Assignment for Section 2.6: Factorization $A = LU$

(1) Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{bmatrix}$.

(a) Carry A to an upper triangular U by a series of elementary row operations.

(b) Find a matrix E such that $EA = U$.

(c) Multiply by $E^{-1} = L$ to factor A into LU.

(2) Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix},$$

such that $A = LU$.

(3) Let

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

(a) Solve $L\boldsymbol{c} = \boldsymbol{b}$ to find $\boldsymbol{c}$.

(b) Then solve $U\boldsymbol{x} = \boldsymbol{c}$ to find $\boldsymbol{x}$.

(c) Compute $A = LU$, and verify $\boldsymbol{x}$ obtained in (b) is a solution to $A\boldsymbol{x} = \boldsymbol{b}$.