] \leq H[c]) \wedge h[p] \leq W[c] \wedge w[p] \leq H[c] \Big) \end{aligned}$$

Additionally, every two posters *post*[i] and *post*[j] should have no overlap. This is expressed by the formula

$$\begin{aligned} (z_{c,i} \wedge z_{c,j}) \Rightarrow \Big(& (x_{c,i} + w_eff_i \leq x_{c,j}) \vee (x_{c,j} + w_eff_j \leq x_{c,i}) \\ & \vee (y_{c,i} + h_eff_i \leq y_{c,j}) \vee (y_{c,j} + h_eff_j \leq y_{c,i})) \end{aligned}$$

Then, we associate *canvase*[c] and *poster*[p]. This is expressed by the formula

$$z_{c,p} \Rightarrow u_c$$

Finally, we set the minimal profit. This is expressed by the formula

$$total\_profit \geq mininal\_profit$$

1.4 Calculation Function

The calculation function of $w\_eff\_i,\ h\_eff\_i,\ w\_eff\_j,\ h\_eff\_j$ is expressed by the formula

$$w\_eff\_i = \begin{cases} h_i, & \text{if } r_{c,i} = 1 \\ w_i, & \text{if } r_{c,i} = 0 \end{cases} \quad h\_eff\_i = \begin{cases} w_i, & \text{if } r_{c,i} = 1 \\ h_i, & \text{if } r_{c,i} = 0 \end{cases}$$

$$w\_eff\_j = \begin{cases} h_j, & \text{if } r_{c,j} = 1 \\ w_j, & \text{if } r_{c,j} = 0 \end{cases} \quad h\_eff\_j = \begin{cases} w_j, & \text{if } r_{c,j} = 1 \\ h_j, & \text{if } r_{c,j} = 0 \end{cases}$$

The calculation function of $total\_profit$ is expressed by the formula

$$\sum_{\substack{c=1,\ldots,N_{\text{canvas}} \\ p=1,\ldots,N_{\text{poster}} \\ z_{c,p}=\text{true}}} price_p - \sum_{\substack{c=1,\ldots,N_{\text{canvas}} \\ u_c=\text{true}}} cost_c$$

2 Solver Implementation using Z3

- In part (a), a solver instance s using s = Solver() is used to store and solve the constraints).
- In part (b), an optimization solver instance using s = Optimize() is used to store constrains and handle objective functions to maximize variables.
- Constraints mentioned in 1.3 are added to the solver using s.add().
- The satisfiability of the constraints is checked by calling s.check().
- The specific values of the variables can be checked by calling s.model().evaluate

3 Results Part

3.1 posters Assignments with three canvases

Table 1: posters Assignments with three canvases for Part (a)

canvas	p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	price	cost	profit
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	10	14	0	15	0	0	0	16	0	0	0	0	55	30	
2	0	0	13	0	0	17	21	0	16	23	19	17	126	90	
Total													181	120	61

It is possible to obtain profit at least 60.

3.2 posters Assignments with two small canvases

Table 2: posters Assignments with three canvases for Part (a)

canvas	p0	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	price	cost	profit
0	10	14	0	0	0	0	0	0	0	0	0	0	19	43	30
1	0	0	0	15	0	0	0	16	0	0	0	17	48	30	
Total													91	60	31

The highest profit created by the two small canvases is 31.

3.3 Solver Performance and Optimization Results

- In Part (a), the run time is around 900ms.
- In Part (a), the run time is 2s.