

Morning



ϵ -NFA and Minimization of DFA

- ◆ Definition
- ◆ Minimize DFA
- ◆ Exercises



Formal Definition

ε - NFA is a five-tuple ,

such as $M = (Q, \Sigma, \delta, q_0, F)$

Where Q is a finite set of *states* ,

Σ is a finite set of *input symbols* ,

q_0 is a *start state* ,

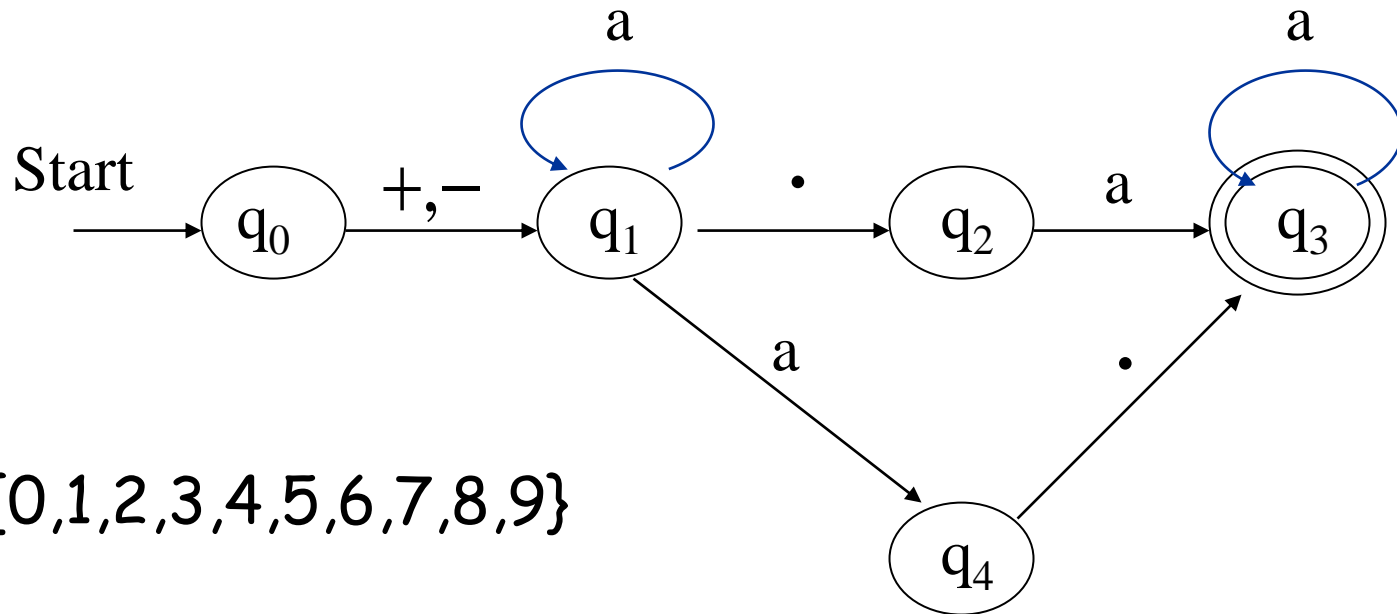
F is a set of *final state* ,

δ is *transition function* , which is a mapping

from $Q \times (\Sigma \cup \{\varepsilon\})$ to 2^Q .

Example 1

Describe the language accepted by this NFA :

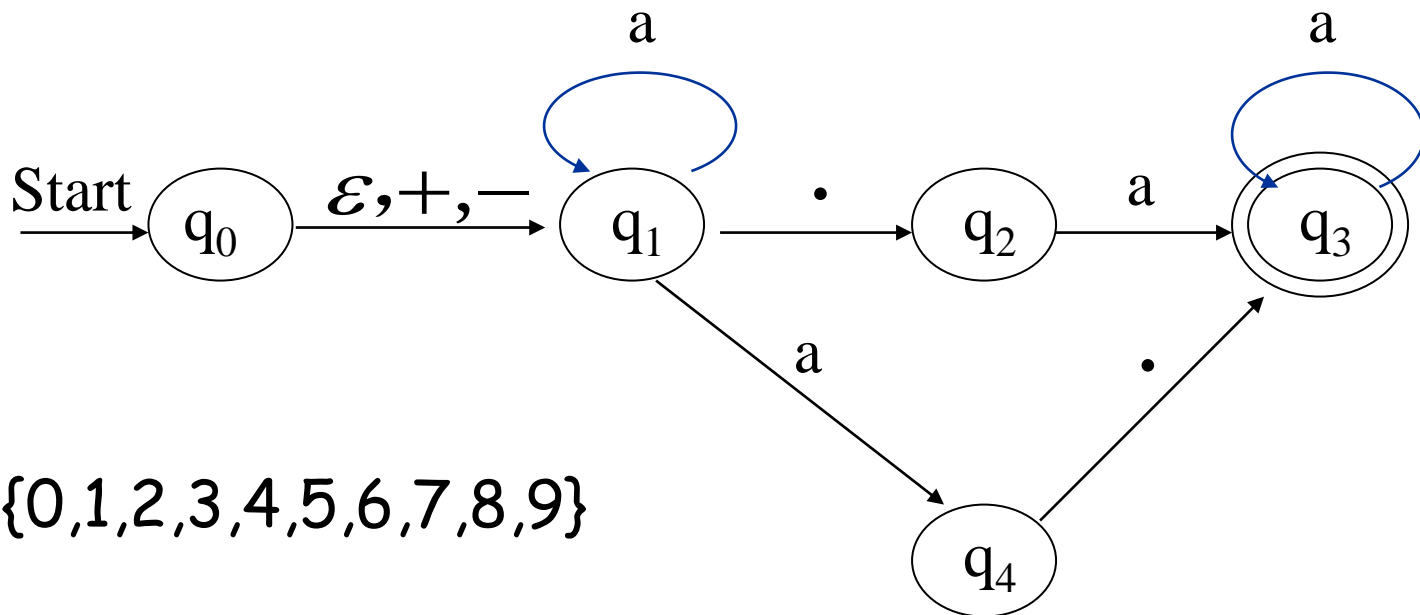


$\{ +23.01, -69.0, +.0, -10., +00.000, \dots \}$

23.01 / 69.0 / .0 / 10. ?

Example 1

Describe the language accepted by this NFA :



$a \in \{0,1,2,3,4,5,6,7,8,9\}$

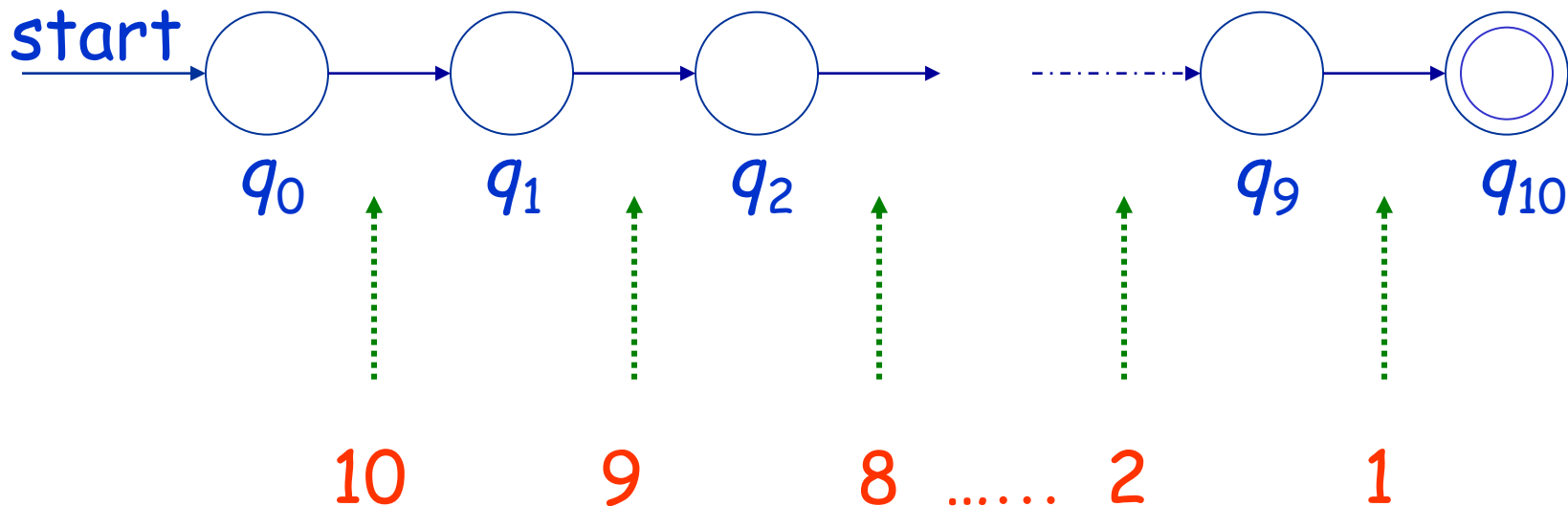
$\{ +23.01, -69.0, +.0, -10., +00.000, \dots \}$

23.01 / 69.0 / .0 / 10. !

Example 2 ϵ -NFA for

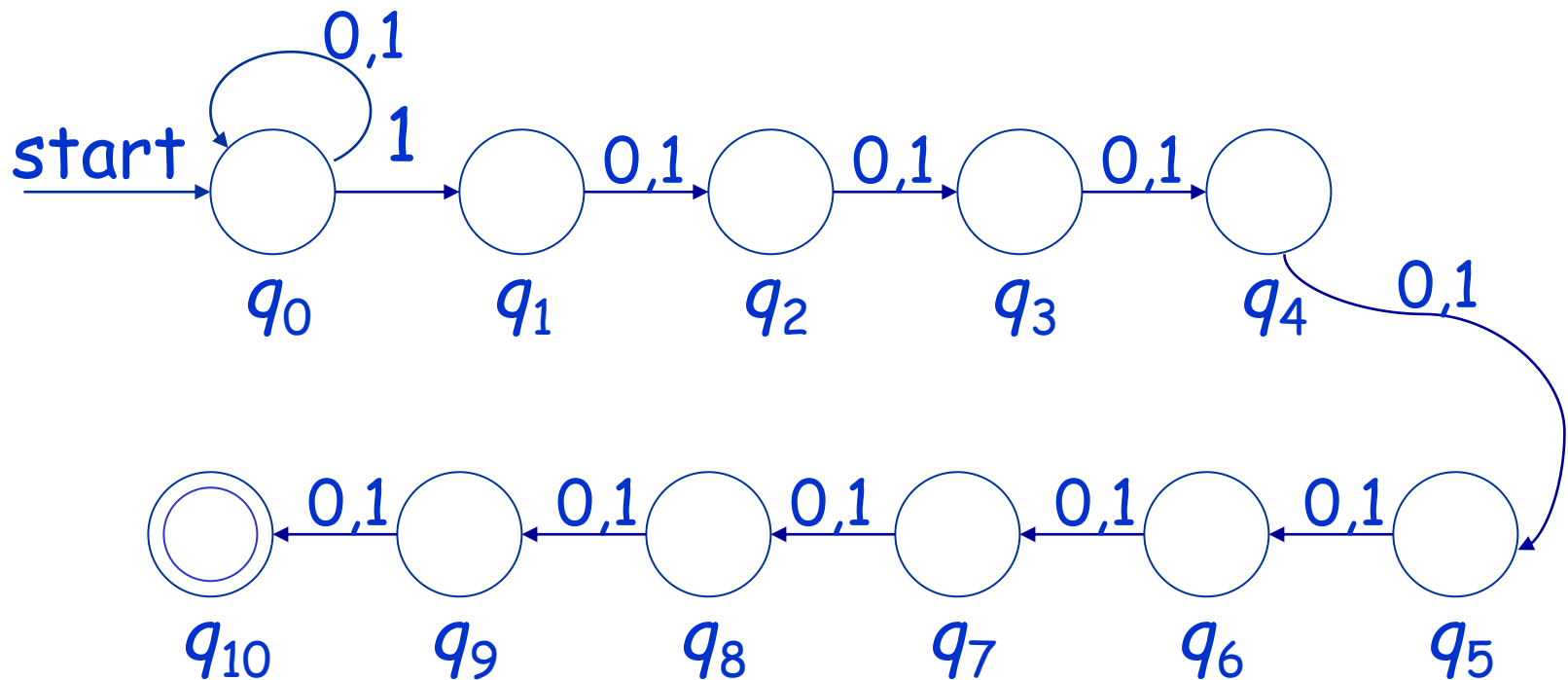
The set of strings of 0's and 1's such that **at least one of the last ten positions is a 1**.

1



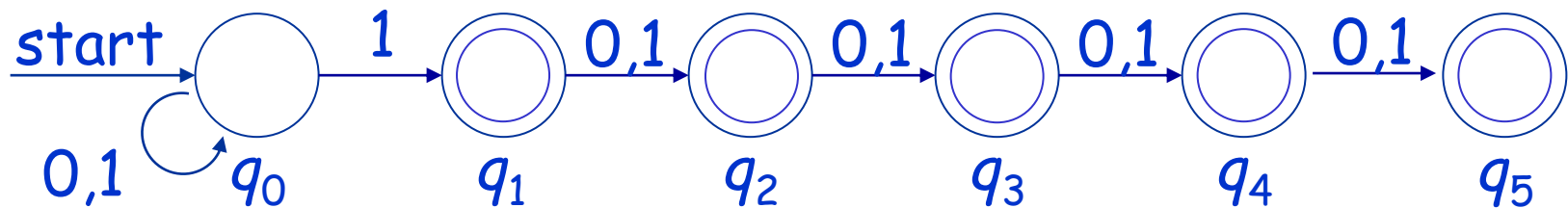
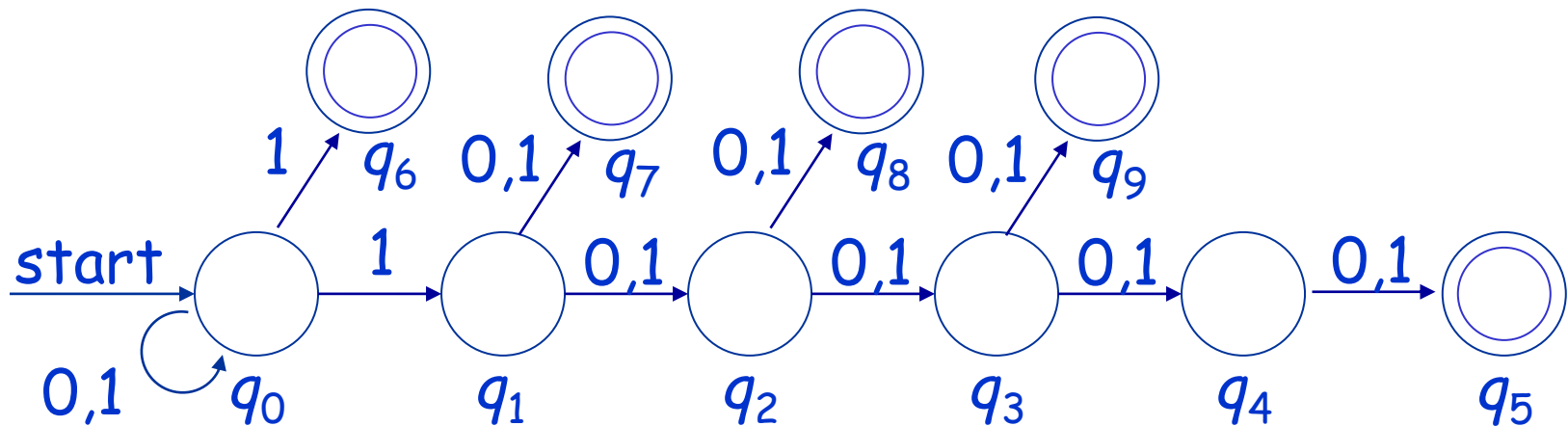
Example 2 NFA for

The set of strings of 0's and 1's such that **at least one of the last ten positions is a 1**.



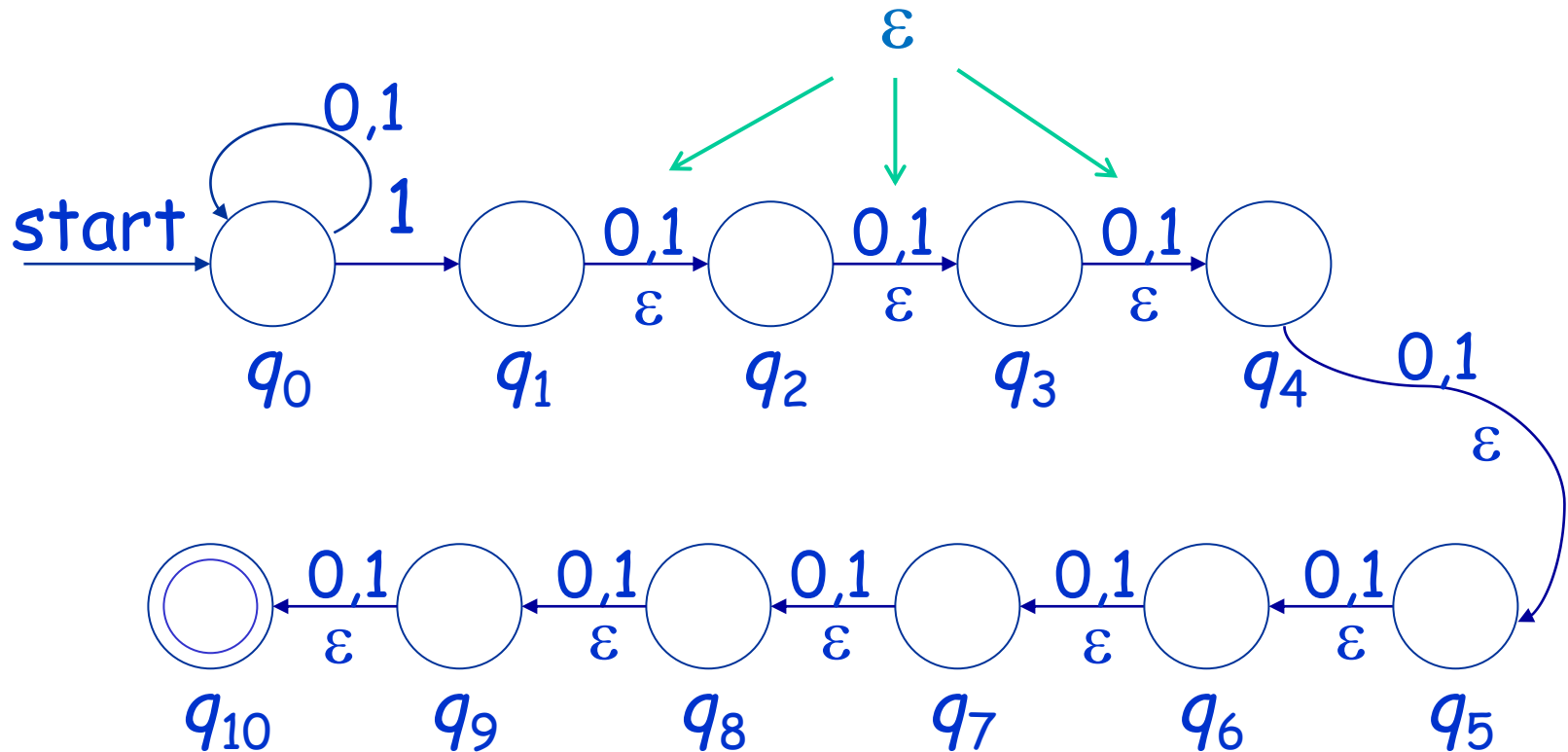
Example 2 NFA for

The set of strings of 0's and 1's such that **at least one of the last five positions is a 1**.



Example 2 ϵ -NFA for

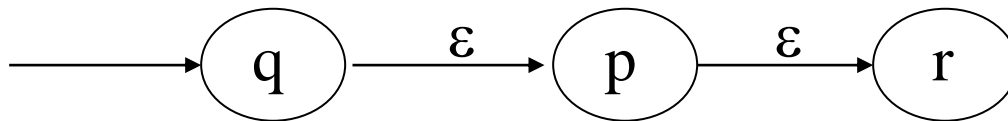
The set of strings of 0's and 1's such that at least one of the last ten positions is a 1.



ε - closure

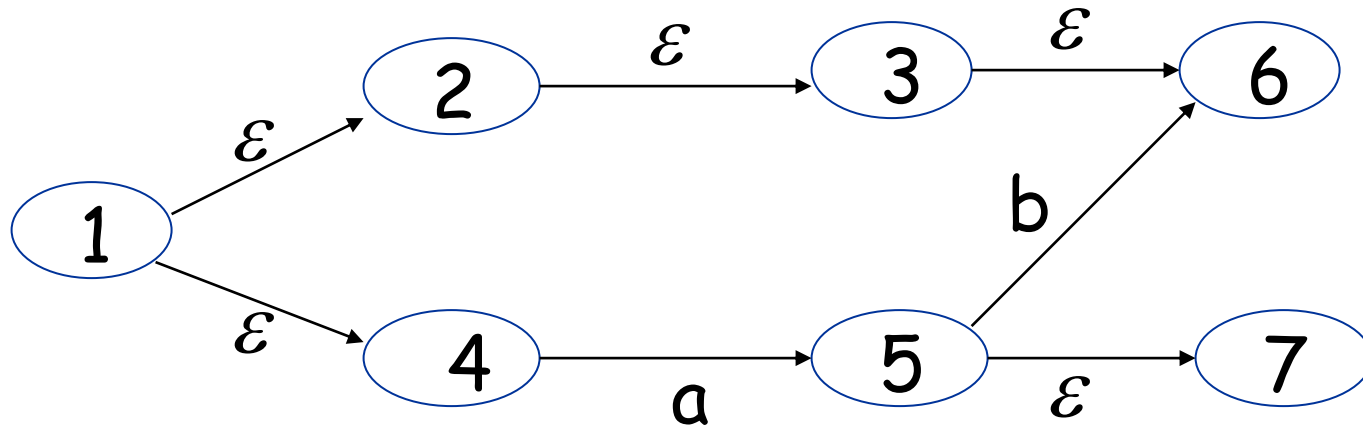
BASIS : State q is in $ECLOSE(q)$

INDUCTION : If state p is in $ECLOSE(q)$, and there is a transition from state p to state r labeled ε , then r is in $ECLOSE(q)$.



$$E(r) = \{ r \}, \quad E(p) = \{ p, r \}, \quad E(q) = \{ p, q, r \}$$

ε - closure

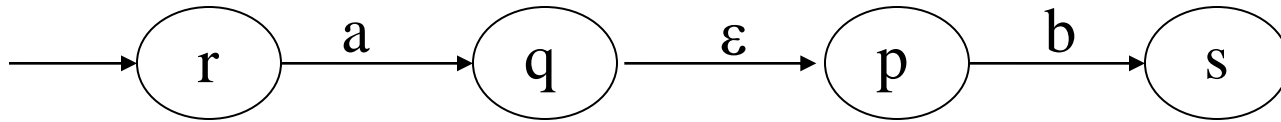


$$E(7) = \{ 7 \} , \quad E(6) = \{ 6 \} , \quad E(5) = \{ 5, 7 \}$$

$$E(4) = \{ 4 \} , \quad E(3) = \{ 3, 6 \} ,$$

$$E(2) = \{ 2, 3, 6 \} , \quad E(1) = \{ 1, 2, 4, 3, 6 \}$$

ϵ - transition



$$\delta(r, a) = ?$$

$$\delta(q, b) = ?$$

To which from state r with input symbol a ?

To which from state q with input symbol b ?

Extending δ to string

BASIS : $\hat{\delta}(q, \varepsilon) = \text{Eclose}(q).$

INDUCTION :

Suppose $w = xa$, $\hat{\delta}(q, x) = \{p_1, p_2, \dots, p_k\}$

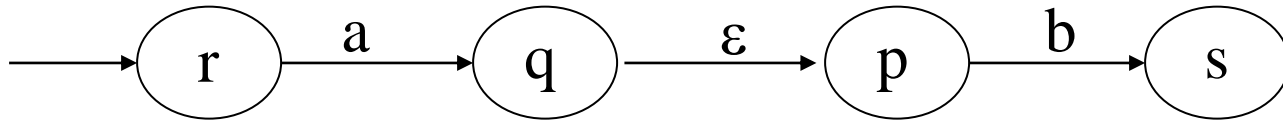
Let $\bigcup_{i=1}^k \delta(p_i, a) = \{r_1, r_2, \dots, r_m\}$

NFA

Then $\hat{\delta}(q, w) = \bigcup_{i=1}^m \text{Eclose}(r_i)$

ε -NFA

Extending δ to string



$w = ab$

$$\text{Eclose}(r) = \{ r \},$$

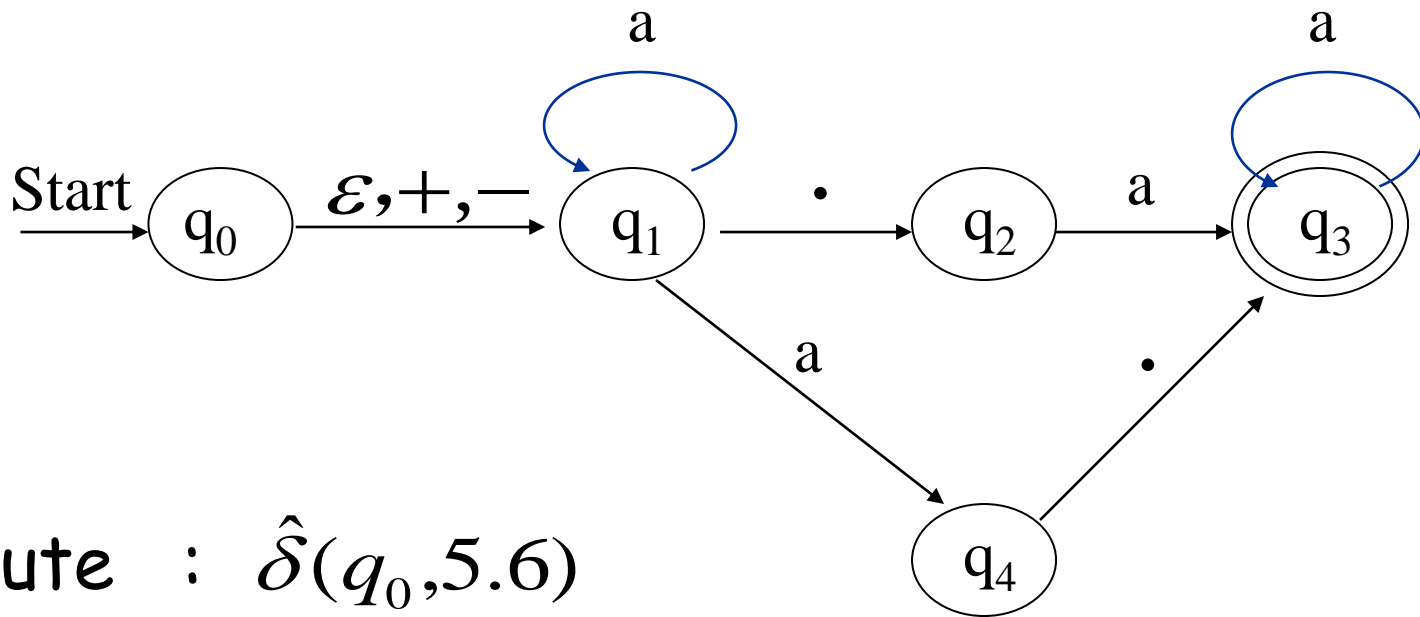
$$\delta(r, a) = \{ q \}$$

$$\text{Eclose}(q) = \{ q, p \} \Rightarrow \hat{\delta}(r, a) = \{ q, p \}$$

$$\delta(\hat{\delta}(r, a), b) = \delta(\{ q, p \}, b) = \{ s \}$$

$$\text{Eclose}(s) = \{ s \} \Rightarrow \hat{\delta}(r, w) = \hat{\delta}(r, ab) = \{ s \}$$

Example 3



Compute : $\hat{\delta}(q_0, 5.6)$

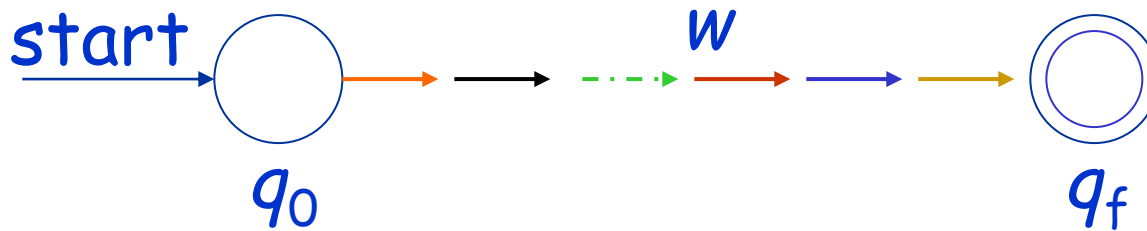
$$\hat{\delta}(q_0, 5.6) = \left\{ \text{Eclose}(q) \mid q \in \bigcup_{p \in \hat{\delta}(q_0, 5.)} \delta(p, 6) \right\}$$

$$\text{Eclose}(q_0) = \{q_0, q_1\}, \quad \hat{\delta}(q_0, 5) = \hat{\delta}(\text{Eclose}(q_0), 5)$$

Language of ε -NFA

Definition The *language of an ε -NFA* A is denoted $L(A)$, and defined by

$$L(A) = \{w \mid \hat{\delta}(q_0, w) \cap F \neq \emptyset\}$$



There is at least a path, labeled with w , from start state to final state.

Equivalence of states

- ◆ equivalent states

$$\forall w \in \Sigma^*, \hat{\delta}(p, w) \in F \Leftrightarrow \hat{\delta}(q, w) \in F$$

- ◆ notice

We never mentioned $\hat{\delta}(p, w) = \hat{\delta}(q, w)$

- ◆ distinguishable states

$$\exists w \in \Sigma^*, \hat{\delta}(p, w) \in F \Leftrightarrow \neg \hat{\delta}(q, w) \in F$$

Equivalence of states

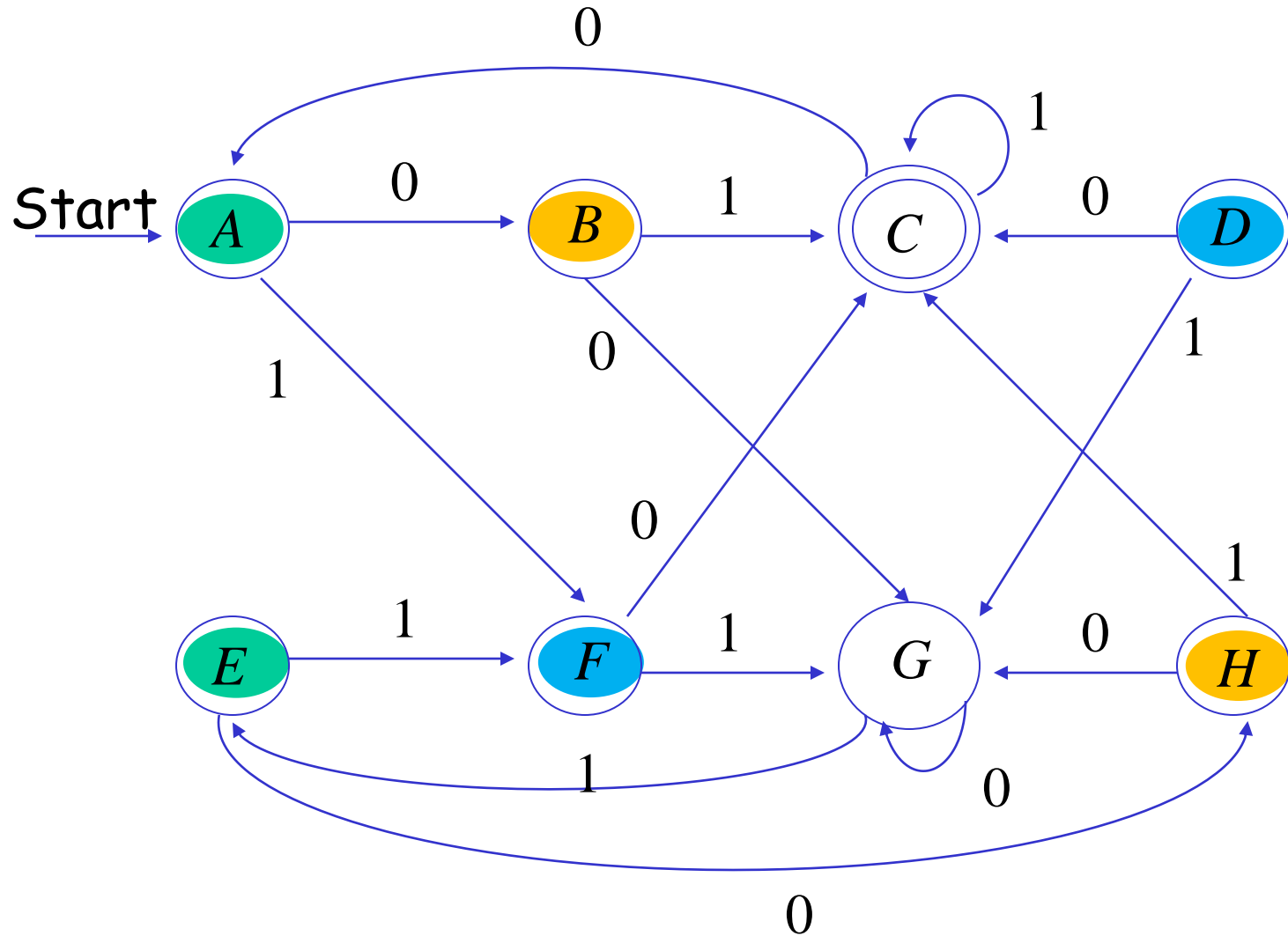


Table-filling algorithm

- ◆ **Basis** If p is accepting and q is not accepting, then p and q are distinguishable.
- ◆ **Induction** Let $r = \delta(p, a)$, $s = \delta(q, a)$, r and s are distinguishable. Then p and q are distinguishable.

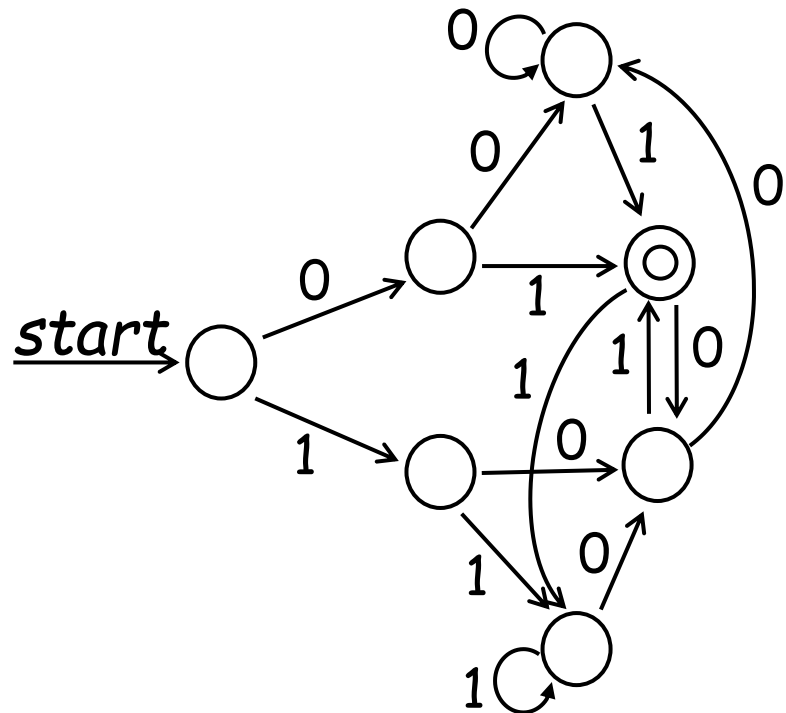
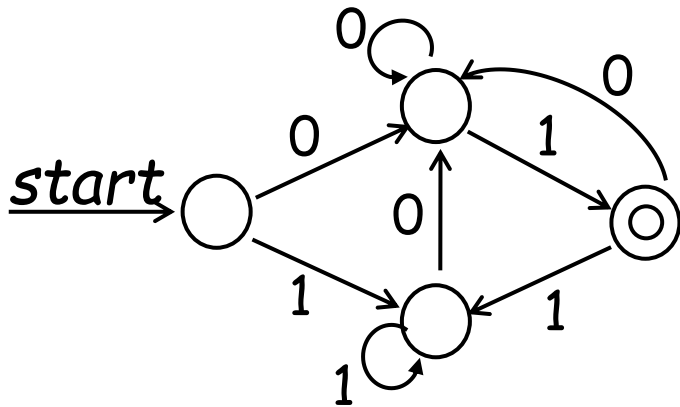
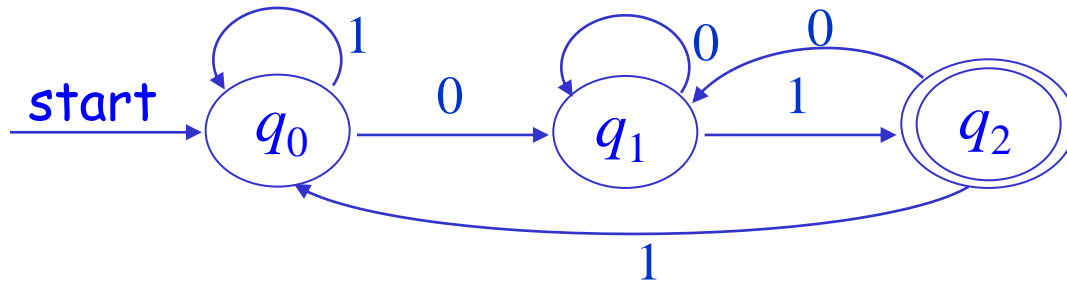
- ◆ **Example**

B	×						
C	×	×					
D	×	×	×				
E		×	×				
F	×	×	×		×		
G	×	×	×	×	×	×	
H	×		×	×	×	×	×
	A	B	C	D	E	F	G

Minimization of DFA's

- ◆ What is minimization of DFA
- ◆ Algorithm for minimization
 - partition remaining states into equivalent blocks
 - take blocks as states
- ◆ Minimum-state DFA for a regular language is unique

Example 4 Minimize DFA



Good good study
day day up!