Morning

Deterministic Finite Automata(DFA)

1. Definition
2. Notation
3. Construction
4. Language accepted by a DFA
5. Regular language

Formal Definition of DFA

Deterministic finite automaton is a five-tuple ,

such as $M = (Q, \Sigma, \delta, q_0, F)$

Where Q is a finite set of *states* ,

Σ is a finite set of *input symbols* ,

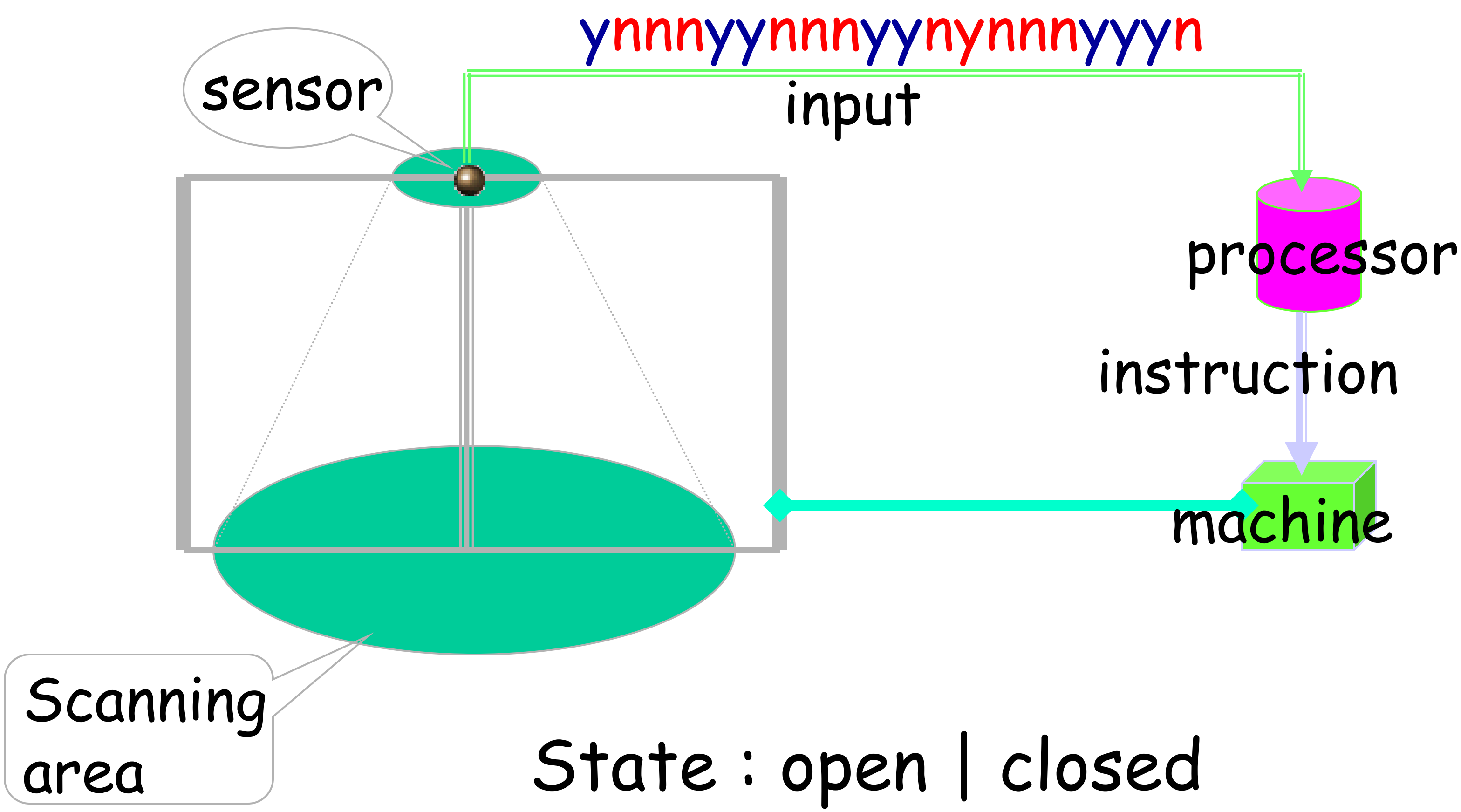
q_0 is a *start state* ,

F is *a set of final state* ,

δ is *transition function* , which is a mapping

from $Q \times \Sigma$ to Q .

Example 2.1 DFA for Auto-gate



State : open | closed

Action : open | close

input : y | n

Example 2.1 DFA for Auto-gate

Input symbols : { 0 , 1 } State : { Closed , Open }

State transition :

$$
\begin{aligned}
(\text{Closed}, 0) &\Rightarrow \text{Closed} \\
(\text{Closed}, 1) &\Rightarrow \text{Open} \\
(\text{Open}, 1) &\Rightarrow \text{Open} \\
(\text{Open}, 0) &\Rightarrow \text{Closed}
\end{aligned}
$$

Start state : Closed

Final state : Closed

Example 2.1 DFA for Auto-gate

Input symbols : $\{0, 1\}$ State : $\{q, p\}$

State transition function:

$$\delta(q, 0) = q$$
$$\delta(q, 1) = p$$
$$\delta(p, 1) = p$$
$$\delta(p, 0) = q$$

Start state : q

Final state : q

Automaton

example 2.1 DFA for Auto-gate

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{p, q\}, \quad \Sigma = \{0, 1\}$

$q_0 = q, \quad F = \{q\}$

δ:

$\delta(q, 0) = q$

$\delta(q, 1) = p$

$\delta(p, 0) = q$

$\delta(p, 1) = p$

example 2.1 DFA for Auto-gate

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q = {closed, open}, Σ = {n,y}

q_0 = closed, F = { closed }

δ :

δ(*closed* , n) = closed

δ(closed , y) = open

δ(*open* , n) = closed

δ(open , y) = open

example 2.1 DFA for Auto-gate

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1\}$, $\Sigma = \{0,1\}$

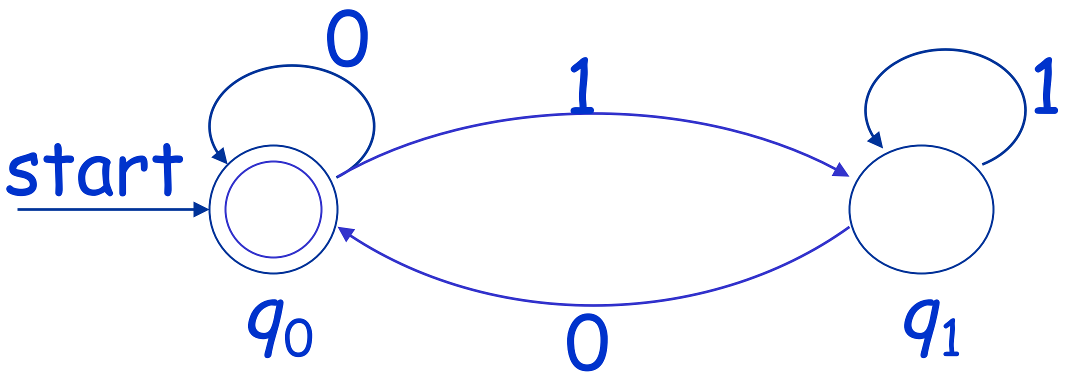
$q_0 = q_0$, $F = \{q_0\}$

δ :

$\delta(q_0, 0) = q_0$, $\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_0$, $\delta(q_1, 1) = q_1$

Diagram Notation of DFA



$$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$$

δ :

$$\delta(q_0, 0) = q_0, \quad \delta(q_0, 1) = q_1$$
$$\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_1$$

Table Notation of DFA

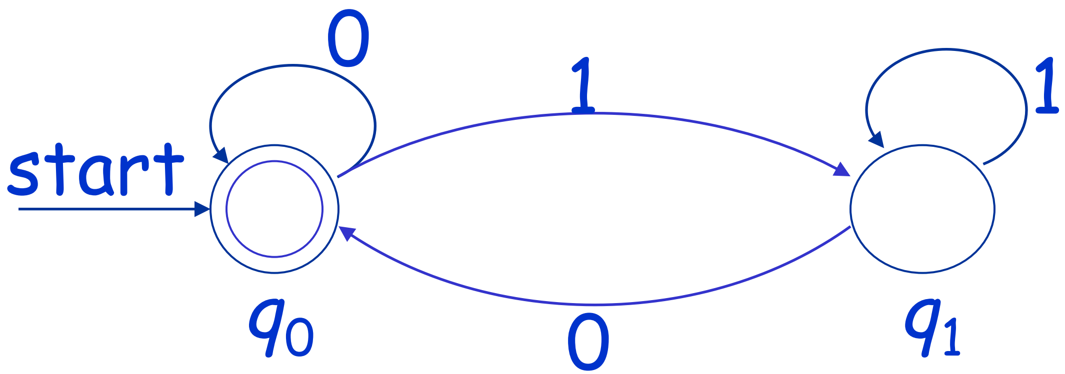
	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_0	q_1

$$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_0\})$$

δ:

$$\delta(q_0, 0) = q_0, \quad \delta(q_0, 1) = q_1$$
$$\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_1$$

Partition Strings

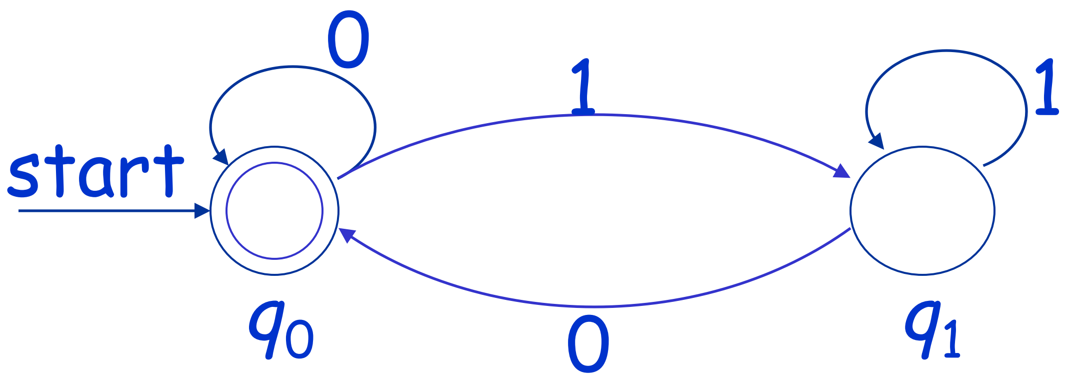


M partitions all strings into two groups:

$$L_1 = \{ w \in \{0,1\}^* \mid w \text{ end with } 0 \} \cup \{ \varepsilon \}$$

$$L_2 = \{ w \in \{0,1\}^* \mid w \text{ end with } 1 \}$$

DFA as a recognizer of language



M “recognize” the following language :

$$L = \{ w \in \{0,1\}^* \mid w \text{ end with } 0 \} \cup \{ \varepsilon \}$$

With the language L, and a string $w \in \{0,1\}^*$

M tell us whether w belongs to L, or not

Decision problem

Given a language L, and a string w

Is w belong to L ?

Example 2.2

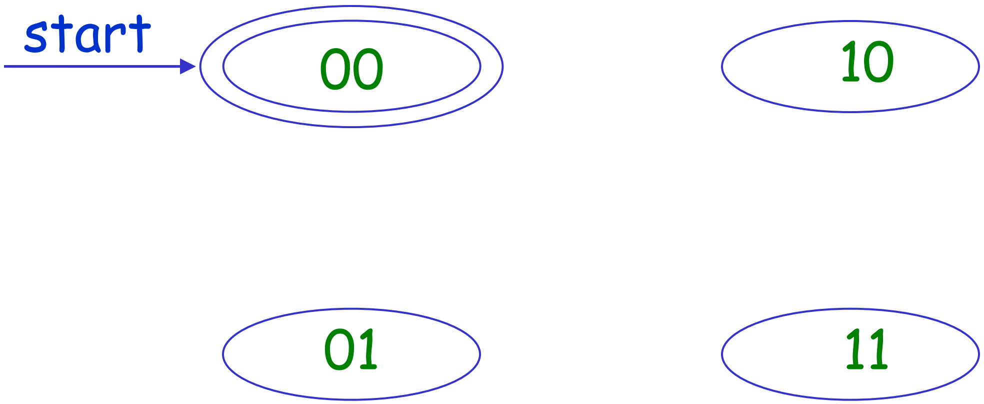
$L = \{w \in \{0,1\}^* \mid w$ has both an even number of 0's and an even number of 1's $\}$

- Partition strings into four groups
  - 00 : even 0 and even 1
  - 01 : even 0 and odd 1
  - 10 : odd 0 and even 1
  - 11 : odd 0 and odd 1

Example 2.2

$L = \{w \mid w$ has both an even number of 0's and an even number of 1's $\}$

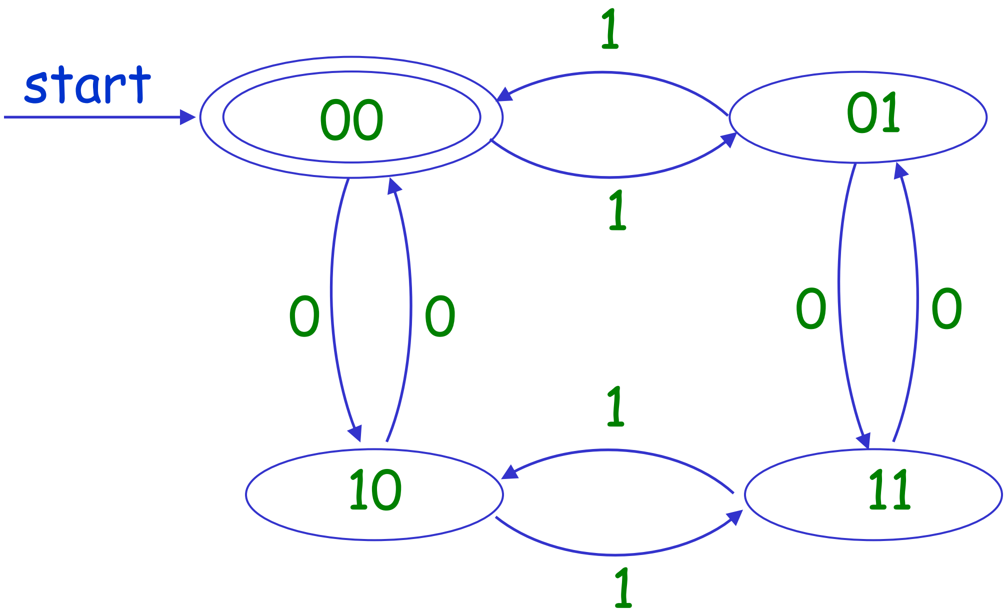
- Set states corresponding to partitions



Example 2.2

$L = \{w \mid w$ has both an even number of 0's and an even number of 1's $\}$

- Put transition arcs between states



Example 2.3

L = {w | w consists of 0's and 1's , and contains
sub-string 01}

or {x01y | x and y are consists of any number
of 0's and 1's }

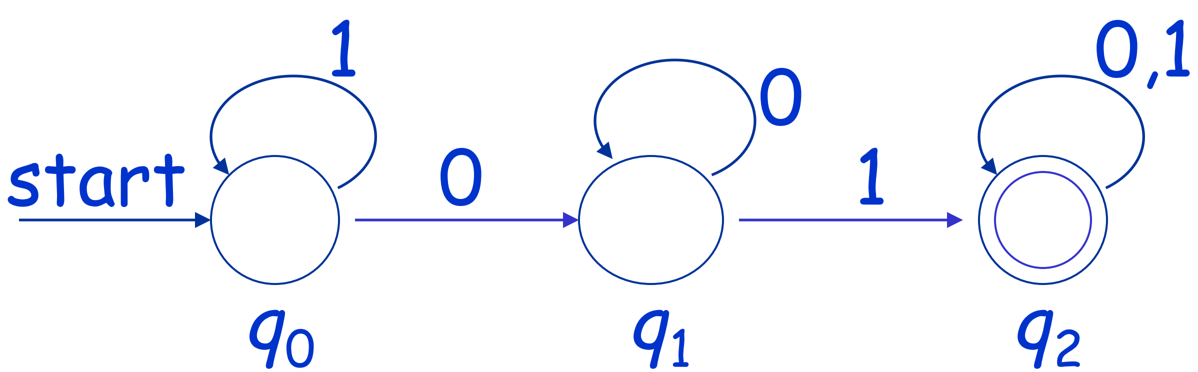
Problem :

How to decide whether a given string w belongs to L ?

Construction of DFA

How to start our work ?

- What is the meaning " w belongs to L "
- Partition strings by properties of L
- Set states which correspond to the partitions
- Put transition arcs between states



Extending transition function to string

BASIS

$$\hat{\delta}(q, \varepsilon) = q.$$

INDUCTION

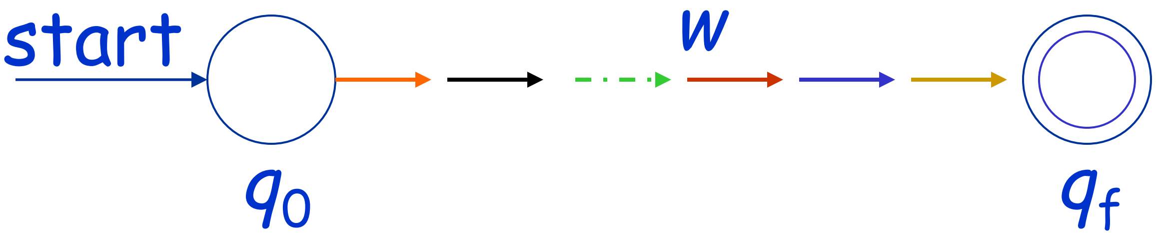
Suppose *w* is a string of the form *xa* , that is , *a* is the last symbol of *w*, and *x* is the string consisting of all but the last symbol. Then

$$\hat{\delta}(q, w) = \delta(\hat{\delta}(q, x), a)$$

The language of a DFA

Definition The *language of a DFA* *A* is denoted *L*(*A*), and defined by

$$L(A) = \{w \mid \hat{\delta}(q_0, w) \ is \ in \ F\}$$



Note : one language accepted by one DFA

Regular language

Definition

If L is $L(A)$ for some DFA A, then we say L is a *regular language*.

$$RL = \{ L \mid \text{There is a DFA to accept } L \}$$

Note : a kind of languages accepted by DFA's

Example 2.4

Construct DFA for following languages :

a) $\{ 0 \}^*$

b) $\{w \mid w \in \{0,1\}^*$ and begin with 0 $\}$

c) $\{w \mid w$ consists of any number of 0's followed by any number of 1's $\}$

d) $\{ \varepsilon \}$

e) ϕ

Good good study

day day up !