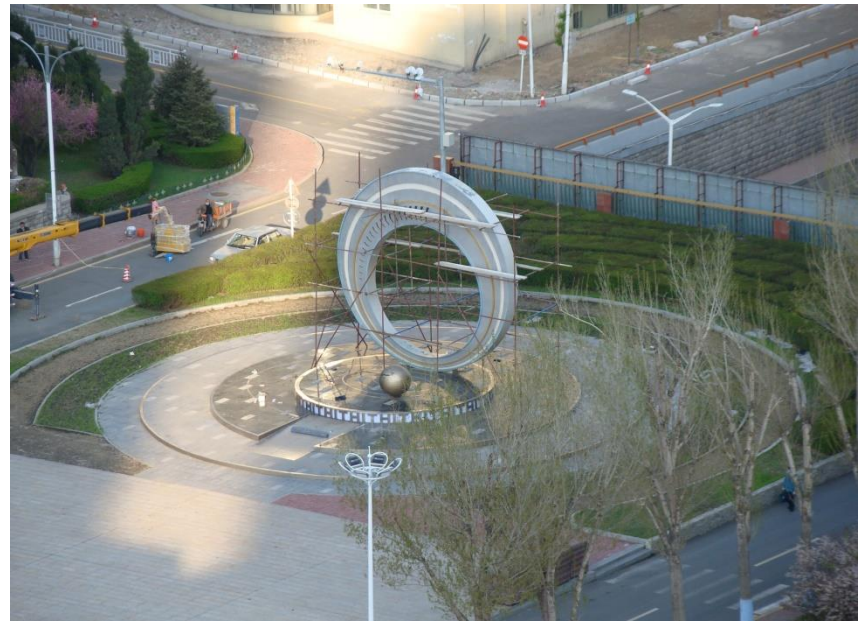


Morning



Regular Expression

- ◆ *Definition*
- ◆ *Design*
- ◆ *Equivalence with FA*



Arithmetical Expression

$0, 1+2, 3 \times (5-2), (56-7)^2, \dots$

- Formal definition
- Inductive definition
 - Any number is a arithmetical expression
 - If a and b are arithmetical expressions , then
so is $a+b, a-b, a \div b, a \times b, a^n, (a)$.

Building Regular Expressions

BASIS

1. ε is a RegExp, denoting the languages $\{\varepsilon\}$.
2. ϕ is a RegExp, denoting the languages ϕ .
3. For each a in Σ , a is a RegExp and denotes the language $\{a\}$.

Notice : language is a set of strings.

Building Regular Expressions

INDUCTION

1. If E and F are RegExp, denoting the language $L(E)$ and $L(F)$, then $E + F$, EF and E^* are RegExp that denote the languages $L(E) \cup L(F)$, $L(E)L(F)$ and $(L(E))^*$.

2. If E is a RegExp, then so is (E) .

Example 1 Language for

$$r = (a + b)^* (a + bb)$$

$$a \rightarrow \{ a \}, b \rightarrow \{ b \}$$

$$a + b \rightarrow \{ a \} \cup \{ b \} = \{ a, b \}$$

$$bb \rightarrow \{ b \} \{ b \} = \{ bb \}$$

$$a + bb \rightarrow \{ a \} \cup \{ bb \} = \{ a, bb \}$$

$$(a + b)^* \rightarrow \{ a, b \}^*$$

$$(a + b)^* (a + bb) \rightarrow \{ a, b \}^* \{ a, bb \}$$

$$L(r) = \{ a, bb, aa, abb, ba, bbb, \dots \}$$

Example 2 Language for

$$r = (aa)^* (bb)^* b$$

$$L(r) = (\{a\}\{a\})^* (\{b\}\{b\})^* \{b\}$$

$$= (\{aa\})^* (\{bb\})^* \{b\}$$

$$= \{aa\}^* \{bb\}^* \{b\}$$

$$= \{ a^{2n} b^{2m+1} \mid n \geq 0, m \geq 0 \}$$

Example 3 RegExp for

$\{ w \mid w \text{ consists of alternating 0's and 1's} \}$

Partition

010101...0101	→	$(01)^*$
101010...1010	→	$(10)^*$
0101010...1010	→	$0(10)^*$ or $(01)^*0$
101010...10101	→	$(10)^*1$ or $1(01)^*$

The regular expression

$(01)^* + (10)^* + 0(10)^* + (10)^*1$

Example 3 RegExp for

$\{ w \mid w \text{ consists of alternating 0's and 1's} \}$

$\{\varepsilon, 1\} \{ 010101\dots 0101 \} \{\varepsilon, 0\}$



$\{\varepsilon, 1\} \{ 01 \}^* \{\varepsilon, 0\}$

$(\varepsilon + 1) (01)^* (\varepsilon + 1)$

$$\begin{aligned} (01)^* + (10)^* + 0(10)^* + (10)^*1 &\Leftrightarrow (\varepsilon + 1)(01)^*(\varepsilon + 0) \\ &\Leftrightarrow (\varepsilon + 0)(10)^*(\varepsilon + 1) \end{aligned}$$

Example 4 RegExp for

$L = \{w \mid w \in \{0, 1\}^* \text{ and } w \text{ has no pair of consecutive 0's} \}$

Partition

no 0	$\{1\}^*$	\longrightarrow	1^*
one 0	$\{1\}^*\{0\}\{1\}^*$	\longrightarrow	1^*01^*
more 0's	$\{1\}^*\{ \{0\}\{1\}\{1\}^* \}^*\{0, \varepsilon\}$		$\{0110111\ldots\}$



$1^* (0 1 1^*)^* (0 + \varepsilon)$

Example 4 RegExp for

$L = \{w \mid w \in \{0, 1\}^* \text{ and } w \text{ has no pair of consecutive 0's} \}$

$\{01\}^* \quad \{\varepsilon, 01, 0101, 010101, 01010101, \dots\}$

$\{1, 01\}^* \quad \{\varepsilon, 1, 01, 11, 101, 011, 0101, 111, 1101, 1011, \dots\}$

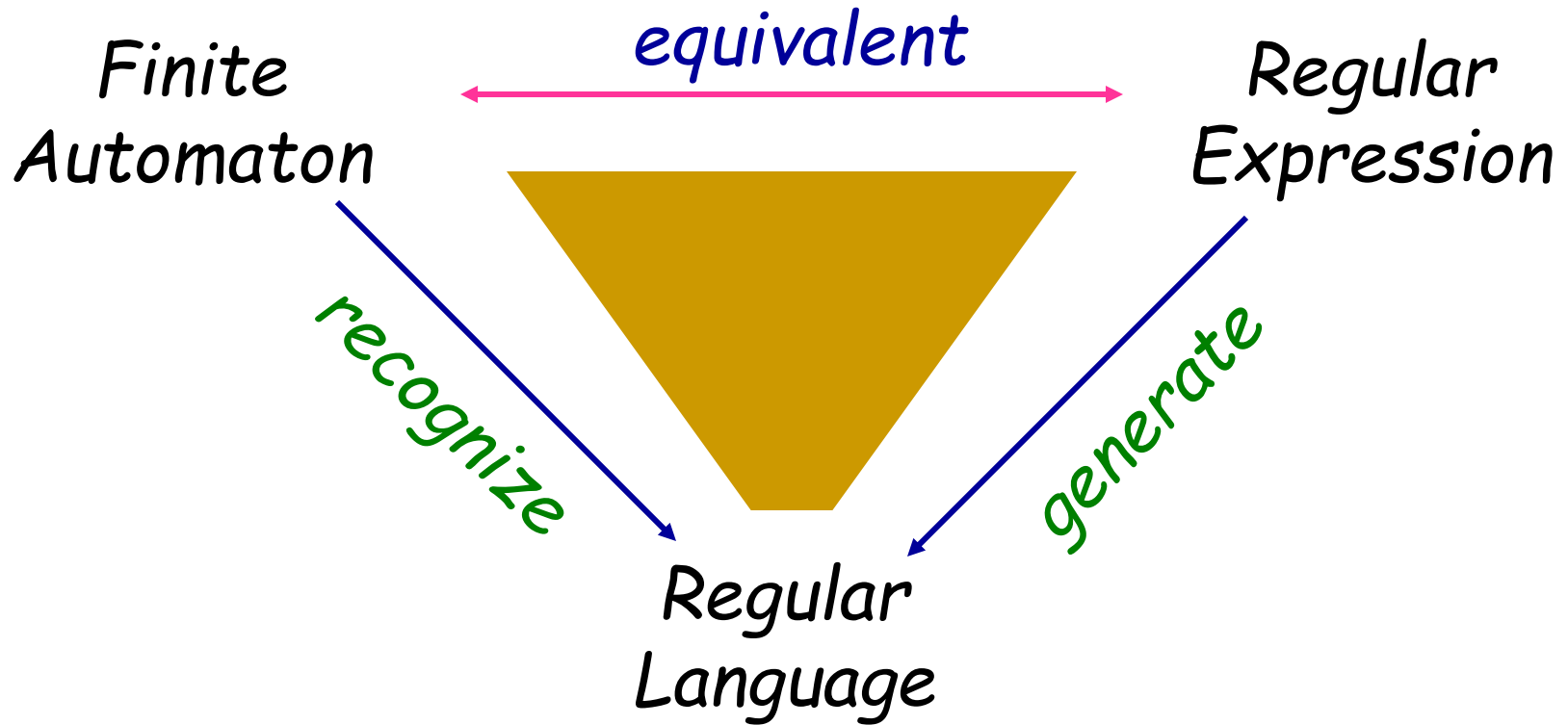


$(1+01)^* \Rightarrow (1+01)^*(0 + \varepsilon)$

Exercises RegExp for ($\Sigma=\{0,1\}$)

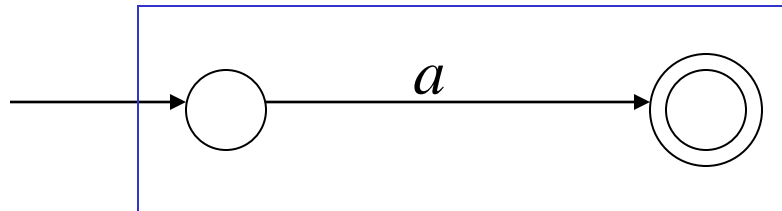
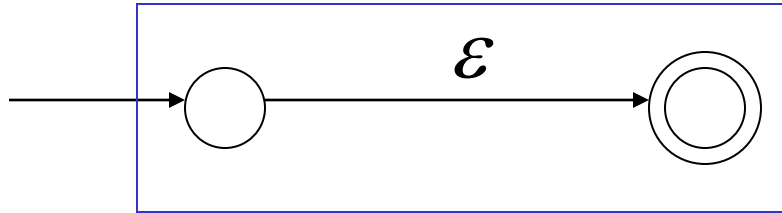
1. $\{w \mid w \text{ has exactly a single } 1\}$
2. $\{w \mid w \text{ contains } 001\}$
3. $\{w \mid \text{length}(w) \geq 3 \text{ and the third symbol is } 0\}$
4. What language does the RegEsp ϕ^* represent ?

FA & RegExp



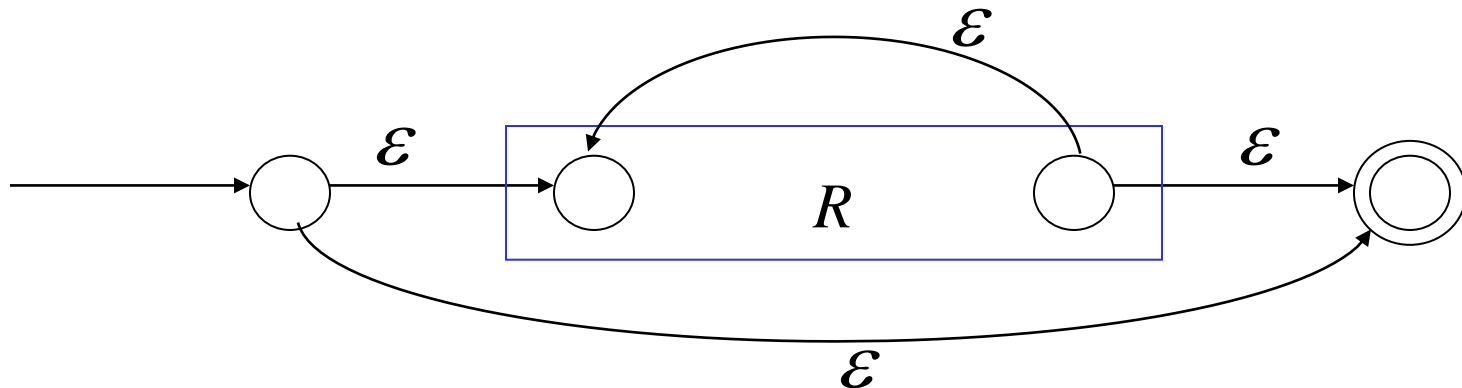
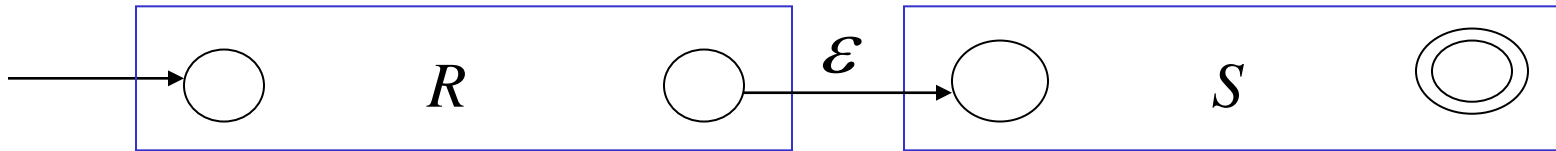
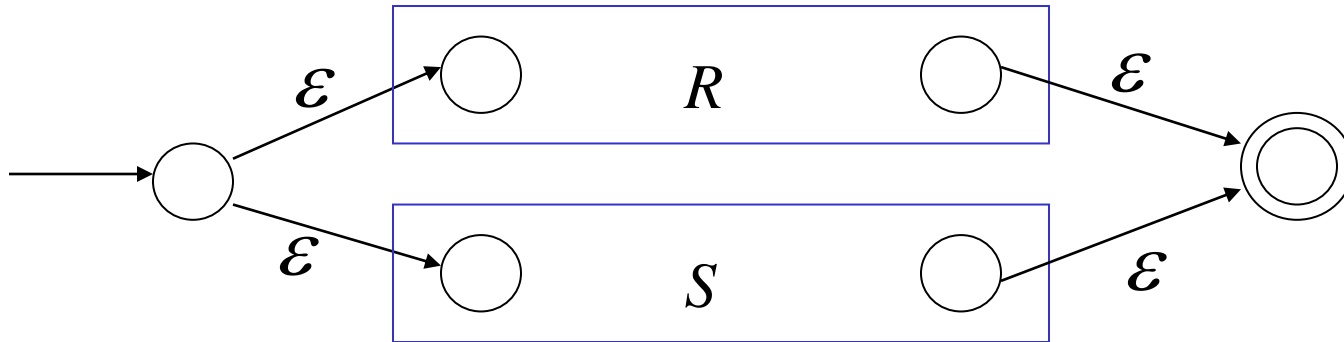
RegExp \Rightarrow FA

Basis :



RegExp \Rightarrow FA

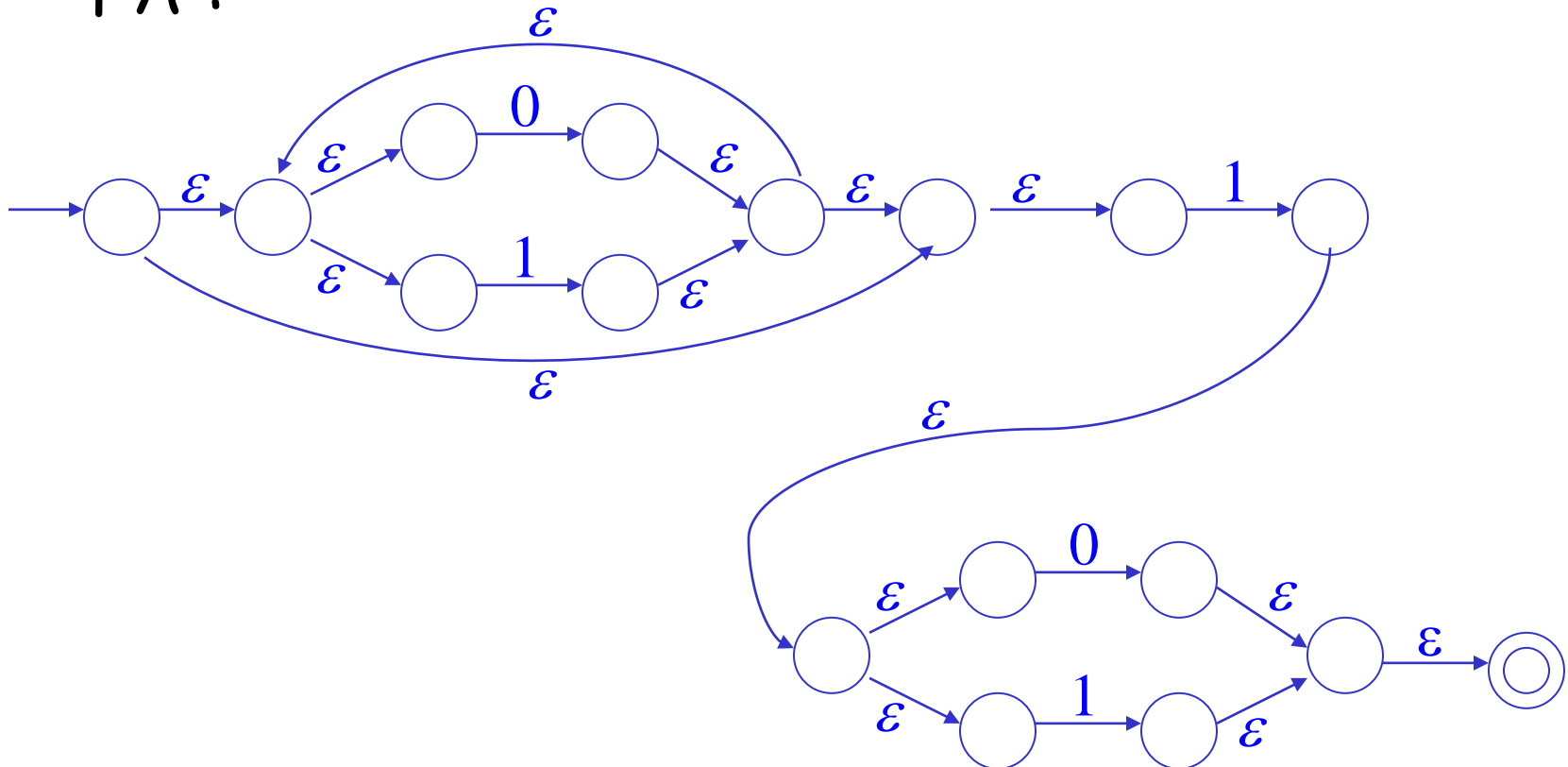
Induction :



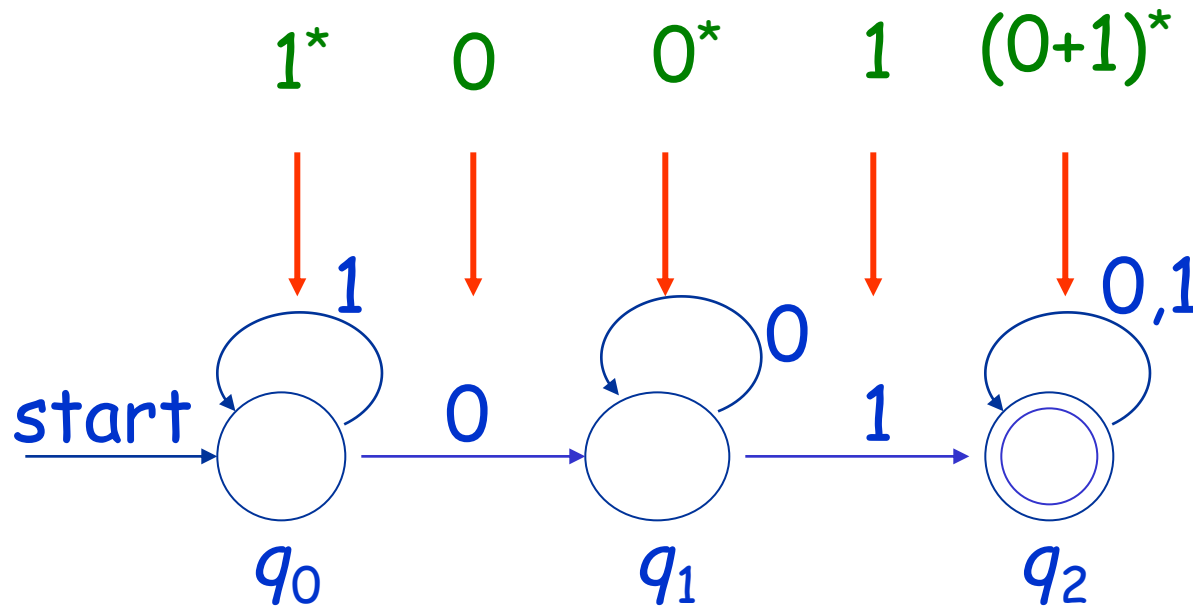
Example 5 RegExp \Rightarrow FA

RegExp : $(0+1)^*1(0+1)$

FA :



Example 6 FA \Rightarrow RegExp

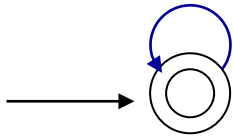


$L = \{w \mid w \in \{0, 1\}^* \text{ and } w \text{ contains } 01\}$

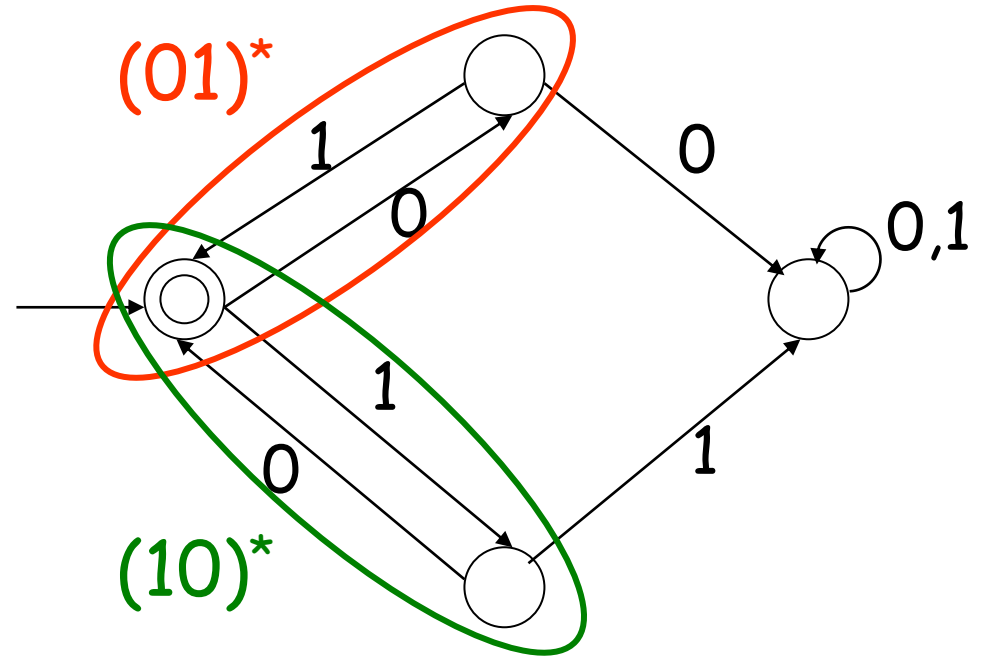
RE: $(0+1)^*01(0+1)^* \Rightarrow 1^*00^*1(0+1)^*$

Example 7 FA \Rightarrow RegExp

$01+10$

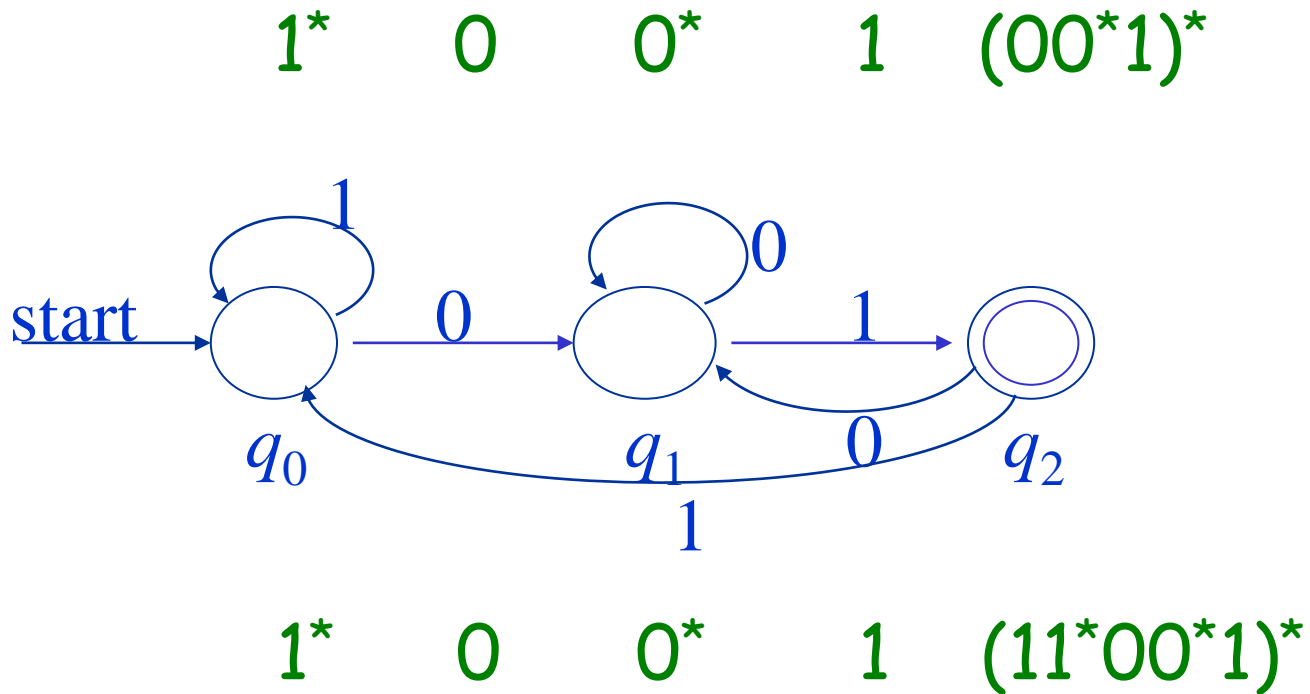


(just notation)



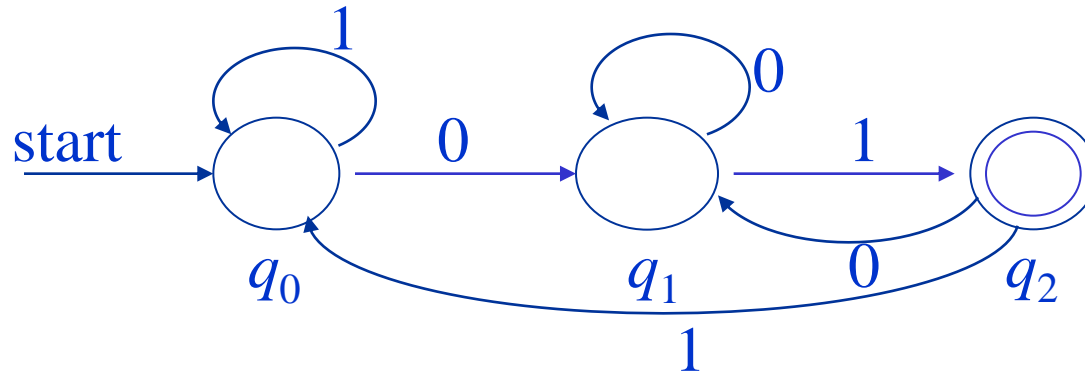
$(01+10)^*$

Example 8 $FA \Rightarrow RegExp$

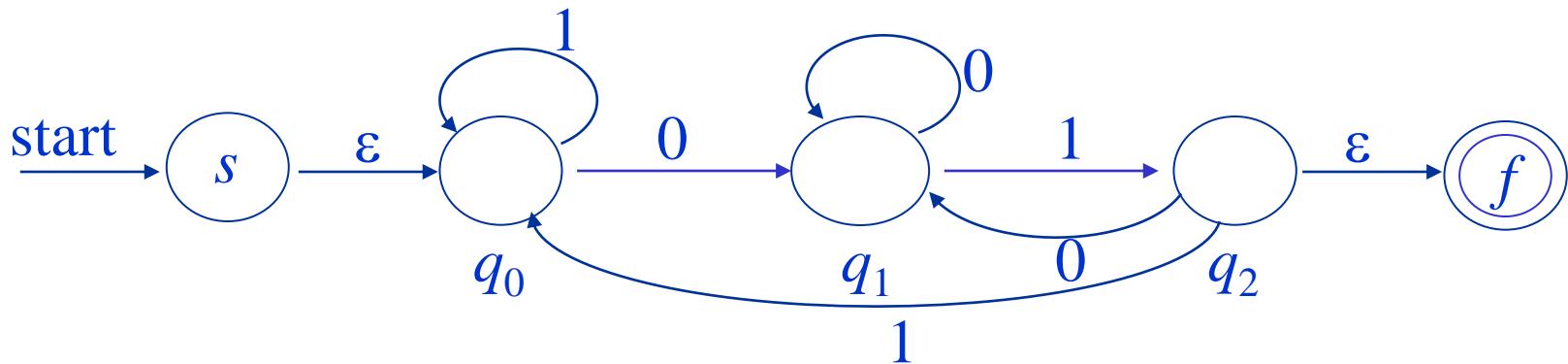


$$1^*00^*1 (00^*1 + 11^*00^*1)^*$$

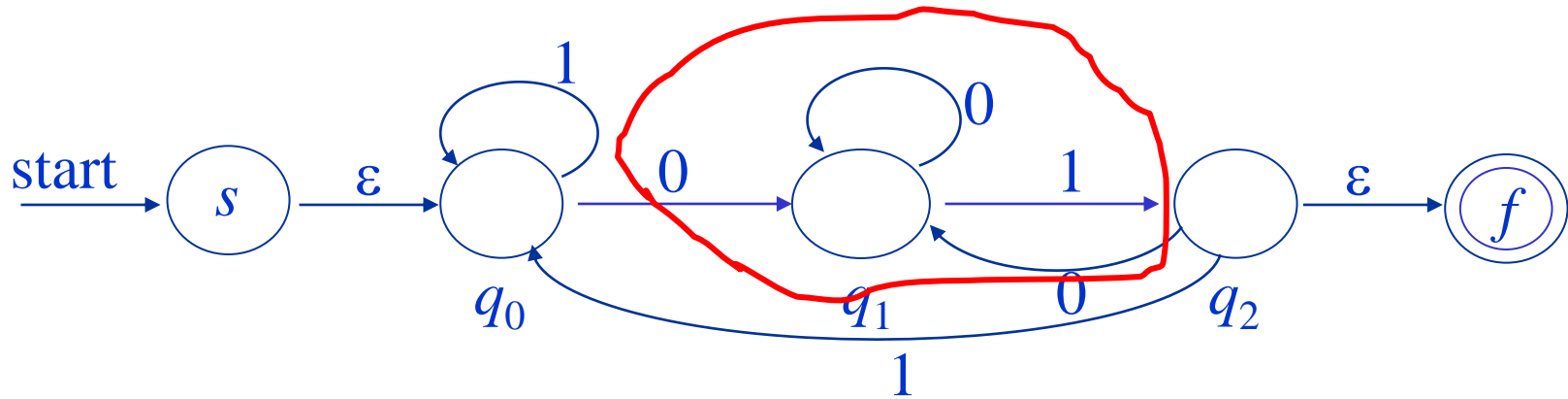
FA \Rightarrow RegExp - Delete states



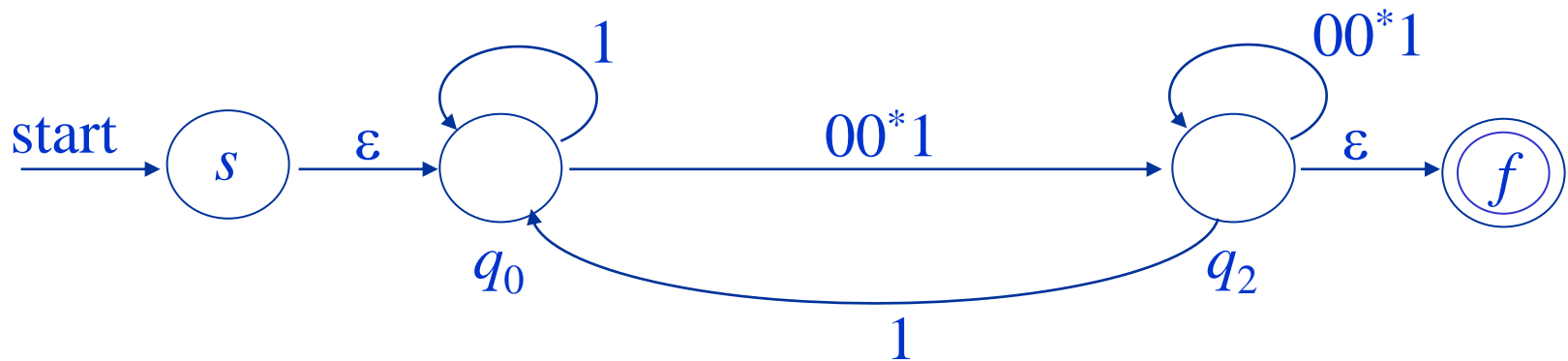
add two states, s and f :



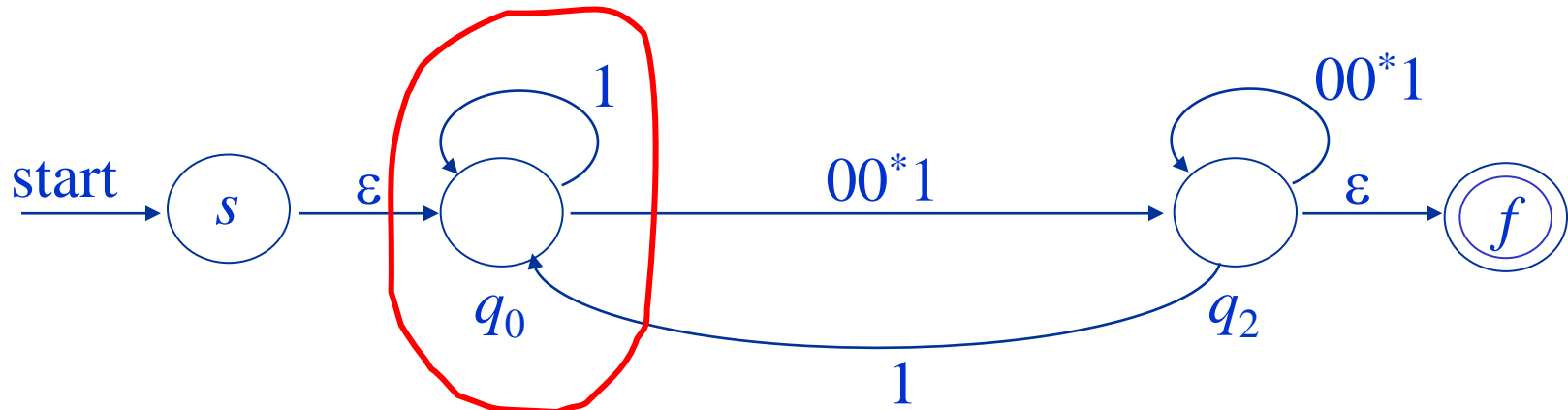
FA \Rightarrow RegExp - Delete states



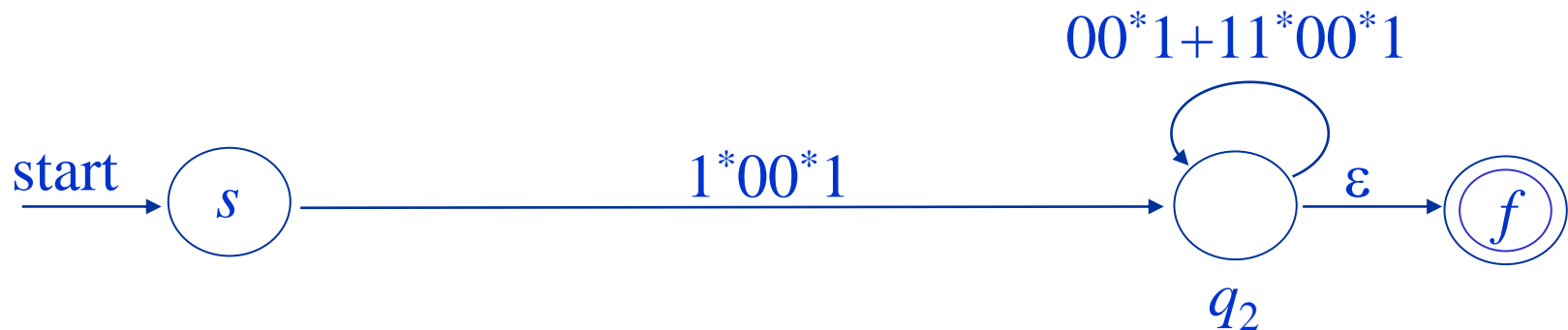
delete q_1 :



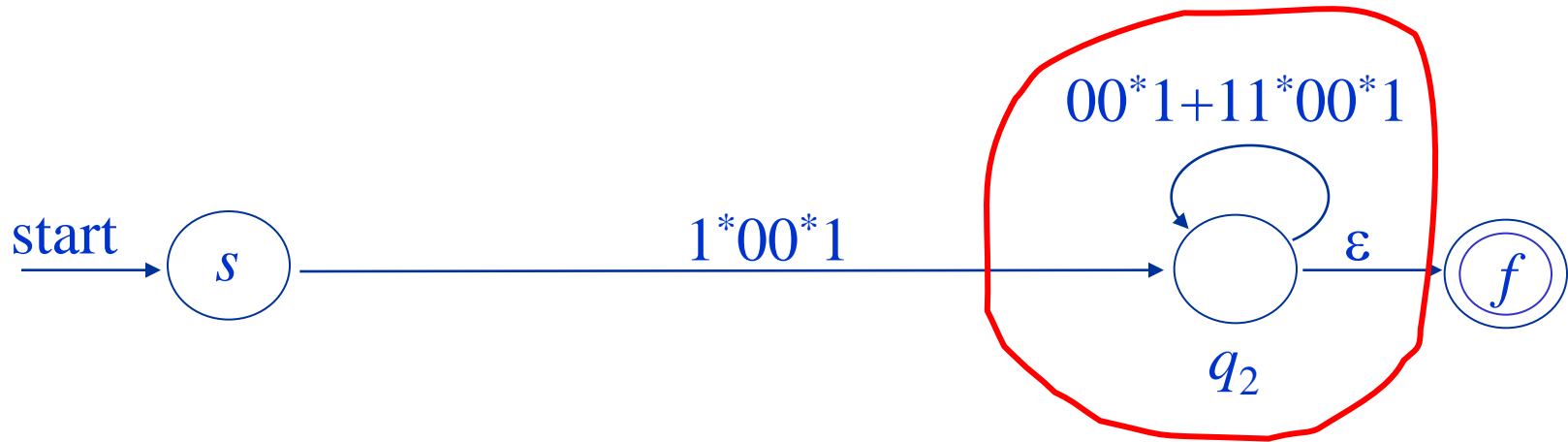
FA \Rightarrow RegExp - Delete states



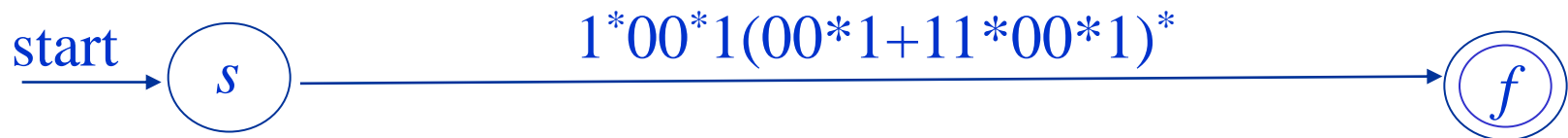
delete q_0 :



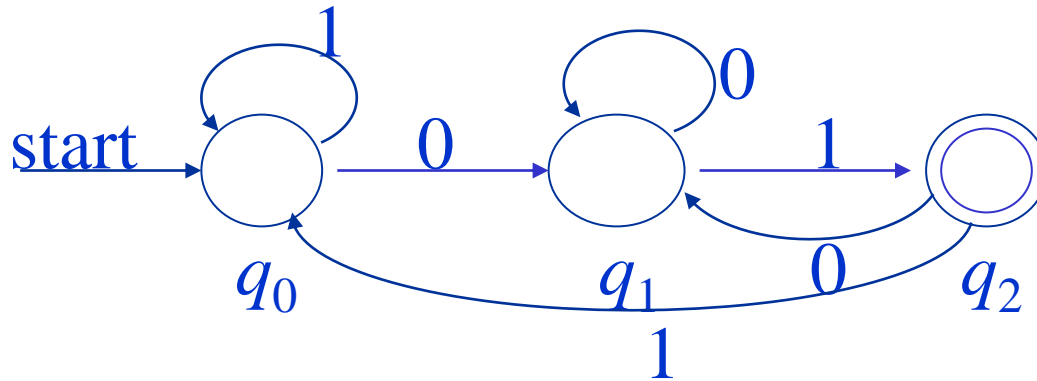
FA \Rightarrow RegExp - Delete states



delete q_2 :



FA \Rightarrow RegExp - Induction



- ◆ Pick every label on the path from q_0 to q_2
---- one by one
- ◆ Form every regexp on the path from q_0 to q_2
---- one by one

FA \Rightarrow RegExp - Induction

- $Q = \{1, 2, 3, \dots, n\}$
- $R_{ij}^{(k)} : 0 \leq k \leq n$
 - regular expression of path from i to j
 - no inner node is greater than k



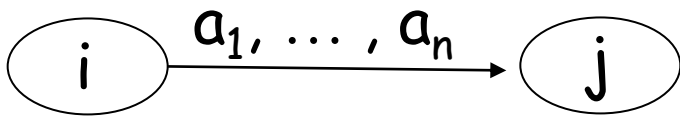
$$R_{ij}^{(k)} \Rightarrow w$$

FA \Rightarrow RegExp - Induction

Basis $k = 0, i \neq j$

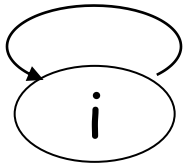
 $\Rightarrow R_{ij}^{(0)} = \phi$

 $\Rightarrow R_{ij}^{(0)} = a$

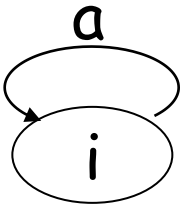
 $\Rightarrow R_{ij}^{(0)} = a_1 + a_2 + \cdots + a_n$

FA \Rightarrow RegExp - Induction

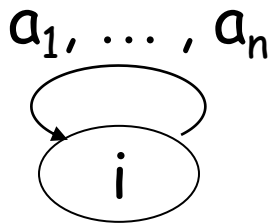
Basis $k = 0, i = j$



$$\Rightarrow R_{ij}^{(0)} = \varepsilon + \phi$$



$$\Rightarrow R_{ij}^{(0)} = \varepsilon + a$$

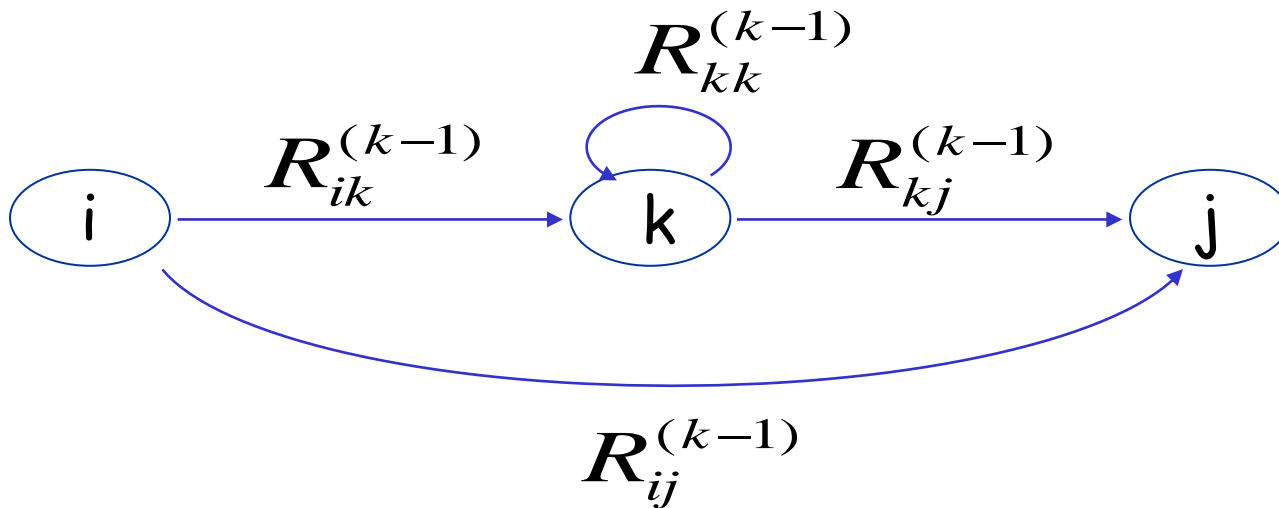


$$\Rightarrow R_{ij}^{(0)} = \varepsilon + a_1 + a_2 + \dots + a_n$$

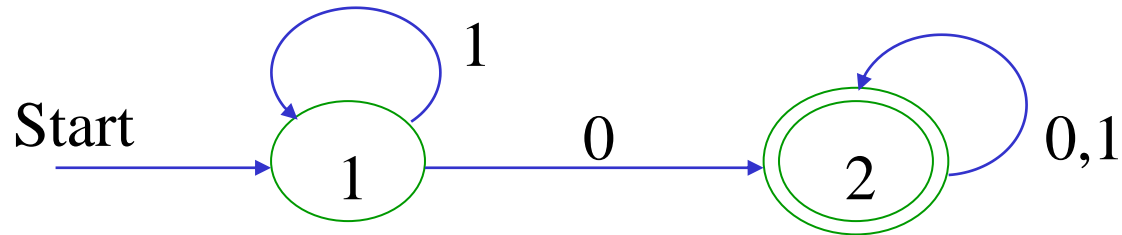
FA \Rightarrow RegExp - Induction

Induction $k \geq 1$

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$



Example 9 $FA \Rightarrow RegExp$



$$R_{11}^{(0)} = \varepsilon + 1, \quad R_{12}^{(0)} = 0, \quad R_{21}^{(0)} = \phi, \quad R_{22}^{(0)} = \varepsilon + 0 + 1$$

Example 9 FA \Rightarrow RegExp

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$

$$R_{11}^{(0)} = \varepsilon + 1, \quad R_{12}^{(0)} = 0, \quad R_{21}^{(0)} = \phi, \quad R_{22}^{(0)} = \varepsilon + 0 + 1$$

$$R_{11}^{(1)} = \varepsilon + 1 + (\varepsilon + 1)(\varepsilon + 1)^*(\varepsilon + 1) = 1^*$$

$$R_{12}^{(1)} = 0 + (\varepsilon + 1)(\varepsilon + 1)^* 0 = 1^* 0$$

$$R_{21}^{(1)} = \phi + \phi = \phi$$

$$R_{22}^{(1)} = \varepsilon + 0 + 1 + \phi = \varepsilon + 0 + 1$$

Example 9 $FA \Rightarrow \text{RegExp}$

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$

$$R_{11}^{(1)} = 1^*, R_{12}^{(1)} = 1^*0, R_{21}^{(1)} = \phi, R_{22}^{(1)} = \varepsilon + 0 + 1$$

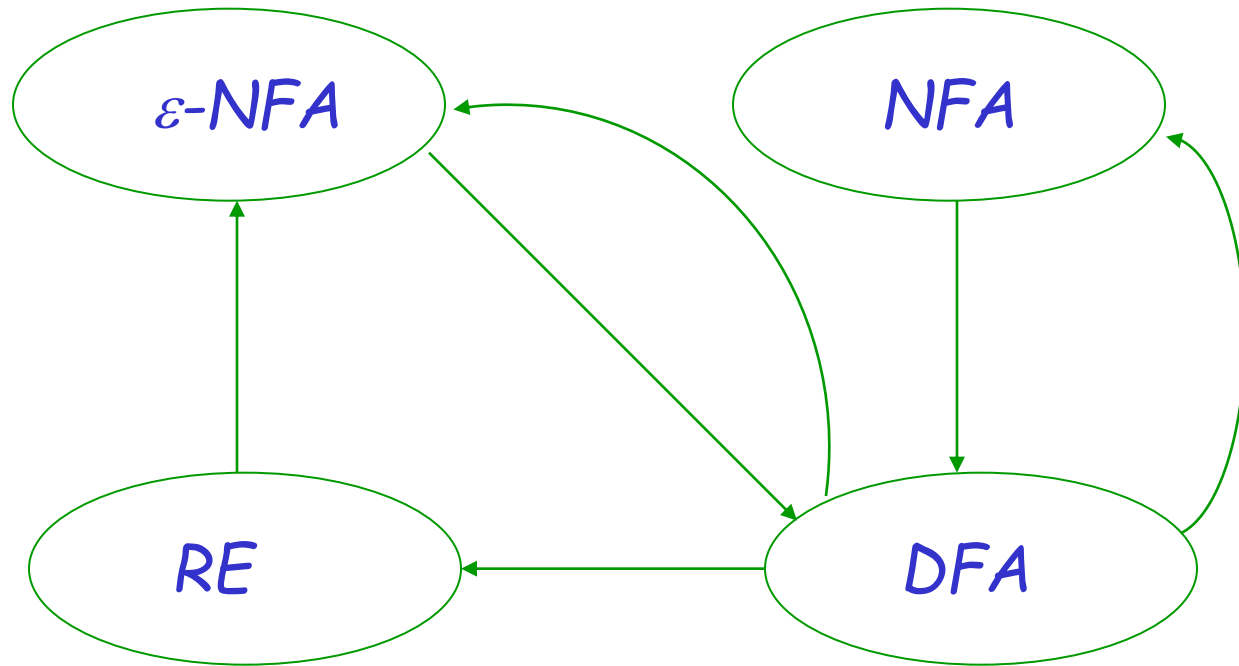
$$R_{11}^{(2)} = 1^* + 1^*0(0 + 1)^* \phi = 1^*$$

$$R_{12}^{(2)} = 1^*0 + 1^*0(\varepsilon + 0 + 1)^* (\varepsilon + 0 + 1) = 1^*0(0 + 1)^*$$

$$R_{21}^{(2)} = \phi + \phi = \phi$$

$$R_{22}^{(2)} = \varepsilon + 0 + 1 + (\varepsilon + 0 + 1)^* = (0 + 1)^*$$

FA & RE



Good good study
day day up!