Morning

Regular Expression

1. Definition
2. Designing
3. Equivalence with FA

Arithmetical Expression

$$0,\ 1+2,\ \ 3\times(5-2),\ (56-7)^2\ ,\ \ldots\ldots$$

- Formal definition

- Inductive definition

  - Any number is a arithmetical expression

  - If **a** and **b** are arithmetical expressions , then

    so is $a+b, a-b, a\div b, a\times b, a^n, (a)$.

Building Regular Expressions

BASIS

1. ε is a regular expression, denoting the languages $\{\varepsilon\}$.

2. ϕ is a regular expression, denoting the languages ϕ.

3. For each a in Σ, a is a regular expression and denotes the language $\{a\}$.

Building Regular Expressions

INDUCTION

1. If E and F are regular expressions, denoting the language $L(E)$ and $L(F)$, then $E+F$, EF and E^* are regular expressions that denote the languages $L(E)\cup L(F)$, $L(E)L(F)$ and $(L(E))^*$.

2. If E is a RE then so is (E) .

Example What is the language defined by r

$$r = (a+b)^*(a+bb)$$

$a \rightarrow \{ \text{a} \}$, $b \rightarrow \{ \text{b} \}$

$a + b \rightarrow \{ \text{a} \} \cup \{ \text{b} \} = \{\text{a, b}\}$

$bb \rightarrow \{ \text{b} \}\{ \text{b} \} = \{ \text{bb} \}$

$a + bb \rightarrow \{ \text{a} \} \cup \{ \text{bb} \} = \{ \text{a, bb} \}$

$(a+b)^* \rightarrow \{ \text{a ,b} \}^*$

$(a+b)^*(a+bb) \rightarrow \{ \text{a ,b} \}^* \{ \text{a, bb} \}$

$L(r) = \{ a, bb, aa, abb, ba, bbb, \ldots\ldots \}$

Example What is the language defined by r

$$r = (aa)^* (bb)^* b$$

$L(r) = (\{a\}\{a\})^* (\{b\}\{b\})^* \{b\}$

$= (\{aa\})^* (\{bb\})^* \{b\}$

$= \{aa\}^* \{bb\}^* \{b\}$

$= \left\{ a^{2n} b^{2m+1} \mid n \geq 0, m \geq 0 \right\}$

Suppose Σ={ 0, 1 }

1. $\{w \mid w$ has exactly a single 1 }

2. $\{w \mid w$ contains 001 }

3. $\{w \mid w$ has length $\geq$ 3 and its third symbol is 0 }

4. What language does the regesp ϕ^* represent ?

Example

Write a regular expression for the set of strings that consist of alternating 0's and 1's.

Partition :

$010101\ldots0101 \longrightarrow (01)^*$

$101010\ldots1010 \longrightarrow (10)^*$

$0101010\ldots1010 \longrightarrow 0(10)^* \text{ or } (01)^*0$

$101010\ldots10101 \longrightarrow (10)^*1 \text{ or } 1(01)^*$

The regular expression :

$$(01)^*+(10)^*+0(10)^*+(10)^*1 \Rightarrow (\varepsilon+0)(10)^*(\varepsilon+1)$$

Example Design regular expression for L

$L = \{ w \mid w \in \{0, 1\}^*$ and w has no pair of consecutive 0's $\}$

Partition :

no 0 $\longrightarrow$ 1^*

one 0 $\longrightarrow$ 1^*01^*

more 0's $\longrightarrow$ $1^*(011^*)^*(0+\varepsilon)$

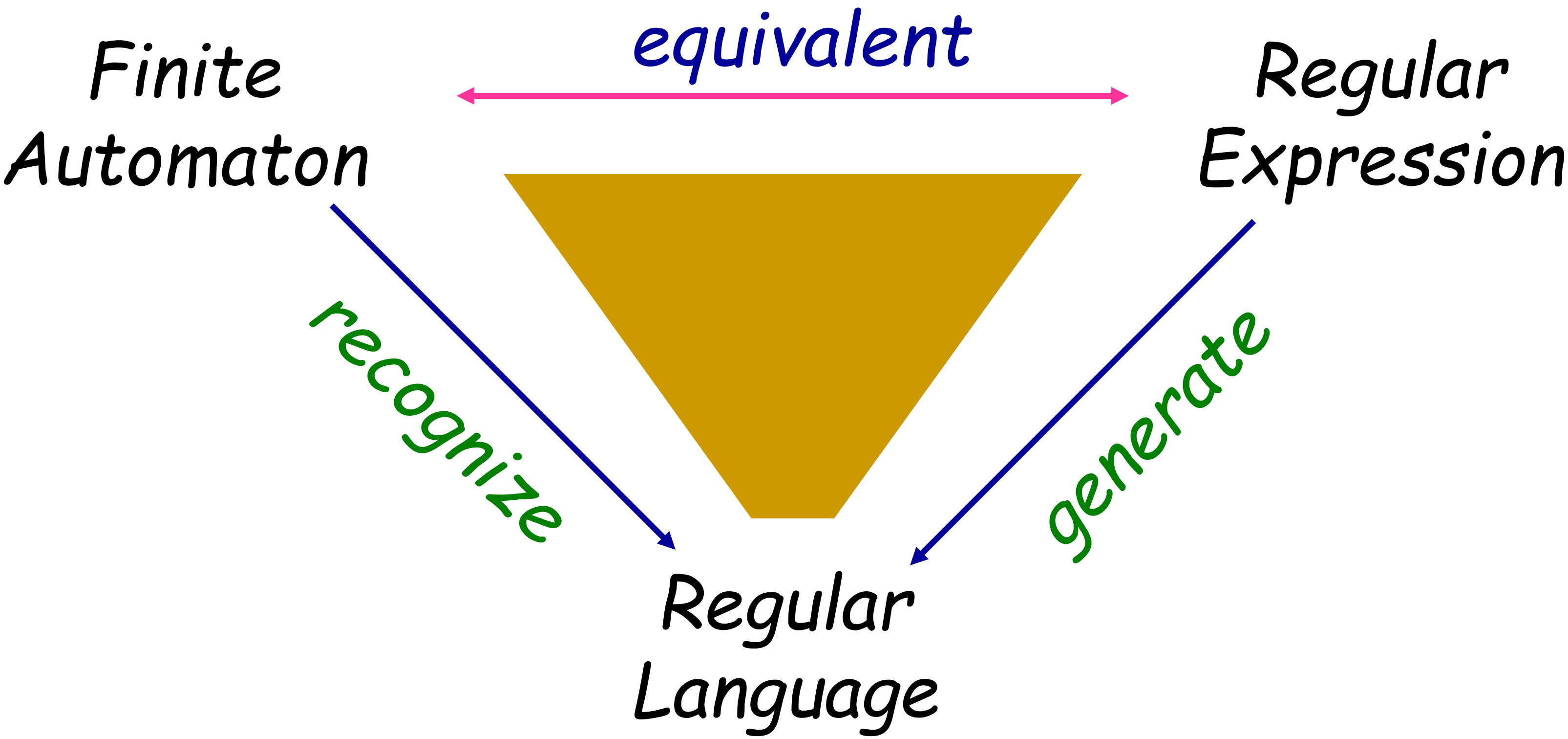
Example Design regular expression for L

$L=\{w \mid w \in \{0, 1\}^*$ and w has no pair of consecutive 0's $\}$

$$r = (1^* 011^*)^* (0+\varepsilon) + 1^* (0+\varepsilon)$$

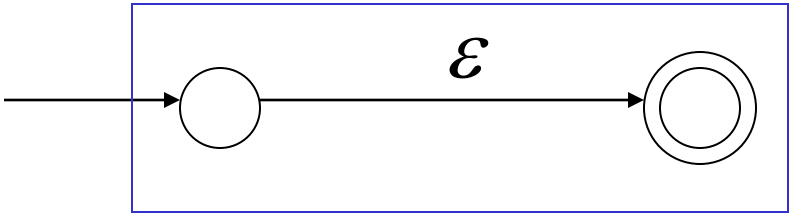
$$r = (1+01)^* (0+\varepsilon)$$

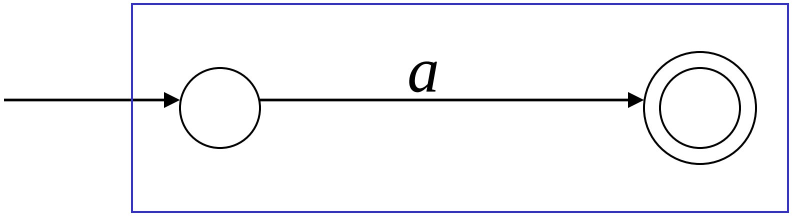
FA & regexp


Finite
Automaton
equivalent
Regular
Expression
recognize
generate
Regular
Language

Construct FA from regexp

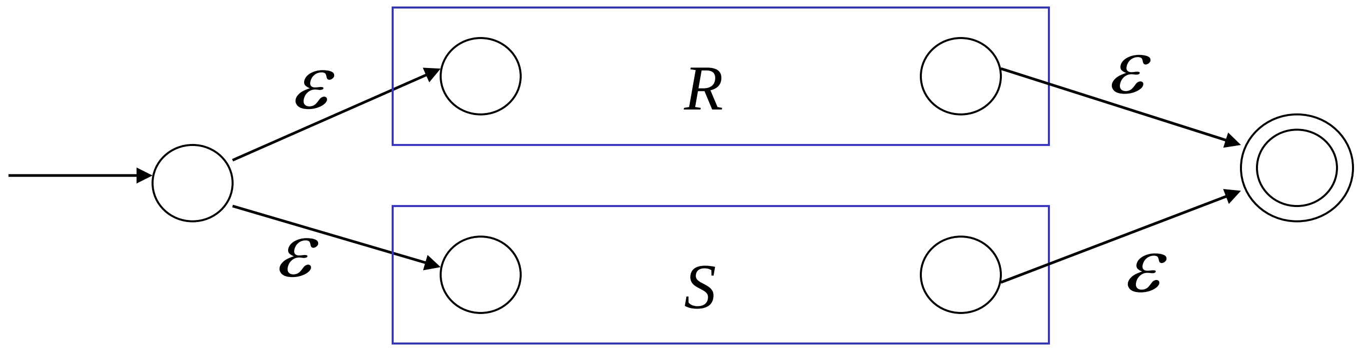
Basis :

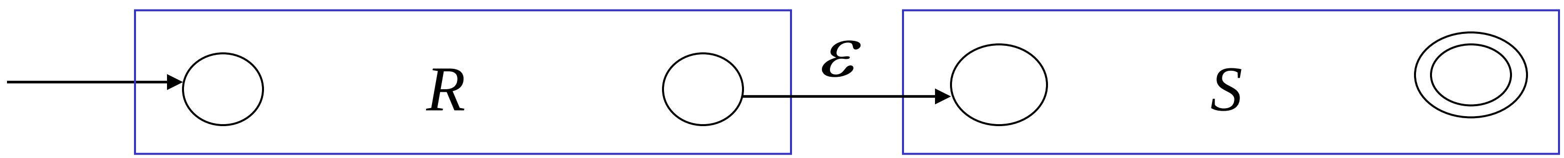


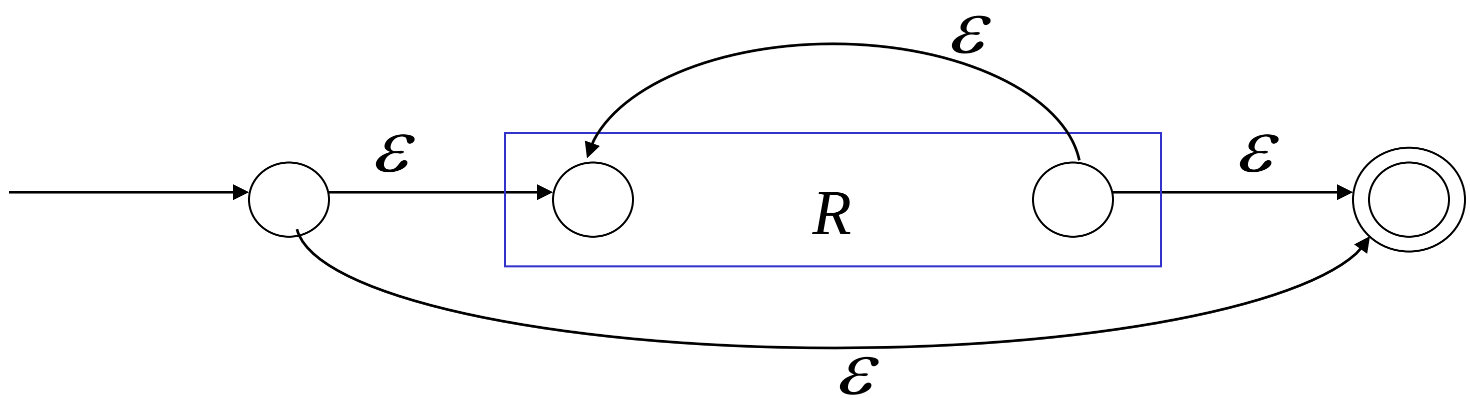


Construct FA from regexp

Induction :



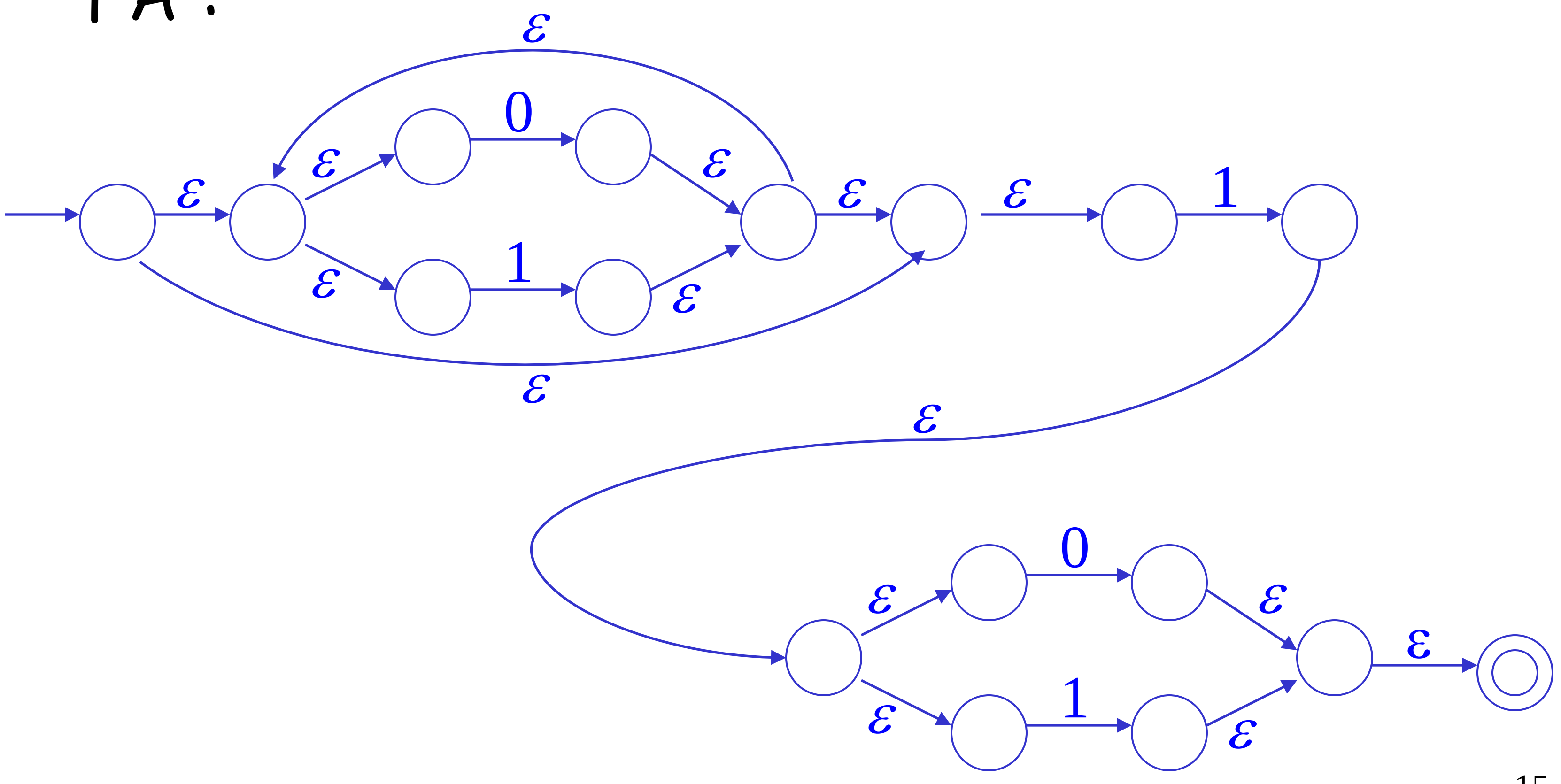




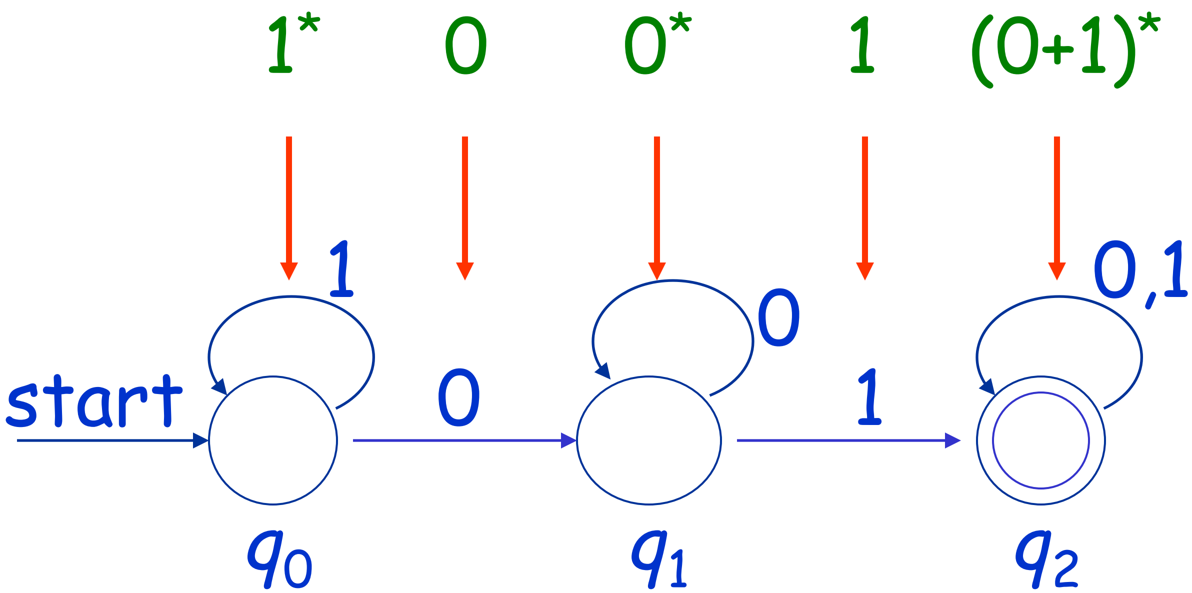
Example Construct FA from regular expression

regexp : $(0+1)^*1(0+1)$

FA :



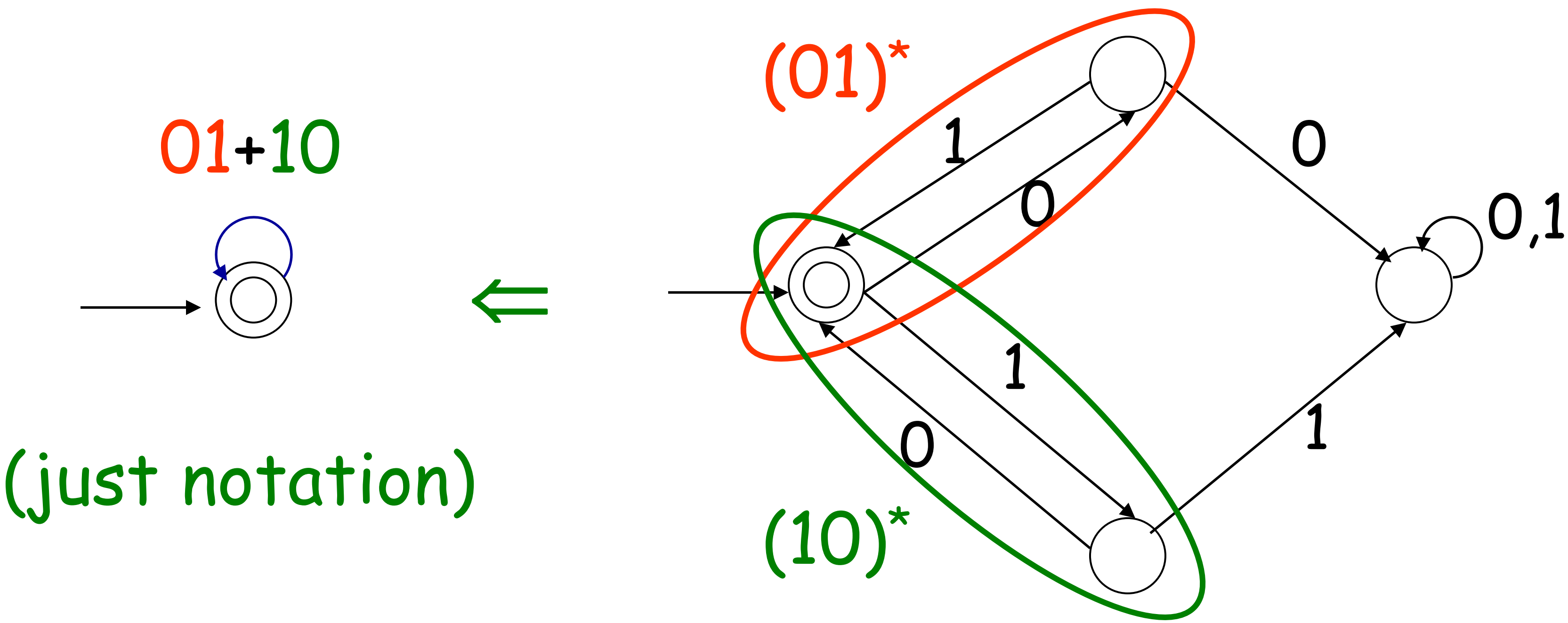
Construct regexp from FA



$L=\{w \mid w \in \{0, 1\}^* \text{ and } w \text{ contains } 01\}$

RE : $(0+1)^*01(0+1)^* \Rightarrow 1^*00^*1(0+1)^*$

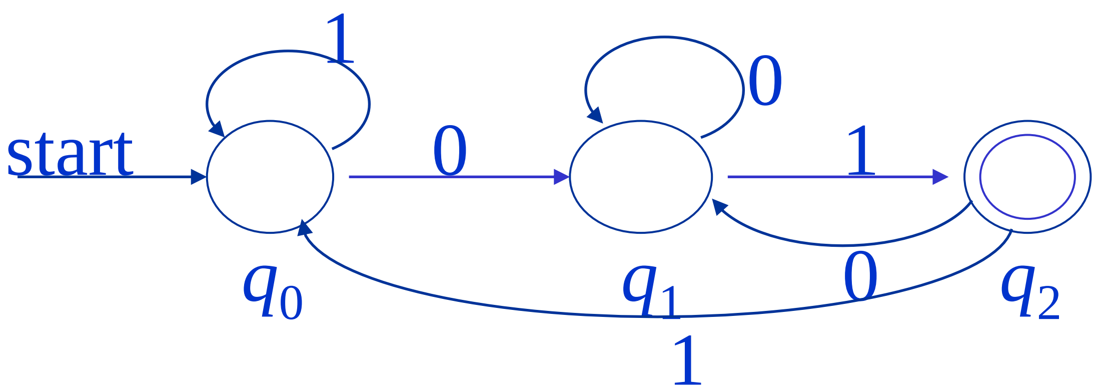
Construct RE from FA



$(01+10)^*$

Construct regexp from FA

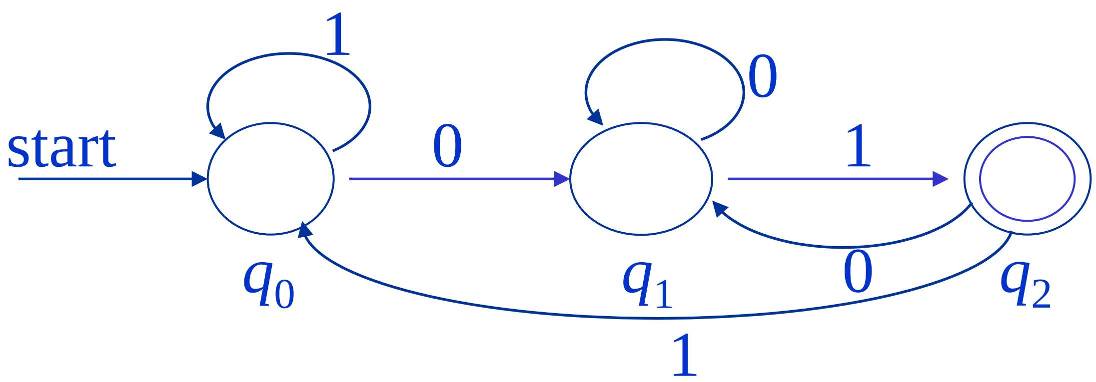
$1^* \quad 0 \quad 0^* \quad 1 \quad (00^*1)^*$



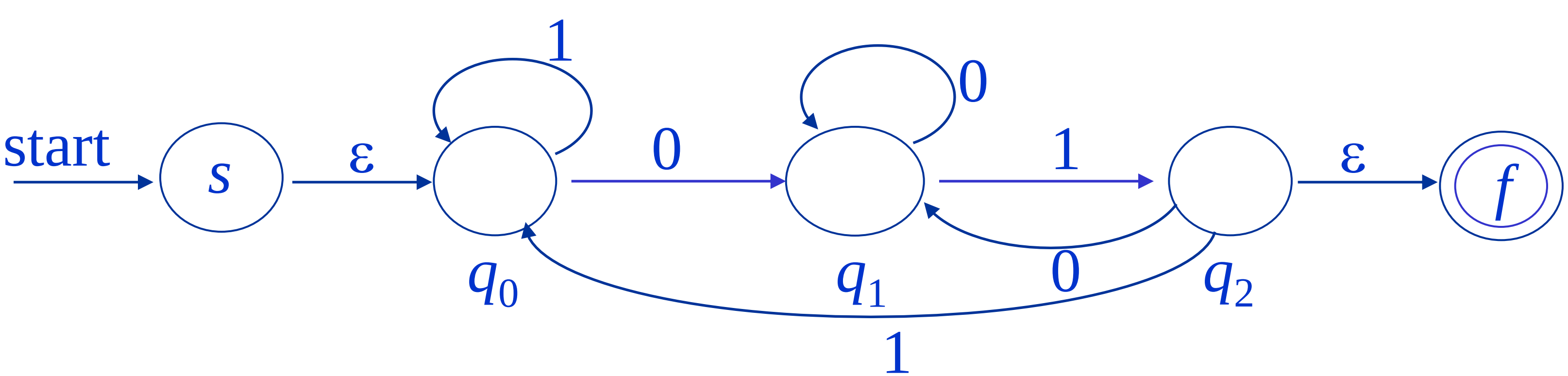
$1^* \quad 0 \quad 0^* \quad 1 \quad (11^*00^*1)^*$

$$1^*00^*1\ (00^*1 + 11^*00^*1)^*$$

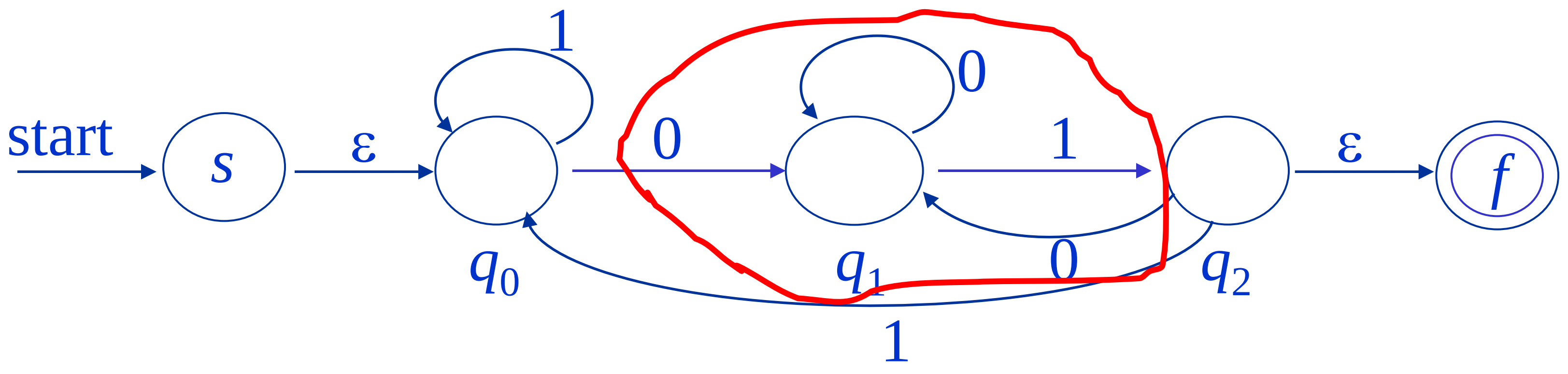
Construct regexp by deleting states



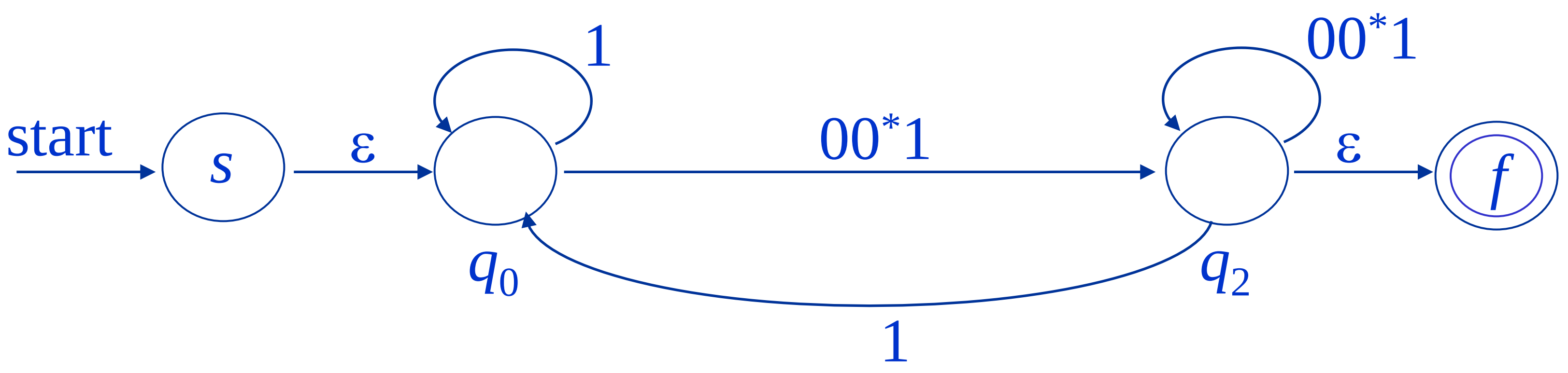
add two states, s and f :



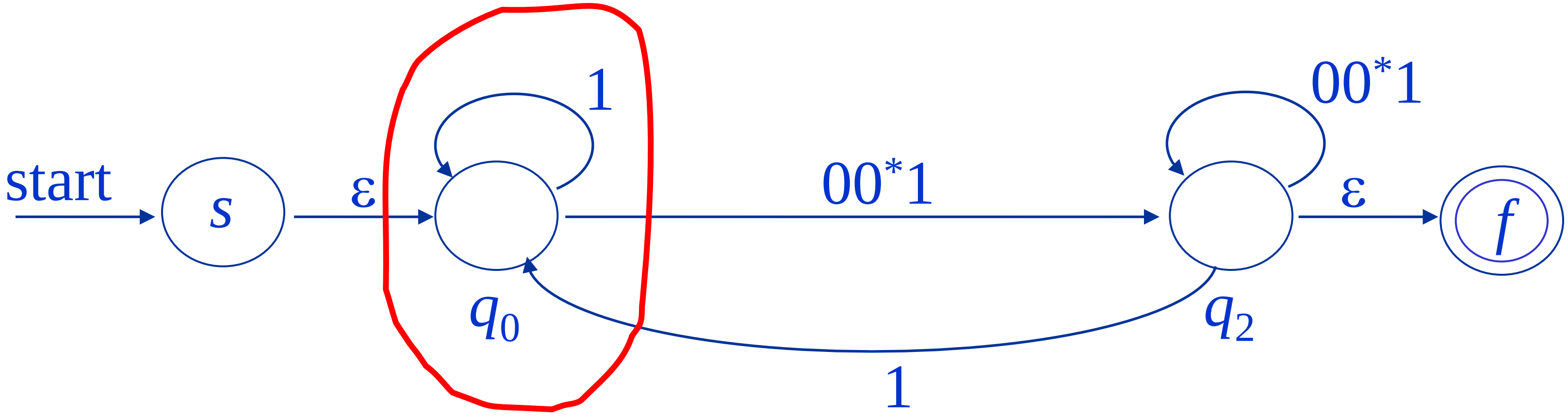
Construct regexp by deleting states



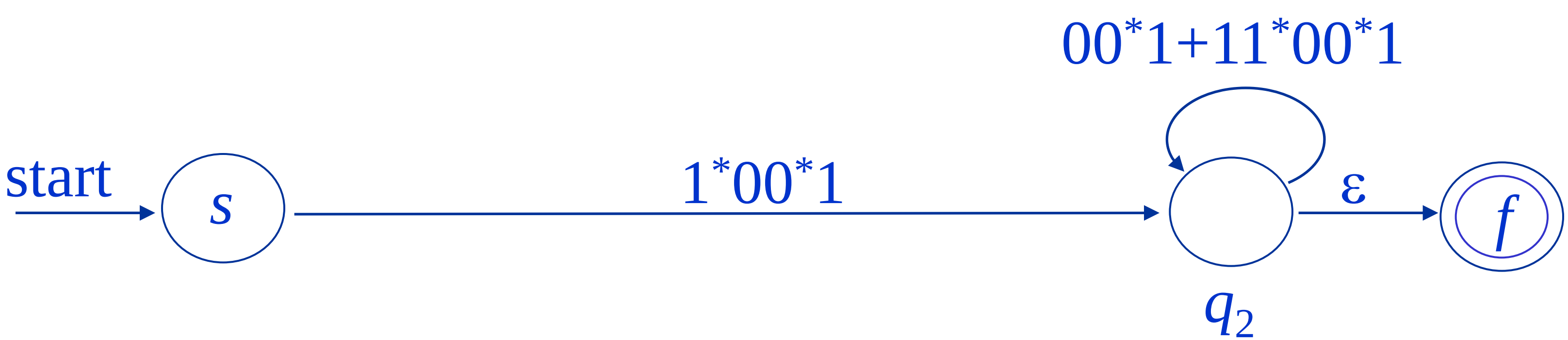
delete q_1 :



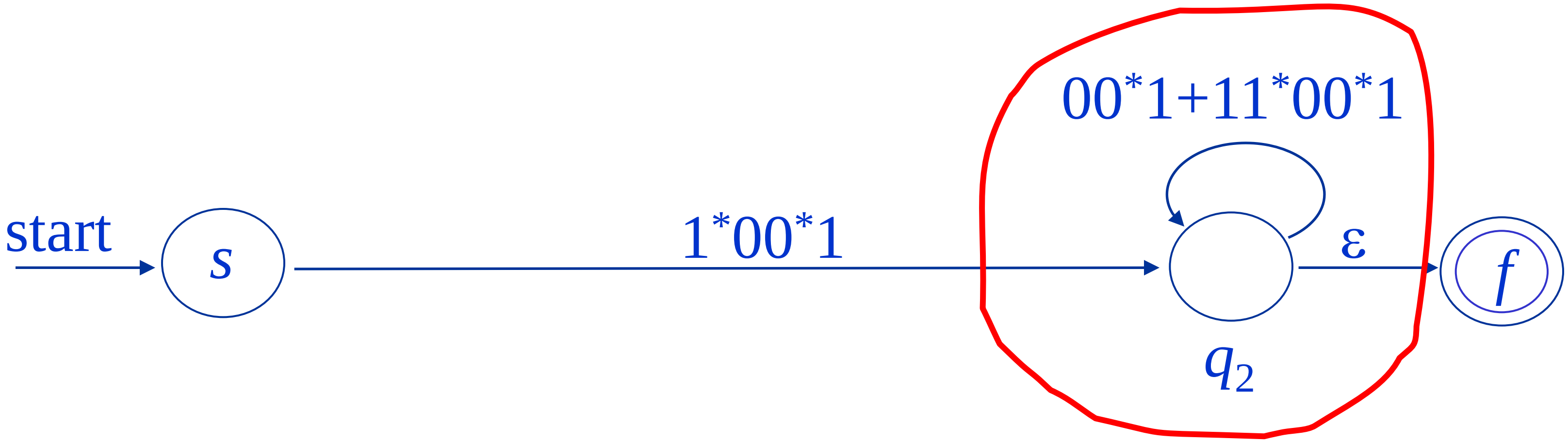
Construct regexp by deleting states



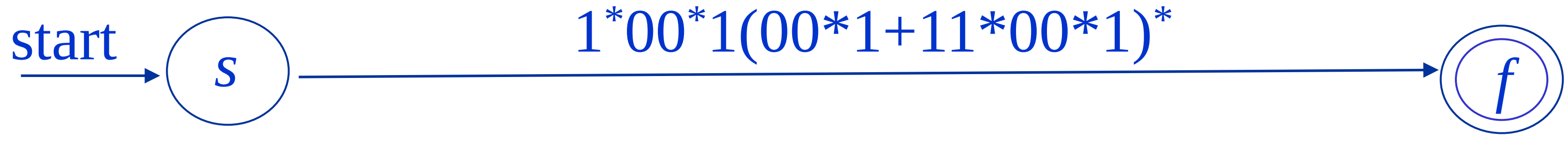
delete q_0 :



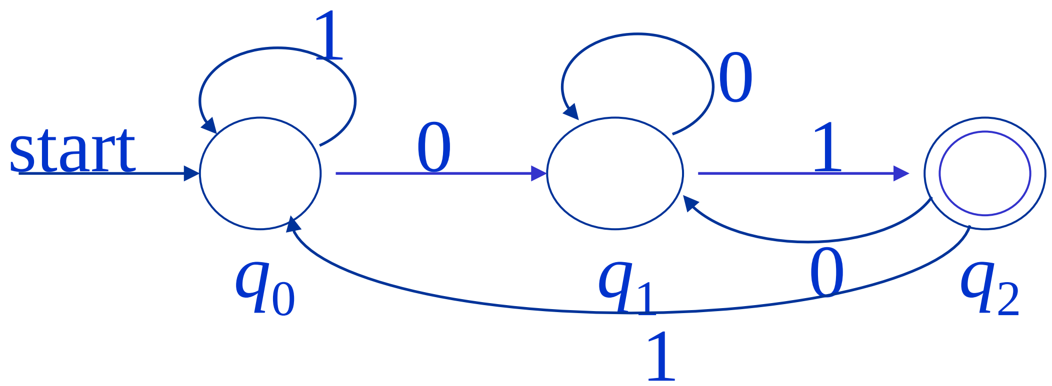
Construct regexp by deleting states



delete q_2 :



Construct regexp by Induction



- Pick every label on the path from q_0 to q_2

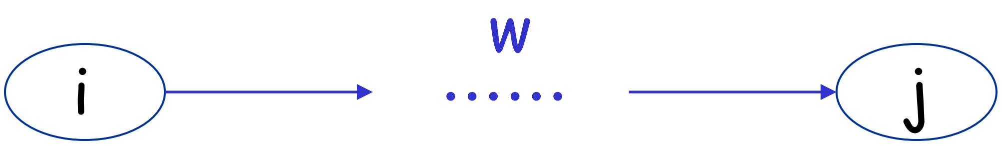
---- one by one

- Form every regexp on the path from q_0 to q_2

---- one by one

Construct regexp by Induction

- Q={1,2,3,……,n}
- $R_{ij}^{(k)} : 0 \le k \le n$
  - regular expression of path from *i* to *j*
  - no inner node is greater than *k*

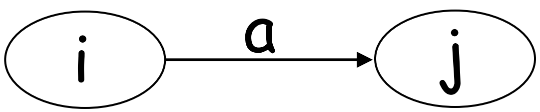


$$R_{ij}^{(k)} \Rightarrow w$$

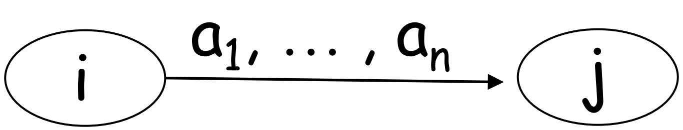
Construct regexp by Induction

Basis $k = 0, \; i \neq j$

$$\Rightarrow \quad R_{ij}^{(0)} = \phi$$



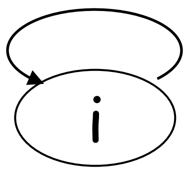
$$\Rightarrow \quad R_{ij}^{(0)} = a$$



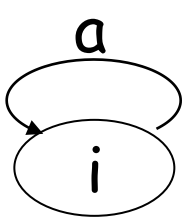
$$\Rightarrow \quad R_{ij}^{(0)} = a_1 + a_2 + \cdots + a_n$$

Construct regexp by Induction

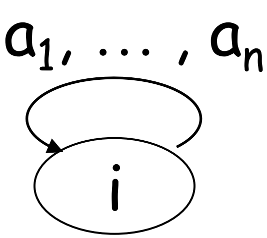
Basis $k = 0,\ i = j$



$$\Rightarrow \quad R_{ij}^{(0)} = \varepsilon + \phi$$



$$\Rightarrow \quad R_{ij}^{(0)} = \varepsilon + a$$

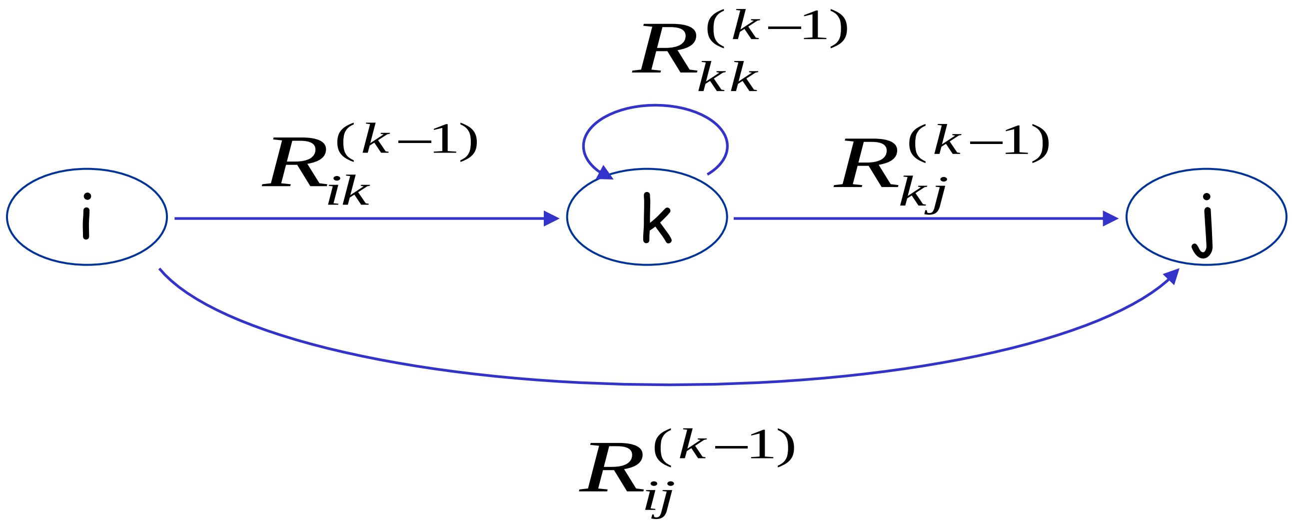


$$\Rightarrow \quad R_{ij}^{(0)} = \varepsilon + a_1 + a_2 + \cdots + a_n$$

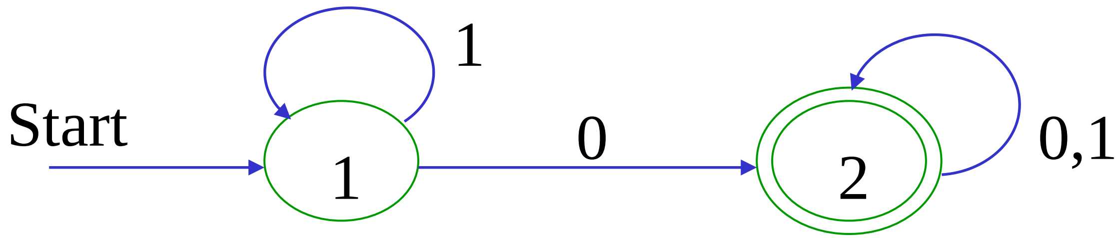
Construct regexp by Induction

Induction $k \geq 1$

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$



Example Convert FA into regular expression



$$R_{11}^{(0)} = \varepsilon + 1,\quad R_{12}^{(0)} = 0,\quad R_{21}^{(0)} = \phi,\quad R_{22}^{(0)} = \varepsilon + 0 + 1$$

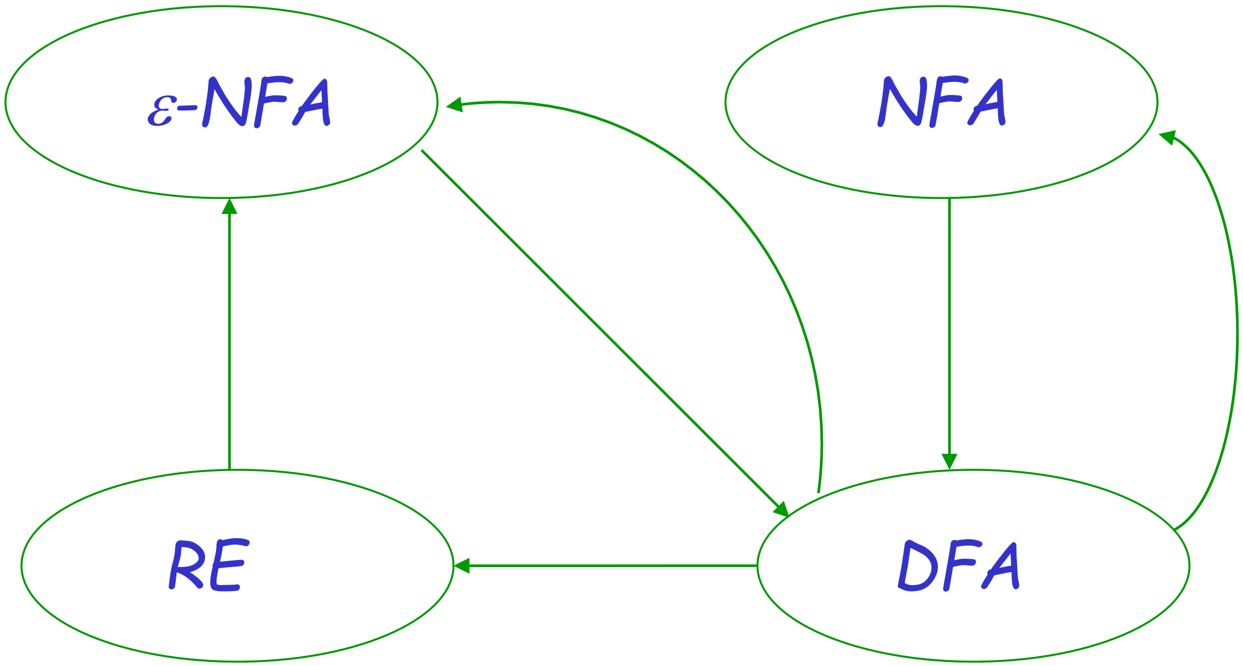
$$R_{11}^{(1)} = 1^*,\quad R_{12}^{(1)} = 1^*0,\quad R_{21}^{(1)} = \phi,\quad R_{22}^{(1)} = \varepsilon + 0 + 1$$

$$R_{11}^{(2)} = 1^*,\quad R_{12}^{(2)} = 1^*0(0+1)^*,\quad R_{21}^{(2)} = \phi,\quad R_{22}^{(2)} = (0+1)^*$$

What we need is :

$$R_{12}^{(2)} = 1^*0(0+1)^*$$

FA & RE



What is the equivalence of FAs and REs?

Good good study

day day up !