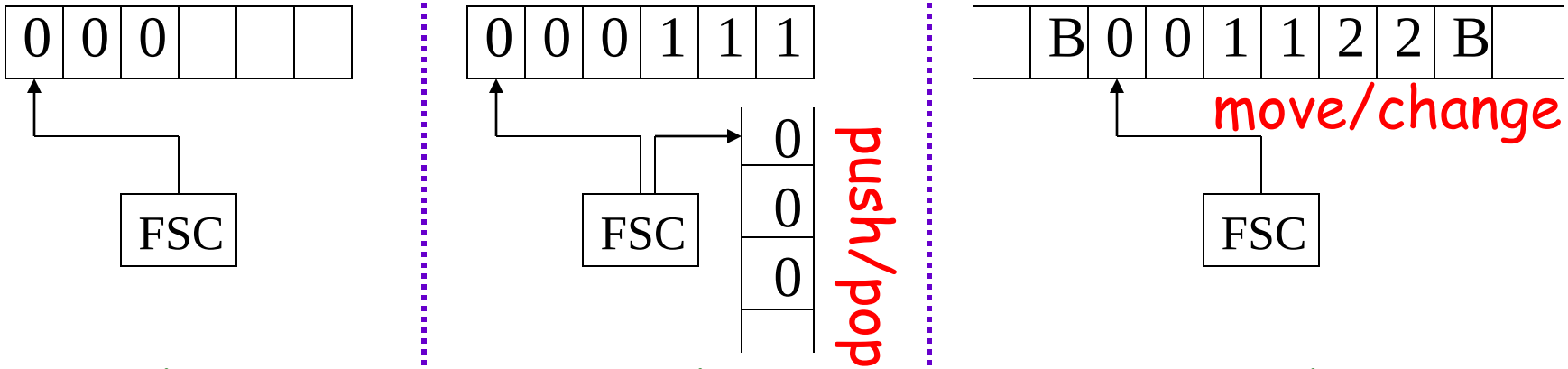
Morning

Turing Machine

1. Definition
2. Construction
3. Language accepted by TM

Turing Machine

- FA ⇒ PDA ⇒ TM



$\{0^n \mid n \geq 1\}$ $\{0^n 1^n \mid n \geq 1\}$ $\{0^n 1^n 2^n \mid n \geq 1\}$

$(Q, \Sigma, \delta, q_0, F)$ $(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ $(Q, \Sigma, \Gamma, \delta, q_0, B, F)$

Definition of Turing Machine

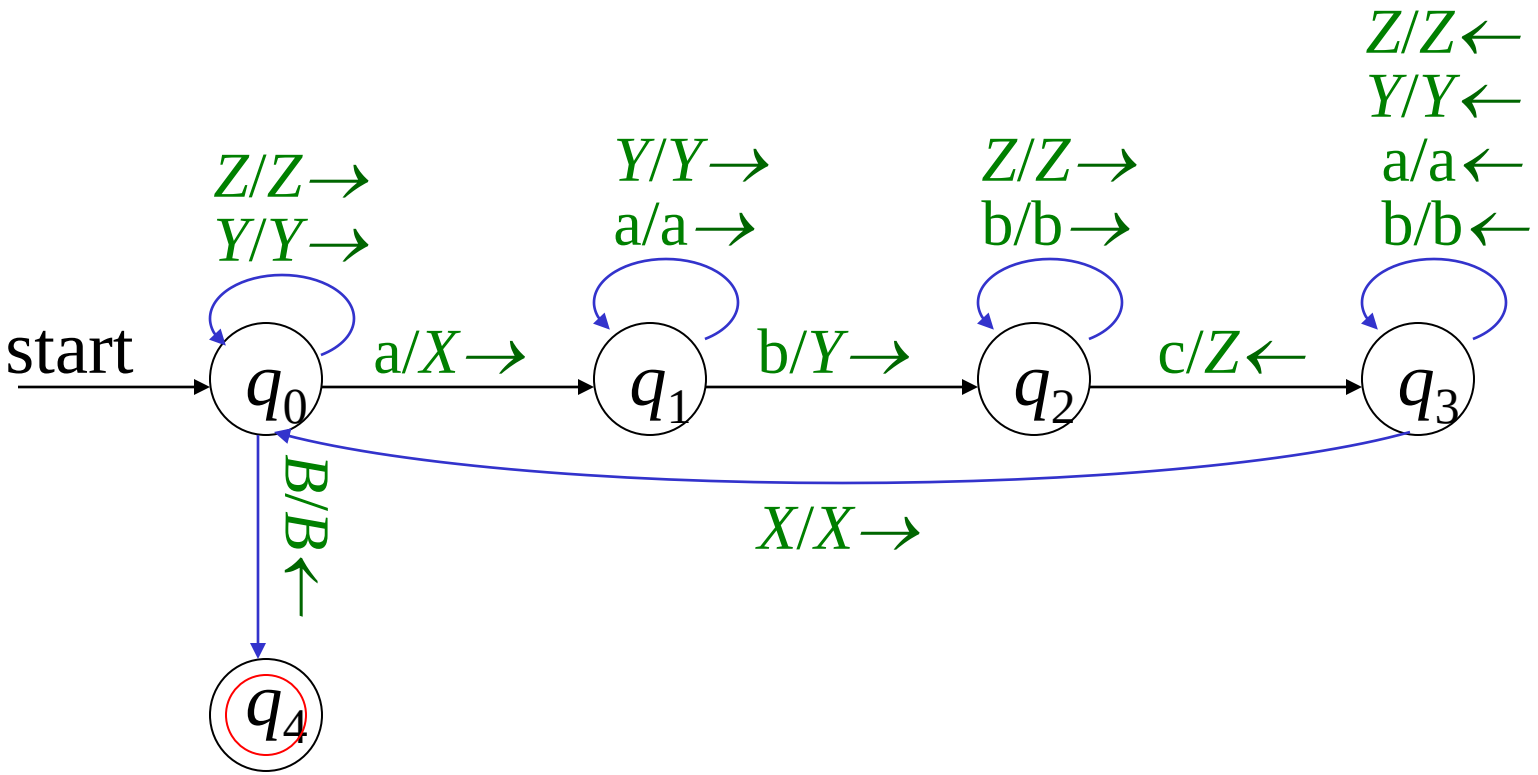
TM is a seven-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

- Q is finite set of states
- Σ is finite set of input symbols
- Γ is finite set of tape symbols
- δ is transition function : $Q \times \Gamma \Rightarrow Q \times \Gamma \times \{R, L\}$

  $\delta(q, X) = (p, Y, D)$
- q_0 is start state
- B is blank symbol
- F is finite set of final state

Example $L = \{ a^n b^n c^n \mid n \geq 1 \}$

$M=(\{q_0,q_1,q_2,q_3,q_4\},\{a,b,c\},\{a,b,c,B,X,Y,Z\},\delta,q_0,B,\{q_4\})$



Instantaneous

- how to describe the configuration of TM
  - sequence of symbols in tape
  - state of TM
  - read/wtite head of TM
  - $X_1 \ldots X_{i-1} q X_i X_{i+1} \ldots X_n$

Instantaneous

- ID of the above TM for w=aabbcc$\in\{a^nb^nc^n \mid n\geq 1\}$

$q_0aabbcc \vdash Xq_1abbcc \vdash Xaq_1bbcc \vdash XaYq_2bcc$

$XaYbq_2cc \vdash XaYq_3bZc \vdash Xaq_3YbZc \vdash Xq_3aYbZc \vdash$

$q_3XaYbZc \vdash Xq_0aYbZc \vdash XXq_1YbZc \vdash XXYq_1bZc \vdash$

$XXYYq_2Zc \vdash XXYYZq_2c \vdash XXYYq_3ZZ \vdash \ldots$

Language of TM

- language accepted by a TM

$$\{w \mid q_0 w \vdash^* \alpha p \beta \ , \ p \in F \ , \ \alpha, \beta \in \Gamma^*\}$$

- The language accepted by TM is called *recursively enumerable(RE)* language.

Example Construct a Turing Machine for

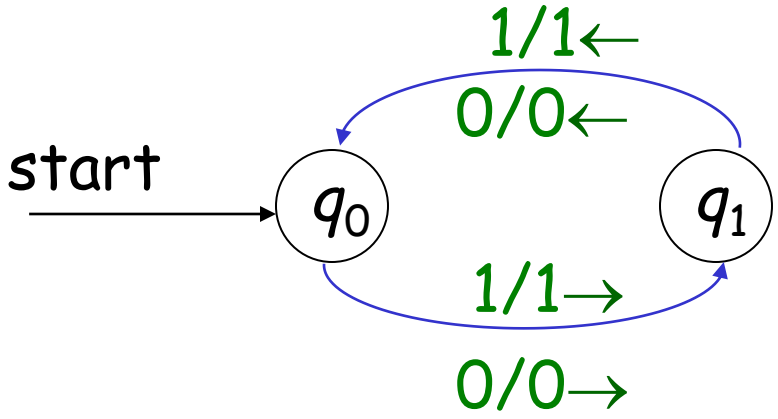
$$L = \{ a^n b^n a^n b^n \mid n \geq 0 \}$$

Example Construct a Turing Machine for

$$L = \{ w \mid w \in \{0,1\}^* \text{ and } |w| \text{ is even} \}$$

Halting

· We say a TM halts if it enters a state q, scanning a tape symbol X, and there is no move in this situation .



Example

Given two positive integers x and y, design a TM to compute $x+y$.

- notation for x and y

$$x \Rightarrow w(x) \in \{1\}^{+} \text{ and } |w(x)|=x$$

$$x+y \Rightarrow w(x+y) \in \{1\}^{+} \text{ and } |w(x+y)|=x+y$$

Example

Compute the function *nomus* $(m, n)=\max(m-n,0)$

- put 0^m10^n into tape as input
- delete one 0 from 0^m and one 0 from 0^n
- three cases :

  $m>n \Leftrightarrow 0^{m-n-1}$ at the left of 1

  $m=n \Leftrightarrow$ no 0

  $m<n \Leftrightarrow 0^{n-m}$ at the right of 1

Example

Construct a TM to compute the function

$$f(w)=ww$$

where $w \in \{1\}^+$.

Example

Construct a TM to compute the function

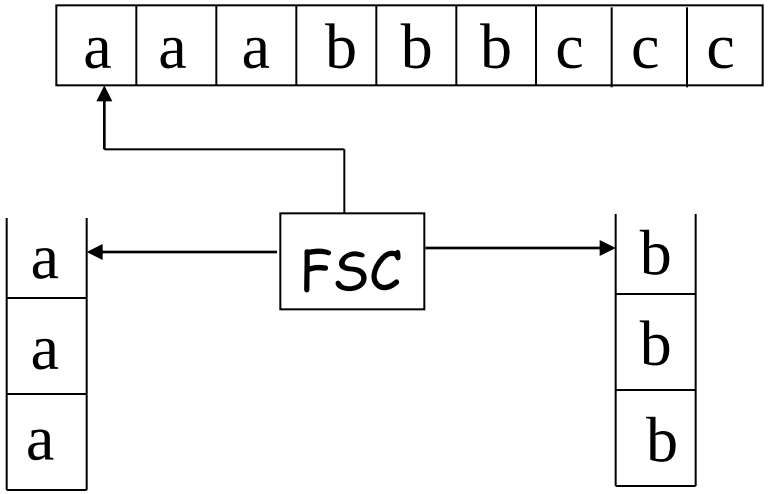
$$f(w)=ww$$

where $w \in \{0,1\}^+$.

Example

Construct a TM to compute $m \times n$.

Two Stack Machine



$$\delta(q, a, X, Y) = (p, \alpha, \beta)$$

Example Construct a two stack machine for

$$L = \{ a^n b^n c^n \mid n \geq 0 \}$$

How about the two stack machine for

$$L = \{ a^n b^n c^n d^n e^n \mid n \geq 0 \}$$

Good good study
day day up !