Assignment for Section 2.5: Inverse matrices

(1) Suppose A has row 1+ row 2 = row 3.

(a) Explain why $A\boldsymbol{x} = (1, 0, 0)$ cannot have a solution.

(b) Which right sides (b_1, b_2, b_3) might allow a solution to $A\boldsymbol{x} = \boldsymbol{b}$?

(c) In elimination, what happens to equation 3?

Then based on either (a) or (b) or (c), explain why A is not invertible.

(2) Multiply $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ times $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. What is the inverse of each matrix if $ad \neq bc$.

(3) If B is the inverse of A^2, show that AB is the inverse of A.

(4) Invert the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

by the Gauss-Jordan method starting with $[A\ I]$.

(5) (a) Prove that $A = \begin{bmatrix} a & b & b \\ a & b & b \\ a & a & a \end{bmatrix}$ is invertible if $a \neq 0$ and $a \neq b$.

(b) Find three numbers c so that $C = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$ is not invertible.