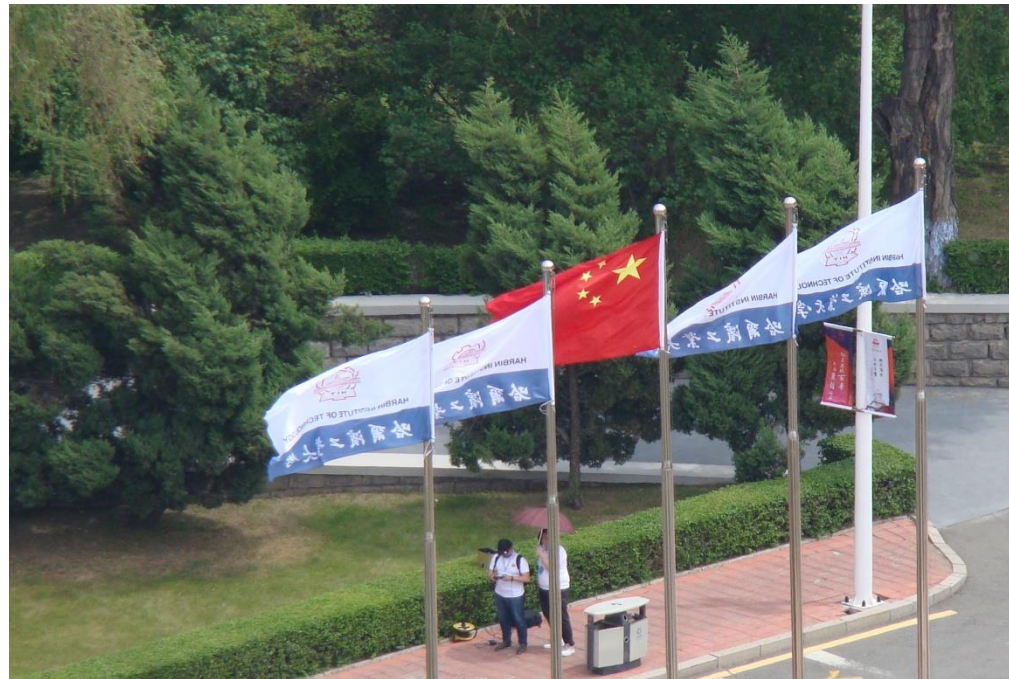


Morning

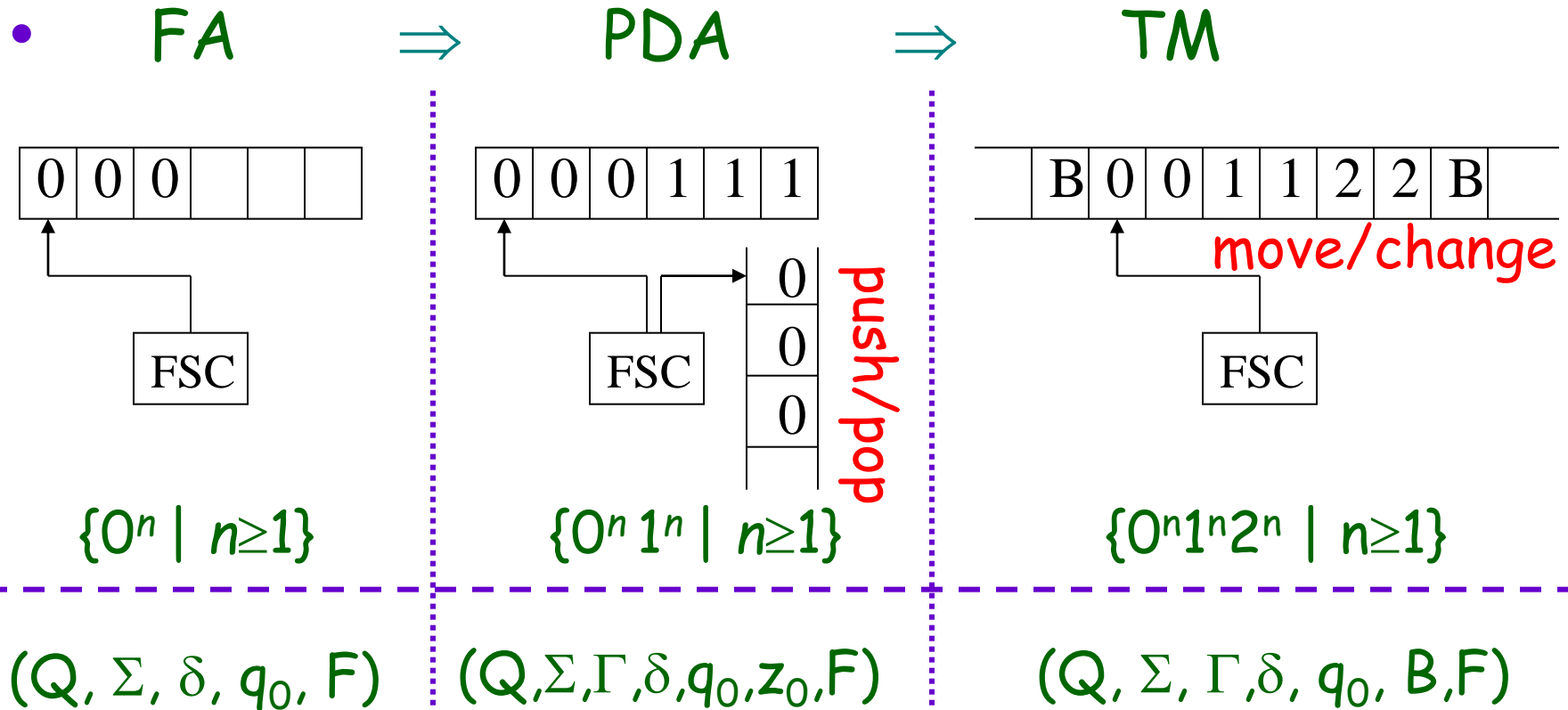


Turing Machine

- ◆ *Definition*
- ◆ *Construction*
- ◆ *Language*



Different automata



Definition

TM is a seven-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

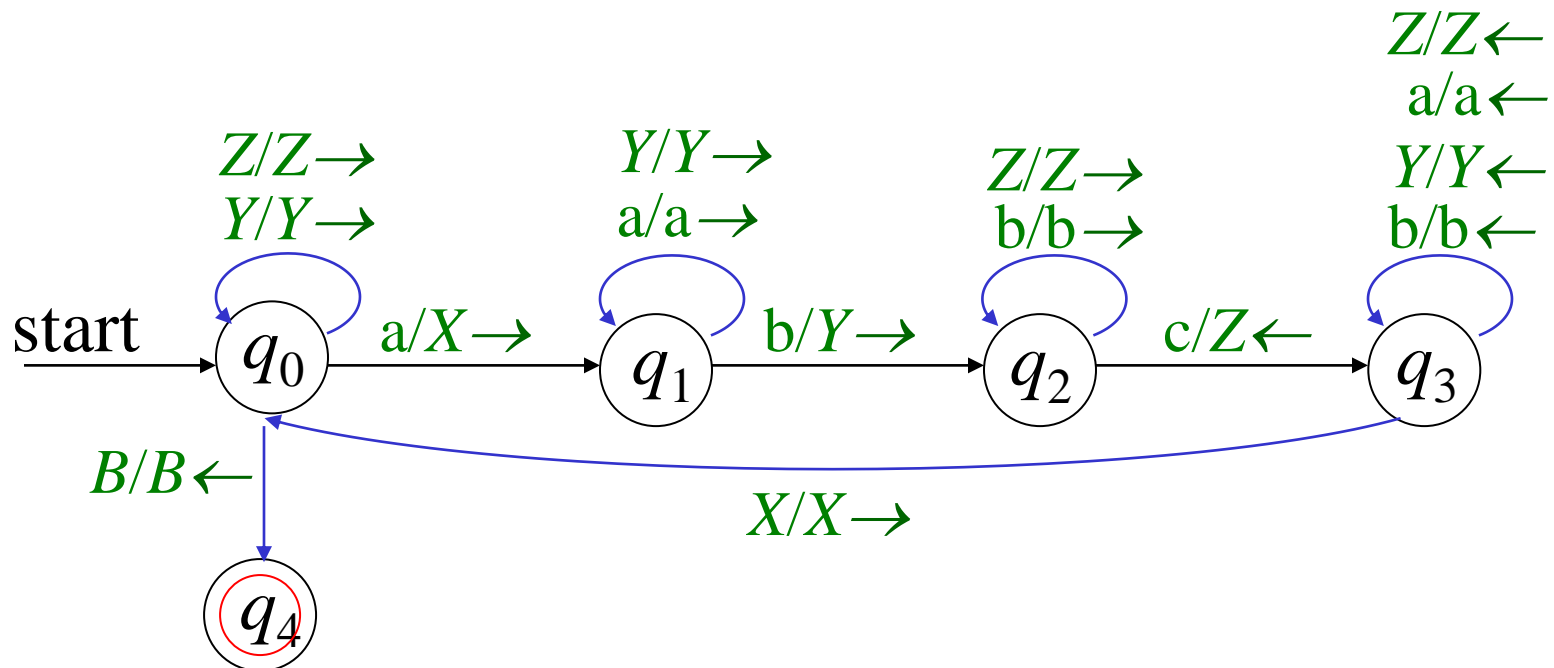
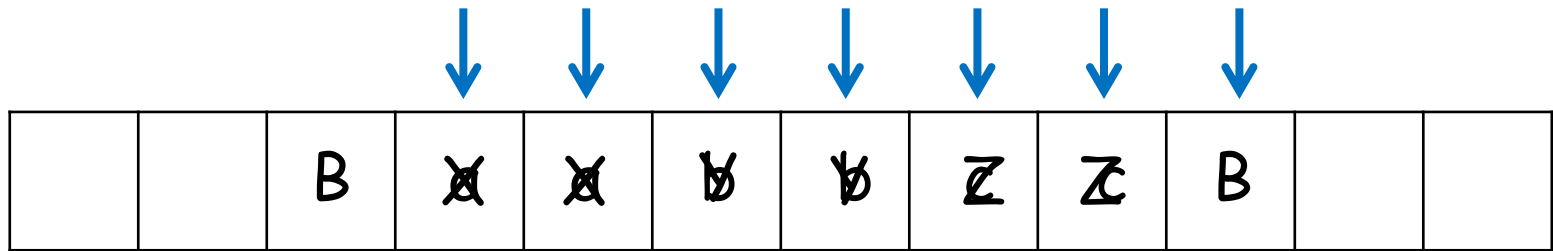
- Q is finite set of **states**
- Σ is finite set of **input symbols**
- Γ is finite set of **tape symbols**
- δ is **transition function** : $Q \times \Gamma \Rightarrow Q \times \Gamma \times \{R, L\}$

$$\delta(q, X) = (p, Y, D)$$

- q_0 is **start state**
- B is **blank symbol**
- F is finite set of **final state**

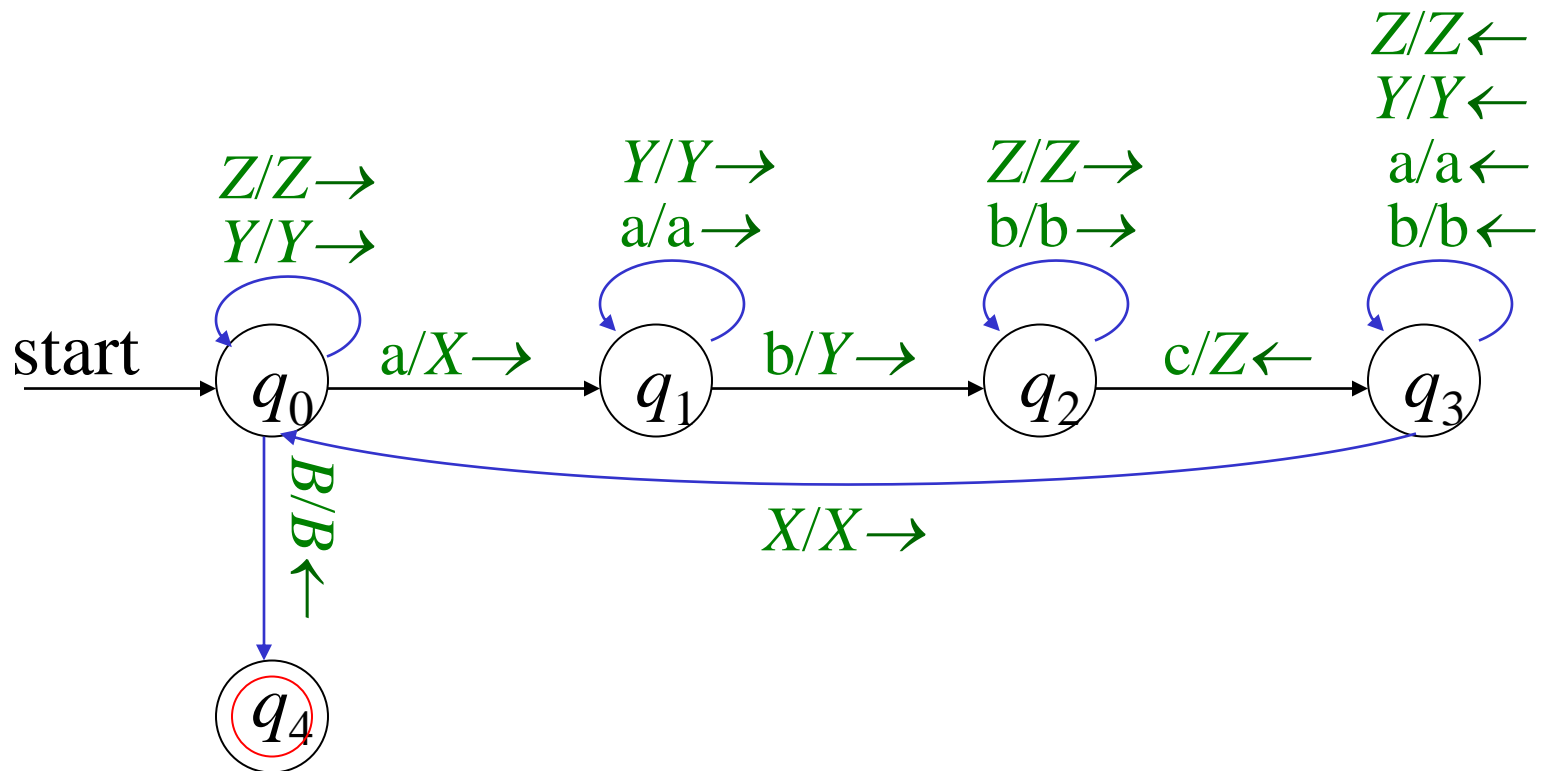
Example 1 TM for

$$L = \{ a^n b^n c^n \mid n \geq 0 \}$$



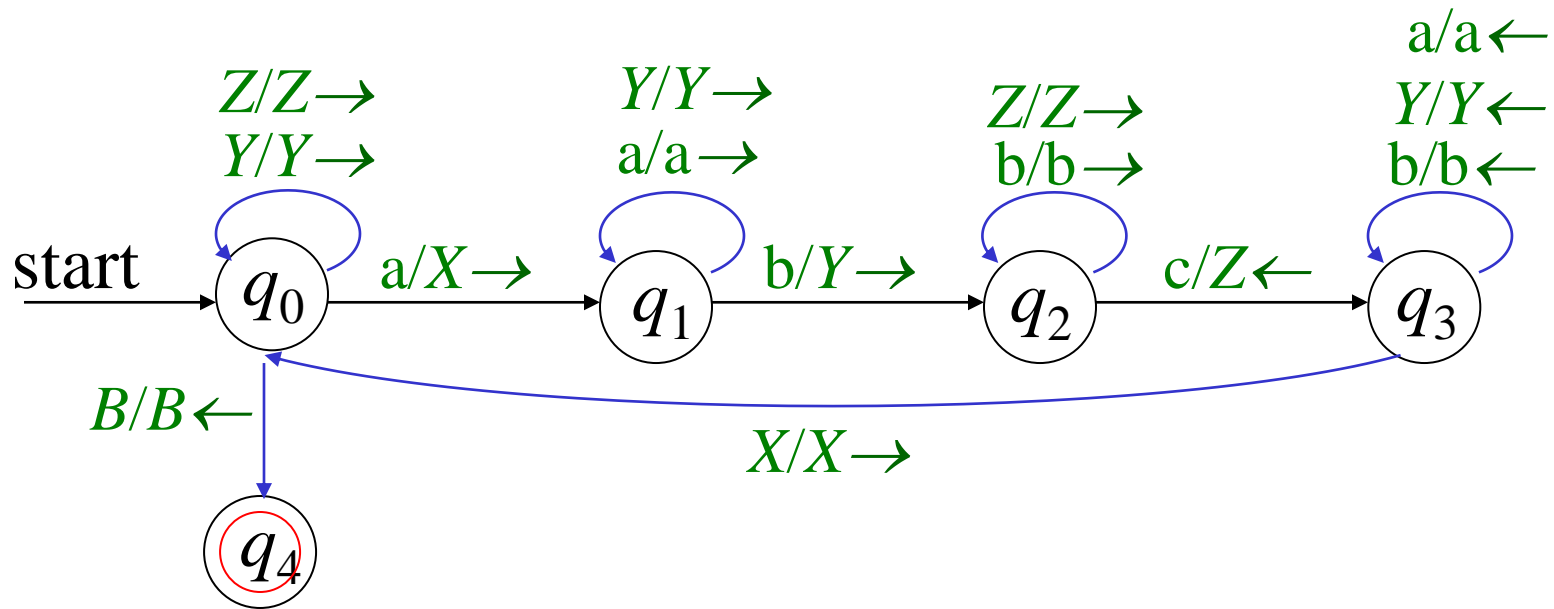
Example 1 TM for ' $a^n b^n c^n$ '

$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b, c\}, \{a, b, c, B, X, Y, Z\}, \delta, q_0, B, \{q_4\})$$

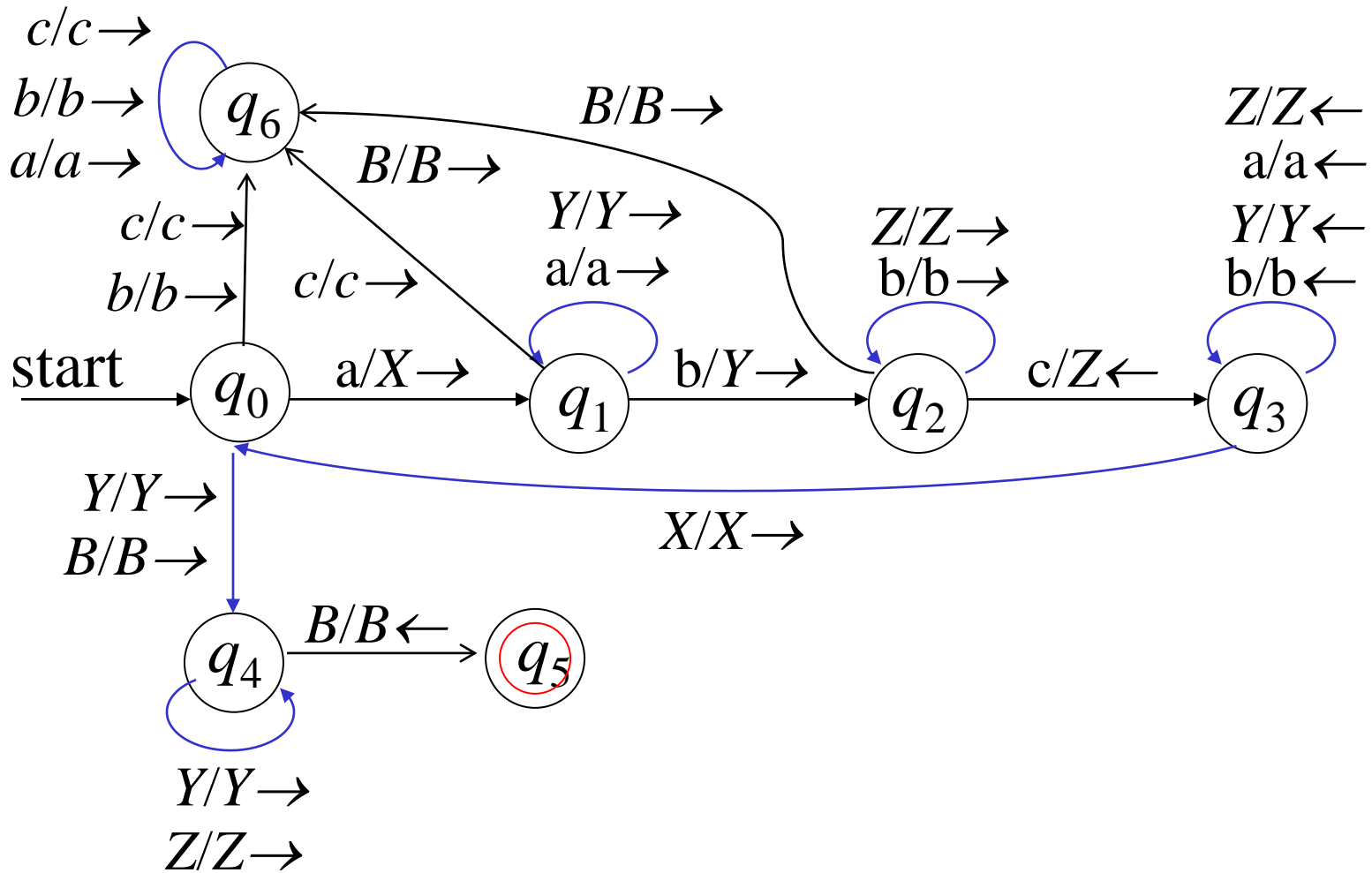


Example 1 TM for ' $a^n b^n c^n$ '

- ◆ ε ?
- ◆ $a^2 b^2 c^2 a^3 b^3 c^3$??
- ◆ $a^2 b^3 c^4$, ac , bbc ???



Example 1 TM for ' $a^n b^n c^n$ '



Instantaneous

- ◆ how to describe the configuration of TM
 - sequence of symbols in tape
 - state of TM
 - read/write head of TM
 - $X_1 \dots X_{i-1} q X_i X_{i+1} \dots X_n$

Instantaneous

- ◆ ID of the above TM for $w = aabbcc \in \{a^n b^n c^n \mid n \geq 1\}$

$q_0 aabbcc \vdash X q_1 abbcc \vdash X a q_1 bbcc \vdash X a Y q_2 bcc$

$X a Y b q_2 cc \vdash X a Y q_3 b Zc \vdash X a q_3 Y b Zc \vdash X q_3 a Y b Zc \vdash$

$q_3 X a Y b Zc \vdash X q_0 a Y b Zc \vdash X X q_1 Y b Zc \vdash X X Y q_1 b Zc \vdash$

$X X Y Y q_2 Zc \vdash X X Y Y Z q_2 c \vdash X X Y Y q_3 Z Z \vdash \dots$

Language of TM

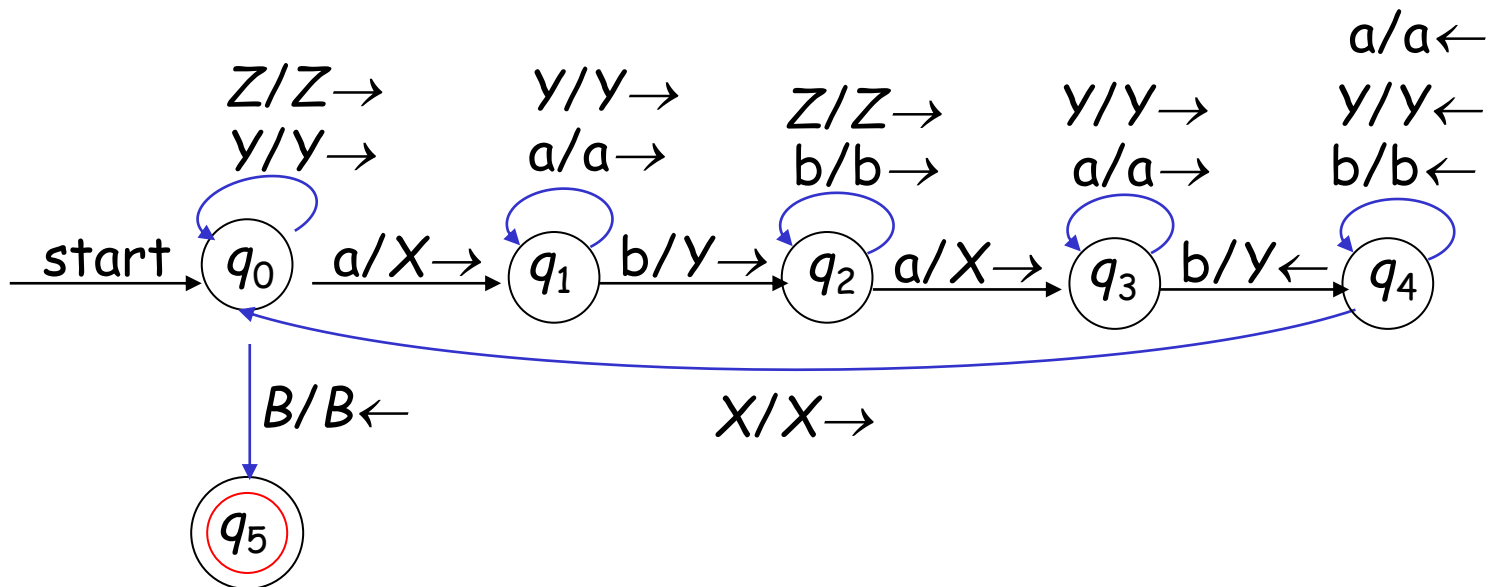
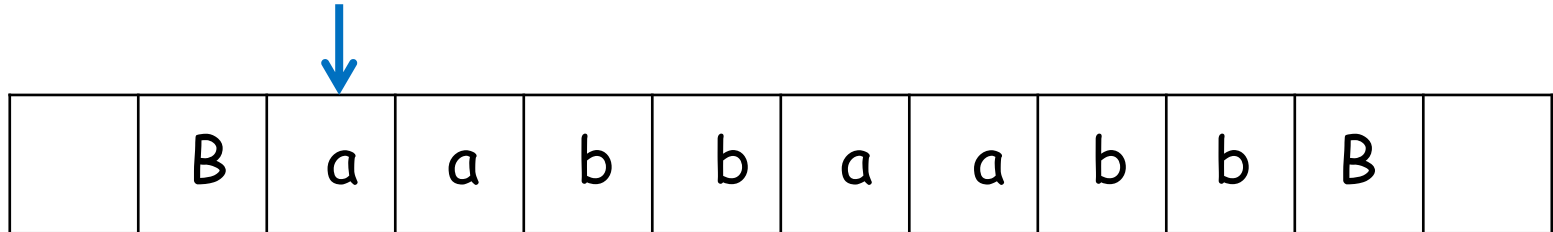
- ◆ language accepted by a TM

$$\{w \mid q_0 w \vdash^* \alpha p \beta, p \in F, \alpha, \beta \in \Gamma^*\}$$

- ◆ The language accepted by TM is called *recursively enumerable(RE)* language.

Example 2 TM for

$$L = \{ a^n b^n a^n b^n \mid n \geq 0 \}$$

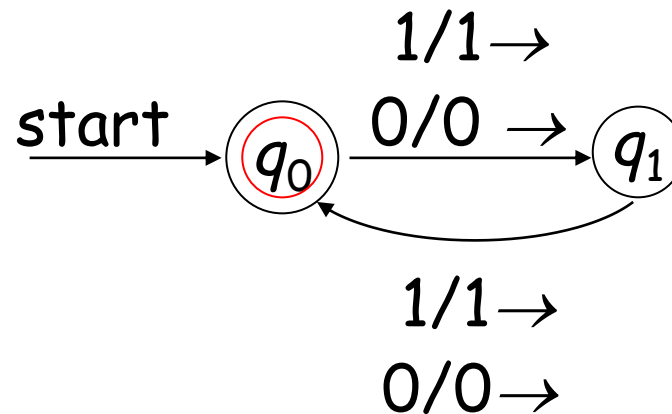


Example 3 TM for

$$L = \{ w \mid w \in \{0,1\}^* \text{ and } |w| \text{ is even} \}$$

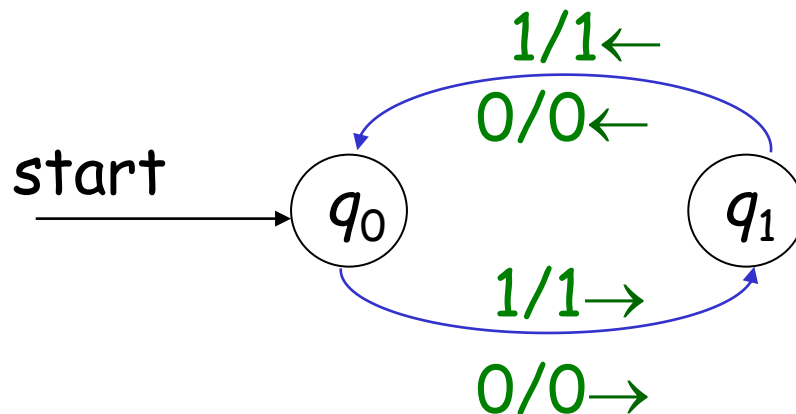


	B	0	0	1	1	1	0	1	1	B	
--	---	---	---	---	---	---	---	---	---	---	--



Halting

We say a TM halts if it enters a state q , scanning a tape symbol X , and there is no move in this situation .



Example 4

Given two positive integers x and y , design a TM to compute $x + y$.

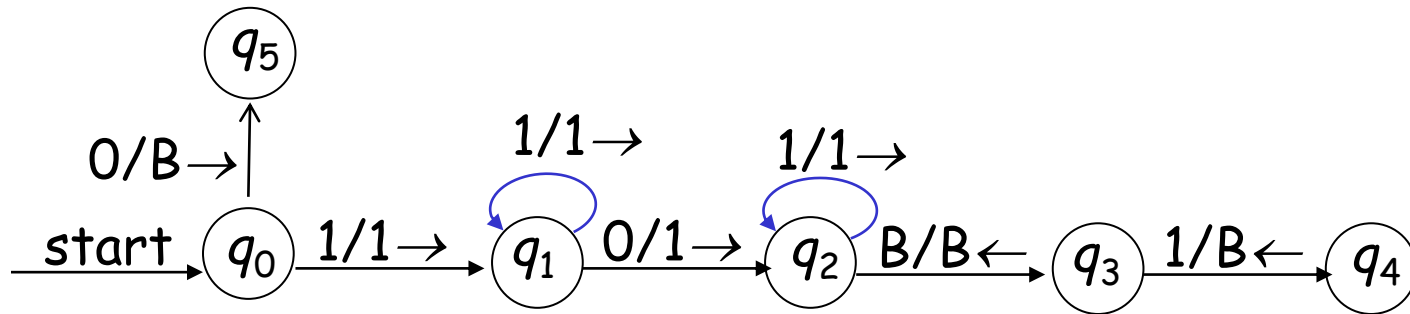
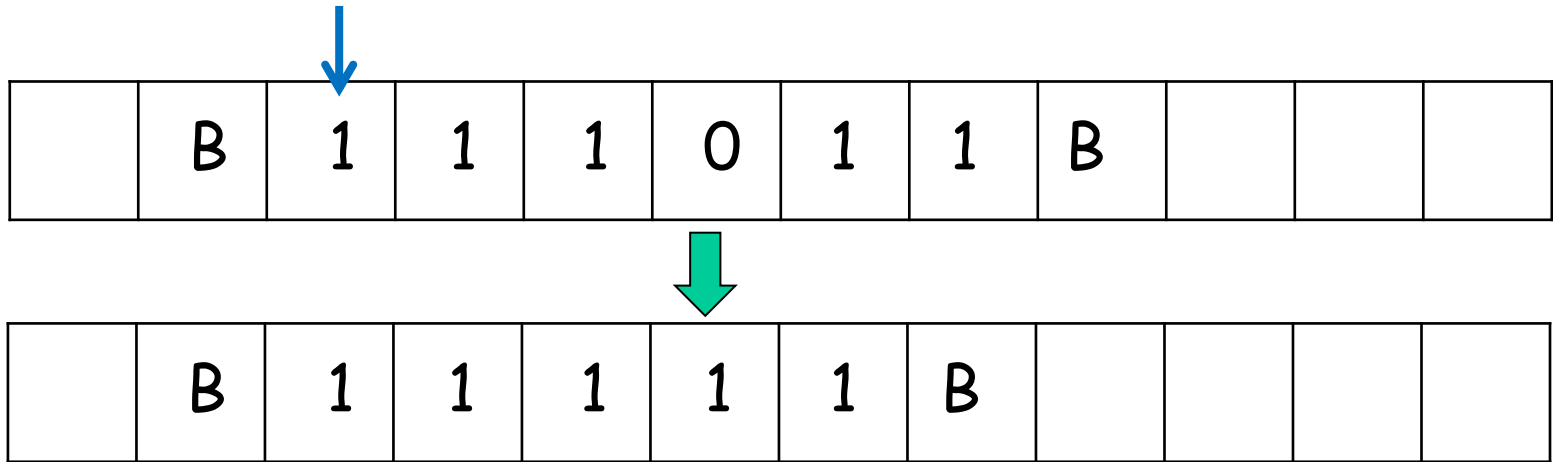
Notation for x and y

$$x \Rightarrow w(x) \in \{1\}^+ \text{ and } |w(x)| = x$$

$$x + y \Rightarrow w(x + y) \in \{1\}^+ \text{ and } |w(x + y)| = x + y$$

$$3 - 111, \quad 2 - 11, \quad 3 + 2 - 11111$$

Example 4



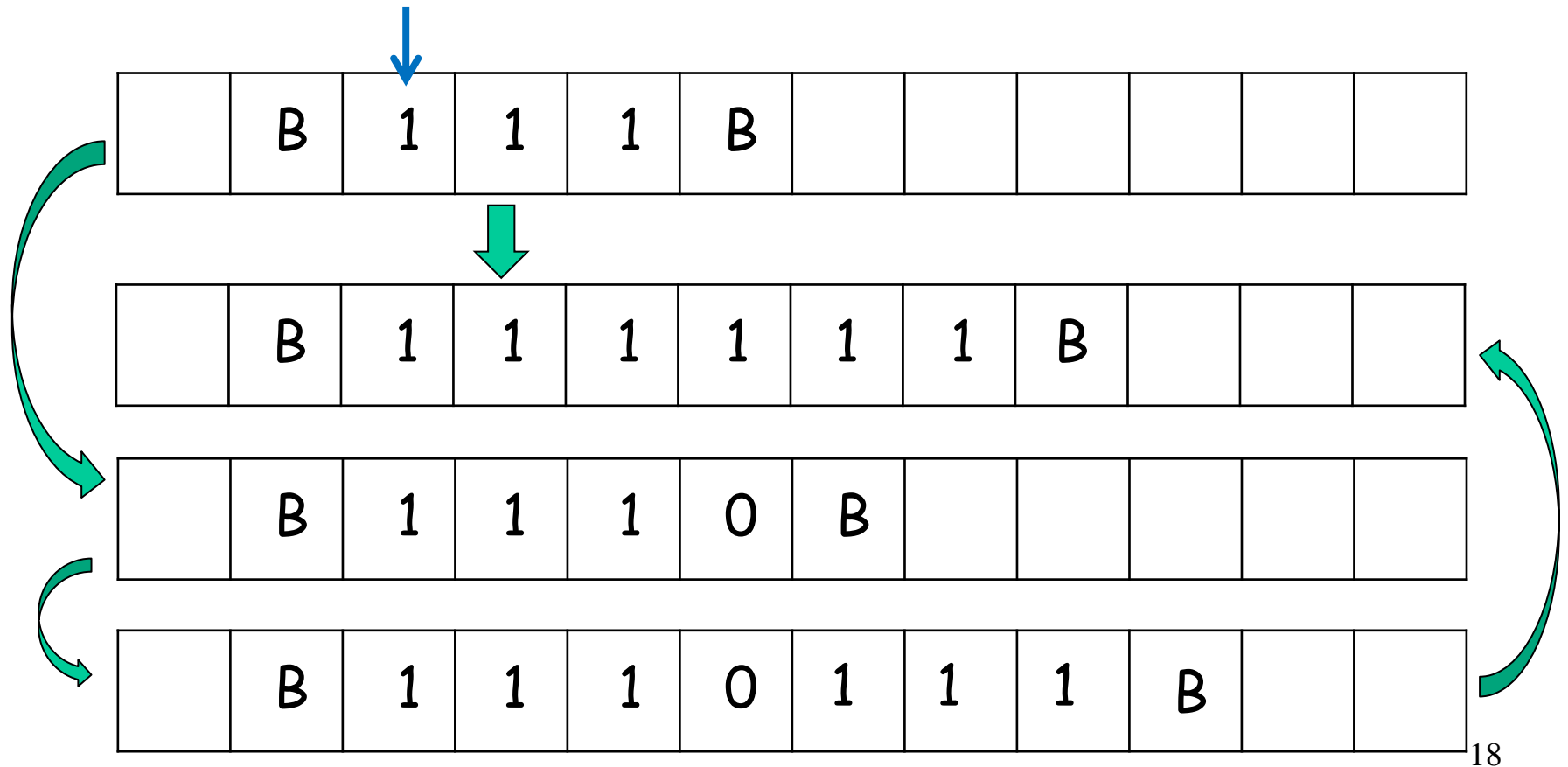
Example 5

Compute the function $nomus(m, n) = \max(m - n, 0)$

- ◆ put $1^m 0 1^n$ into tape as input
- ◆ delete a 1 from 1^m and a 1 from 1^n
- ◆ three cases :
 - $m > n \Leftrightarrow 1^{m-n-1}$ at the left of 1
 - $m = n \Leftrightarrow$ no 1
 - $m < n \Leftrightarrow 1^{n-m}$ at the right of 1

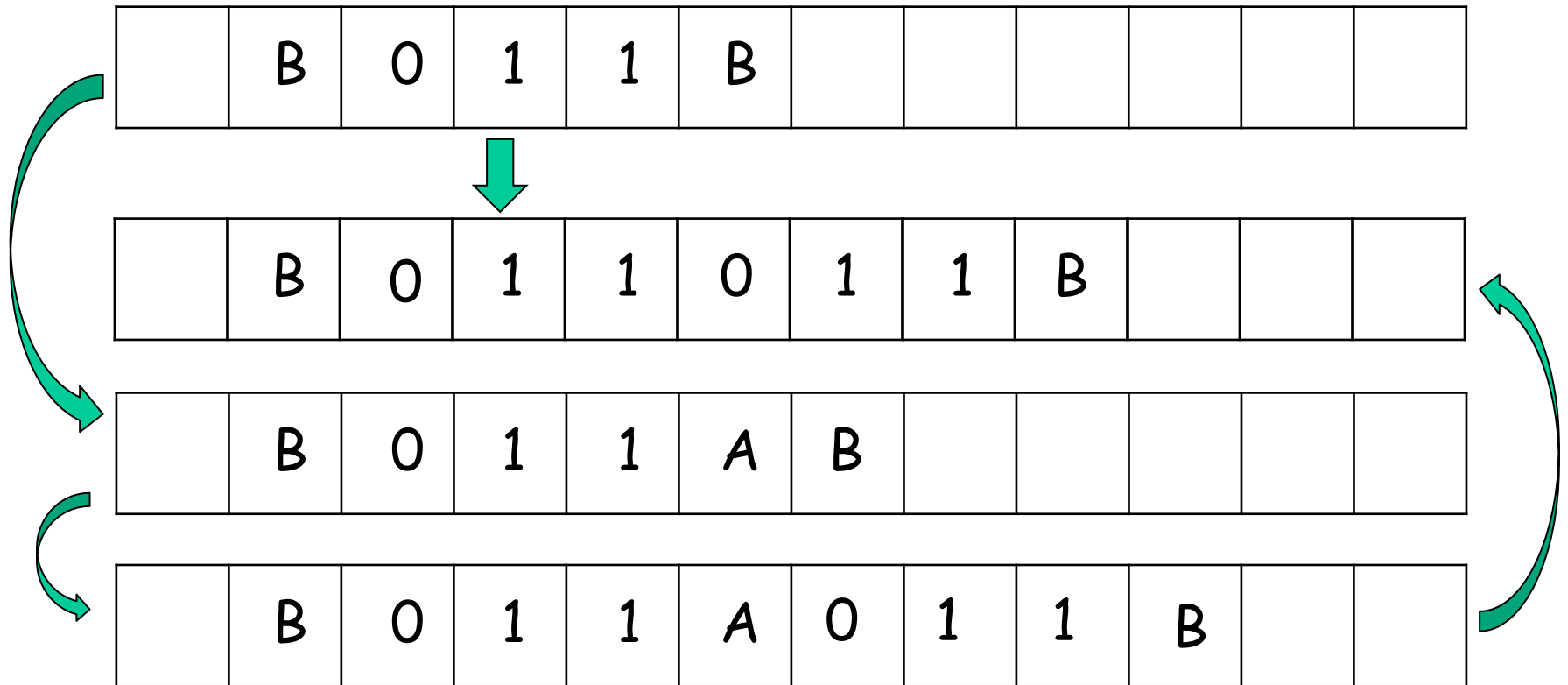
Example 6

Construct a TM to compute the function $f(w) = ww$ where $w \in \{1\}^+$.



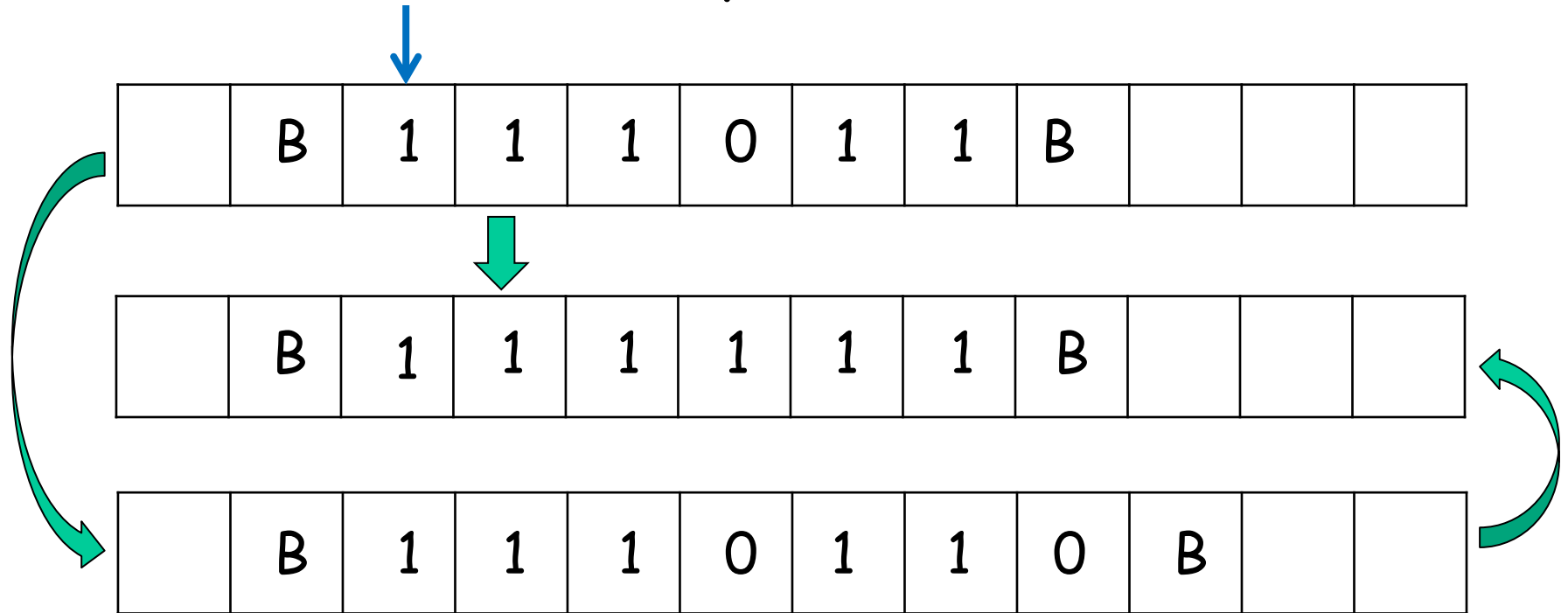
Example 7

Construct a TM to compute the function $f(w) = ww$ where $w \in \{0, 1\}^+$.



Example 8

Construct a TM to compute $m \times n$.



$$3 \times 2 \Rightarrow 2 + 2 + 2$$

Example 8

B	1	1	1	0	1	1	0	B			
---	---	---	---	---	---	---	---	---	--	--	--

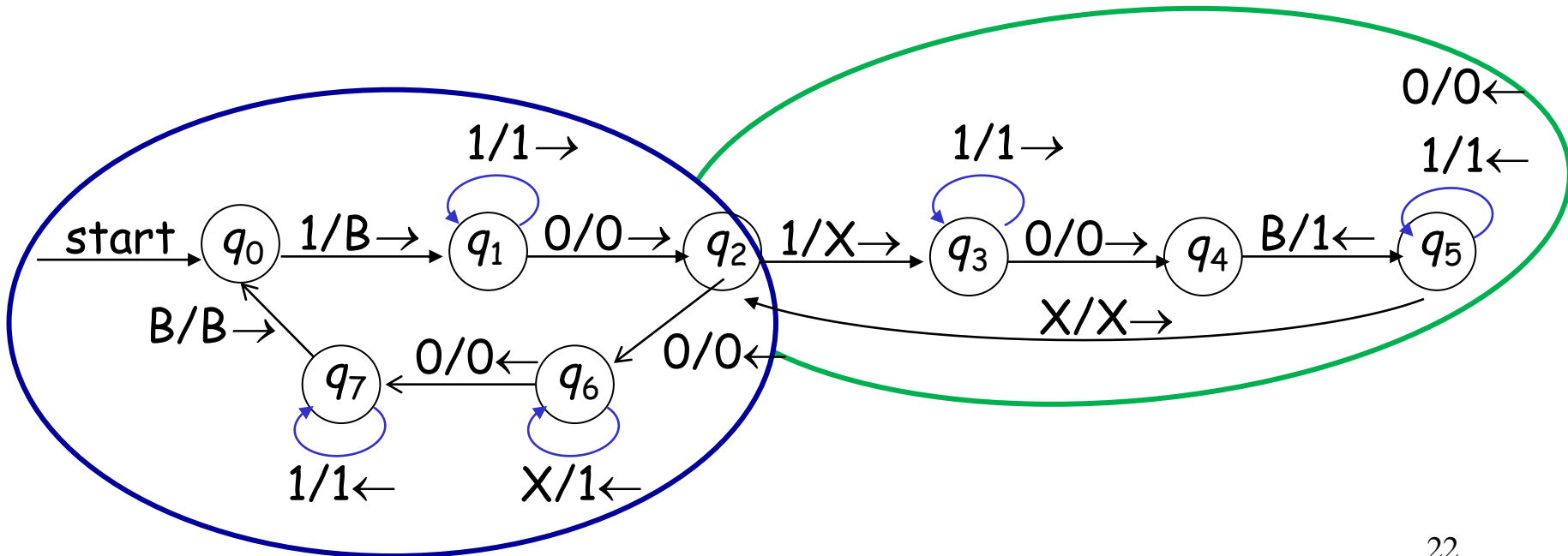
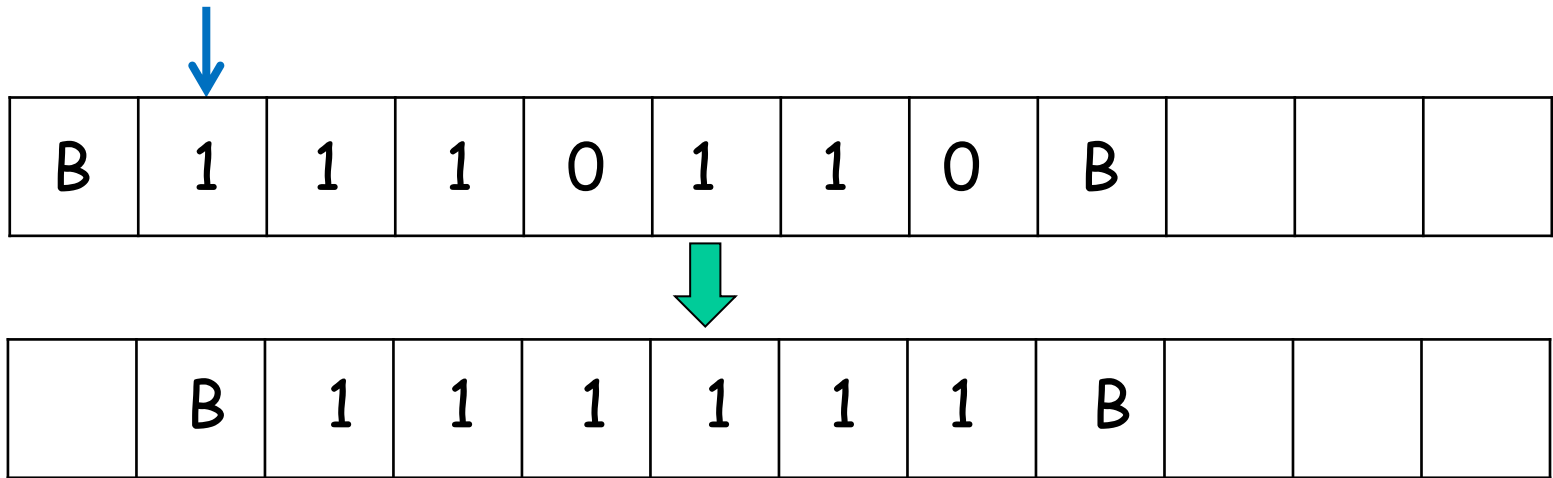
	B	1	1	0	1	1	0	1	1	B	
--	---	---	---	---	---	---	---	---	---	---	--

		B	1	0	1	1	0	1	1	1	1	B
--	--	---	---	---	---	---	---	---	---	---	---	---

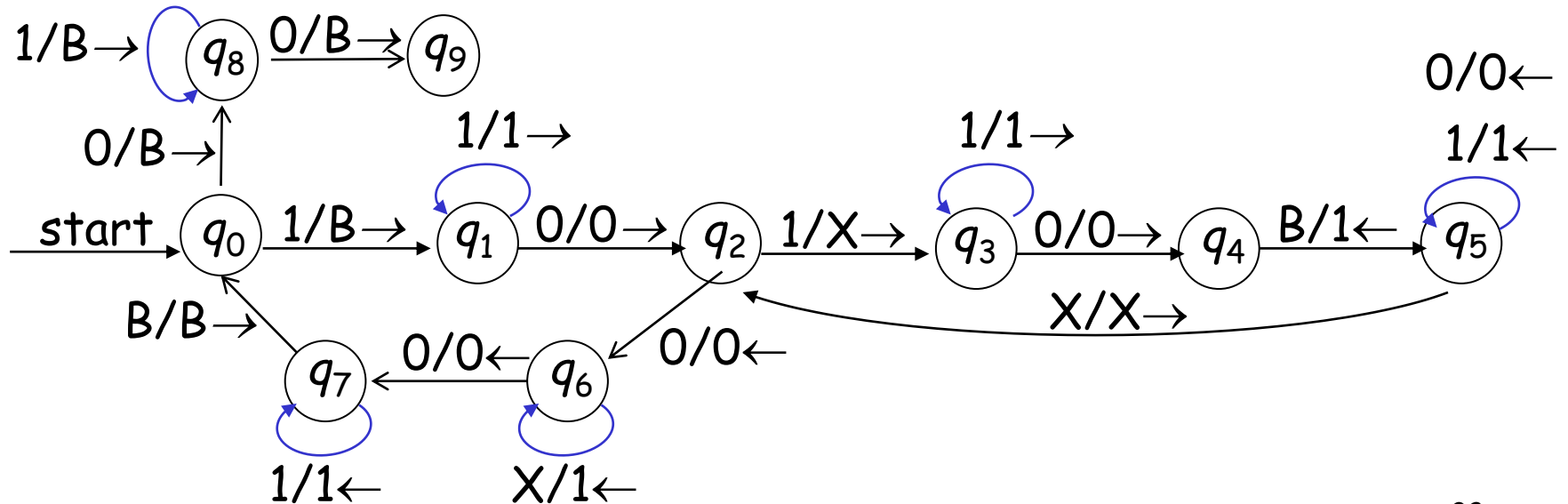
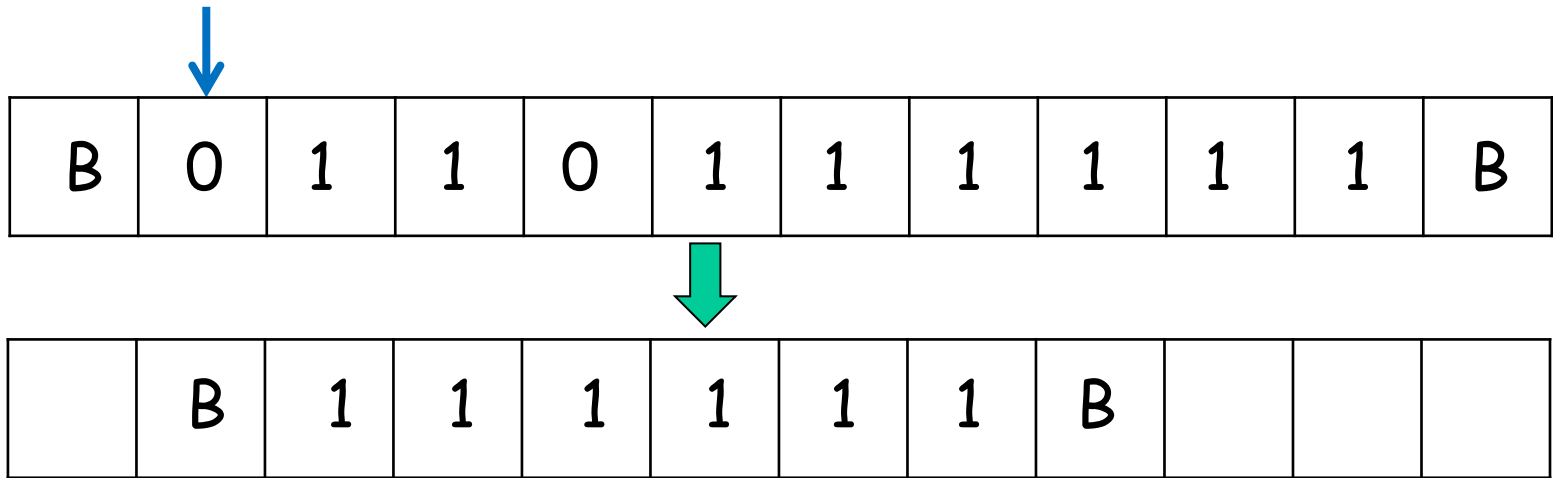
	B	0	1	1	0	1	1	1	1	1	1	B
--	---	---	---	---	---	---	---	---	---	---	---	---

					B	1	1	1	1	1	1	B
--	--	--	--	--	---	---	---	---	---	---	---	---

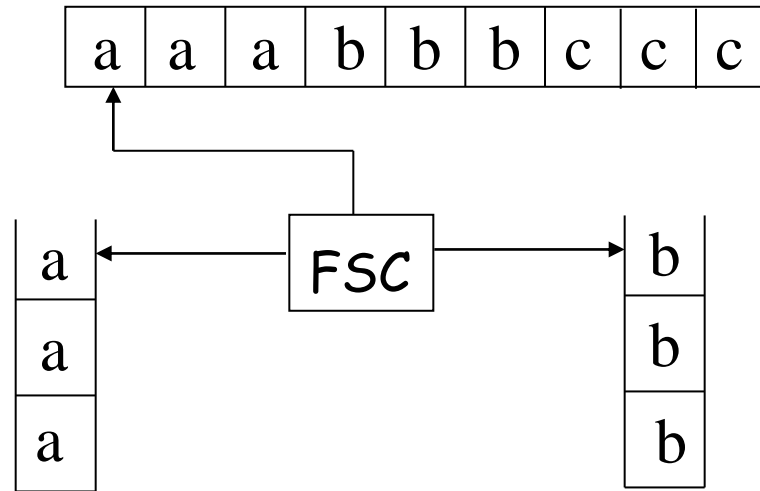
Example 8



Example 8



Two Stack Machine

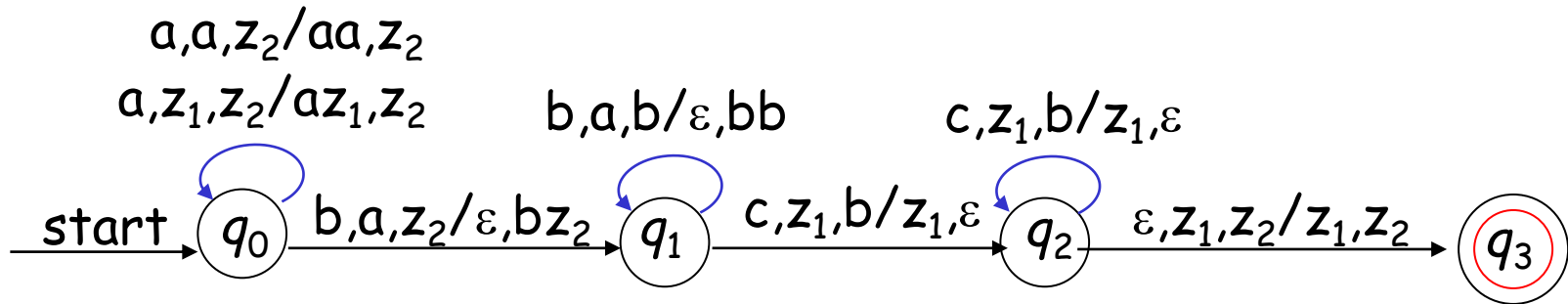
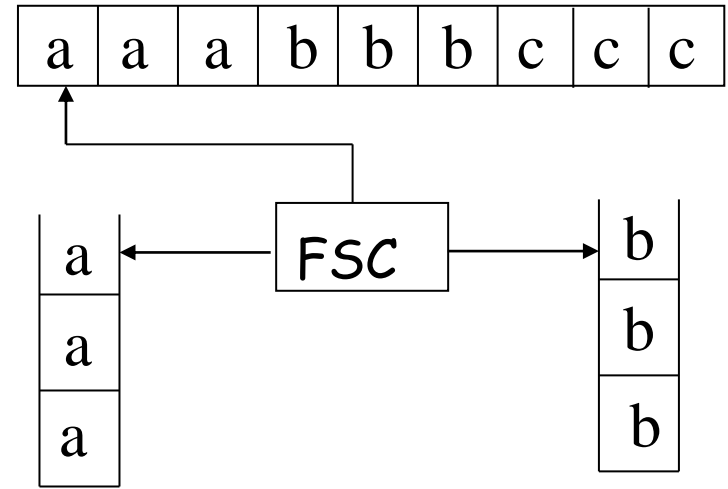


$$\delta(q, a, X, Y) = (p, \alpha, \beta)$$

Example 9

Construct a two stack machine for

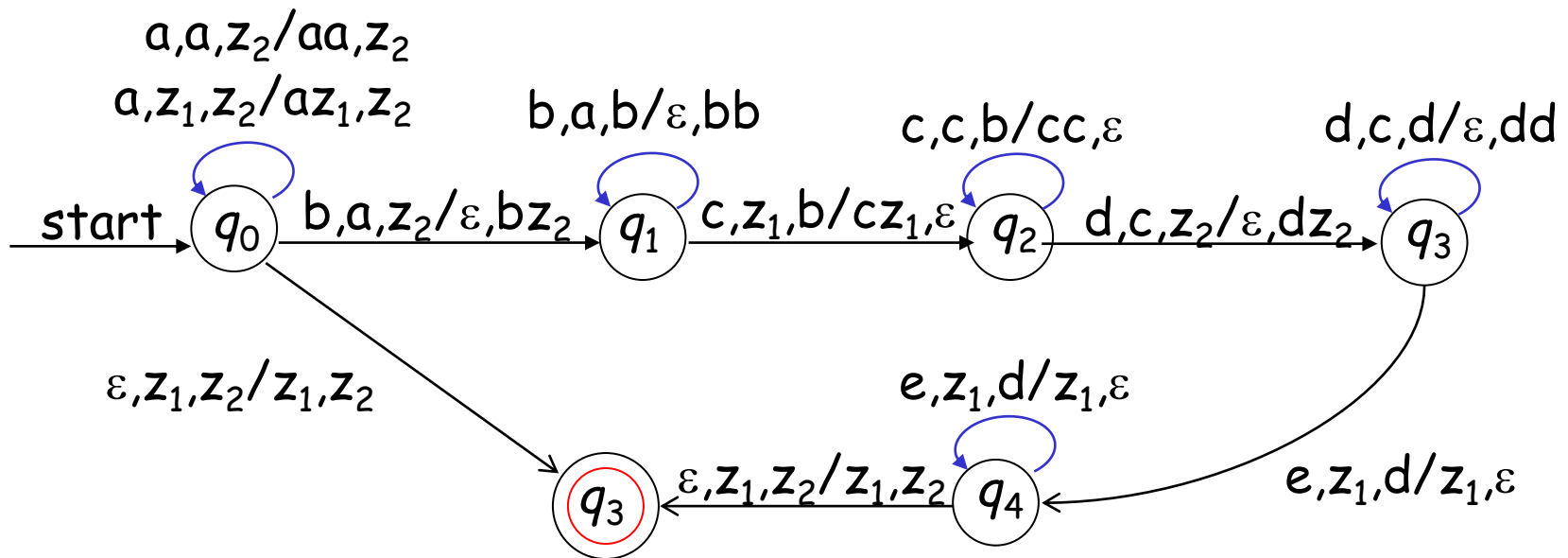
$$L = \{ a^n b^n c^n \mid n \geq 0 \}$$



Example 10

Construct a two stack machine for

$$L = \{ a^n b^n c^n d^n e^n \mid n \geq 0 \}$$



Good good study
day day up!

Never confuse education with intelligence.

Intelligence isn't ability to remember and repeat like they teach you in school.

Intelligence is the ability to learn from experience, solve problems, and use our knowledge to adapt to new situation.

—— Richard Feynman