Morning

Equivalence of CFG & PDA

$$\begin{tikzcd}
\text{PDA} \arrow[r, "\textit{recognize}"] \arrow[rr, leftrightarrow, bend right, "\text{equivalent}"'] & \text{CFL} & \text{CFG} \arrow[l, "\textit{generate}"']
\end{tikzcd}$$

- With a given CFL L , there is a CFG to generate L, and a PDA to recognize L .
- So they are equivalent .

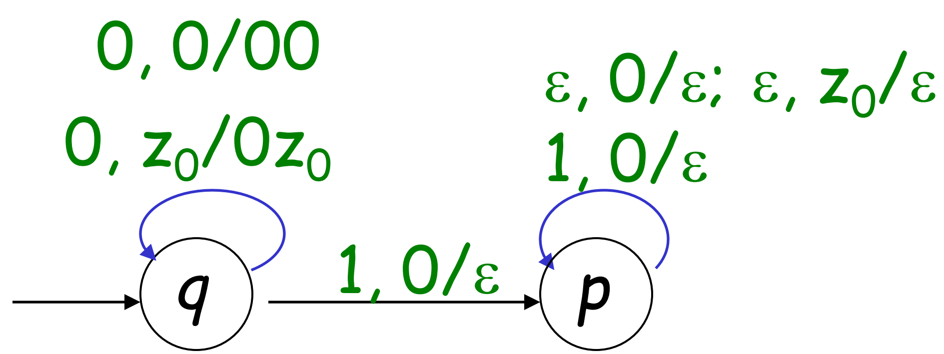
Equivalence of CFG & PDA

Example $L = \{\, 0^n 1^m \mid n \geq m \geq 1 \,\}$

- CFG : $S \rightarrow AB,\ A \rightarrow 0A \mid \varepsilon,\ B \rightarrow 0B1 \mid 01$

  GNF : $S \rightarrow 0SC \mid 0S \mid 0C,\ C \rightarrow 1$

- PDA

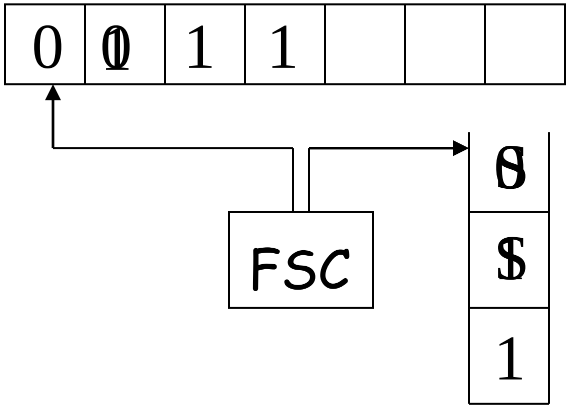


CFG $\Rightarrow$ PDA

Let CFG $G = (V, T, S, P)$

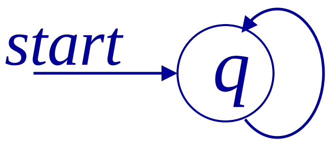
$$\Rightarrow B = (\{q\}, T, V \cup T, \delta, q, S, \{\ \})$$

- $\delta(q, \varepsilon, A) = \{(q, \alpha) \mid A \to \alpha \in P\}$
- $\delta(q, a, a) = (q, \varepsilon)$



Example

$R(L) = (\{S\}, \{0,1\}, \{S \to 0S1,\ S \to SS,\ S \to \varepsilon\}, S)$



$\varepsilon, S \,/\, \varepsilon \longleftarrow S \to \varepsilon$

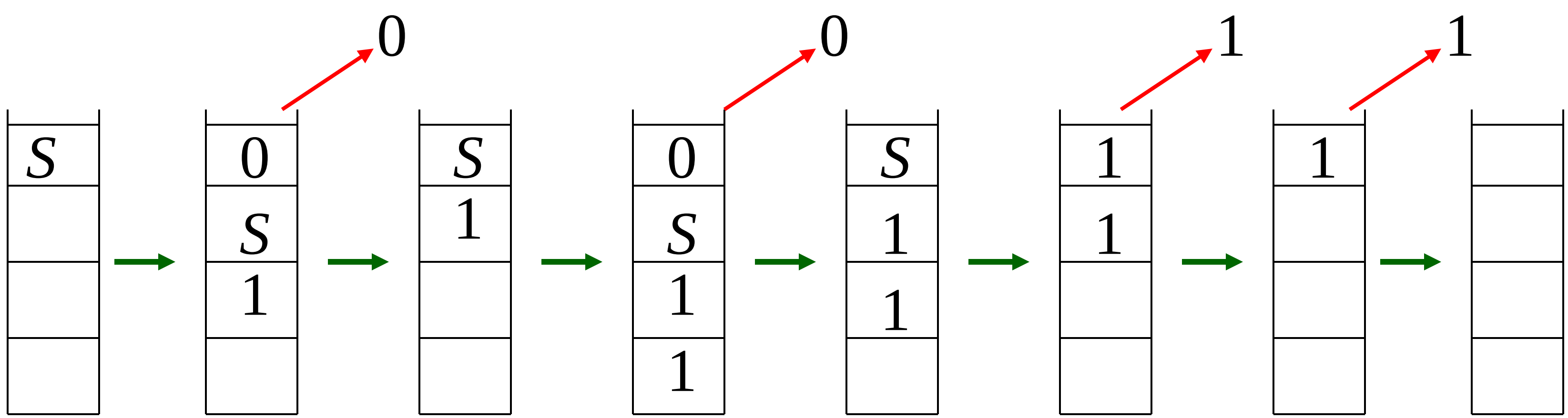
$\varepsilon, S \,/\, SS \longleftarrow S \to SS$

$\varepsilon, S \,/\, 0S1 \longleftarrow S \to 0S1$

$0,0 \,/\, \varepsilon$

$1,1 \,/\, \varepsilon$

Let $w =$ 0 0 1 1



Proof

Let GNF $G = (V, T, S, P)$

P : $A \to a\alpha \quad (A \in V, a \in T, \alpha \in V^*)$

For $w \in L(G)$, let $w = a_1 a_2 \ldots, a_n$

$$S \Rightarrow a_1 \alpha_1$$
$$\Rightarrow a_1 a_2 \alpha_2$$
$$\Rightarrow a_1 a_2 a_3 \alpha_3$$
$$\Rightarrow \ldots\ldots$$
$$\Rightarrow a_1 a_2 \ldots a_{n-1} \alpha_{n-1}$$
$$\Rightarrow a_1 a_2 \ldots a_{n-1} a_n$$

$\alpha_1, \ldots, \alpha_{n-1} \in V^*$

$\alpha_i \Rightarrow a_{i+1} \alpha_{i+1}$

$\alpha_{n-1} \to a_n$

We have PDA $P=(\{q\}, T, V \cup T, \delta, q, S, \{\ \})$

$$
\begin{aligned}
(q,w,S) &\vdash (q, a_1a_2\ldots a_n, a_1\alpha_1) \\
&\vdash (q, a_2\ldots a_n, \alpha_1) \\
&\vdash \ldots\ldots \\
&\vdash (q, a_{n-1}a_n, a_{n-1}\alpha_{n-1}) \\
&\vdash (q, a_n, \alpha_{n-1}) \\
&\vdash (q, a_n, a_n) \\
&\vdash (q, \varepsilon, \varepsilon)
\end{aligned}
$$

- $\delta(q,\varepsilon,S) = (q, a_1\alpha_1)$
- $\delta(q, a_1, a_1) = (q, \varepsilon)$
- $\delta(q, a_{n-1}, a_{n-1}) = (q, \varepsilon)$
- $\delta(q, \varepsilon, \alpha_{n-1}) = (q, a_n)$
- $\delta(q, a_n, a_n) = (q, \varepsilon)$

$$
\begin{array}{llll}
(q,w,S) & \vdash (q,\ a_1a_2\ldots a_n,\ a_1\alpha_1) & S & \Rightarrow a_1\alpha_1 \\
 & \vdash (q,\ a_2\ldots a_n,\ \alpha_1) & & \Rightarrow a_1a_2\alpha_2 \\
 & \vdash \ldots\ldots & & \Rightarrow \ldots\ldots \\
 & \vdash (q,\ a_{n-1}a_n,\ a_{n-1}\alpha_{n-1}) & & \Rightarrow a_1a_2\ldots a_{n-1}\alpha_{n-1} \\
 & \vdash (q,\ a_n,\ \alpha_{n-1}) & & \Rightarrow a_1a_2\ldots a_{n-1}a_n \\
 & \vdash (q,\ a_n,\ a_n) & & \\
 & \vdash (q,\ \varepsilon,\ \varepsilon) & &
\end{array}
$$

PDA simulate the derivations of CFG in the stack

PDA $\Rightarrow$ CFG

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F) \Rightarrow G = (V, \Sigma, S, R)$$

1. pop X from stack
2.transition from q to p

V :

- start symbol S
- all symbols like $[qXp]$

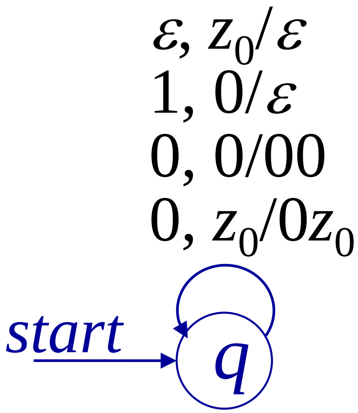
R :

- $S \rightarrow [q_0 z_0 p]$ for all $p \in Q$
- $[qXr_k] \rightarrow a[rY_1r_1][r_1Y_2r_2]\ldots[r_{k-1}Y_kr_k]$

  for $(r, Y_1Y_2\ldots Y_k) \in \delta(q, a, X)$

Example $L=\{\, w \mid w$ contains equal number of 0's and 1's, and no prefix has more 1's than 0's $\}$

PDA



ID's

for $w = 0011$

$$(q, 0011, z_0) \vdash (q, 011, 0z_0)$$
$$\vdash (q, 11, 00z_0) \vdash (q, 1, 0z_0)$$
$$\vdash (q, \varepsilon, z_0) \vdash (q, \varepsilon, \varepsilon)$$

$$(q, 0011, z_0) \vdash^{*} (q, \varepsilon, \varepsilon)$$
$$\updownarrow \qquad\qquad \updownarrow$$
$$S \stackrel{*}{\Rightarrow} 0011$$

ID's and derivation

$\varepsilon, z_0/\varepsilon$: $[qz_0q] \rightarrow \varepsilon$

$0, z_0/0z_0$: $[qz_0q] \rightarrow 0[q0q][qz_0q]$

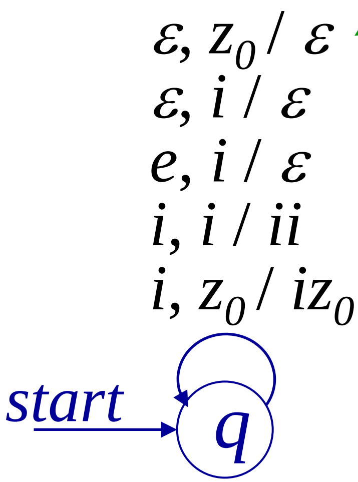
$0,0/00$: $[q0q] \rightarrow 0[q0q][q0q]$

$1,0/\varepsilon$: $[q0q] \rightarrow 1$

rules

Example $L = \{w \mid w \text{ is if-else structure}\}$

PDA



CFG

$G = (V, \Sigma, S, P)$

$V = \{S, [qz_0q], [qiq]\}$

P:

$S \rightarrow [qz_0q]$

PDA		CFG
$\delta(q, \varepsilon, z_0) = (q, \varepsilon)$	$\rightarrow$	$[qz_0q] \rightarrow \varepsilon$
$\delta(q, i, z_0) = (q, iz_0)$	$\rightarrow$	$[qz_0q] \rightarrow i\,[qiq][qz_0q]$
$\delta(q, \varepsilon, i) = (q, \varepsilon)$	$\rightarrow$	$[qiq] \rightarrow \varepsilon$
$\delta(q, i, i) = (q, ii)$	$\rightarrow$	$[qiq] \rightarrow i\,[qiq][qiq]$
$\delta(q, e, i) = (q, \varepsilon)$	$\rightarrow$	$[qiq] \rightarrow e$

Good good study
day day up！