Circularity Evaluation Using Machine Learning SVR Algorithm

ME5300

Report Presentation

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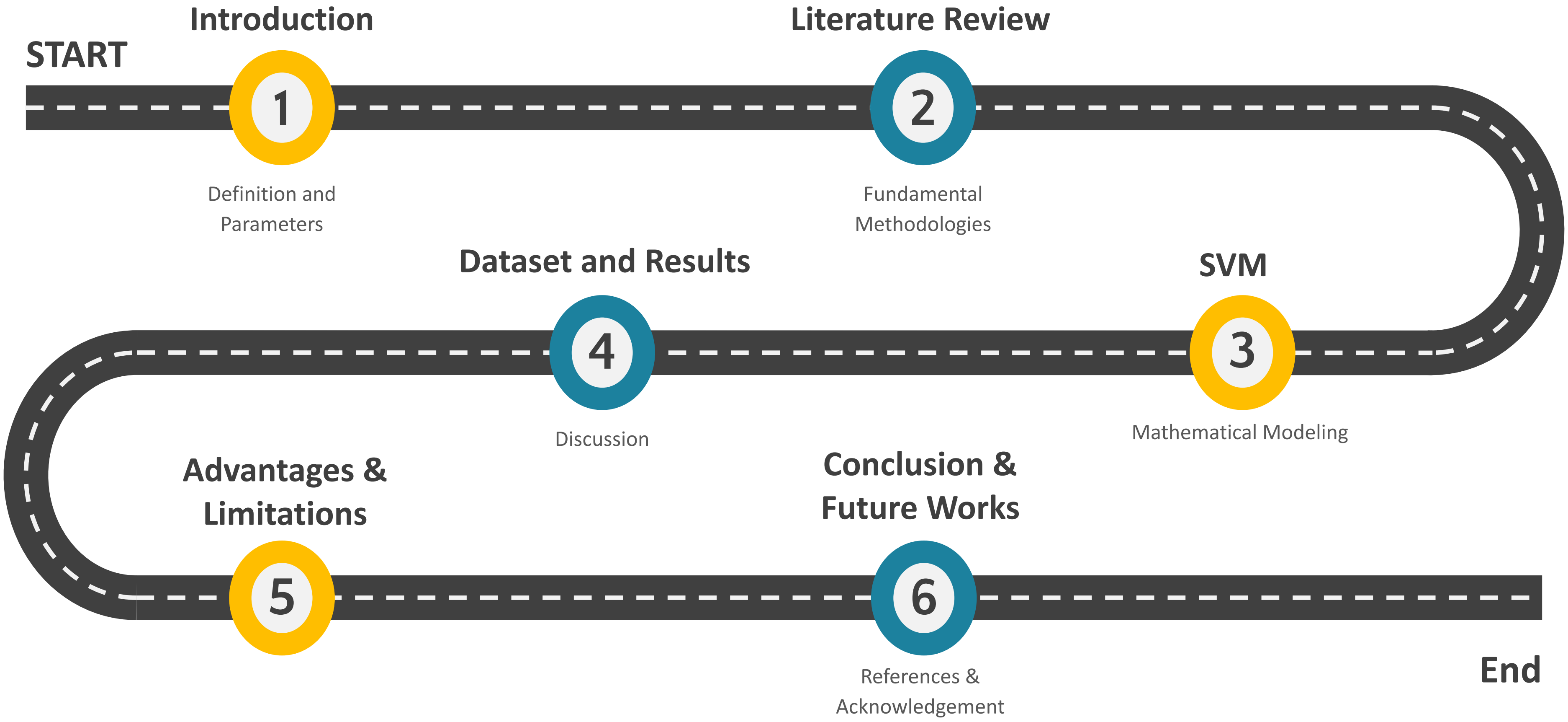
Roadmap of Presentation



Introduction

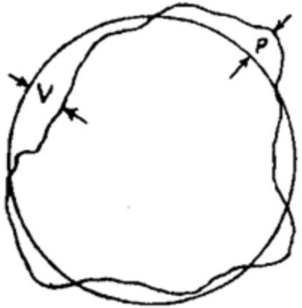
Circularity Evaluation

The ***error of circularity*** is defined as the radial distance between the minimum circumscribing circle and the maximum inscribing circle, which contain the profile of the surface at a section perpendicular to the axis of rotation

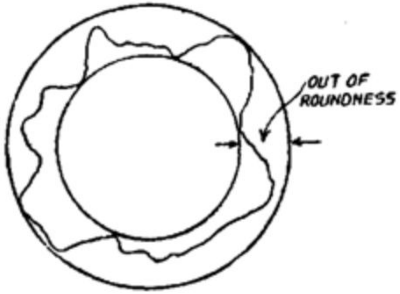
1. Poor Bearings
2. Shaft Misalignment
3. Deflection of WP or Tool
4. Lobing

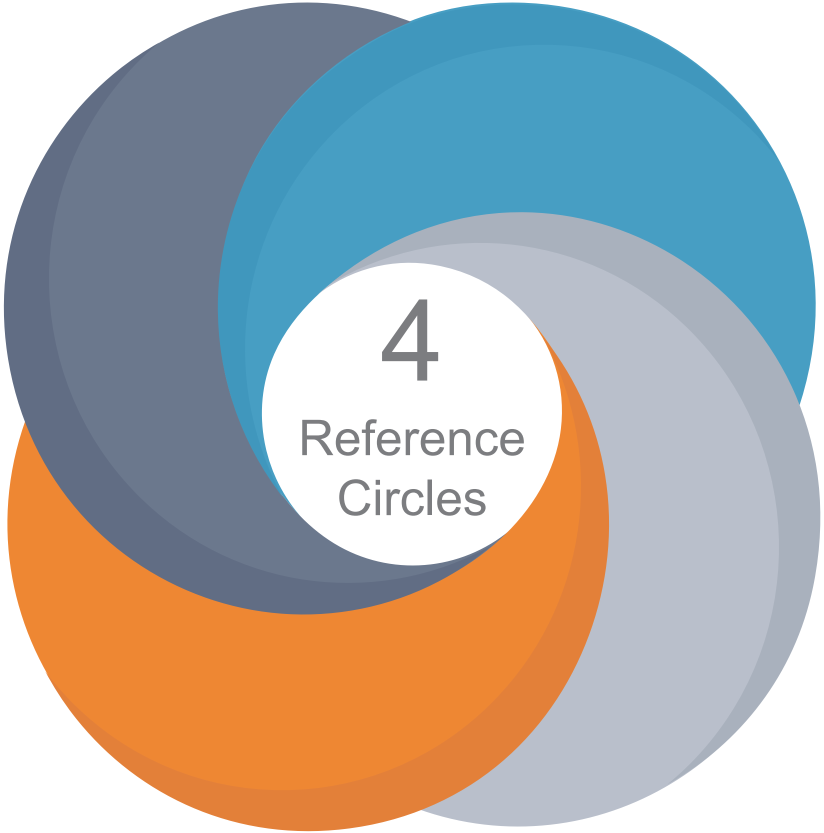
Reference Circles

.01
Least Squares Reference Centre

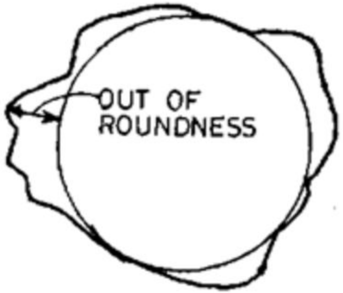


.02
Minimum Radial Separation Circles

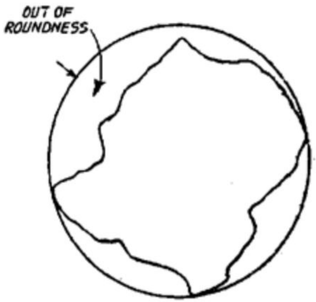




.03
Maximum Inscribed Circle



.04
Minimum Circumscribed Circle



Parameters

Least Squares Circle

This circle represents the average of all the peaks and valleys, and can be defined mathematically as:

“The sum of the squares of a sufficient number of equally spaced radial ordinates, measured from the circle to the profile has minimum value”

Minimum Zone/ Minimum Radial Separation Circles

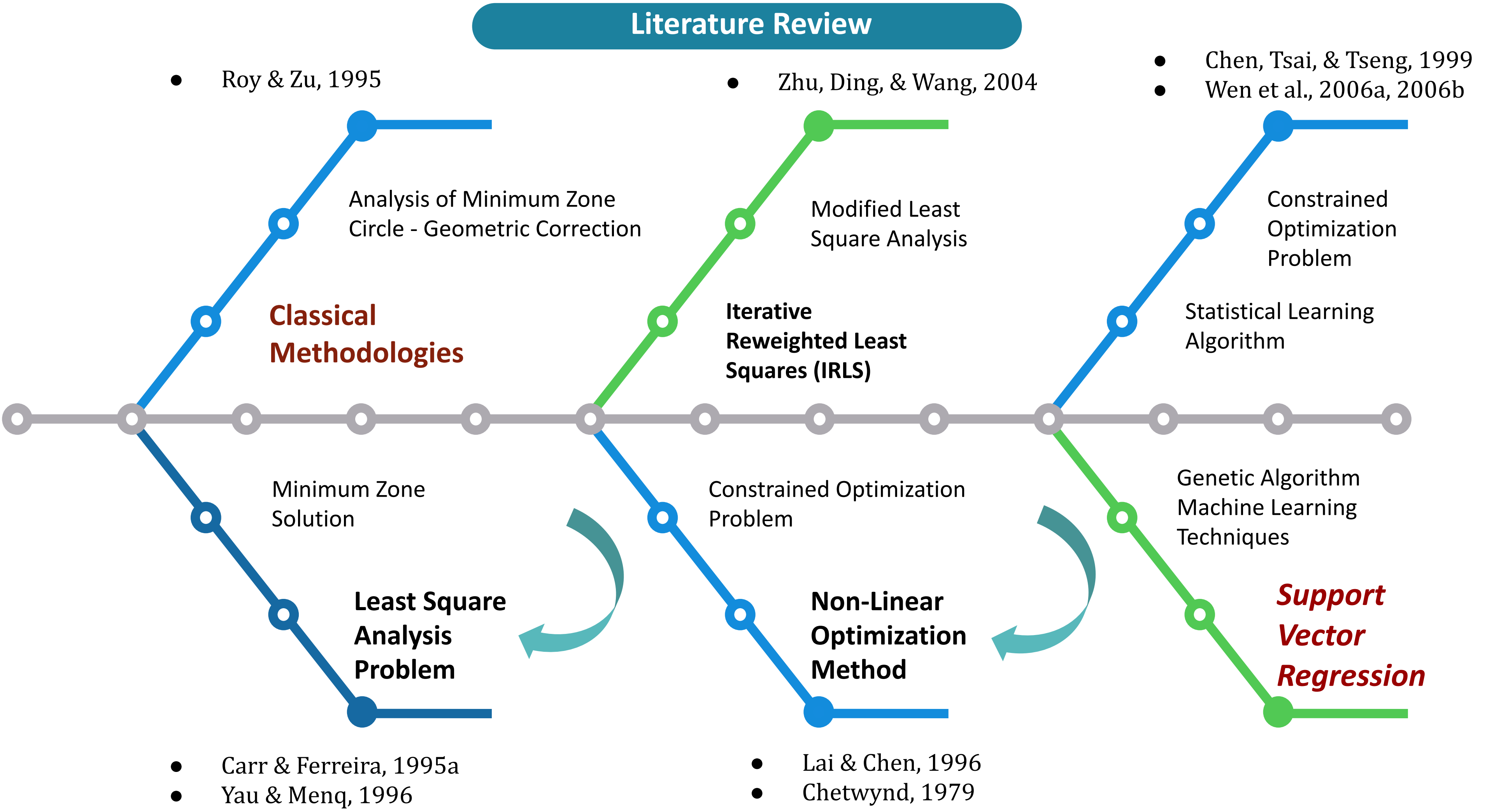
- These are two concentric circles that just enclose the profile and which have minimum radial separation
- The value of the out-of-roundness is the radial distance between the two circles
- The centre of such a circle is termed as the minimum zone centre.

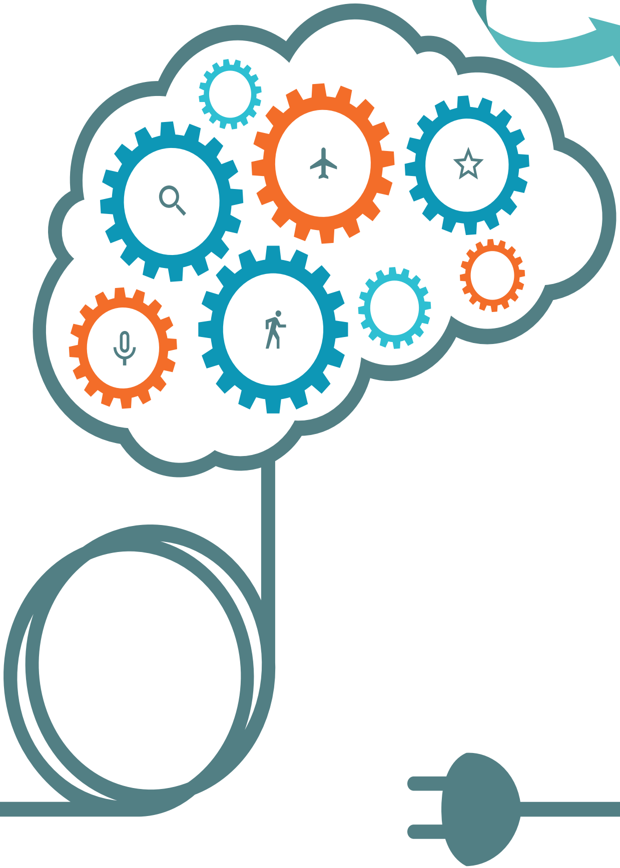
Maximum Inscribed Circle

- This is the largest circle that can be drawn inside the profile without cutting it
- Its centre and radius can be found by trial-and-error using compasses, by template or computer

Minimum Circumscribed Circle

- This is the smallest circle that will completely enclose the profile without cutting it
- Its centre and radius can be found in a similar manner to that of the inscribed circle


Literature Review
Roy & Zu, 1995
Analysis of Minimum Zone Circle - Geometric Correction
Classical Methodologies
Minimum Zone Solution
Least Square Analysis Problem
Carr & Ferreira, 1995a
Yau & Menq, 1996
Zhu, Ding, & Wang, 2004
Modified Least Square Analysis
Iterative Reweighted Least Squares (IRLS)
Constrained Optimization Problem
Non-Linear Optimization Method
Lai & Chen, 1996
Chetwynd, 1979
Chen, Tsai, & Tseng, 1999
Wen et al., 2006a, 2006b
Constrained Optimization Problem
Statistical Learning Algorithm
Genetic Algorithm Machine Learning Techniques
Support Vector Regression



Drawbacks

Least-Square Analysis

1. Tendency to reject good parts due to over fitting
2. Inherent nature of approximation over actual data
3. Difficult to be employed for real-time vision based applications

Nonlinear Programming

1. Deficiency lies in Computing Time
2. Calculations which depend on initial conditions
3. Other modifications have to be done like Limacon approximation and Transforming matrices

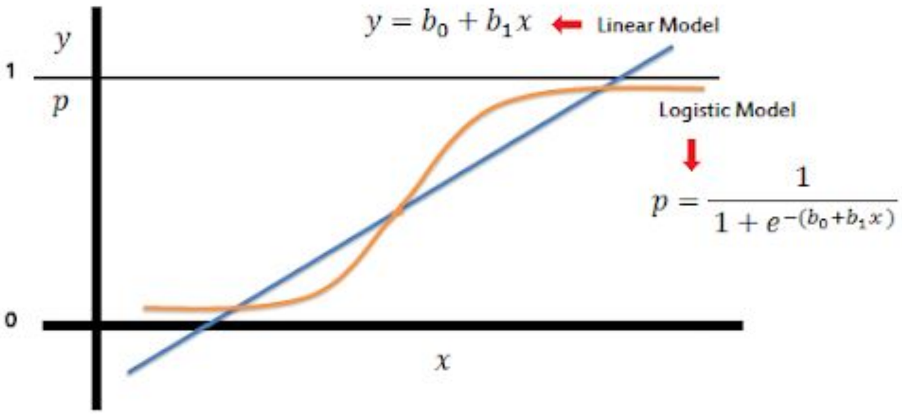
Logistic Regression

In statistics, the **logistic model** (or **logit model**) is used to model the probability of a certain class or event existing such as pass/fail, win/lose, alive/dead or healthy/sick.

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X$$

$$ln\left(\frac{P}{1-P}\right) = \beta_0 + \beta_1\, x$$

$$=> P = \frac{e^{\beta_0 + \beta_1\, x}}{1 + e^{\beta_0 + \beta_1\, x}}$$



- This can be extended to model several classes of events such as determining if images contain cat, dog, lion, etc.
- Each object being detected in the image would be assigned a probability between 0 and 1, with a sum of one.

Polynomial LSQ

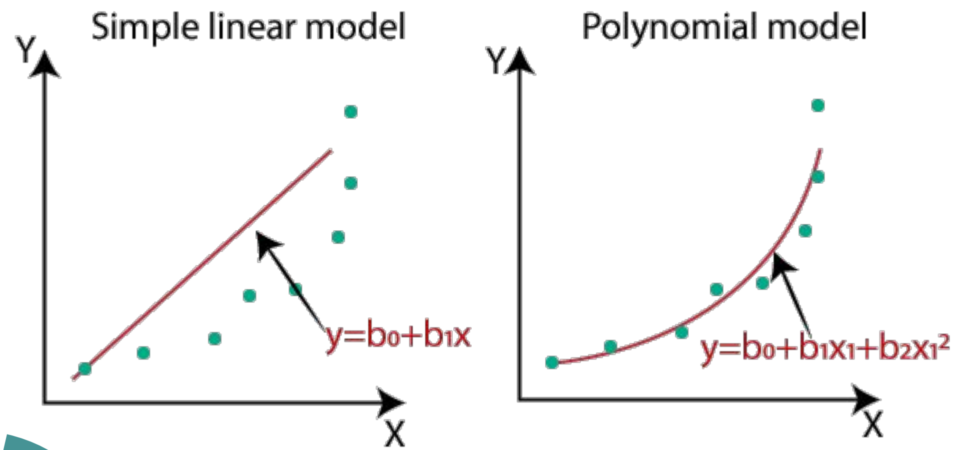
In statistics, **polynomial LSQ** is a form of regression analysis in which the relationship between the independent variable *x* and the dependent variable *y* is modelled as an *n*th degree polynomial in *x*. Polynomial regression fits a nonlinear relationship between the value of *x* and the corresponding conditional mean of *y*, denoted E(*y* |*x*).

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^m \\ 1 & x_2 & x_2^2 & \dots & x_2^m \\ 1 & x_3 & x_3^2 & \dots & x_3^m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^m \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{bmatrix},$$

Simple Linear Regression $y = b_0 + b_1 x_1$

Multiple Linear Regression $y = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_n x_n$

Polynomial Linear Regression $y = b_0 + b_1 x_1 + b_2 x_1^2 + \ldots + b_n x_1^n$



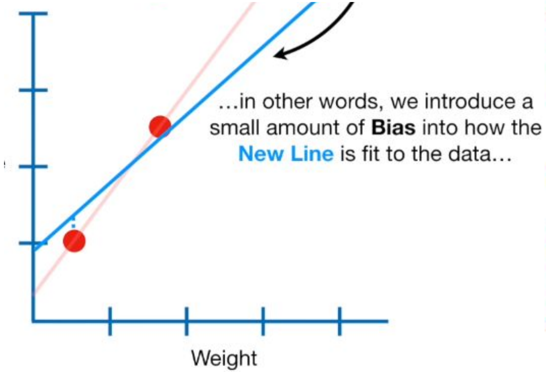
Polynomial LSQ is considered to be a special case of **multiple linear regression**

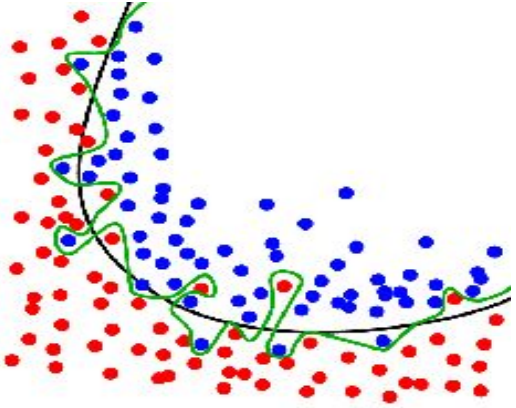
Ridge Regression

Ridge regression is a way to create a parsimonious model when the number of predictor variables in a set exceeds the number of observations, or when a data set has multicollinearity (correlations between predictor variables).

$$Cost\,(W) = RSS(W) + \lambda * (sum\ of\ squares\ of\ weights)$$

$$= \sum_{i=1}^{N} \left\{ y_i - \sum_{j=0}^{M} w_j\, x_{ij} \right\}^2 + \lambda \sum_{j=0}^{M} w_j^2$$





- Ridge Regression is a technique for analyzing multiple regression data that suffer from multicollinearity
- When multicollinearity occurs, least squares estimates are unbiased, but their variances are large so they may be far from the true value
- By adding a degree of bias to the regression estimates, ridge regression reduces the standard errors

Ridge Regression

Advantages

- Ridge regression can reduce the variance (with an increasing bias) – travel along the bias variance tradeoff curve
- Works best in situations where the OLS estimates have high variance
- Can improve predictive performance of the model (Generalization power increases)
- Works in situations where (p) < (n)

Disadvantages

- Ridge regression is not able to shrink coefficients to exactly zero
- As a result, it cannot perform variable selection

Code - Ridge Regression

$$Y^2 = -683.1 + 11.73*X + 14.5*X^2$$

```
# Ridge Regression
from sklearn.model_selection import GridSearchCV
from sklearn.linear_model import Ridge
ridge = Ridge()

parameters = {"alpha":[1e-15, 1e-10, 1e-8, 1e-4, 1e-3, 1e-2, 1, 5, 10, 20]}
ridge_regression = GridSearchCV(ridge, parameters,
scoring='neg_mean_squared_error', cv=5)
ridge_regression.fit(xs, y)
```

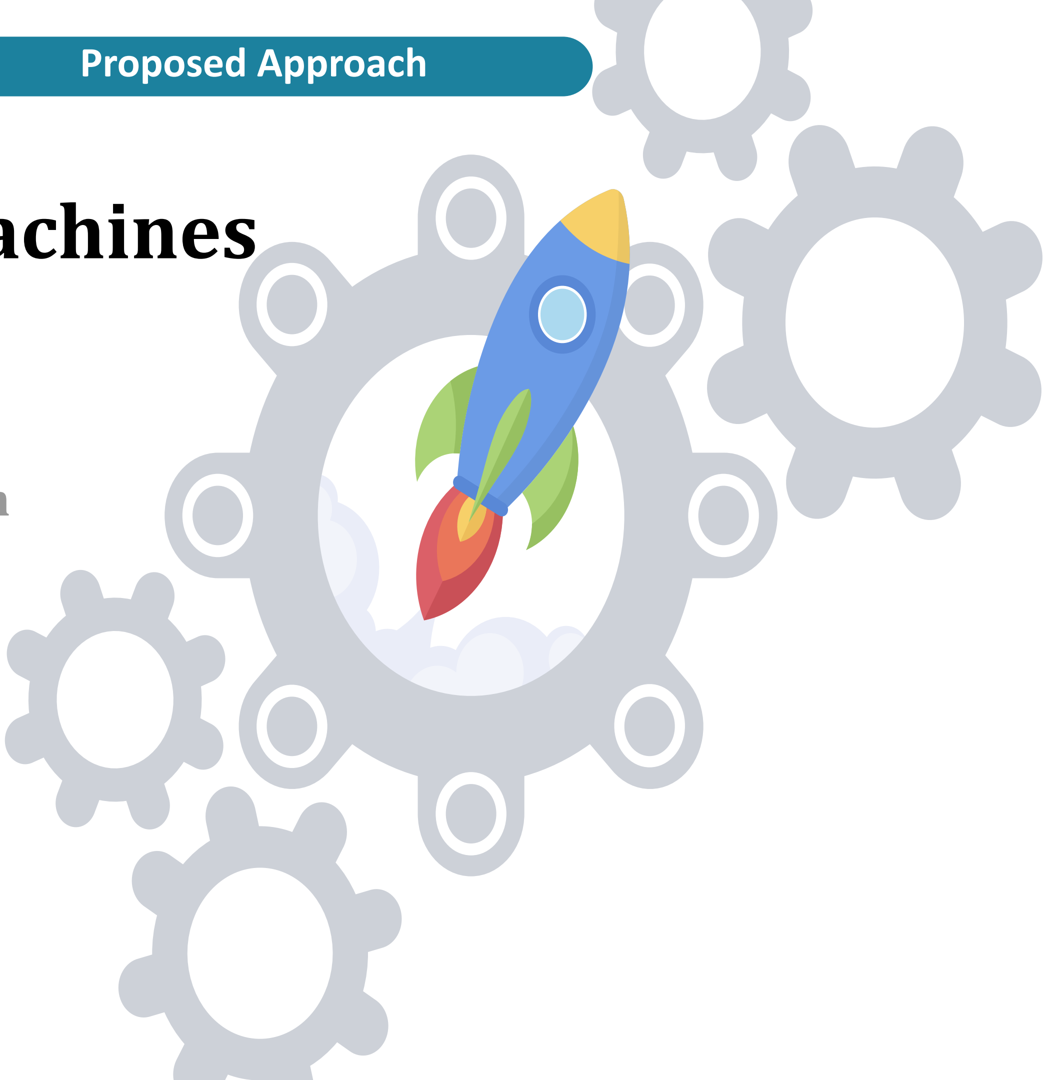
```
print(ridge_regression.best_params_)
print(ridge_regression.best_score_)
```

Proposed Approach

Support Vector Machines (SVM)

Circularity evaluation through

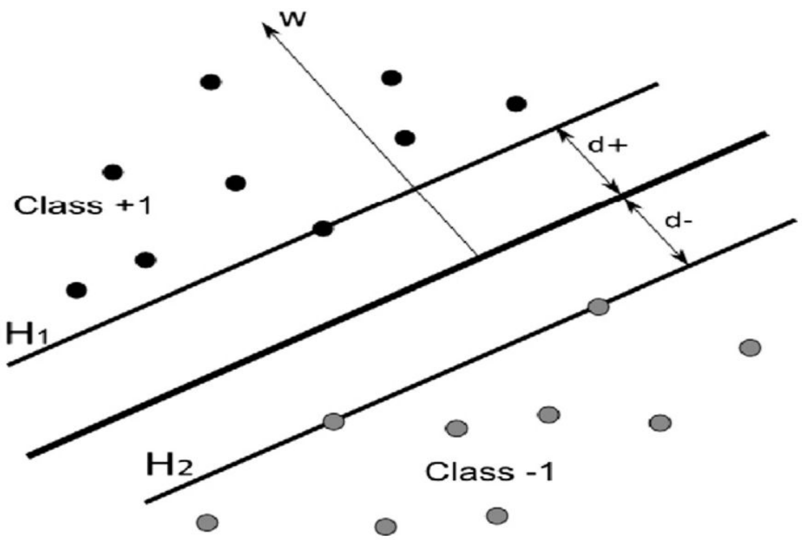
SVR Analysis approach



Support Vector Machines

In machine learning, support-vector machines are supervised learning models with associated learning algorithms that analyze data for classification and regression analysis.

Classification



Prediction

For circularity measurements we would be using ***Support Vector Regression Analysis***

This set of data is classified according to its values of y along the different values of x. They can be separated by a linear indicator, or commonly called a hyperplane.

Applications of SVM

B. Operation Efficiency

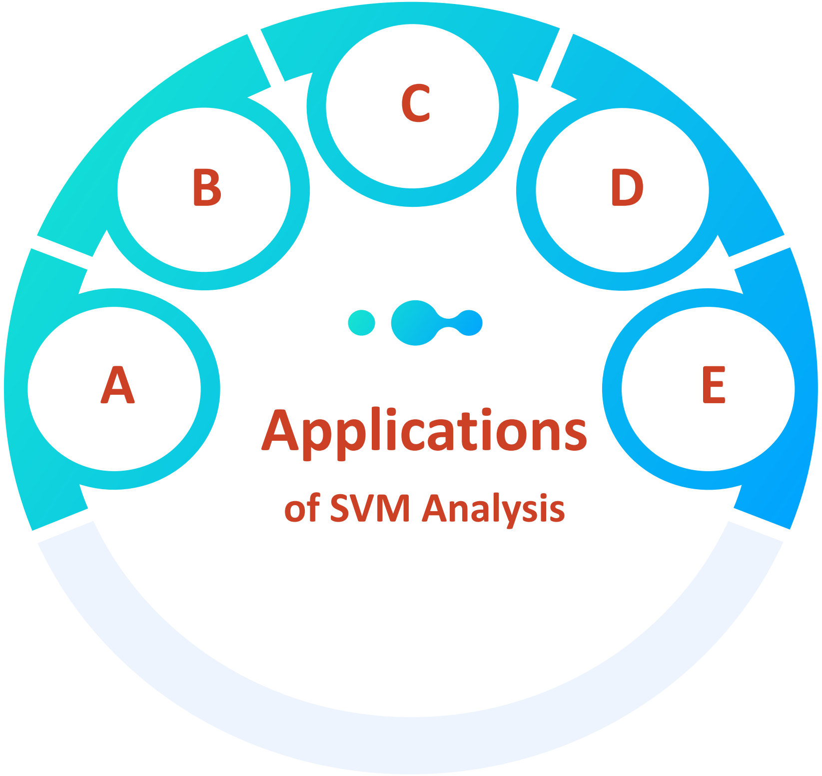
Data-driven decision making eliminates guesswork, hypothesis and corporate politics from decision making. This improves the business performance by highlighting the areas that have the maximum impact on the operational efficiency and revenues.

A. Supporting Decisions Robust Method

Predictive analytics i.e. forecasting future opportunities and risks is the most prominent application of regression analysis in business. Demand analysis, for instance, predicts the number of items which a consumer will probably purchase.

C. Predictive Analytics

This technique acts as a perfect tool to test a hypothesis before diving into execution.



D. Correcting Errors

1. Regression is not only great for lending empirical support to management decisions but also for identifying errors in judgment
2. This analysis can provide quantitative support for decisions and prevent mistakes due to manager's intuitions.

E. New Insights

1. RA techniques can find a relationship between different variables by uncovering patterns that were previously unnoticed.
2. For example, analysis of data from point of sales systems and purchase accounts may highlight market patterns like increase in demand on certain days of the week or at certain times of the year.

Fundamentals of SVM

Equation of Hyperplane:

$$(w \cdot x) + b = 0$$

Margin of Hyperplane

$$d_{+} + d_{-} = 0$$

The Rules for separating Hyperplanes can be defined by following inequalities:

For Y(i) = +1

$$(w \cdot x_i) + b \geqslant 1$$

For Y(i) = - 1

$$(w \cdot x_i) + b \geqslant 1$$

Goal of **SVM** is to **Minimize** this cost function:

$$\frac{1}{2}\|w\|^2$$

Proposed SVR Algorithm

Formation of Kernel:

$$x \rightarrow \phi(x) : \phi(x) = \begin{bmatrix} x^2 & x & y^2 & y \end{bmatrix}$$

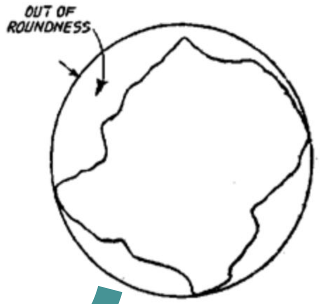
$$w = [w_1 \; w_2 \; w_1 \; w_3]$$

Equation for Plane Vector

Equation for Inner Circle

$$\left(x_1 + \frac{w_2}{2w_1}\right)^2 + \left(y_1 + \frac{w_3}{2w_1}\right)^2 = \frac{w_2^2 + w_3^2}{4w_1^2} + \frac{-1-b}{w_1}$$

$$\left(x_2 + \frac{w_2}{2w_1}\right)^2 + \left(y_2 + \frac{w_3}{2w_1}\right)^2 = \frac{w_2^2 + w_3^2}{4w_1^2} + \frac{1-b}{w_1}$$



Equation for Outer Circle

Shared Centers of Two Circles

$$\bar{x} = -\frac{w_2}{2w_1}, \quad \bar{y} = -\frac{w_3}{2w_1}$$

Goal of **SVR** is to **Minimize** this cost function:

$$\frac{2}{\|w\|^2}$$

Such that

$$-1 \leqslant (w \cdot \phi(x_i)) + b \leqslant +1$$

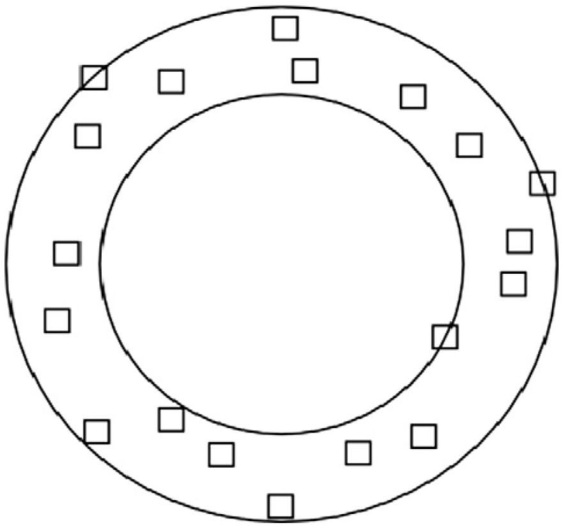
Dataset Specifications

Credible dataset with proper specifications is necessary for **Computational Experiments**

I have used the **12 different datasets** available through different literature

Objectives:

- **Test the proposed algorithm**
- **As well as to compare it with LSQ**



Dataset Specifications:

- Dataset contain **875 sample points**
- One cylindrical measurement with 4 different cross-sections
- Matrix of **400 X 3** (Including Z-axis elevation)

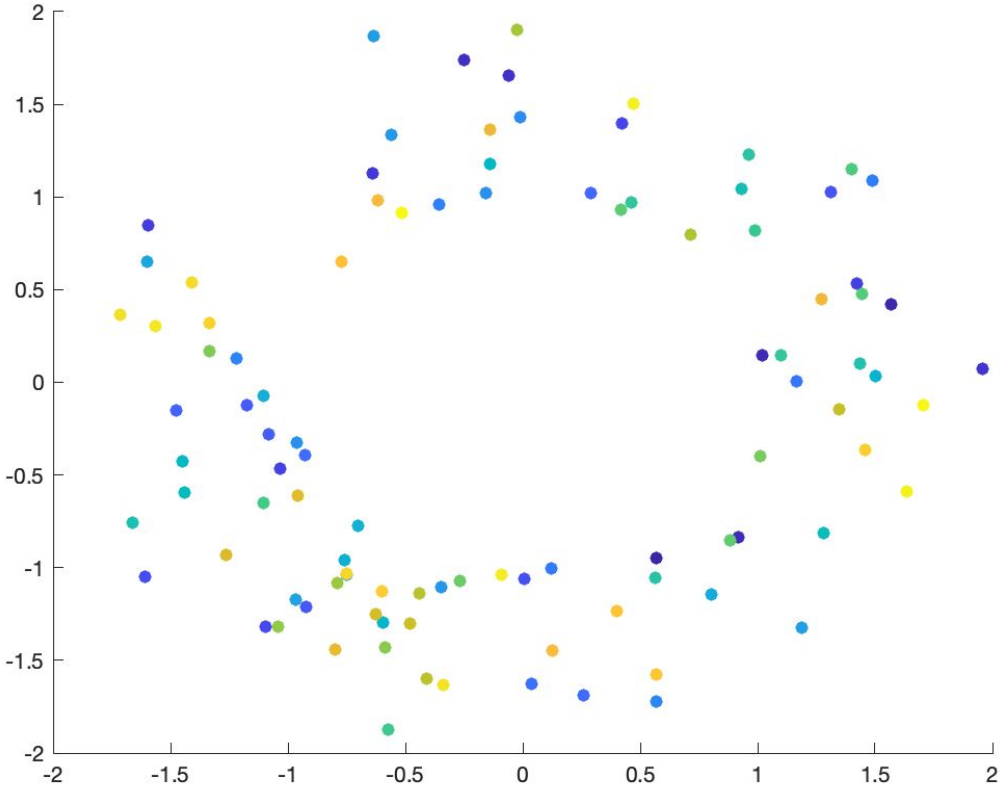
Dataset Visualization

Data Specifications

- **Radius** (Nominal): 1.2
- Number of Cross-sections: 4
- Z-Axis Bound: 50
- Number of Data points per Cross - section: 100
- Axes: X & Y

2-D View of the Sample Data

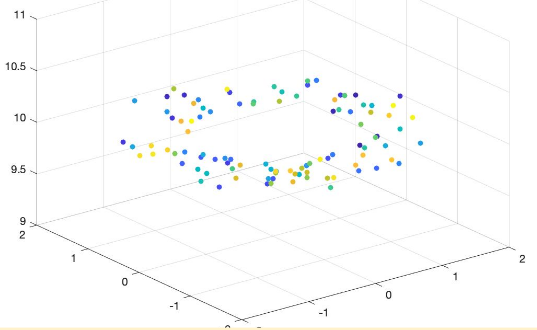
Given figure is the plot of only one cross-section



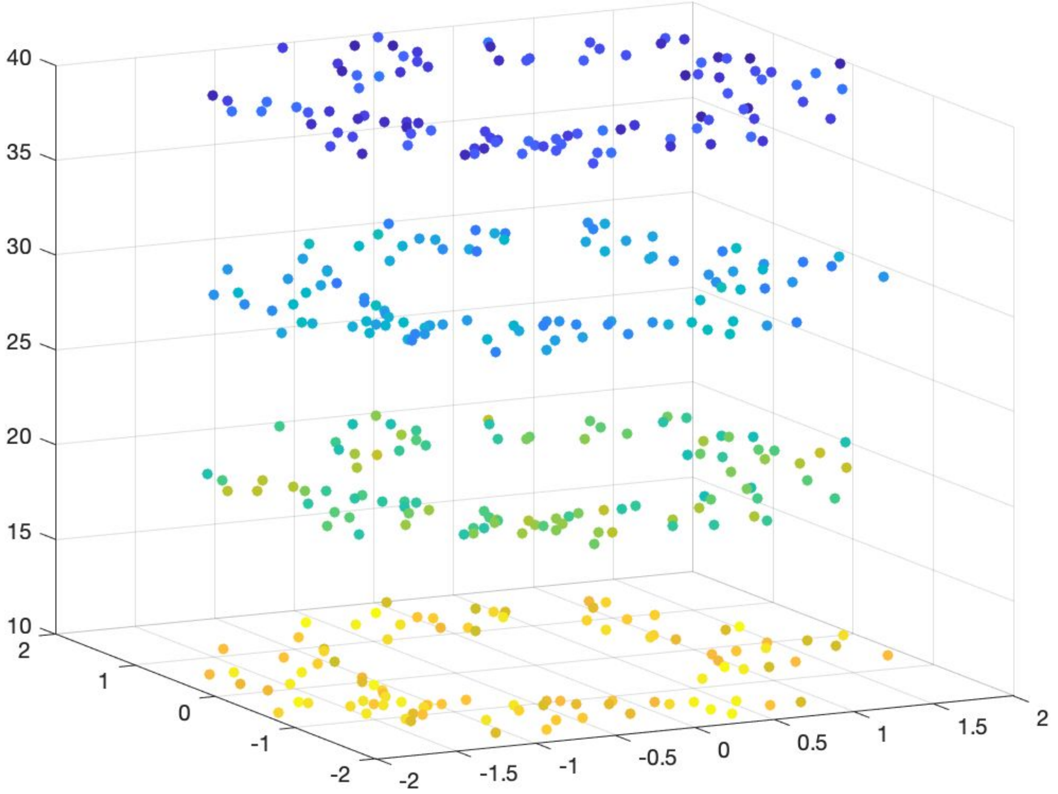
MATLAB Software has been used for Data Visualization and Results

Dataset Visualization

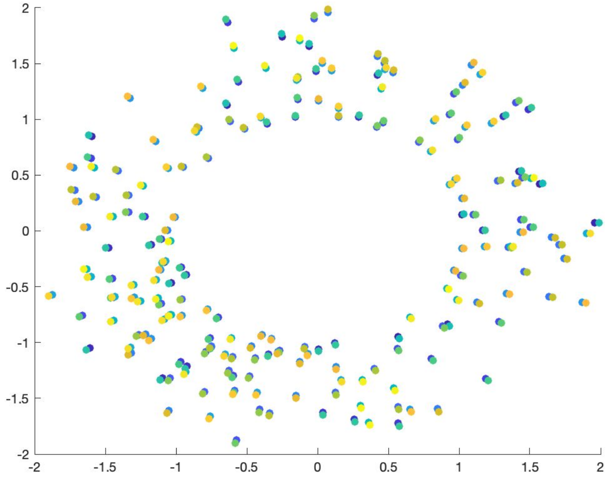
3D-View of Single Cross-Section



Points data for Circularity Evaluation All four cross-sections



2D-View of entire Dataset

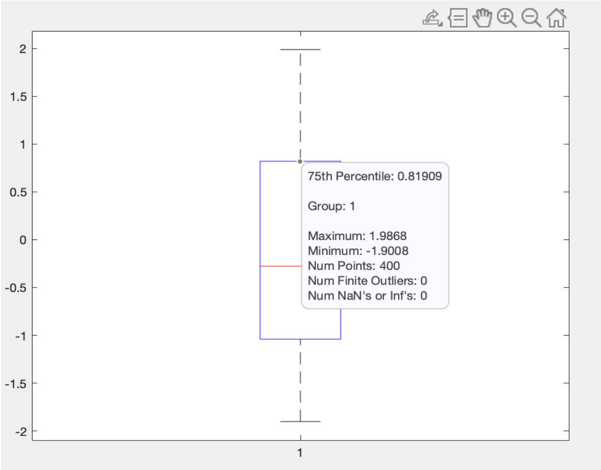


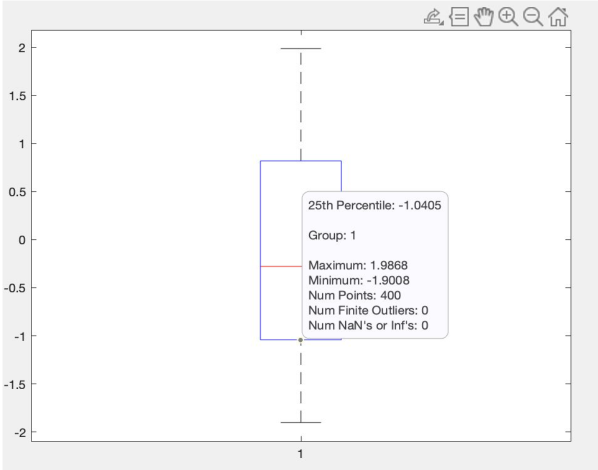
MATLAB Software has been used for Data Visualization and Results

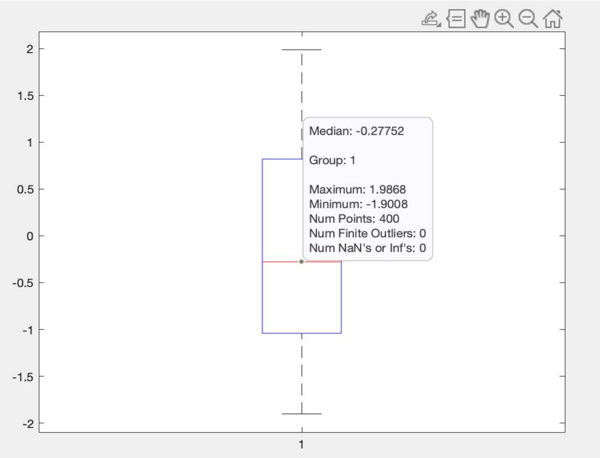
Results & Analysis

Boxplot for X Coordinates

Maximum: +1.9868; Minimum: -1.90068







Median: 0.81909

25th Percentile: -1.0405

75th Percentile: -0.27752

Note: Number of outliers = 0

MATLAB Software has been used for Data Visualization and Results

Results & Analysis

Circularity Analysis

Maximum: +1.9868; Minimum: -1.90068

Cross-Section	LSQ x-coord	LSQ y-coord	SVR x-coord	SVR y-coord
Z = 10	-0.0541068	-0.1783084	-0.4483550	-0.3859560
Z = 20	0.0200256	0.1670919	0.5729632	0.0928081
Z = 30	0.0278199	-0.0310403	-0.0073595	0.0433884
Z = 40	-0.0896733	-0.0639159	0.0794822	0.5145722

MATLAB Command for CPU Time

```
tStart = cputime;
pause(1)
tEnd = cputime - tStart
```

...

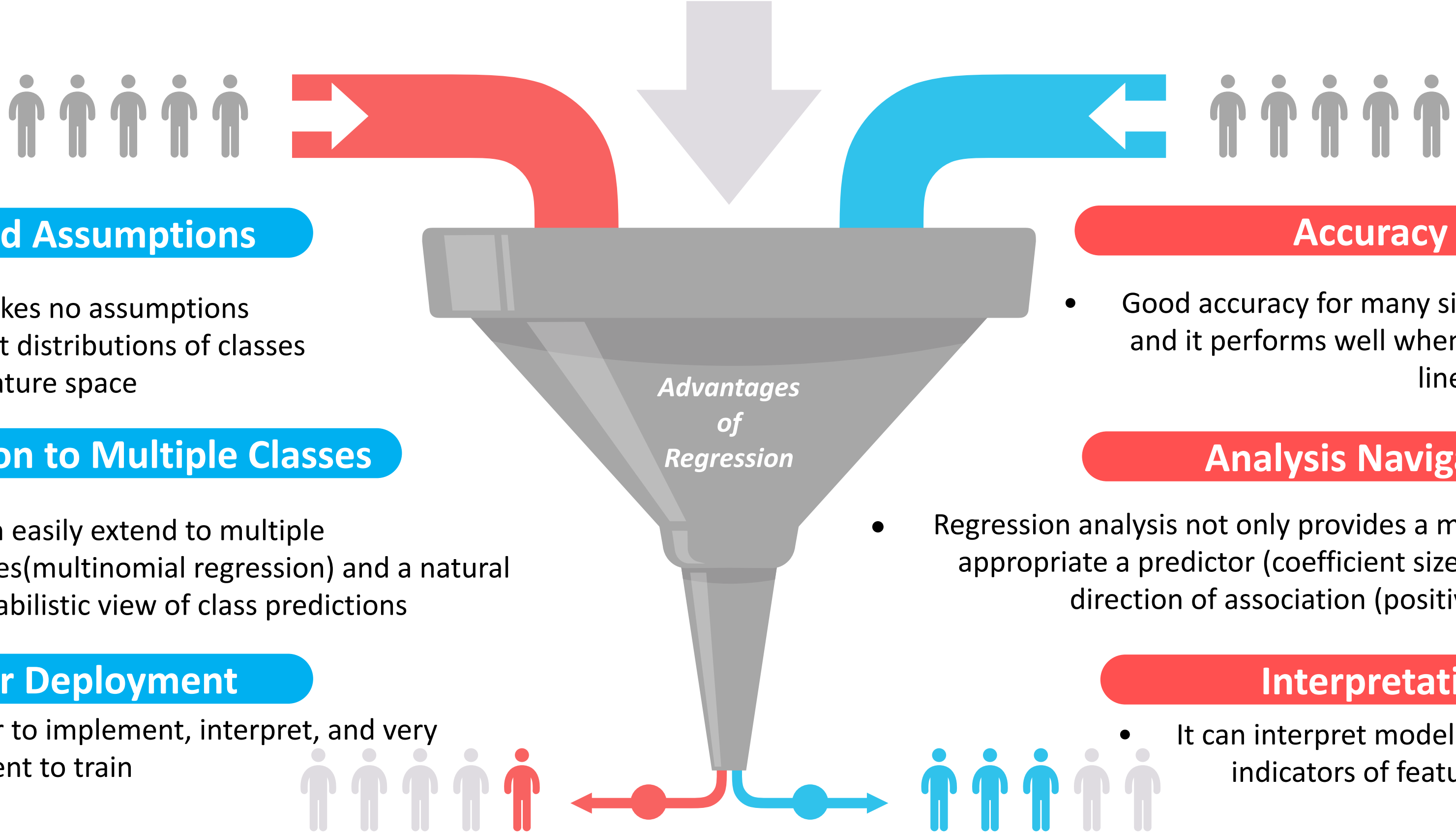
LSQ CPU Time: 0.984375 ms

SVR CPU Time: 0.718125 ms

MATLAB Software has been used for Data Visualization and Results

Advantages of SVR

Several modifications in Support Vector have diverse advantages



Limited Assumptions

- It makes no assumptions about distributions of classes in feature space

Extension to Multiple Classes

- It can easily extend to multiple classes(multinomial regression) and a natural probabilistic view of class predictions

Easier Deployment

- Easier to implement, interpret, and very efficient to train

Accuracy

- Good accuracy for many simple data sets and it performs well when the dataset is linearly separable

Analysis Navigation

- Regression analysis not only provides a measure of how appropriate a predictor (coefficient size)is, but also its direction of association (positive or negative)

Interpretation

- It can interpret model coefficients as indicators of feature importance

Limitations of Theory

01

It might look easy for linear relationships but as not everything is the world is linear, more complex modifications take the stage which is so much more complex

02

Outliers and Overfitting

It stands no chance against outliers are extreme points. It can cause to overfit very easily.

03

Bound on Dependant Variable

Used to predict discrete functions. Dependent variable of **SVR** is bound to discrete set.

04

Fine-Tuning is difficult. Once assumptions are violated they need to be corrected before fine-tuning the model as it becomes unproductive very quickly.

05

Multicollinearity

Requires average or no multicollinearity between independent variables.

06

Complex Relationships

Tough to obtain complex relationships using **support vector regression**. More powerful algos such as Neural Networks can easily outperform.

Ways to detect Overfitting

- The easiest way to avoid overfitting is to **increase sample size** by collecting more data
- If data size is limited, the second option is to reduce the number of predictors in our model — either by combining or eliminating them.
- **Factor Analysis** is one method to identify related predictors that might be candidates for combining.

1. Cross-Validation

Use **cross validation** to detect overfitting: this partitions our data, generalizes our model, and chooses the model which works best. One form of cross-validation is **predicted R-squared**. Most good statistical software will include this statistic, which is calculated by:

- **Removing one observation** at a time from the data
- **Estimating the regression equation** for each iteration
- Using the regression equation **to predict the removed observation**

2. Shrinkage & Resampling

Shrinkage and resampling techniques (like this R-module) can help you to find out how well your model might fit a new sample

3. Automated Methods

Automated stepwise regression shouldn't be used as an overfitting solution for small data sets

Conclusion

- The presentation covers theoretical and experimental aspects of circularity evaluation algorithm based on **support vector machine learning** with a specific kernel function, referred to as SVR, as a viable alternative to traditional least square method (LSQ).
- It is concluded that this outstanding performance is ensured because of **relatively simple structure in the employed kernel function**.
- Using the theory of support vector machine regression, the proposed algorithm provides more accurate results with **less CPU time**.
- This is supported by computational experiments that show that the strength of the proposed algorithm is the ability to always accurately assess the circularity of any sample of data points while **LSQ and NLP have instances of failure**.
- It has been shown that **the failure rate of LSQ increases** when the measured data points have **higher variance**.
- In a real-world setting, this would provide a manufacturer with an algorithm that can be trusted to give the correct answer rather than making a good part rejected because of inaccurate computational results.

Future Work

01

Boxplot Analysis

Boxplot Analysis for circularity has to be done with two cases:

1. **With Outliers**
2. **Without Outliers**

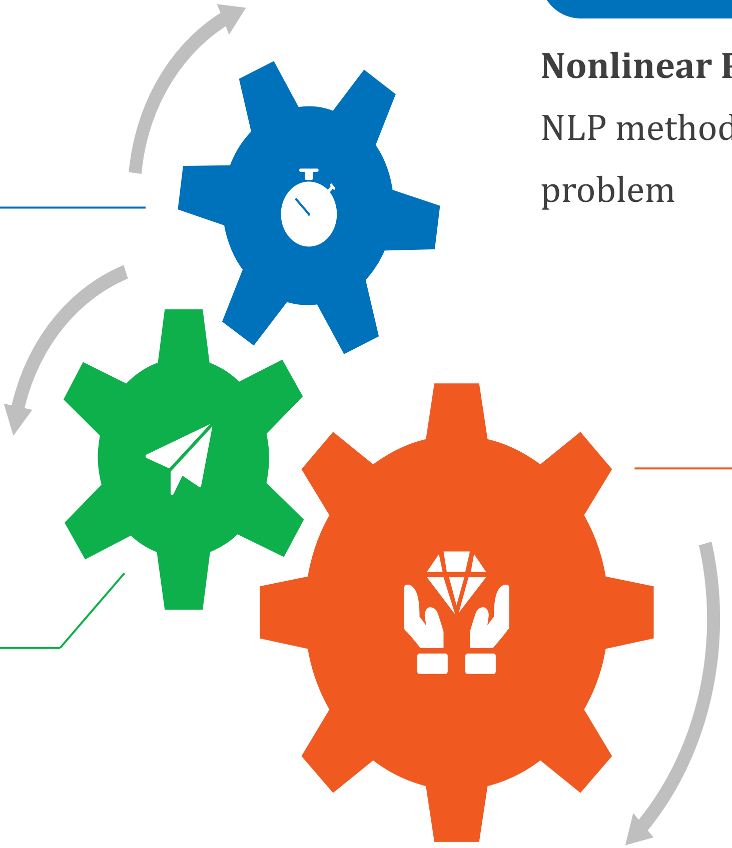
02

CPU Time

Anova Test for CPU Time has to be performed

CPU Time Order:

SVR < LSQ < NLP



NLP

Nonlinear Programming:

NLP method solves following optimization problem

03

NLP Testing

Cost Function: $\min\ r_1 - r_2$

$$r_2 \leqslant \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2} \leqslant r_1$$

Given Constraint

R1: Outer Radius; R2: Inner Radius

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Acknowledgement

I would like to thank **Prof. N. Arunachalam** for conducting this course **ME5300 Metrology And Computer Aided Inspection** and giving us this opportunity to present report presentation. The lectures throughout the semester helped us understand and build a strong grasp on the subject. I'd also like to thank our Teaching Assistants and fellow coursemates who helped us throughout this entire course.

Note

All the MATLAB Codes, the Report, the Dataset and the Result images/ plots in .png format have been attached in the file which was submitted successfully on 20/02/2021 through the Google Form.