Explanations / Elementary Linear Algebra

Exercise 21a

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Elementary Linear Algebra

ISBN: 9781118473504

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Explanation

Verified

Step 1

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We have

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix}$$

$T_A : R^3 \to R^3$ multiplication with A and $\{e_1, e_2, e_3\}$, where

$$e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)$$

Show that the

$$S = \{T_A(e_1), T_A(e_2), T_A(e_3)\}$$

is linearly independent in R^2.

First find $T_A(e_1), T_A(e_2), T_A(e_3)$

$$T_A(e_1) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$T_A(e_2) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \quad \begin{bmatrix} 2 \end{bmatrix}$$

$$T_A(e_3) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

According to **Definition 1 Section 4.3**, the given set is linearly independent if the vector equation

$$a \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + c \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

it only has a trivial solution. When multiplying the matrices we get a system of equations

$$a + b + c = 0 \tag{2}$$

$$b - 3c = 0 \tag{3}$$

$$-a + 2b = 0 \tag{4}$$

The corresponding matrix form of a system of equations is

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Reveal next step

Reveal all steps

Exercise 20

Exercise 21b