



Math Prerequisites: Practice Exercises

Functions

Exercise 1

Verbal definitions

1. Define what a relation is
2. Define what a function is
3. What is the difference between a relation and a function?
4. What is the difference between the input and the output of a function?

Exercise 2

Algebraic

Determine whether the relations below represent a function. (x, y) stand for x is in relation with y

1. $\{(a, b), (c, d), (a, c)\}$
2. $\{(a, b), (b, c), (c, c)\}$
3. $\{(Mbog Bassong, Cosmogenese 1), (Cheikh Anta Diop, Nation Naigre et Culture), (Cheikh Anta Diop, Civilisation ou Barbarisme) \}$

Exercise 3

Algebraic

Determine whether the relations below between x and y represent functions.

1. $x = y^3$
2. $y = \pm\sqrt{(1 - x)}$
3. $y = x^3$
4. $y^2 = x^2$

Exercise 4

Evaluating functions



Evaluate the function $f(x) = 2x - 5$; $f(x) = \sqrt{2 - x} + 5$; $f(x) = \frac{6x-1}{5x+2}$ (apply the function to an argument) at the indicated value

1. $f(-3)$
2. $f(2)$
3. $f(-a)$
4. $-f(a)$
5. $f(a + h)$
6. $f(\frac{-2}{5})$

Exercise 5

Images and Antecedents

Given the function $k(t) = 2t - 1$

1. Evaluate $K(2)$
2. Solve $K(t) = 3$
3. What can we deduce from the above results?

Exercise 6

Domain of a function

Find the domain of definition for each of the following functions:

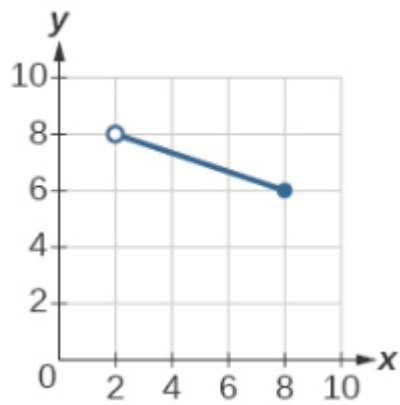
1. $f(x) = 5 - 2x^2$
2. $f(x) = \sqrt{4 - 3x}$
3. $f(x) = \frac{9}{(x-6)}$

Exercise 7

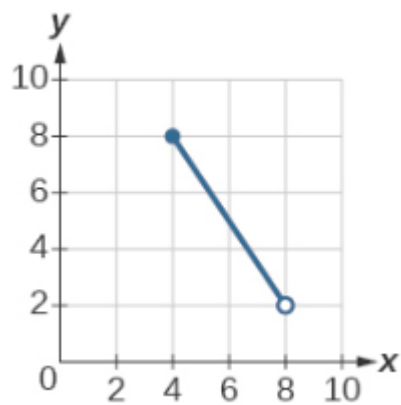
Domain and Range of a function

Find the domain of definition and range for each of the following functions:

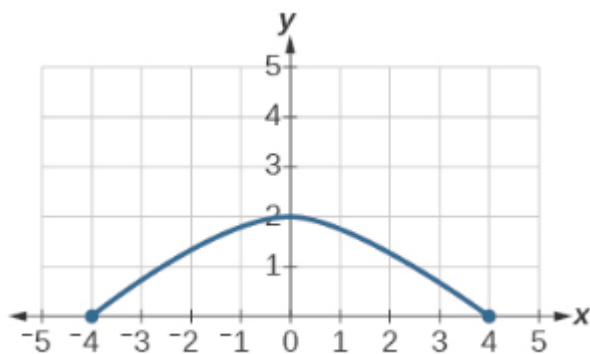
1. Function 1



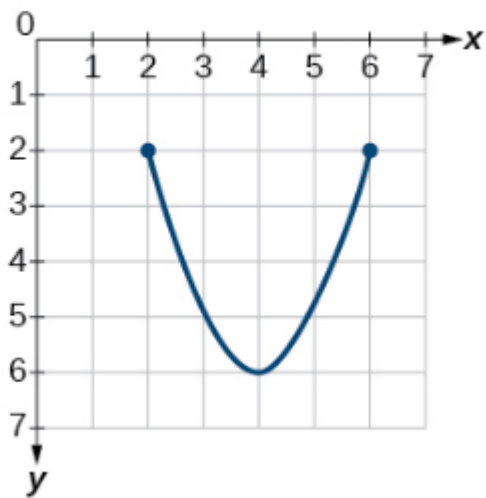
2. Function 2



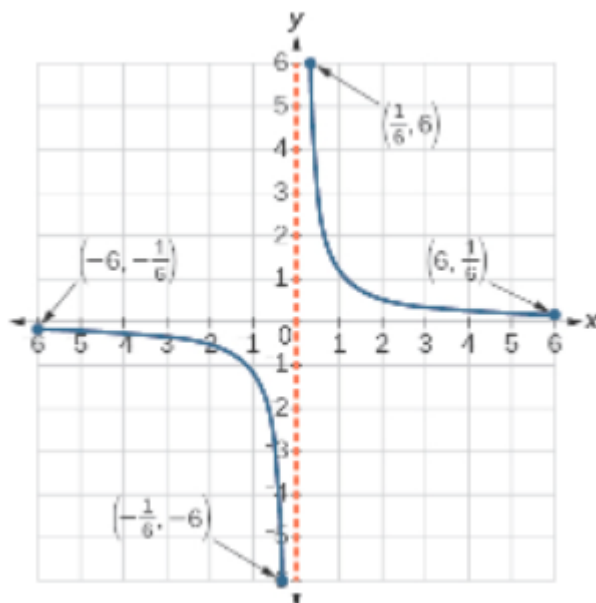
1. Function 3



2. Function 4



3. Function 5



Exercice 8

Real World Application

The height h of a projectile is a function of the time t it is in the air. The height in feet for seconds is given by the function: $h(t) = -16t^2 + 96t$

1. What is the domain of the function?
2. What does the domain mean in the context of the problem?



Exercise 9

Real World Application

The cost in MaatCoin (a local African token) for digitizing (into a NFArt) a stolen artwork is given by the function: $cost(numberOfPieces) = 10 * numberOfPieces + 500$

1. Calculate the fixed cost. The fixed cost is determined when zero items are produces
2. What is the cost for making 12 items?
3. How many items could I make when given 600 MatCoins?
4. Let suppose the maximum production cost allowed is 15000 MaatCoins. What is the domain and range of the cost function?

Exercise 10

Real World Application: Parametric functions

The cost in MaatCoin (a local African token) in digitizing (into a NFArt) a stolen artwork is given by the function: $cost\ m\ of\ (numberOfPieces) = m * numberOfPieces + 500$ where m represents the unitary price of a given piece of artwork.

1. Redo Exercise 7 with m fixed to:
 - a. m is 20
 - b. m is 3
 - c. m is 7

Exercise 11

Local Context Application

The amount of garbage (plastic wrappers, putrefied food, ...etc), G , produced by a city in Cameroon with population p is given by $G = f(p)$. G is measured in tons per week, and p is measured in millions of people.

The town of Douala has a population of 5,000,000 and produces 2,500 tons of garbage each week. Express this information in terms of the function f . Douala has a population of 5,000,000 and produces 2,500 tons of garbage each week. Express this information in terms of the function f .

Explain the meaning of the statement $f(5) = 2$.

How can we turn this problem into a parametric function where the parameter represents the amount of garbage per city?



Exercise 12

Local Context Application

The number of cubic meters of dirt, D , needed to cover a garden with an area a in square meters is given by $D = g(a)$.

A garden with area 5000 m^2 requires 50 m^3 of dirt.

1. Express this information in terms of the function g .
2. Explain the meaning of the statement $g(100) = 1$.

Exercise 13

Defining Operation / Composition of Functions

Assuming f and g are well defined functions,

1. How do you define the function $\frac{f}{g}$ given f and g are functions
2. How do you define the function $f - g$ given f and g are functions
3. How do you define the function $f + g$ given f and g are functions
4. How do you define the function $f * g$ given f and g are functions
5. How do you define $f \circ g$?
6. How do you find the domain for the composition of two functions ?

Exercise 14

Composition of Functions

Given $f(x) = 2x^2 + 1$ and $g(x) = 3x + 5$, find the following:

1. $f(g(2))$
2. $f(g(x))$
3. $g(f(x))$
4. $(g \circ g)(x)$
5. $(f \circ f)(-2)$

Exercise 15

Composition of Functions: Application



1. The function $D(p)$ gives the number of items that will be demanded when the price is p . The production cost is $C(x)$ the cost of producing x items. To determine the cost of production when the price is 6 MaatCoins, which of the following should we use?
 - a. Evaluate $D(C(6))$
 - b. Evaluate $C(D(6))$
 - c. Solve $D(C(x)) = 6$
 - d. Solve $D(C(p)) = 6$
2. The function $A(d)$ gives the pain level on a scale of 0 to 10 experienced by a patient with d milligrams of a pain-reducing drug in her system. The milligrams of the drug in the patient's system after t minutes is modeled by $m(t)$. Which of the following would you do in order to determine when the patient will be at a pain level of 4?
 - a. Evaluate $A(m(4))$
 - b. Evaluate $m(A(4))$
 - c. Solve $A(m(t)) = 4$
3. A store offers customers a 30% discount on the price x of selected items. Then, the store takes off an additional 15% at the cash register.
 - a. Write a price $P(x)$ function that computes the final price of the item in terms of the original price x .
(Hint: Use function composition to find your answer.)
4. A raindrop hitting a lake makes a circular ripple. If the radius, in inches, grows as a function of time in minutes according to $r(t) = 25\sqrt{t} + 2$,
 - a. Find the area of the ripple as a function of time.
 - b. Find the area of the ripple at $t = 2$
5. A forest fire leaves behind an area of grass burned in an expanding circular pattern. If the radius of the circle of burning grass is increasing with time according to the formula $r(t) = 2t + 1$, express the area burned as a function of time, t (minutes).
6. The radius, in inches, of a spherical balloon is related to the volume V , by $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$. Air is pumped into the balloon, so the volume after t seconds is given by $V(t) = 10 + 20t$.
 - a. Find the composite function $r(V(t))$.
 - b. Find the exact time when the radius reaches 10 inches
7. The number of bacteria in a refrigerated food product is given by $N(t) = 23T - 56T + 1$, $23 < T < 33$, where T is the temperature of the food. When the food is removed from the refrigerator, the temperature is given by $T(t) = 5t + 1.5$, where t is the time in hours.
 - a. Find the composite function $N(T(t))$
 - b. Find the time (round to two decimal places) when the bacteria count reaches 6752.