

# 12 Geometrical wavefront sensing - Shack Hartmann sensor

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## 1 Objective and overview

The practical work should introduce the following subjects to you:

- Concept of wavefront detection with a Shack-Hartman sensor
- Description of deformations and aberrations by Zernike polynomials
- Evaluation of wavefront deformations by spot detection and wavefront reconstruction with MATLAB

Please read the reference documents provided. They will help you answer the questions under Appendix A of this document.

## 2 Safety Issues

In this experiments laser sources and low power electrical equipment are used. The laser is of class II. Class II: low-power visible lasers that emit a radiant power below 1 mW. The concept is that the human aversion reaction to bright light will protect a person. Only limited controls are specified. (<http://www.osha.gov>, Laser Hazards)

The laser is safe because the blink reflex of the eye will limit the exposure to less than 0.25 seconds. It only applies to visible-light lasers (400–700 nm). Intentional suppression of the blink reflex could lead to eye injury. In our experiment the laser sources are collimated and should be handled with care. A strongly divergent beam will not be focussed on the eyes retina and represents often no danger. Collimated beams will lead to small focus spots onto the retina special care is needed. ***Do not stare into a collimated beam!***



The electrical equipment used in the experiments is based on USB power (5V, 0.5 A, 2.5 W) and not subjected to any particular security issues. Nevertheless you should **not produce short circuits** on the printed circuit board (PCB) or to the computers USB connection to avoid damage to the material. Make proper use of screwdrivers. Do not force any mechanical parts.

## 3 Background

### 3.1. Wavefront of optical fields

In optics, different models exist to describe the propagation of light. A classical model is linked to optical rays that propagate through components changing their intensity and direction. Such rays travel in space and carry phase information that depends on the distance travelled and the materials in which the light propagates. In geometrical optics this phase information is rarely used except for the definition of the **wavefront**. The wavefront is the surface of equal phase and can be constructed considering rays by assuming that the rays are the normal vectors to a small plane surface. The local plane surfaces can then be unified to a continuous surface – the wavefront. In many problems only paraxial (close to the axis) rays are considered. In this sense, one can also define a paraxial wave if its wavefront normal are formed by paraxial rays.

The idea becomes clearer when looking at the illustrations below.

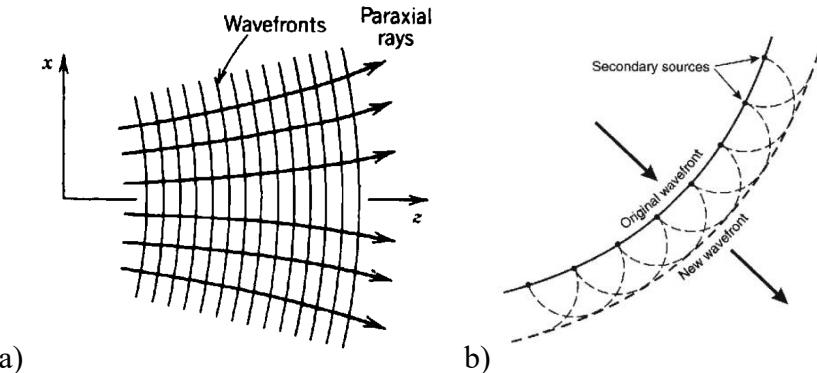


Figure 1 a) A wave is propagating along  $z$  and paraxial rays are considered. The normal to the ray bundle forms the wavefront. (Saleh, Teich, Photonics, Wiley 1991) b) Another possibility to construct a wavefront from point sources is illustrated.

There are other ways to construct the wavefront based on basic optical principles such as the Huygens-Fresnel principle that are less interesting for our work to be accomplished here. Wavefronts can have different shapes and forms. Imagine a wave produced by a point source is propagating freely in space. The wavefront is a sphere but can be approximated with different functions along its propagation. At infinity the wavefront curvature is infinity too and the wavefront would be a flat (planar) – a plane wave would result. Figure 2 illustrates this. If we come closer to the source on the left (here a point source) the curvature increases and is best modelled with parabola or even a sphere.

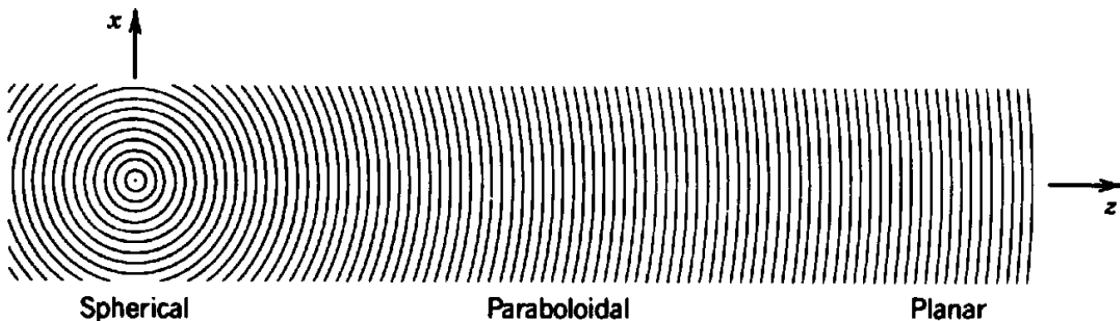


Figure 2 A spherical wave may be approximated at points near the  $z$  axis and sufficiently far from the origin by a paraboloidal wave. For very far points, the spherical wave approaches the plane wave. (Saleh, Teich, Photonics, Wiley 1991)

Any disturbance of such a wave leads to local wavefront distortion. The aim of this experience is to measure the wavefront using the Shack-Hartmann sensor.

### 3.2 Shack Hartman sensor

A lens transforms a flat wavefront into a spherical wavefront. That is, it focuses the collimated light propagating parallel to the optical axis into a bright spot in a plane at the back focal distance from the lens. The spot position in that plane depends on the wavefront angle relative to the optical axis as shown in the Figure below.

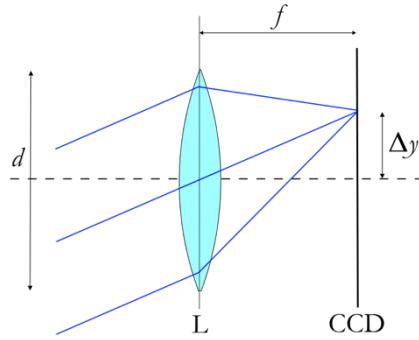


Figure 3 A lens focuses collimated light (a plane wave) at a point. The focusing position depends on the focal lengths and the angle of incidence.

The spot position  $\Delta y$  is related to the wavefront angle  $\phi$  and the focal length  $f$  through

$$\Delta y = f \tan(\phi) \quad \text{Eq.1}$$

This is the basic relation used in the Shack-Hartman wavefront sensor. In the Shack-Hartman sensor, instead of a single lens, the wavefront is sampled by an array of lenses. Each lens measures the local slope of the wavefront as shown in the Figure 4 below. One of its advantages over interferometric measurement techniques is that it does not require light with a high degree of coherence. The Shack-Hartman sensor is used in most adaptive optics systems for astronomy and ophthalmology.

In astronomy, it senses the wavefront distortion induced by the air turbulence in the atmosphere. This information is used to drive a fast deformable mirror to compensate in real time the optical aberrations. This is needed because already for a telescope diameter of 10 cm fluctuations of the refractive index caused by air turbulences would limit the performance of the optics.

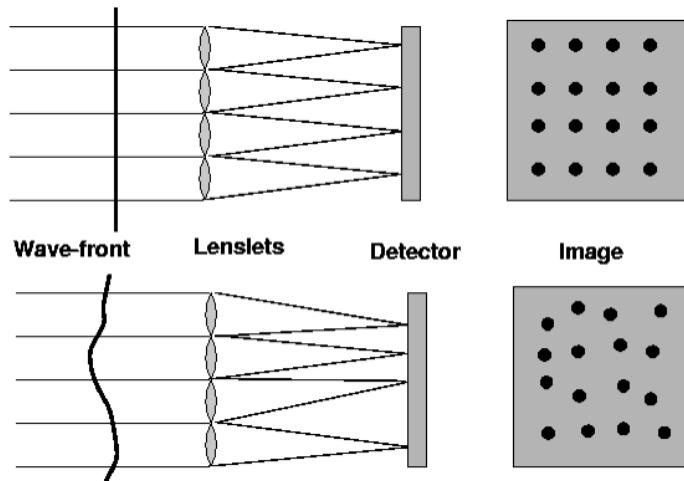


Figure 4 A microlens array can be used to locally sample the wavefront. The upper image shows the situation for an undistorted plane wave where all high intensity positions arrive at an array with symmetry similar to the microlens array. If the wavefront is distorted the position of the focal points is changed. This position change can be used to reconstruct the local angles and therefore the wavefront.

(<http://www.ctio.noao.edu/~atokovin/tutorial/part3/wfs.html>)

In ophthalmology, wavefront sensing is used to measure the aberrations of the eyes. It gives a high-resolution prescription (beyond focus and astigmatism). This information is used in research laboratory to correct the eye aberration for retinal imaging or for super-vision experiments.

The measurement range depends on the microlens array geometry and its focal length. Because the precision is given by geometric considerations it is easy to discuss the basic dependence of parameters. A long focal length/pitch ratio gives high angular resolution but a very small measurement range (wavefront tilt), because it should be avoided measuring the wrong or neighbouring spot. Larger microlenses at a fixed focal length lead to larger measurement range but to a bad spatial sampling and only a few points over the field could be detected. A Shack-Hartmann sensor needs to be designed with greatest care for each application considering field size, resolution and sampling.

### **Wavefront evaluation principle**

The Shack Hartmann sensor is calibrated by taking a first image with spots where a plane wave entrance is assumed. The software system performs the following steps before measurement

- Find the spots,
- Attribute at each spot a measurement area
- Calibrate their positions and
- Define the maximal spot displacement.

For measurement the change of position for each point is evaluated in both directions x and y. In Figure 5 the procedure is illustrated.

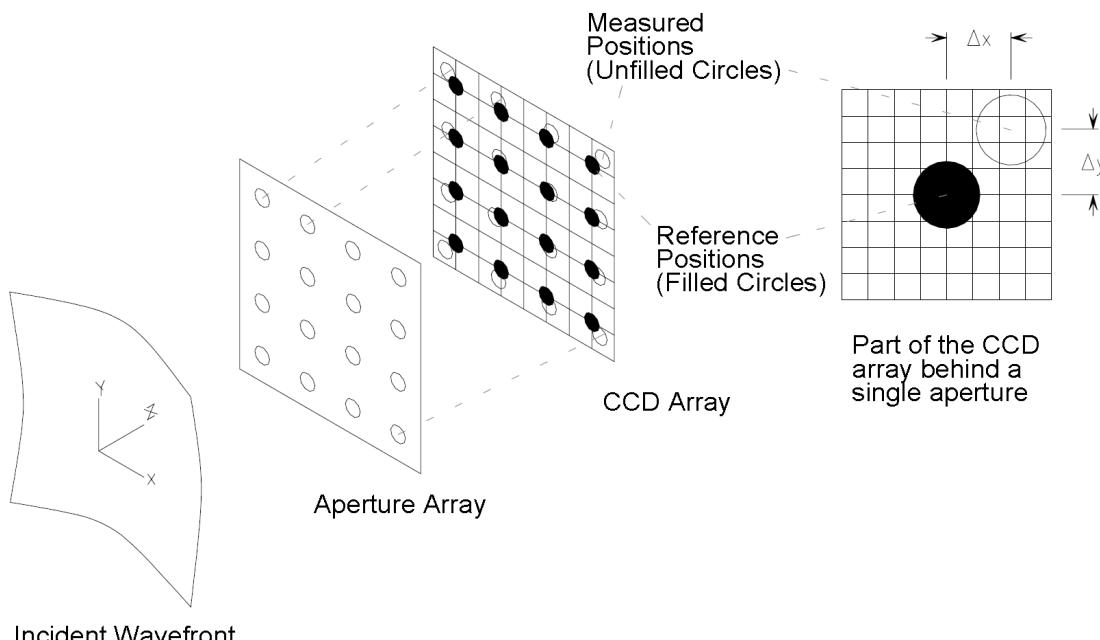


Figure 5 Drawing of the Shack-Hartmann Wavefront Sensor operation. Showing an incident wavefront travelling along the z-axis illuminating an aperture (of microlens) array and the diffracted spots illuminating a camera and the displacement of a single spot from the reference location. ([http://www.ophiropt.com/user\\_files/laser/beam\\_profilers/tutorial-hartman.pdf](http://www.ophiropt.com/user_files/laser/beam_profilers/tutorial-hartman.pdf))

The maximum wavefront deviation a Shack-Hartmann sensor can measure is determined by the maximum shift of the position of the diffracted spots. Figure 5 illustrates the maximum motion of a spot within an integration area. Assume that the maximum shift of the spots is limited by to the position when the spot outer rim touches the edge of its integration area, which is equal in size to spacing given by the microlens pitch. The minimum spot size has to be considered too.

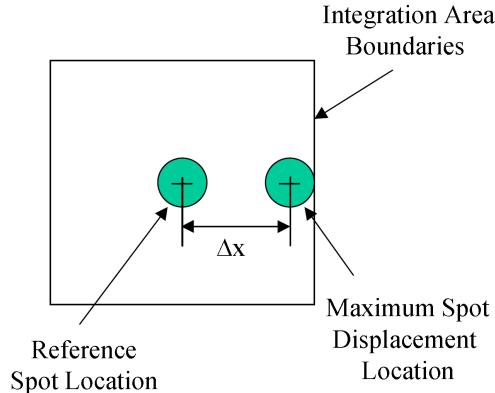


Figure 6 Diagram illustrating maximum diffracted spot motion in a given integration area.

Assuming a Shack-Hartmann sensor composed out of a microlens array of circular apertures with diameter  $D$  and a spacing (pitch)  $p$  being illuminated by light with wavelength  $\lambda$  separated from the detector by a distance  $f$  (the focal distance), the maximum wavefront slope can be written as,

$$\frac{\Delta\phi}{\Delta x} = \frac{\left(\frac{p}{2} - 1.22 \frac{f\lambda}{D}\right)}{f} \quad \text{Eq. 2}$$

Here we assume that the microlenses are diffraction limited and create a spot size accordingly. For example, consider a Shack-Hartmann array with an lens separation of  $p=250$  microns, a focal lengths of 3.77 mm, being illuminated by light with a wavelengths of  $\lambda=633\text{nm}$  and having a corresponding diffraction limited spot diameter of  $2.44 f\lambda/D = 1.22 \cdot 3770 \cdot 0.632 / 250 = 21 \mu\text{m}$ , the maximum wavefront slope that the sensor can observe is

$$\frac{\Delta\phi}{\Delta x} = \frac{\left(\frac{p}{2} - 1.22 \frac{f\lambda}{D}\right)}{f} = \frac{(125 \mu\text{m} - 11.5 \mu\text{m})}{3770 \mu\text{m}} = 2.88 \text{ mrad} \quad \text{Eq. 3}$$

There are different techniques to reconstruct the complete wavefront for piecewise available slopes over the field. In a strict sense it is a two dimensional integration over the measurements surface area (field). The two primary types of **wavefront reconstruction algorithms are zonal and modal**. The zonal wavefront reconstruction is a type of numerical integration. As the name indicates the reconstruction is based on zones and does not treats the wavefront over the whole aperture.

The modal wavefront reconstruction fits the data to a set of orthogonal surface polynomials that are orthogonal over the circular area that is considered. The polynomials used for this task are called Zernike polynomials. The Zernike polynomials are particular popular to describe aberrations in optical systems that have mainly circular apertures. The chapter below gives some more details on this subject. Our example program uses this approach.

### 3.3 Zernike polynomials and aberration

Aberrations of optical systems have historically been described, characterized, and catalogued by power series expansions. The majority of optical systems have circular pupils and a set of polynomials for circular openings will be preferential. In real applications experimental results are typically fitted to data and a set of polynomials should consider this particularity. It is, therefore, desirable to expand the wave aberration in terms of a complete set functions that are orthogonal over the interior of a circle.

Zernike polynomials form a complete set of functions or modes that are orthogonal over a circle of unit radius (but not for other geometries like rectangles!!). Zernike polynomials are convenient for serving as a set of basis functions. They can be expressed in polar coordinates or Cartesian coordinates. They are scaled so that non-zero order modes have zero mean and unit variance. This puts the modes in a common reference frame for meaningful relative comparison between different modes and aberrations. Wave aberrations in an optical system with a circular pupil can be accurately described by a weighted sum of Zernike polynomials. Because of its common use, the orthonormal set of Zernike polynomials is recommended for describing wave aberration functions and for data fitting of experimental measurements. The different contributions or terms are normalized so that the coefficient of a particular term or mode represents the RMS (or effective) contribution of that term. The set of formulas as shown below can be used to calculate the polynomials.

$$Z_n^m(\rho, \theta) = N_n^m R_n^{|m|}(\rho) \cos(m\theta) \quad \text{for } m \geq 0, \quad 0 \leq \rho \leq 1, \quad 0 \leq \theta \leq 2\pi \\ = -N_n^m R_n^{|m|}(\rho) \sin(m\theta) \quad \text{for } m < 0, \quad 0 \leq \rho \leq 1, \quad 0 \leq \theta \leq 2\pi$$

for a given  $n$ :  $m$  can only take on values of  $-n, -n+2, -n+4, \dots, n$

$N_n^m$  is the normalization factor

$$N_n^m = \sqrt{\frac{2(n+1)}{1 + \delta_{m0}}} \quad \delta_{m0} = 1 \text{ for } m=0, \quad \delta_{m0} = 0 \text{ for } m \neq 0$$

$R_n^{|m|}(\rho)$  is the radial polynomial

$$R_n^{|m|}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s (n-s)!}{s! [0.5(n+|m|)-s]! [0.5(n-|m|)-s]!} \rho^{n-2s}$$

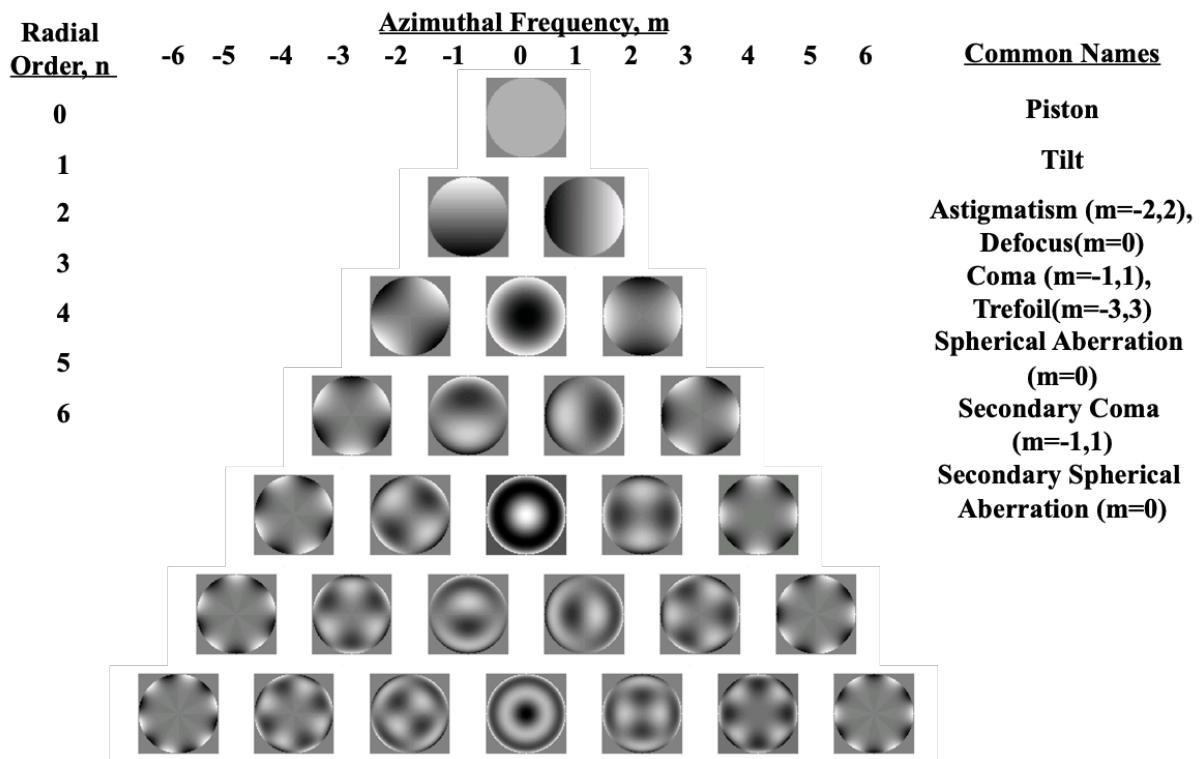
The MATLAB script that corresponds to this implementation is named

- “factorial.m” and
- “Zernike.m”.

(credits to Patrick MAEDA for the implementation of this code).

The table below describes the different polynomials and their principle implication to aberrations.

mode j	order n	frequency m	$Z_n^m(\rho, \theta)$	Meaning
1	0	0	1	Constant term, or Piston
2	1	1	$2\rho \cos(\theta)$	Tilt in x-direction, Distortion
3	1	-1	$2\rho \sin(\theta)$	Tilt in y-direction, Distortion
4	2	0	$\sqrt{3}(2\rho^2 - 1)$	Field curvature, Defocus
5	2	-2	$\sqrt{6}\rho^2 \sin(2\theta)$	Astigmatism with axis at $\pm 45^\circ$
6	2	2	$\sqrt{6}\rho^2 \cos(2\theta)$	Astigmatism with axis at $0^\circ$ or $90^\circ$
7	3	-1	$\sqrt{8}(3\rho^3 - 2\rho) \sin(\theta)$	Coma along y-axis
8	3	1	$\sqrt{8}(3\rho^3 - 2\rho) \cos(\theta)$	Coma along x-axis
9	3	3	$\sqrt{8}\rho^3 \sin(3\theta)$	
10	3	-3	$\sqrt{8}\rho^3 \cos(3\theta)$	
11	4	0	$\sqrt{5}(6\rho^4 - 6\rho^2 + 1)$	Spherical Aberration, Defocus
12	4	-2	$\sqrt{10}(4\rho^4 - 3\rho^2) \cos(2\theta)$	Secondary Astigmatism
13	4	2	$\sqrt{10}(4\rho^4 - 3\rho^2) \sin(2\theta)$	Secondary Astigmatism
14	4	4	$\sqrt{10}\rho^4 \cos(4\theta)$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	



The wave aberrations can be expressed as a weighted sum of Zernike polynomials and one finds

$$\begin{aligned}
 W(\rho, \theta) &= \sum_n^k \sum_{m=-n}^n W_n^m Z_n^m(\rho, \theta) \\
 &= \sum_n^k \left\{ \sum_{m=-n}^{-1} W_n^m (-N_n^m R_n^{|m|}(\rho) \sin(m\theta)) + \sum_{m=0}^n W_n^m (N_n^m R_n^{|m|}(\rho) \cos(m\theta)) \right\} \text{Eq. 4} \\
 W(x, y) &= \sum_{j=0}^{j_{\max}} W_j Z_j(x, y)
 \end{aligned}$$

This is available as “**WaveAberration.m**”.

Above is a representation if common Zernike double index Polynomials and a visualisation Implementation in MATLAB is given by the script “**ZernikePolynomial.m**”.

### 3.4 Example of wavefront evaluation for a microlens

The usefulness of the approach becomes clear if one discusses a real example. The microlens used in our experiment have been fabricated by reflow technique. Photoresist lenses are then immersed into an optical glue to reduce the optical power (the index difference becomes very small) and achieve a long focal length. The characterization of such microlenses was done by a Mach-Zehnder interferometer available for this purpose. The measurement result is presented below. There are two measurements, one with plane wave and one with a spherical wave illumination. The plan wave illumination measurement shows a series of interference fringes that indicate the lens curvature. The measurement is used to find the focal length. Its value is written in the table on the right:

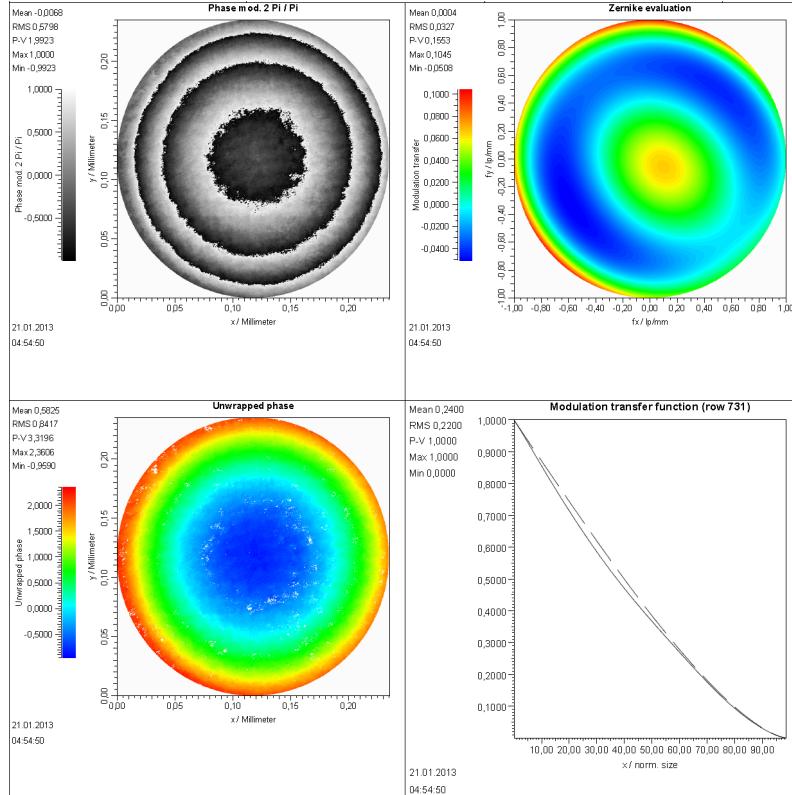
**Focal length = 3.766 mm**

A more precise testing can be done if the lens is illuminated with a spherical wave and deviations of the form from this spherical wave are measured. This is a differential measurement and delivers usually best results. The figure in the lower part shows the results. One recognizes that less than one fringe is present and the image “Unwrapped phase” on the left bottom the RMS value is given as low as 0.11. This is a very good value for a wavefront deviation from the ideal. An analysis is done by fitting Zernike polynomials to this wavefront and the details of the coefficients are given in the table to the right. One sees that all values are very small proofing the high quality of the lens.

In the section “Lens Parameters” one finds Strehl ratio and spot size. The spot size corresponds to the diffraction limited spot size calculated according to equations used in Eq. 2. Another parameter qualifying a lens is the Strehl ratio. The Strehl ratio is a measure of the quality of a lens used for image formation, and was originally proposed by Karl Strehl (1864-1940). It is based on the idea to compare the light collected into the diffraction limited spot diameter for the ideal situation (collection efficiency and Strehl ratio 1) and a real situation with aberrations where the collection efficiency and Strehl ratio is smaller than 1. The Strehl ratio has a value between 0 and 1 and if it is larger than 0.8 the imaging system is called diffraction limited.

Microlenses of the kind used in this experiment are a SWISS product. (<http://www.suss-microoptics.com>) See the attached datasheets for more details.

## Plane wave illumination measurement



### Polynom Coefficients:

$$a + bx + cy + d(x^2+y^2)$$

- a: Constant = -0,8632
- b: Tilt x direction = -0,2065
- c: Tilt y direction = -0,0097
- d: Defocus = 2,9020

### Zernike Coefficients:

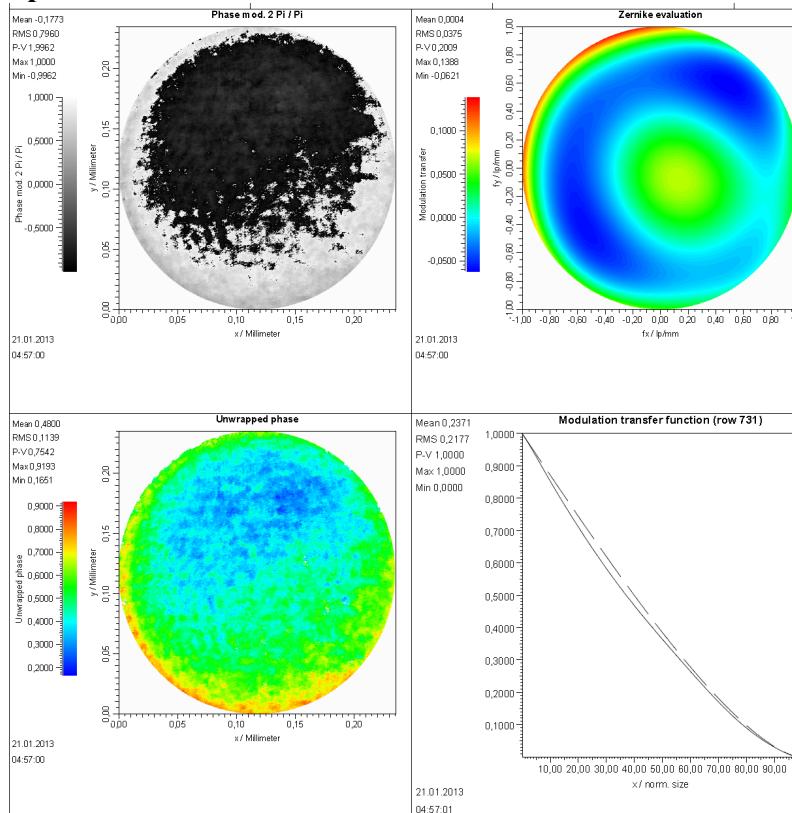
#### (deviations)

- Astigm. 1. ord. 0° = -0,0103
- Astigm. 1. ord. 45° = -0,0057
- Trifoil 30° = -0,0072
- Trifoil 0° = -0,0008
- Coma x = -0,0331
- Coma y = 0,0158
- Tetrafoil 22,5° = -0,0021
- Tetrafoil 0° = -0,0041
- Astigm. 2. ord. 0° = 0,0245
- Astigm. 2. ord. 45° = -0,0082
- spher. aber. = 0,0634

### Lensparameter

Diam. micro lens = 235 x 235  $\mu\text{m}^2$   
 Focal length: 3766,0  $\mu\text{m}$   
 Numerical aperture: 0,03  
 Strehl ratio = 0,96  
 Spot size (diff. lim.) = 24,7  $\mu\text{m}$   
 Cutoff frequency (diff. lim.) = 98,7 lp/mm

## Spherical wave illumination measurement.



### Polynom Coefficients:

$$a + bx + cy + d(x^2+y^2)$$

- a: Constant = 0,3574
- b: Tilt x direction = -0,0044
- c: Tilt y direction = -0,1385
- d: Defocus = 0,2460

### Zernike Coefficients:

#### (deviations)

- Astigm. 1. ord. 0° = 0,0031
- Astigm. 1. ord. 45° = -0,0285
- Trifoil 30° = -0,0071
- Trifoil 0° = 0,0128
- Coma x = -0,0472
- Coma y = 0,0263
- Tetrafoil 22,5° = 0,0036
- Tetrafoil 0° = 0,0201
- Astigm. 2. ord. 0° = 0,0131
- Astigm. 2. ord. 45° = -0,0028
- spher. aber. = 0,0639

### Lensparameter

Diam. micro lens = 235 x 235  $\mu\text{m}^2$   
 Focal length: 3766,0  $\mu\text{m}$   
 Numerical aperture: 0,03  
 Strehl ratio = 0,95  
 Spot size (diff. lim.) = 24,7  $\mu\text{m}$   
 Cutoff frequency (diff. lim.) = 98,7 lp/mm

## 4 Setup and equipment

### 4.1 Materials and setup

- The camera (1600x1200 pixels, colour, pixel size 2.835 um) C600 from Logitech
- Three different light sources (Halogen, LED, laser diode) USB driven
- Sheet polarizer for intensity regulation
- Objective lens for the Logitech C600
  - Mechanical setup
  - Microlens array

**Mechanical setup:**

Start with the breadboard and mount the translation stage.

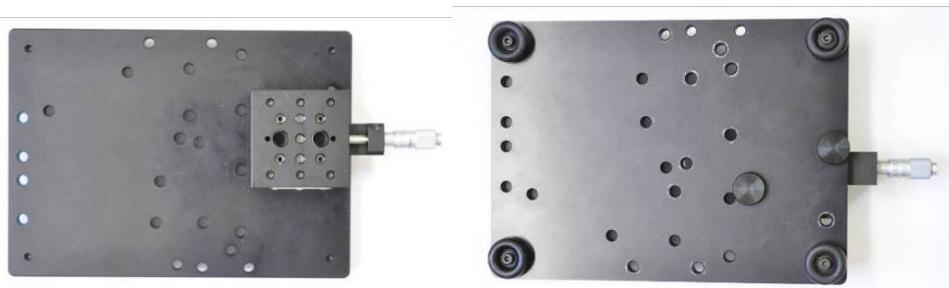


Figure 7: Breadboard with translation stage

The stage is fixed with screws arranged asymmetric (see the right picture). Continue by mounting the adapter plate and the intermediate piece as below.



Figure 8: Adapter plate and intermediate fixing have to be screw together.

The cameras PCB (printed circuit board) special holder is mounted as shown below.

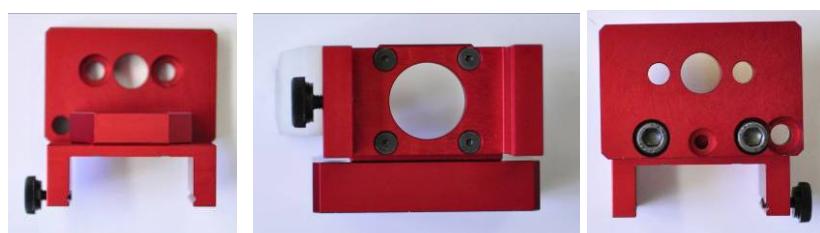


Figure 9: Camera holder (left) mounted on the adapter plate with the intermediate piece.  
Right: as seen from below.

The assembly has to be fixed on the translation stage before the camera PCB is put into place.

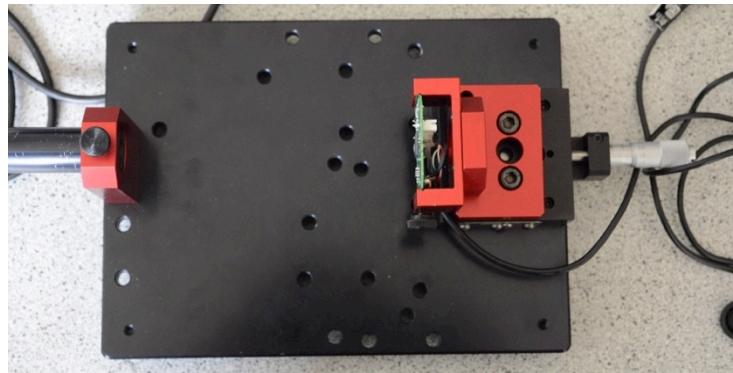


Figure 10 the setup containing the source holder with source at the right and the camera fix on the left. Please note the position of the camera holder on the linear stage (**middle screws!!**)

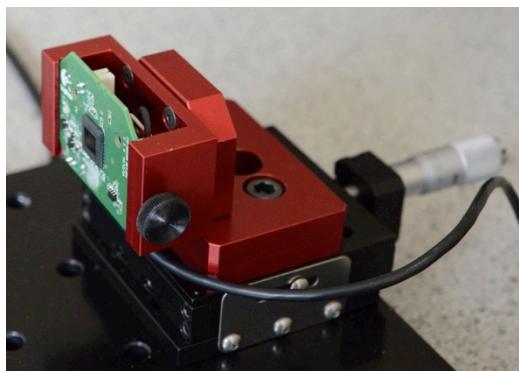


Figure 11: The camera is mounted without objective.

Mount the source holder at the other side (here left) of the breadboard so that the source is in front of the camera and put the laser in place.

Next comes the mounting of the microlens array onto the precision alignment stage. The microlens array is shown below and need to be mounted do that it sticks out of its mount. Only under this condition access to the focal points can be guaranteed without damage.

Mount the system correctly positioning the red adapter piece at the x-y stage as shown below.



Figure 12: Microlens array (left) is mounted into the x-y stage. The ring inside the mount can be used to adjust the position of the array when applied. The final position is shown on the right where the microlens array sticks out by its thickness.

The xy stage is than set in front of the linear stage.

Please be careful to not damage the camera and use the right screws to fix the x-y stage at the optical axis of the system (there are two wholes almost at the same position!!!!)

You should have a setup similar to that shown below.

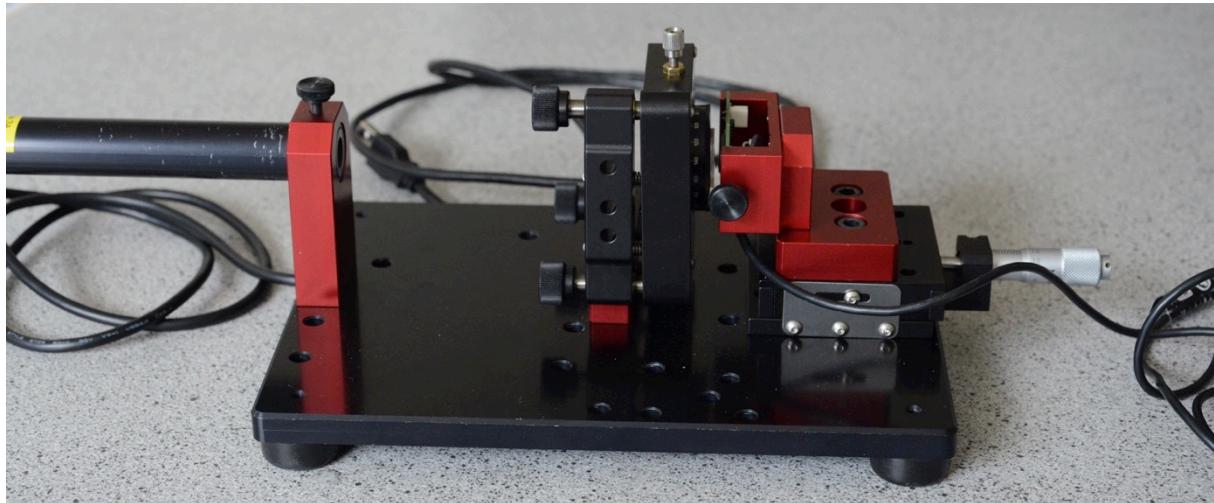


Figure 13: The x-y stage is mounted in front of camera at an distance of approximately 3mm.  
**DO NOT TOUCH THE CAMERA** during manipulation. The laser is on the left.

Please carefully check the position of the x-y stage and control if the mount is really on one axe with the linear stage and the source holder. To confirm an image of the back site of the setup with the positions of screws is shown below.

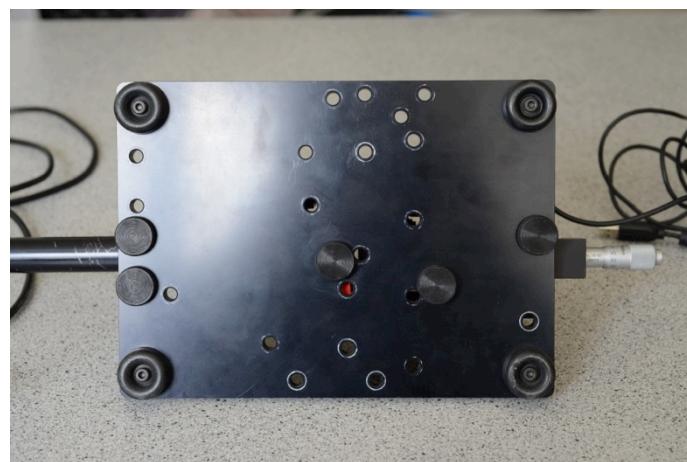


Figure 14. Breadboard with all components seen from the back side. The particularity is the screw in the middle that fixes the x-y stage. Please assure that you use the correct fixing point.

### ***Adjustment of the setup***

To use the sensor effectivly adjustment are needed. First place the source at its extreme position as shown below.

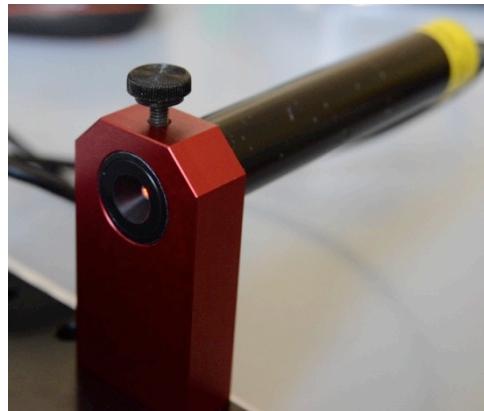


Figure 15 The source is put at the extreme position to have the largest possible distance between source and sensor.

The extreme position will be our reference and simulates a nearly flat wavefront. We will start to measure from this position.

- Fix the source at its extreme position
- Switch on the laser and the camera.

A typical image you will get looks like the one below on the right. Adjust the exposure conditions to manual and set the minimum conditions as shown below. Use a polarizer which you can directly fix at the x-stage as shown on the image below.

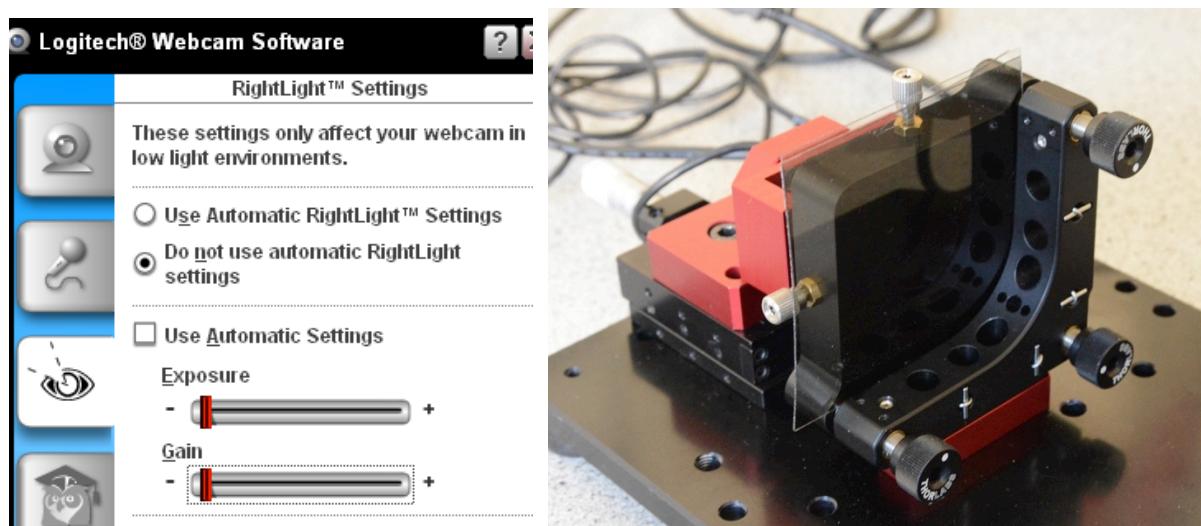


Figure 16 Exposure setting and the employlemt of the polarizer at the x-y stage.

By rotating the laser (and because of the polarizer) your can adjust the intensity further to find a series of spots as shown in the second picture to the right below.

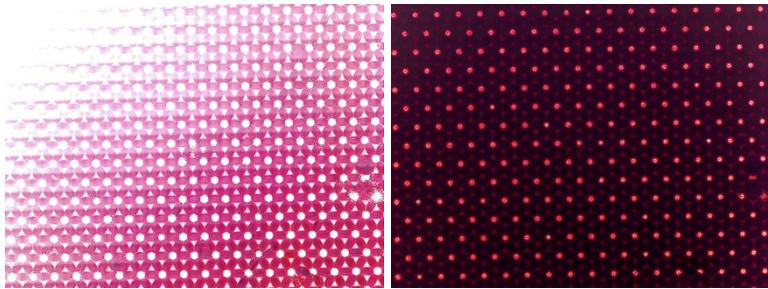


Figure 17 Initial configuration (left) and after adjustment of light intensity (right).

The adjustment of intensity is subjected to different problems:

- Straylight might overwhelm your image because we work with the open sensor and do not have the IR filter mounted. The image in Figure 17 on the left shows very nicely this effect. **Turn the setup away from strong light sources (window) or cover it with the black tissue. Switch off the control LED of the camera**
- The laser might have different power from model to model

Next we adjust the spot quality and alignment by moving the linear stage and the x-y stage and make an angle adjustment. A possible sequence is given below.

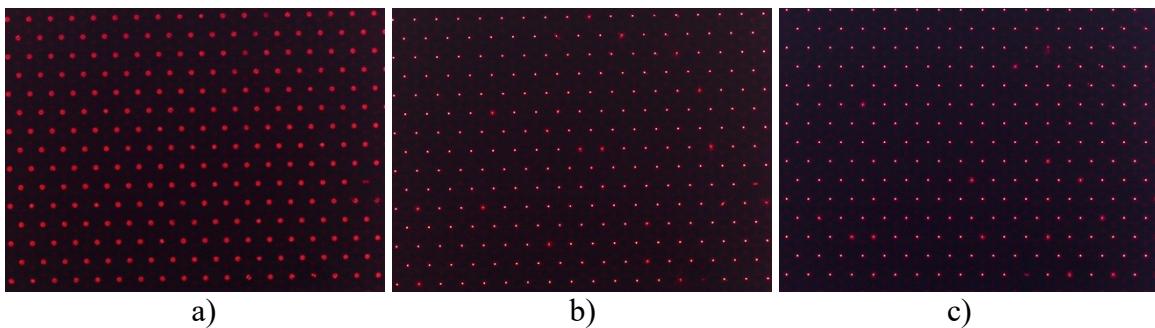


Figure 18. The initial situation is shown in a). Focussing of spot is done by changing the linear stage position and one finds an image as given in b) when well focussed. In c) the spots are aligned with the edged of the field of view.

- Check that your x-y stage is almost parallel to the detector and the angular adjustment screws are at a middle position.
- Adjust the linear movement adjustment (x-y) screws of the x-y stage to a middle position.
- Focus the spots onto the detector with the help of the linear stage.
- Rotate the microlens array, observe the movement of spots and adjust the centre of rotation to the centre of the visible field. You might need to move the camera and use the x-y adjustment.
- Align the spots as shown in Figure 17 c) to find a row of spots perfectly aligned vertically and a line of spots perfectly aligned horizontally.
- Control the focussing and readjust so that all spots are in focus. The angular adjustment screws of the stage do this. You might need to realign the stage because it is only fixed with one screw, which leads to instabilities if not well fixed.

Note, in the images in Figure 17 we have microlenses that show defects. This is due to fabrication issues of some of the samples. You will not find this in your experimental setup where lenses are produced with greatest care.

The setup is now adjusted and ready to use. If you have doubts let check the setup by an assistant please.

### 4.3 Calibration with point source at “infinity”

The measurement is heavily based on image processing and need careful preparation and calibration. In a first experiment we want to calibrate the system and control the functionality of the evaluation algorithm. The evaluation gives a rather complex script, which we adapted, from the following source at TU Delft (Netherlands) downloaded in 2012:

<http://www.desc.tudelft.nl/~jantonello/code.html>

Parameters that have to be set:

- The image pixels need to be normalized with values between zero and one
- A circular area need to be chosen
- The centre point has to be defined to have high intensity spot at the centre. This **centre point coordinates** have to be used in the evaluation script

We use the script “imagepreparation.m” to do this task.

```
function I = imagepreparation(fn,r,xc,yc);
%function I = imagepreparation(fn,r,xc,yc);
%
% Prepare image for Hartman-Shack wavefront sensor scripts
%
% require
%   fn    file name string
%   r     pupil radius, pixel
%   xc   pupil center x-coordinate, pixel
%   yc   pupil center y-coordinate, pixel
%
% return
%   I     the image
%
% Eric Logean
% Mai 13, 2013

adumax = 255; %maximum number of counts needed for normalization

I = imread(fn);
I = double(I(:,:,:,1))./adumax; %data conversion and normalization

% circular are definition with radius r
[ymax xmax] = size(I);
[X Y] = meshgrid([1:1:xmax]-xc,[1:1:ymax]-yc);
[T R] = cart2pol(X,Y);
I(find(R>=r)) = 0;

imagesc(I); axis equal; colormap (gray);
```

This a function that can be called by writing the following in the command line

**imagepreparation ('Picture 325 0.jpg',400,800,600);**

Parameters:

- '**Picture 325 0.jpg'** is the image name and need to be changed with the name of your image
- **400** is the radius of the circle used for exploitation. Please take care that your image is 1600x1200 pixel to assure full coverage.
- **800** is the x value of a possible centre point and need to be changed. Please take care that your image is 1600x1200 pixel
- **600** is the y value of a possible centre point and need to be changed. Please take care that your image is 1600x1200 pixel

When called this function with the abovementioned parameters you will find the image below and you can identify the centre point.

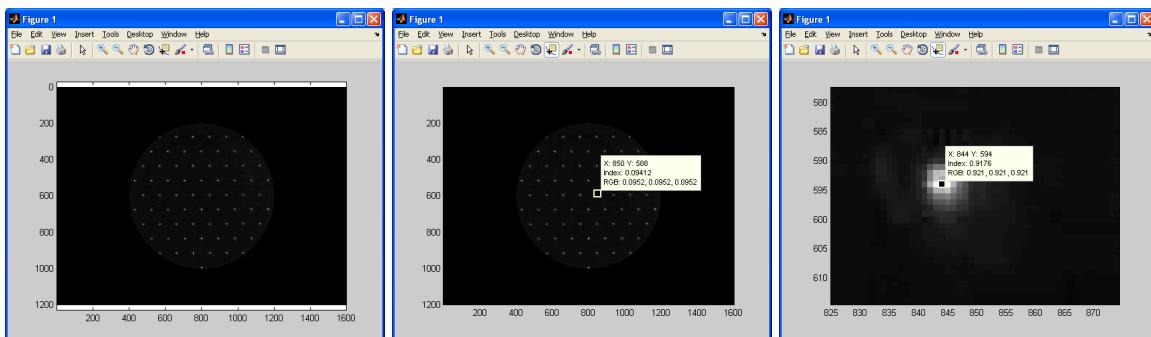


Figure 19 Identification of a possible centre spot. Original image – identification of a possible centre spot - zoom to find the exact coordinates of the centre spot – here 844 and 594

- Use the cursor tool to identify a possible centre point.
- Use the zoom tool to go closer to the centre area
- Use the cursor tool to identify as precise as possible the centre point.
- **Note the coordinates**

Next we need to do the calibration procedure. The script that will perform this task is called

### **shwfs\_calibrate.m**

The program has different sections:

```
%% parameters
%% lenslet documentation
%% camera documentation and image preparation
%% flat mirror reference
%% coarse grid with image processing
%% finer grid
%% Dai modal wavefront estimation
%% save results
```

Parts of this scripts have to be modified and adapted. We give below some comments on the script. The parameters in bold are parameters that have to be revised when evaluation fails.

Definition of basic parameters – the threshold for binary images depends on the image quality and might need to be set to a different value

```
%% parameters
shstruct = struct();
shstruct.calibration_date = datestr(now, 'yyyymmddHHMMSS');
shstruct.K = 1; % gain scaling image intensity
shstruct.use_bg = 0; % subtract background from images
shstruct.thresh_binary_img = 0.5; % threshold for binary image
shstruct.npixsmall = 8; % remove objects less than npixsmall
shstruct.strel_rad = 8; % strel ratio (image processing)
shstruct.coarse_grid_radius = 16; % radius for coarse grid
shstruct.percent = 0.2; % used when computing the centroid
shstruct.multiply_est_radius = 1/sqrt(2); % scale estimated radius in the
fine grid
shstruct.lambda = 632.8e-9; % operation wavelength
```

In this part the parameters found in the step before need to be filled in. One needs to specify the name of the reference picture (parameter fn) and the centre point for x and y (named xc and yc)

```
%% camera documentation and image preparation
shstruct.camera_pixsize = 2.835*1e-6;
shstruct.maxinteger = 2^8-1; % maximum integer value for the image

% ADDED
r = 400;
fn = 'Picture 325 0.jpg';
xc = 844;
yc = 594;
```

The other part of the script does not need any modifications. We can go forward to calibrate and analyse the different steps.

- Open the script **shwfs\_calibrate.m**
- Under `%% camera documentation and image preparation` replace the name of your file `fn = 'Your filename.jpg';` in line 34
- Under `%% camera documentation and image preparation` replace the centre position for xc in line 35 `xc = yourvalue(number);`
- Under `%% camera documentation and image preparation` replace the centre position for yc in line 36 `yc = yourvalue(number);`
- Check if the filename and positions are corrects and run the script.
- If successfully launched a first figure is plotted showing the flat reference. The sample image is given below in Figure 20 a)
- Press **enter** to continue

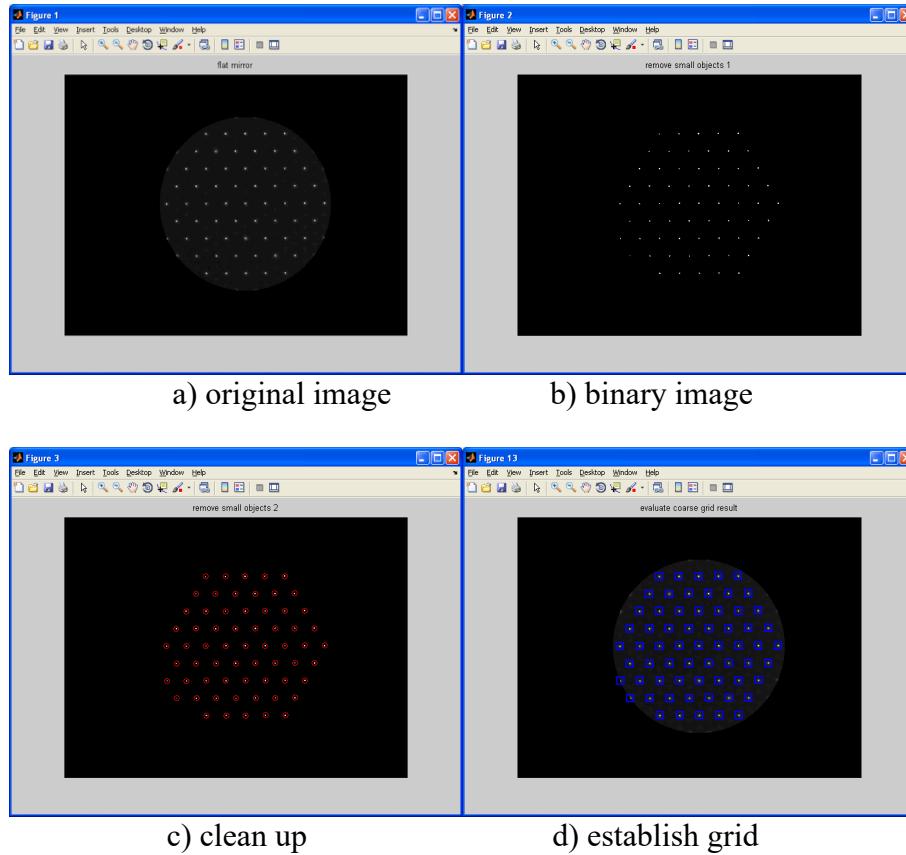
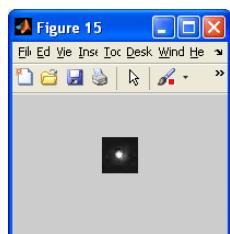


Figure 20 Different processing steps that are done during evaluation. It is important that the red circles under c) do not overlap!!



During processing a small window (Figure 16) pops up that shows the evaluation result for each spot. This is a normal procedure. **The red circle in c) should not overlap.** Please adjust the threshold parameter to avoid such situations. You will find it under

```
%% parameters ...
shstruct.thresh_binary_img = 0.5; % threshold for binary image
```

You can stop the script to do so by pressing CTRL-C!

- Press **enter** to continue

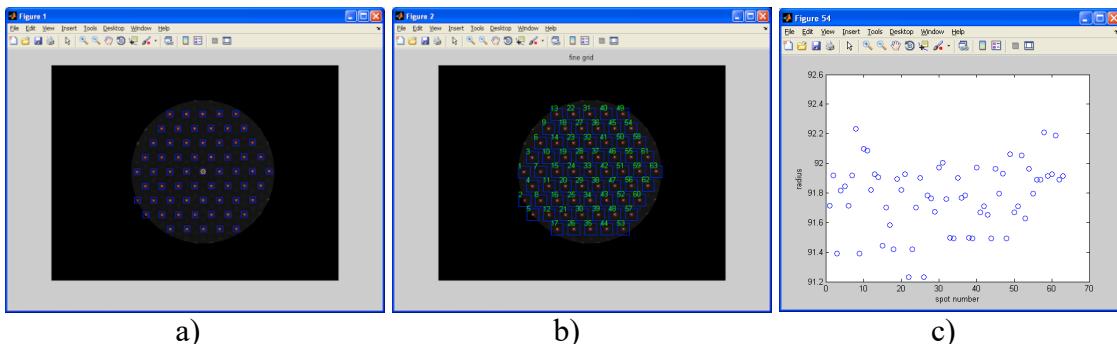


Figure 21 Definition of the grid and point numbers and measure of the spot size.

At this point the grid is further refined and the spots are numbered. A Figure 54 (Matlab) presents the spot size for each spot in pixels. In the best case all spots have the same size. The major work of calibration is done and the result will be displayed again and then saved.

- Press **enter** to continue

For each spot a sub-aperture that is the measurement area will be established. The image below shows the final result.

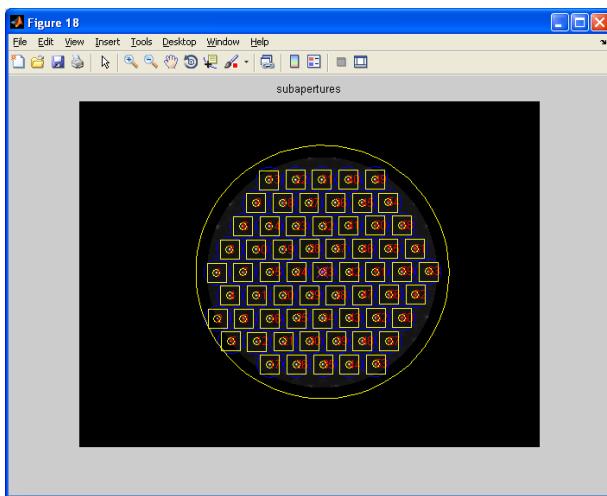


Figure 22 The final calibration result with the spot number and sub-apertures.

- Press **enter** to continue to finally save the calibration matrix called “shstruct.mat”

**After the calibration the sensor (camera and microlens array) must not be moved anymore. The calibration is done for a particular position of the laser source. ONLY MOVE THE SOURCE IF ESPECIALLY MENTIONED BECAUSE IT IS PART OF THE CALIBRATION.**

We will do now a first measurement using the script “test\_dai\_estimation.m”

- Open the script
- In line 9 change the file name `fn = 'Yourfilename.jpg'`; **Use the same file as applied for the calibration!**
- In line 10 and 11 change the positions for the centre point
 

```
xc = 845; %centre position x to be taken from the evaluation before
yc = 594; %centre position y to be taken from the evaluation before
```
- Run the script – two Figures will be opened, one giving the results of the Zernike coefficients (Figure 1) and a second with the surface plot of the wavefront.

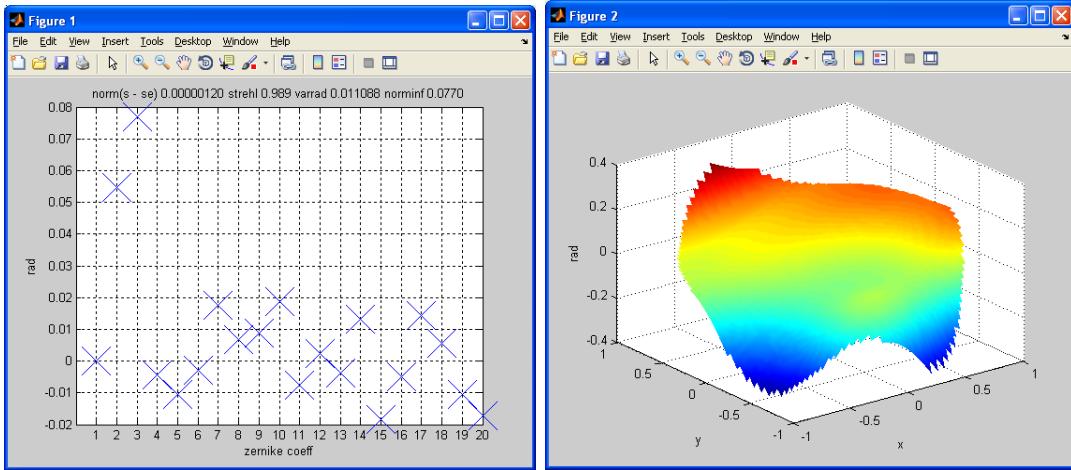


Figure 23 Left: Zernike coefficients order along the table given below (mode corresponds to Zernike coefficients x axis). The values are rather small. The reconstructed surface shows a maximum deviation of 0.8 rad over the whole aperture.

mode	order	frequency	$Z_n^m(\rho, \theta)$	Meaning
j	n	m	$Z_n^m(\rho, \theta)$	
1	0	0	1	Constant term, or Piston
2	1	1	$2\rho \cos(\theta)$	Tilt in x-direction, Distortion
3	1	-1	$2\rho \sin(\theta)$	Tilt in y-direction, Distortion
4	2	0	$\sqrt{3}(2\rho^2 - 1)$	Field curvature, Defocus
5	2	-2	$\sqrt{6}\rho^2 \sin(2\theta)$	Astigmatism with axis at $\pm 45^\circ$
6	2	2	$\sqrt{6}\rho^2 \cos(2\theta)$	Astigmatism with axis at $0^\circ$ or $90^\circ$
7	3	-1	$\sqrt{8}(3\rho^3 - 2\rho) \sin(\theta)$	Coma along y-axis
8	3	1	$\sqrt{8}(3\rho^3 - 2\rho) \cos(\theta)$	Coma along x-axis
9	3	3	$\sqrt{8}\rho^3 \sin(3\theta)$	
10	3	-3	$\sqrt{8}\rho^3 \cos(3\theta)$	
11	4	0	$\sqrt{5}(6\rho^4 - 6\rho^2 + 1)$	Spherical Aberration, Defocus
12	4	-2	$\sqrt{10}(4\rho^4 - 3\rho^2) \cos(2\theta)$	Secondary Astigmatism
13	4	2	$\sqrt{10}(4\rho^4 - 3\rho^2) \sin(2\theta)$	Secondary Astigmatism
14	4	4	$\sqrt{10}\rho^4 \cos(4\theta)$	
:	:	:	:	

To evaluate the precision one has to consider the real space dimensions of the measurement. The x and y are relative coordinates and represent in our case a real space dimension of 400 pxls. (Set value in the script) The diameter of the field is  $2 \times 400 \times 2.835$  micron = 2.268 mm. Over this diameter we have a maximum wavefront error of  $\Delta\phi=0.8$  rad which represents an real absolute deviation of

$$\Delta\phi[\mu\text{m}] = \Delta\phi[\text{rad}] \frac{\lambda}{2\pi} = 0.8[\text{rad}] \frac{0.635\mu\text{m}}{2\pi} = 0.08085\mu\text{m} \approx 81 \text{ nm} \quad \text{Eq.5}$$

This value represents the error of the processing and fitting and should be as small as possible. It depends on the image quality and hence on exposure conditions etc. With this final step we are ready for real measurements.

**After the calibration the sensor (camera and microlens array) must not be moved anymore. The calibration is done for a particular position of the laser source. ONLY MOVE THE SOURCE IF ESPECIALLY MENTIONED BECAUSE IT IS PART OF THE CALIBRATION.**

The Zernike coefficients of the measurement are stored in the file “**zcs.mat**”. You can access the values by double clicking on the file in the workspace or import it into a program to read the file (txt). The file “**zcs.mat**” contains 21 entries of which 14 are identified by the table above! Our interest is in the first 3 coefficients (1 - piston, 2 - x-tilt, 3 – y-tilt ). **Please note that the x and y definition is arbitrary!**

**TO BE DONE FOR THE REPORT:** Run through the calibration procedure and plot the following images of **YOUR** calibration: Figure 20 c) (red circles that should not overlap), Figure 21 c) (size of dots that should be as uniform as possible, look at the scale). Make measurement of the image used for calibration and present images as shown in Figure 23, the Zernike coefficients and the wavefront. Calculate the maximum wavefront deviation with equation 5.

#### **4.4 Measurement of tilts in different direction**

The measurement is done by using the script “**test\_dai\_estimation.m**”. We will use the same calibration for all upcoming measurements. Please be careful to not accidentally change the mechanical setup. The task here is to measure the tilt angle induced by turns of the knobs at the x-y stage and find the tilt angle, observe the value of the Zernike coefficients and interpret the result.

- Take a first image with the initial situation
- Take images for different turns of one knob of your x-y stage at  $\frac{1}{4}$ ,  $\frac{1}{2}$  and  $\frac{3}{4}$  turns (3 images)
- Turn the knob back to its original position and take an image (1 image)
- Take images for different turns of a second knob of your x-y stage at  $\frac{1}{4}$ ,  $\frac{1}{2}$  and  $\frac{3}{4}$  turns (3 images)
- Turn the knob back to its original position and take an image (1 image)
- In total you need 8 images. Evaluate the images and plot the sequence as given below.

The tilt gives values between 0 and 300 rad. To visualize best the effect it might be necessary to rotate the image before copy it. This can be done in the image toolbar with the rotate



Prepare a table with the Zernike coefficients and tilt values for x and y tilt which are a result of the evaluation. An example is given below.

The Zernike coefficients of the measurement are stored in the file “**zcs.mat**”. You can access the values by **double clicking** on the file in the workspace or import it into a program to read the file (txt). The file “**zcs.mat**” contains 21 entries of which 14 are identified by the table

above! Our interest is in the first 3 coefficients (1 - piston, 2 - x-tilt, 3 – y-tilt ). **Please note that the x and y definition is arbitrary!**

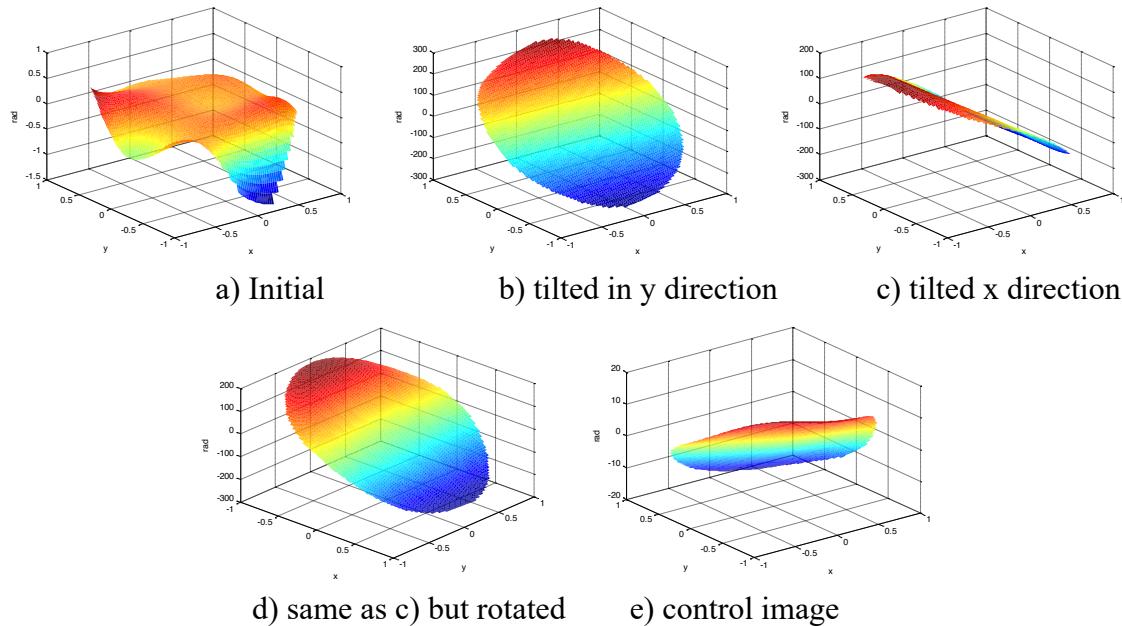


Figure 24 Images for different tilts. b) and c) are results of tilts in two different directions. d) is rotated and in a) and e) the tilt angle should be the same because these are the control images.

Image number	Tilt setting	Estimated maximum wavefront shift (rad)	Estimated maximum wavefront shift (micron)	Zernike coefficient X tilt	Zernike coefficient y tilt
1	0	2	0.2	-0.0126	0.0922
2	$\frac{1}{4}x$	To be filled in	To be filled in	To be filled in	To be filled in
3	$\frac{1}{2}x$	To be filled in	To be filled in	To be filled in	To be filled in
4	$\frac{3}{4}x$	500	50	13.14	118.5
5	$\frac{1}{4}y$	To be filled in	To be filled in	To be filled in	To be filled in
6	$\frac{1}{2}y$	To be filled in	To be filled in	To be filled in	To be filled in
7	$\frac{3}{4}y$	400	40	111.5	-4.896
8	0	40	4	-4.073	-9.018

**TO BE DONE FOR THE REPORT:** Measure the tilt in two different directions by using the Shack-Hartmann sensor. Give four images similar to those shown in Figure 24 for the situations: initial, maximum tilt ( $3/4$  turns) in x and y direction and a control image when turned back to the original position. Establish a table with eight (8) measurements like shown above and fill in all numbers!

#### 4.5 Measurement of different beam propagation characteristics.

In this part of the experiment we would like to measure the change of the beam parameters when the source is set at different distances. The measurement needs to be made with care to keep the calibration valid. When the source is moved it should **not be rotated!** This would change the intensity setting. The measurement should be done for 6 different source settings as illustrated in the image below.

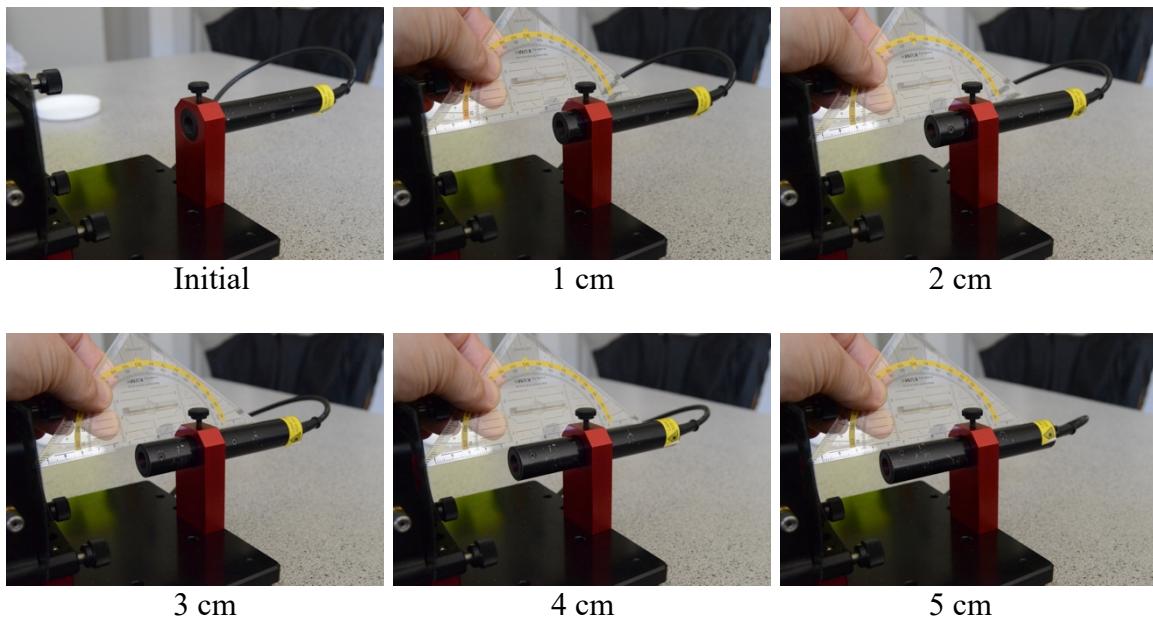
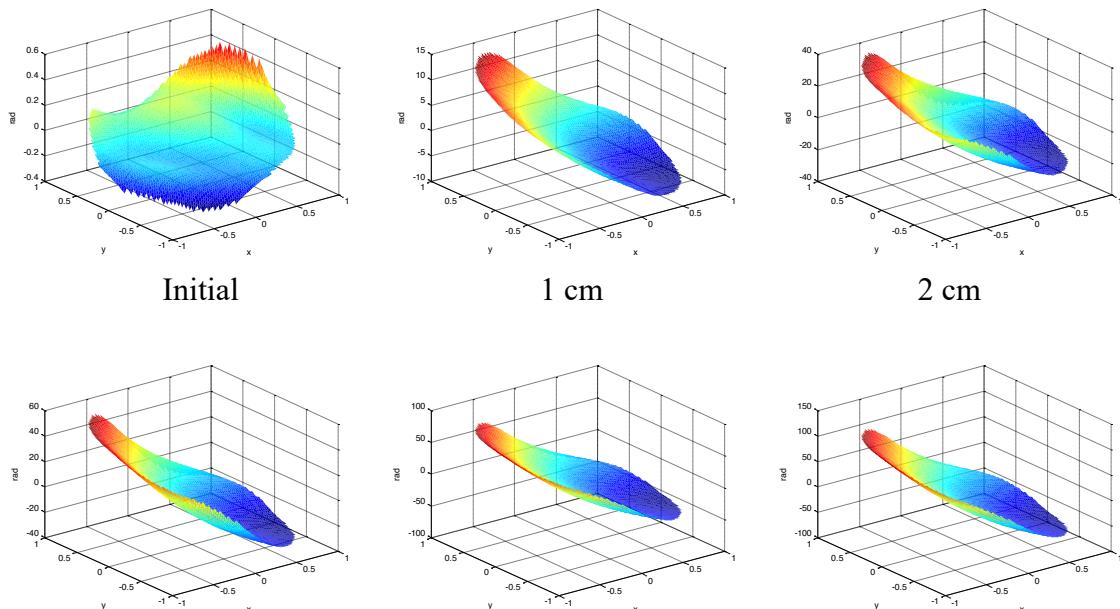


Figure 25 Source setting for six different positions with different distances from the support.

For each position the measurement of the wavefront should be done. Additionally the Zernike coefficients should be documented in a table as shown below. You can copy (CTRL-C) them directly from MATLAB into an Excel table for instance.

The results are presented by the following plots and table.



3 cm

4 cm

5 cm

Figure 26 Wavefront deviations for different positions of the source. The scale in each image changes and indicates the increase of tilt.

Zernike coefficient	description	0 cm	1cm	2cm	3cm	4cm	5cm
1	Piston	0	0	0	0	0	0
2	Tilt x	0.0452	-4.4425	-13.1498	-20.9979	-33.1874	-46.3992
3	Tilt y	0.07575	3.40595	7.74974	9.84109	10.1877	16.5007
4	Defocus - curvature	0.00080	1.38661	3.20930	5.30514	8.2862	11.3306
5	Astigmatism	0.00762	-0.02434	-0.25012	-0.30643	-0.04322	0.00210
6	Astigmatism	0.00475	0.13455	0.64225	0.93083	0.93443	1.07198
7	Coma y	0.00297	-0.05608	-0.66598	-0.78691	-0.73778	-0.92638
8	Coma x	0.01436	-0.01391	-0.44394	-0.40838	-0.43833	-0.33888
9		0.02219	0.10156	0.66987	0.62428	0.90769	1.11935
10		-0.00764	0.12232	0.62506	0.75396	0.68263	0.70183
11	Spherical - defocus	0.00711	-0.081	-0.35923	-0.38649	-0.372	-0.48813

**TO BE DONE FOR THE REPORT:** Establish a graph like Figure 26 and plot the 6 (six) images of the wavefront deviation. Make a table like shown above with all Zernike coefficients up to the 11<sup>th</sup> order. Comment on the values for the coefficient 1,2 and 3.

#### 4.6 Application example

Shack Hartmann sensors are used extensively in astronomy, ophthalmology and optical components qualifications and measurement. Please search the web and pick up an example of your choice. Try to find an example that you find particular interesting.

**TO BE DONE FOR THE REPORT:** Find an application example for a Shack Hartmann sensor and describe on ONE (1) page (including images and citations) its principle use and characteristics. TAKE CARE TO CITE THE IMAGES AND SOURCES CORRECTLY ESPECIALLY IF YOU USE WEB SOURCES.

## 5. Summary of tasks of the experimental work

### 4.3 Calibration with point source at “infinity” (45 min)

Run through the calibration procedure and plot the following images of **YOUR** calibration: Figure 20 c) (red circles that should not overlap), Figure 21 c) (size of dots that should be as uniform as possible, look at the scale). Make measurement of the image used for calibration and present images as shown in Figure 23, the Zernike coefficients and the wavefront. Calculate the maximum wavefront deviation with equation 5.

### 4.4. Measurement of tilts in different direction (30 min)

Measure the tilt in two different directions by using the Shack-Hartmann sensor. Give four images similar to those shown in Figure 24 for the situations: initial, maximum tilt (3/4 turns) in x and y direction and a control image when turned back to the original position. Establish a table with eight (8) measurements like shown above and fill in all numbers!

### 4.5 Measurement of different beam propagation characteristics (30 min)

Establish a graph like Figure 26 and plot the 6 (six) images of the wavefront deviation. Make a table like shown above with all Zernike coefficients up to the 11<sup>th</sup> order. Comment on the values for the coefficient 1,2 and 3.

### 4.6 Application example

Find an application example for a Shack Hartmann sensor and describe on ONE (1) page (including images and citations) its principle use and characteristics. TAKE CARE TO CITE THE IMAGES AND SOURCES CORRECTLY ESPECIALLY IF YOU USE WEB SOURCES.