Optimal Decision Making: Group Project
Economic Dispatch and Unit Commitment

In this project, you will use linear programming, integer programming, and robust optimization to solve practical problems in the energy domain. Please observe the following guidelines:

- You are supposed to hand in a report (as a pdf file) that contains your answers and explains your reasoning. Please provide also a short description of your implementations.
- You must hand in your MATLAB or Python files in a standardized format so that we can automatically test them.
- A skeleton of each MATLAB or Python script you should implement is available on Moodle (Project.zip). Please use these skeletons as a basis for your implementation. The name as well as the structure of the scripts should not be changed. Make sure that each script runs without errors. If your code does not run without errors, we will not be able to give you any points for the corresponding implementation exercise.
- You may work in groups of up to 3 people. Collaborations between groups are not allowed.
- Each team should upload a single zip file containing the report and the code to Moodle. Please do **not** send your files by email. The deadline is on **May 20 at midnight (11:59 pm)**.

Description

Electricity plays an important role in people's lives and in industrial production. To meet an uncertain electricity demand, the grid operator must find a safe dispatch schedule for electricity generation at a low cost. To this end, the grid operator solves economic dispatch and unit commitment problems.

Economic dispatch aims to determine the amount of electricity that each generator should produce in each hour in order to meet the demand at minimal cost. As they produce cheap power without polluting the environment,[1] renewable energy sources are always prioritized irrespective of the production quantity. The part of the demand that exceeds the renewable generation must be covered by traditional generators such as thermal and hydro-power plants.

Unit commitment aims to determine the timing of the start-ups and shut-downs of the traditional generators. It uses binary variables to account for the costs of turning the generators on or off and for the minimum up- or down- times, which are disregarded in economic dispatch.

1. Economic Dispatch Problem [25 points]

Denote by $\mathcal{N}_g := \{1, 2, \ldots N_g\}$ the set of traditional generators, and by $\mathcal{T} := \{t = 1, 2, \ldots, T\}$ the planning horizon. In addition, for any hour $t \in \mathcal{T}$, we denote by $g_i^t \in \mathbb{R}$ the power produced by the traditional generator $i \in \mathcal{N}_g$, by $r^t \in \mathbb{R}$ the power produced by the renewable energy sources (e.g., solar

[1] In this project, we assume there is no cost for renewable energy sources.

power plants and wind farms), and by $d^t \in \mathbb{R}$ the total electricity demand. We further denote by $C_i^g \in \mathbb{R}$ the variable generation cost and by $\bar{g}_i \in \mathbb{R}$ the generation capacity of plant $i \in \mathcal{N}_g$. Each generator also has a ramping rate that limits the amount by which the power output can increase (ramp up) or decrease (ramp down) over one hour. We use $R_i^{up}, R_i^{dn} \in \mathbb{R}$ denote the ramp-up and ramp-down rates of generator $i \in \mathcal{N}_g$. Using the above notation, the economic dispatch problem can be formulated as:

$$\min_{\boldsymbol{g}} \quad \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^g\, g_i^t \tag{1a}$$

$$\text{s.t.} \quad d^t = \sum_{i\in\mathcal{N}_g} g_i^t + r^t \quad \forall t \in \mathcal{T} \tag{1b}$$

$$0 \le g_i^t \le \bar{g}_i \quad \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T} \tag{1c}$$

$$g_i^t - g_i^{t-1} \le R_i^{up} \quad \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T}\setminus\{1\} \tag{1d}$$

$$g_i^{t-1} - g_i^t \le R_i^{dn} \quad \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T}\setminus\{1\}. \tag{1e}$$

Questions

1.1.[10 points] Table 1 summarizes the generator data, and Tables 2 and 3 contain the renewable generation and the electricity demand for the following day, respectively.[2] Please calculate the optimal dispatch plan $\{g_i^t\}_{t\in\mathcal{T}}$ for each generator by implementing and solving the economic dispatch problem.

Identifiers	G1	G2	G3	G4	G5	G6
Generation Cost ($/MW)	15.0	20.0	15.0	20.0	30.0	25.0
Capacity (MW)	10.0	5.0	10.0	10.0	20.0	30.0
Ramp-up Rate (MW)	2.0	5.0	2.0	5.0	10.0	5.0
Ramp-down Rate (MW)	2.0	5.0	2.0	5.0	10.0	5.0

Table 1: Data of traditional generators

Time	1	2	3	4	5	6	7	8	9	10	11	12
r^t	15.2	16.4	16.1	10.9	14.8	7.6	15.6	5.5	9.2	5.7	1.5	12.4
Time	13	14	15	16	17	18	19	20	21	22	23	24
r^t	10.4	4.8	14.3	0.5	6.6	5.7	11.5	11.9	2.8	7.3	6.7	9.7

Table 2: Renewable generation r^t (MW)

Time	1	2	3	4	5	6	7	8	9	10	11	12
d^t	21.3	21.4	17.8	20.9	15.5	17.6	20.2	23.8	27.7	30.1	35.4	39.4
Time	13	14	15	16	17	18	19	20	21	22	23	24
d^t	43.2	47.0	49.3	51.5	52.6	50.3	47.0	43.1	38.8	33.2	28.6	24.3

Table 3: Electricity demand d^t (MW)

[2]We assume here that the renewable generation and the electricity demand of the next day are known upfront.

Energy Storage Device (Battery)

We now enhance the economic dispatch model by including a storage device (e.g., a battery) that can both charge or discharge energy. We expect that the device should be charged when energy production is cheap and discharged when energy production is expensive. Let $S^t \in \mathbb{R}$ be the battery's state-of-charge at the beginning of hour $t \in \mathcal{T}$,[3] and denote by b_c^t, $b_d^t \in \mathbb{R}$ the charging and discharging amounts in hour $t \in \mathcal{T}$, respectively. In addition, denote by η_c, $\eta_d \in \mathbb{R}$ the charging and discharging efficiencies, and by $\bar{S} \in \mathbb{R}$ the capacity of the battery.

1.2.[5 points] Formulate the enhanced economic dispatch model.

1.3.[10 points] Assume that the storage device is a lithium battery with a capacity of $\bar{S} = 20$ MWh and with efficiencies[4] $\eta_c = 0.95$ and $\eta_d = 0.92$. For consistency, we assume that the battery is empty both at the beginning and at the end of the planning horizon (i.e., $S^1 = S^{T+1} = 0$). Please calculate the optimal dispatch plan $\{g_i^t\}_{t\in\mathcal{T}}$ for each generator and the optimal charging and discharging decisions $\{b_c^t, b_d^t\}_{t\in\mathcal{T}}$ of the battery by solving the enhanced economic dispatch model. Compare your results with those of question 1.1 both in terms of optimal costs and generation plans.

[3] By extension, S^{T+1} denotes the battery charge at the end of the planning horizon

[4] This means that if the battery charges 10MWh from the grid, then it will only store 9.5MWh. Similarly, if the battery discharges 10MWh, then only 9.2MWh will be fed back to the grid.

Solutions:

1.1. See attached solution files.

1.2. See the formulation below.

$$
\begin{aligned}
\min_{\boldsymbol{g}} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^g\, g_i^t \\
\text{s.t.} \quad & d^t = \sum_{i\in\mathcal{N}_g} g_i^t + r^t - b_c^t + \eta_d b_d^t && \forall t \in \mathcal{T} \\
& S_t = S_{t-1} + \eta_c b_c^t - b_d^t && \forall t \in \mathcal{T}\setminus\{1\} \\
& 0 \le S_t \le \bar{S} && \forall t \in \mathcal{T} \\
& S_0 = 0 \\
& S_T + \eta_c b_c^T - b_d^T = 0 \\
& 0 \le g_i^t \le \bar{g}_i && \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T} \\
& g_i^t - g_i^{t-1} \le R_i^{up} && \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T}\setminus\{1\} \\
& g_i^{t-1} - g_i^t \le R_i^{dn} && \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T}\setminus\{1\}.
\end{aligned}
$$

or

$$
\begin{aligned}
\min_{\boldsymbol{g}} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^g\, g_i^t \\
\text{s.t.} \quad & d^t = \sum_{i\in\mathcal{N}_g} g_i^t + r^t - b_c^t + b_d^t && \forall t \in \mathcal{T} \\
& S_t = S_{t-1} + \eta_c b_c^t - \frac{1}{\eta_d} b_d^t && \forall t \in \mathcal{T}\setminus\{1\} \\
& 0 \le S_t \le \bar{S} && \forall t \in \mathcal{T} \\
& S_0 = 0 \\
& S_T + \eta_c b_c^T - \frac{1}{\eta_d} b_d^T = 0 \\
& 0 \le g_i^t \le \bar{g}_i && \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T} \\
& g_i^t - g_i^{t-1} \le R_i^{up} && \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T}\setminus\{1\} \\
& g_i^{t-1} - g_i^t \le R_i^{dn} && \forall i \in \mathcal{N}_g, \quad \forall t \in \mathcal{T}\setminus\{1\}.
\end{aligned}
$$

The important points are (i) charging and discharging decisions, (ii) updating battery state, (iii) battery limits (0 and $\bar{S}$) and (4) initial and final state.

1.3. Results please see the attached solution files. Based on the result, we could observe that, in the enhanced model, the battery tries to store the energy when production cost is lower, and tries to discharge when the production cost is higher, so that the use of expensive generators can be avoided. Therefore, the enhanced model could reduce the total generation cost and correspondingly adjust the generation plans.

2. Unit Commitment [30 points]

The unit commitment problem is more realistic than the economic dispatch model in that it accounts for the start-up and shut-down costs, for the fixed (no-load) costs of production as well as for the minimum up- and down-times of the generators by using binary variables. The total cost can thus be written as

$$\textit{Total cost} = \textit{generation cost} + \textit{start-up cost} + \textit{shut-down cost} + \textit{no-load cost}.$$

In order to formalize this problem, we denote by C_i^u, C_i^d, and C_i^n the start-up, shut-down, and no-load costs, respectively, and by T_i^{up} and T_i^{down} the minimum up- and down-times of each generator $i \in \mathcal{N}_g$. In addition, we use the following binary decision variables:

- $x_i^t = 1$ if generator i is running at time t, $x_i^t = 0$ otherwise;
- $u_i^t = 1$ if generator i is being turned on at time t, $u_i^t = 0$ otherwise;
- $v_i^t = 1$ if generator i is being turned off at time t, $v_i^t = 0$ otherwise.

We assume that the initial on/off-state x_i^1 of generator $i \in \mathcal{N}_g$ is given (i.e., it is not a decision variable). Without loss of generality, we can thus also set $u_i^1 = v_i^1 = 0$. For simplicity, we also neglect the ramping constraints. Using these conventions, we can define the unit commitment problem as:

$$
\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{g}} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^u u_i^t + C_i^d v_i^t + C_i^n x_i^t + C_i^g g_i^t && && \text{(2a)}\\
\text{s.t.} \quad & d^t = \sum_{i\in\mathcal{N}_g} g_i^t + r^t && \forall t \in \mathcal{T} && \text{(2b)}\\
& 0 \le g_i^t \le \bar{g}_i x_i^t && \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} && \text{(2c)}\\
& x_i^{t-1} - x_i^t + u_i^t \ge 0 && \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T}\setminus\{1\} && \text{(2d)}\\
& x_i^t - x_i^{t-1} + v_i^t \ge 0 && \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T}\setminus\{1\} && \text{(2e)}\\
& x_i^t - x_i^{t-1} \le x_i^\tau && \forall i \in \mathcal{N}_g,\ \forall \tau \in \{t+1,\dots,\min\{t+T_i^{up},T\}\},\ \forall t \in \mathcal{T}\setminus\{1,T\} && \text{(2f)}\\
& x_i^{t-1} - x_i^t \le 1 - x_i^\tau && \forall i \in \mathcal{N}_g,\ \forall \tau \in \{t+1,\dots,\min\{t+T_i^{down},T\}\},\ \forall t \in \mathcal{T}\setminus\{1,T\} && \text{(2g)}\\
& x_i^t, u_i^t, v_i^t \in \{0,1\} && \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T}\setminus\{1\}. && \text{(2h)}
\end{aligned}
$$

<u>**Questions**</u>

2.1. [5 points] Explain the meaning of the constraints (2c)–(2g).
HINT: It may be helpful to enumerate all possible combinations of the values of the binary variables. Remember that all cost coefficients in the objective are positive.

2.2. [10 points] Based on the data given in Tables 1–4, please implement problem (2) and report both its optimal value as well as its optimal decisions.

Identifiers	G1	G2	G3	G4	G5	G6
Start-up Cost ($/MW)	75.0	100.0	75.0	100.0	100.0	125.0
Shut-down Cost ($/MW)	7.5	10	17.5	10.0	10.0	12.5
No-load Cost ($/MW)	10.0	5.0	10.0	10.0	10.0	10.0
Initial State	ON	OFF	OFF	OFF	OFF	OFF
Minimum Up-Time	3	3	3	3	3	3
Minimum Down-Time	2	2	2	2	2	2

Table 4: Additional data of traditional generators

2.3. [5 points] Implement problem (2), where the binary variables u_i^t and v_i^t are replaced with continuous variables valued in $[0, 1]$. Compare the results and the runtime to those of question 2.2.

2.4. [10 points] Explain the differences in the result of questions 2.2 and 2.3 theoretically.
HINT: Consider the constraints containing u_i^t and v_i^t. For all possible values of x_i^t and x_i^{t-1}, characterize all feasible u_i^t and v_i^t. Identify the optimal u_i^t and v_i^t in each case.

<u>Solutions:</u>

2.1. Explain the meaning of constraints:

- (2c): g_i^t is the power produced by traditional generator i at time t. When generator i is running ($x_i^t = 1$), g_i^t should be non-negative but no larger than its capacity $\bar{g}_i$. When generator i is off ($x_i^t = 0$), g_i^t is 0.
- (2d) and (2f): we could enumerate all possible cases for x_i^{t-1} and x_i^t to find the meanings of u_i^t and v_i^t. e.g.,

Case	x_i^{t-1}	x_i^t	u_i^t	v_i^t	Meaning
1	0	0	0	0	Gen i stays unchanged
2	0	1	1	0	Gen i is turned on
3	1	0	0	1	Gen i is turned off
4	1	1	0	0	Gen i stays unchanged

  remark: since the start-up cost and shut-down cost are positive, the objective function is monotonically increasing in u_i^t and v_i^t. So u_i^t will stay 0 when $x_i^{t-1} = x_i^t = 0$.

- (2f) and (2g): enforce the minimum on- and off- times. For instance: we assume generator i is turned on at time 5, which means $x_i^4 = 0$ and $x_i^5 = 1$. If the minimum up time for generator i was 2 ($T_i^{up} = 2$), $x_i^5 - x_i^4 \leq x_i^6$ and $x_i^5 - x_i^4 \leq x_i^7$ guarantee that generator i would keep running at time 6&7 (i.e., $x_i^6 = x_i^7 = 1$)

2.2. See attached solution files.

2.3. See attached solution files.

2.4. There is no difference on the optimal objective value and optimal decisions between question 2.2 and 2.3. In 2.3, optimal u and v are always enforced to be binary as we explained in 2.1. While the computation for 2.3 should be faster than 2.2 as we have fewer binary variables in 2.3.

3. Robust Unit Commitment [45 points]

So far we have assumed that we have access to an exact forecast of the renewable energy production. However, due to the stochastic nature of the weather, the renewable generation is difficult to predict 24 hours in advance. A conservative decision-maker might therefore want to optimize in view of the worst-case realization of the uncertain renewable generation. That is, she might assume that the renewable generation vector $\boldsymbol{r} = (r^1, \ldots, r^T)$ falls within a box uncertainty set of the form:

$$\mathcal{R} := \{\boldsymbol{r} \in \mathbb{R}^T : |r^t - \bar{r}^t| \leq \hat{r}^t \ \forall t \in \mathcal{T}\}.$$

We can now formulate a robust unit commitment model, which treats the binary decision variables $\boldsymbol{x}, \boldsymbol{u}$ and $\boldsymbol{v}$ as here-and-now decisions. They are selected at time 0 before any uncertain parameters are revealed. The generation decisions g_i^t are selected at time t after the uncertain parameters $r^1, \ldots, r^t$ have been observed. We thus model them as non-anticipative decision rules, that is,as functions of the form $g_i^t(r^1, \ldots, r^t)$ for all $t \in \mathcal{T}$ and $i \in \mathcal{N}_g$. The robust unit commitment model is now defined as:

$$\begin{aligned}
\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{g}(\cdot)} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^u u_i^t + C_i^d v_i^t + C_i^n x_i^t + \max_{\boldsymbol{r}\in\mathcal{R}} \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^g g_i^t(r^1, \ldots, r^t) && \text{(3a)}\\
\text{s.t.} \quad & d^t = \sum_{i\in\mathcal{N}_g} g_i^t(r^1, \ldots, r^t) + r^t \qquad \forall \boldsymbol{r} \in \mathcal{R},\ \forall t \in \mathcal{T} && \text{(3b)}\\
& 0 \leq g_i^t(r^1, \ldots, r^t) \leq \bar{g}_i x_i^t \qquad \forall \boldsymbol{r} \in \mathcal{R},\ \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} && \text{(3c)}\\
& (2d)\text{–}(2h).
\end{aligned}$$

Problem (3) differs from the deterministic unit commitment problem in three ways. First, the objective is given by the *worst-case* generation cost, that is, the maximum cost over all uncertainty realizations. In addition, the constraints have to be satisfied for all possible realizations of the renewable energy production. Finally, the production decisions represent non-anticipative decision rules $g_i^t(r^1, \ldots, r^t)$.

Questions

3.1. [5 points] The robust unit commitment problem is, in its current form, not suitable for implementation with standard optimization software. Identify the three problems that need to be overcome. *Hint: Consider the numbers of decision variables and constraints and the form of the objective function.*

3.2. [5 points] Approximate the functional decisions by linear decision rules with limited memory

$$g_i^1(r^1) = a_i^1 r^1 + c_i^1 \quad \text{and} \quad g_i^t(r^1, \ldots, r^t) = a_i^t r^t + b_i^t r^{t-1} + c_i^t \quad \forall t \in \mathcal{T} \setminus \{1\},$$

which are parametrized by a_i^t, b_i^t and c_i^t. From now on, these parameters are treated as decision variables. Using these linear decision rules, simplify the robust optimization problem (3) with infinitely many decision variables as a robust optimization problem with finitely many decision variables.

3.3. [10 points] Reformulate the objective of (3) by dualizing the maximization problem over $\boldsymbol{r}$.

3.4. [10 points] Reformulate the robust constraints in terms of finitely many linear constraints. *Hint: As for constraint* (3b)*, note that a linear function of r^t and r^{t-1} vanishes if and only if its gradient and its intercept vanish. As for constraint* (3c)*, you may want to use duality.*

3.5. [10 points] Implement the mixed-integer linear program derived in questions 3.2, 3.3 and 3.4 and solve it. Set $\bar{r}^t$ to the predicted renewable production from question 1.1, and set $\hat{r}^t = 0.6$ for all $t \in \mathcal{T}$. Compare the resulting optimal value to the one of the deterministic unit commitment problem. In addition, compare the optimal $\boldsymbol{x}$ in the deterministic and robust models, and explain any differences.

3.6. [5 points] Explain how you could improve the linear decision rules in order to approximate the original robust unit commitment model (3) more accurately.

Solutions:

3.1. (1) functional decision variables, (2) nonlinear objective, (3) infinite number of constraints

3.2. The resulting problem is

$$\min_{\boldsymbol{x},\boldsymbol{u},\boldsymbol{v},\boldsymbol{g}(\cdot)} \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^u u_i^t + C_i^d v_i^t + C_i^n x_i^t + \max_{\mathbf{r}\in\mathcal{R}} \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^g (a_i^t r^t + b_i^t r^{t-1} + c_i^t) \tag{4a}$$

$$\text{s.t.} \quad (2d) - (2h)$$

$$d^t = \sum_{i\in\mathcal{N}_g} a_i^t r^t + b_i^t r^{t-1} + c_i^t + r^t \qquad \forall r \in \mathcal{R},\ \forall t \in \mathcal{T} \tag{4b}$$

$$0 \le a_i^t r^t + b_i^t r^{t-1} + c_i^t \le \bar{g}_i x_i^t \qquad \forall r \in \mathcal{R},\ \forall i \in \mathcal{N}_g,\ \forall t \in \mathcal{T} \tag{4c}$$

3.3. We have that

$$\max_{r\in\mathcal{R}} \sum_{t=1}^{T}\sum_{i=1}^{N} C_i^g (a_i^t r^t + b_i^t r^{t-1} + c_i^t)$$

is equivalent to

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} \lambda_t^1(\bar{r}_t + \hat{r}_t) + \lambda_t^1(\bar{r}_t - \hat{r}_t) \\
\text{s.t.} \quad & \sum_{i=1}^{N} C_i^g (a_i^{t-1} + b_i^t) \le \lambda_{t-1}^1 + \lambda_{t-1}^2 \quad \forall t = 1,\ldots,T-1 \\
& \sum_{i=1}^{N} C_i^g (a_i^T) \le \lambda_T^1 + \lambda_T^2 \\
& \lambda_t^1 \ge 0 \quad \forall t \\
& \lambda_t^2 \le 0 \quad \forall t
\end{aligned}$$

3.4. The first constraint has to be satisfied for all $\boldsymbol{r}$: (12b):

$$(a_i^t r^t + b_i^t r^{t-1} + c_i^t) + r^t = d^t$$

is equivalent to

$$\sum_{i=1}^{N} a_i^t = -1 \quad \forall t$$

$$\sum_{i=1}^{N} b_i^t = 0 \quad \forall t$$

$$\sum_{i=1}^{N} c_i^t = d_t \quad \forall t$$

The remaining constraints are reformulated by duality:

$$0 \le \min_{r\in\mathcal{R}} a_i^t r^t + b_i^t r^{t-1} + c_i^t$$

is equivalent to is equivalent to $\exists(\gamma_1^t, \gamma_2^t, \gamma_3^t, \gamma_4^t)$s.t.

$$\begin{aligned}
&\gamma_1^t(\bar{r}^t + \hat{r}^t) + \gamma_2^t(\bar{r}^t - \hat{r}^t) + \gamma_3^t(\bar{r}^{t-1} + \hat{r}^{t-1}) + \gamma_4^t(\bar{r}^{t-1} - \hat{r}^{t-1}) + c_i^t \geq 0 \\
&\gamma_1^t, \gamma_3^t \leq 0 \\
&\gamma_2^t, \gamma_4^t \geq 0 \\
&a_i^t \geq \gamma_1^t + \gamma_2^t \\
&b_i^t \geq \gamma_3^t + \gamma_4^t
\end{aligned}$$

and

$$\max_{r \in \mathcal{R}} a_i^t r^t + b_i^t r^{t-1} + c_i^t \leq \bar{g}_i x_i^t$$

is equivalent to $\exists(\rho_1^t, \rho_2^t, \rho_3^t, \rho_4^t)$s.t.

$$\begin{aligned}
&\rho_1^t(\bar{r}^t + \hat{r}^t) + \rho_2^t(\bar{r}^t - \hat{r}^t) + \rho_3^t(\bar{r}^{t-1} + \hat{r}^{t-1}) + \rho_4^t(\bar{r}^{t-1} - \hat{r}^{t-1}) + c_i^t \leq \bar{g}_i x_i^t \\
&\rho_1^t, \rho_3^t \geq 0 \\
&\rho_2^t, \rho_4^t \leq 0 \\
&a_i^t \leq \rho_1^t + \rho_2^t \\
&b_i^t \leq \rho_3^t + \rho_4^t
\end{aligned}$$

3.5. Use all of the constraints. We obtain a higher loss. But interestingly, we turn on the second generator earlier. This is because the demand at $t = 4$ cannot be met for the worst-case realization of renewable production with only generator 1.
3.6. (1) Use better linear decision rule and/or (2) use better uncertainty set.