Numerical solution of transport equations

Evaluation homework

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Modifications in the text are at the end and noted in bold face.

1 Introduction

The goal of this study is to study heat transfer at the wall in rectangular duct.

2 Problem position

We wish to compute Nusselt number for a laminar flow of a duct of rectangular section. Duct dimensions are $a \times b \times L$ where L is its length. Ratio a/b is noted β (with $\beta < 1$).

A fluid of known properties (dynamic viscosity μ, densityρ, heat conduction coefficient λ) flows in the duct in the z direction, with an average velocity $\bar{u}$.

The velocity profile in a rectangular duct is expressed using an infinite series :

$$V(x, y) = \frac{16\beta^2}{\pi^4} \sum_{nodd} \sum_{modd} \frac{\sin(n\pi x/a)\sin(m\pi y/b)}{nm(\beta^2 n^2 + m^2)}$$

where V is the dimensionless velocity. Reference velocity V_0 is computed by using average velocity $V_a = \frac{Q}{ab}$ where Q is the volumetric flow rate, and the average W of dimensionless velocity given by :

$$W = \frac{64\beta^2}{\pi^6} \sum_{nodd} \sum_{modd} \frac{1}{n^2 m^2 (n^2 + m^2/\beta^2)}$$

Wall temperature is fixed and equal to T_A, inlet temperature is $T_B < T_A$.

The local heat transfer coefficient h is defined by :

$$\phi = h(T_m - T_A)$$

where ϕ is the heat flux density and T_m is the average temperature defined by :

$$T_m = \frac{\int_0^a \int_0^b u(x,y)T(x,y,z)\, dxdy}{\int_0^a \int_0^b u(x,y)\, dxdy}$$

Coefficient h can be computed from temperature profile since flux density at the wall can also be deduced from Fourier's law (wall velocity is zero) :

$$\phi = -\lambda \left. \frac{\partial T}{\partial n} \right|_{\text{wall}}$$

where n is the normal coordinate (either x or y).

Global coefficient $\bar{h}$ is computed by averaging the local h values :

$$\bar{h} = \frac{1}{L} \int_0^L h(z)\, dz$$

The Matlab function `trapz` can be used to compute this integral.

3 Discretization and solving

3.1 Heat balance

Write the heat balance equation. Write the boundary conditions : you can take advantage of the problem symmetries or not, but this has to be discussed.

3.2 Discrétisation en volumes finis

Discretize the heat balance on a given element, and write the corresponding Finite Volume problem (do not forget boundaries). The corresponding coefficients must be explicitly given in the report.

3.3 Résolution numérique

Solve the FV problem using an iterative method. The convergence criterion is the normalized residuals on temperature.

3.4 Hints

- One can first try with uniform velocity profile, before using a laminar one.
- Are there symmetries in the system ?
- Use half (and quarter, and one eighth volumes on boundaries). Any comment ?

— The use of a Matlab function which has $\bar{h}$ as output argument and beta, L, average velocity, number of volumes,... as input arguments will ease the making of a result table.

4 Expected results

For different configurations (various β, L, Reynolds numbers...), compute the temperature profile in the duct.

For each configuration, you must compute the global Nusselt number.

You should present your results using various kind of graphs (e.g. several contour for several z values, plot T at a given (x, y) position vs z, local h vs z,...). Finding a trend for Nu vs Re and β is required.

Numerical validation can include the number of volumes (in each direction), the value of critical residual value, ... Although these tests must be detailed in the report, results must be prensented only for the adequate numerical settings.

5 Conditions

A written report is expected (around 10 pages excluding summary, appendix, ...), which details results and important steps in problem solution. Notation will also take into account an individual interview. The Matlab code is also to be given to the examinator (same date as for written report). Digital versions are preferred.

This work is personal : any plagiarism (either in code or report) will be heavily punished. It is however allowed to discuss about methods and results, as far as any consequent contribution is duely mentioned in the report.

Assignment due date is friday, 13$^{\text{th}}$ of january 2023. You are welcome to ask for help before this date... Send an email to `Francois.Lesage@univ-lorraine.fr` and a Teams or physical meeting will be arranged. Asking for help on the very last days might be unsuccessful...