Morning

Equivalence of CFG & PDA

Equivalence of CFG and PDA

$$\begin{tikzcd} \text{PDA} \arrow[r, "\text{recognize}"] \arrow[rr, bend right, leftrightarrow, "\text{equivalent}"'] & \text{CFL} & \text{CFG} \arrow[l, "\text{generate}"'] \end{tikzcd}$$

- With a given CFL L , there is a CFG to generate L,

  and a PDA to recognize L .

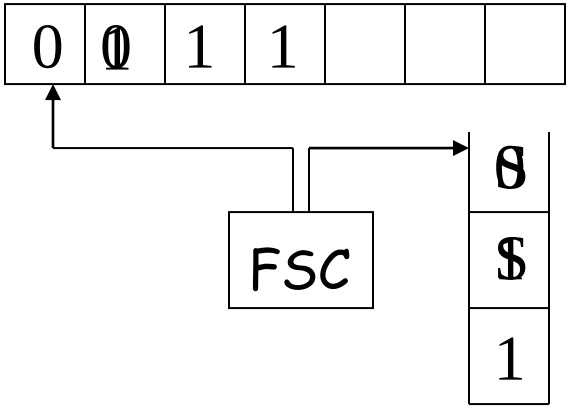
- So they are equivalent .

CFG $\Rightarrow$ PDA

Let CFG $G = (V, T, S, P)$

$$\Rightarrow B = (\{q\}, T, V \cup T, \delta, q, S, \{\ \})$$

- $\delta(q, \varepsilon, A) = \{(q, \alpha) \mid A \rightarrow \alpha \in P\}$
- $\delta(q, a, a) = (q, \varepsilon)$



Example 1 CFG $\Rightarrow$ PDA

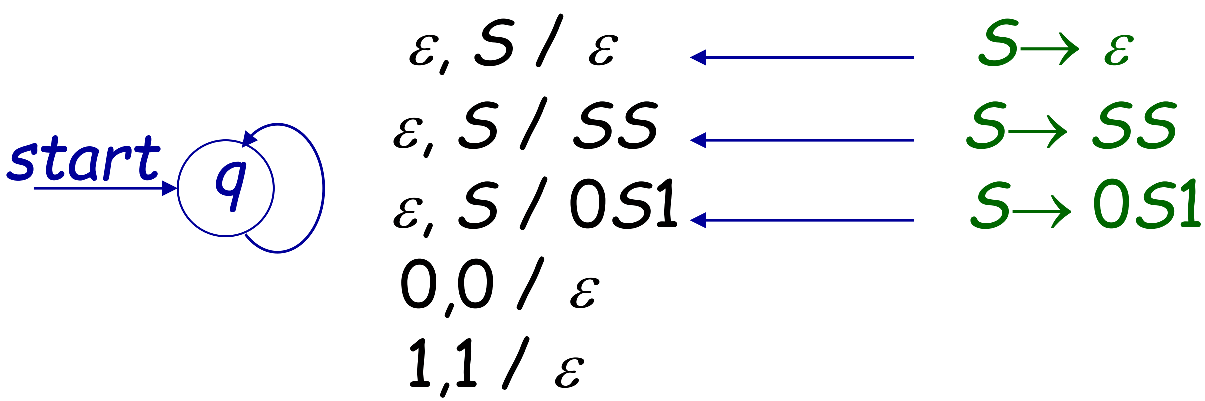
$$G=(\{S\},\{0,1\},\{S\to 0S1,\ S\to SS,\ S\to\varepsilon\},\ S)$$

$$\Rightarrow P=(\{q\},\{0,1\},\{0,1,S\},\ \delta,\ q,\ S,\ \{\ \})$$

δ :

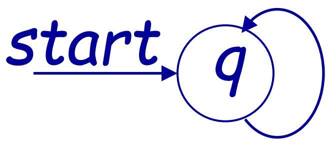
$$\delta(q,\varepsilon,S)=\{(q,0S1),(q,SS),(q,\varepsilon)\}$$

$$\delta(q,0,0)=\{(q,0)\},\ \delta(q,1,1)=\{(q,1)\},$$



Example 1 CFG $\Rightarrow$ PDA

$G=(\{S\},\{0,1\},\{S\rightarrow 0S1,\ S\rightarrow SS,\ S\rightarrow \varepsilon\},\ S)$



$\varepsilon, S / \varepsilon \leftarrow S\rightarrow \varepsilon$

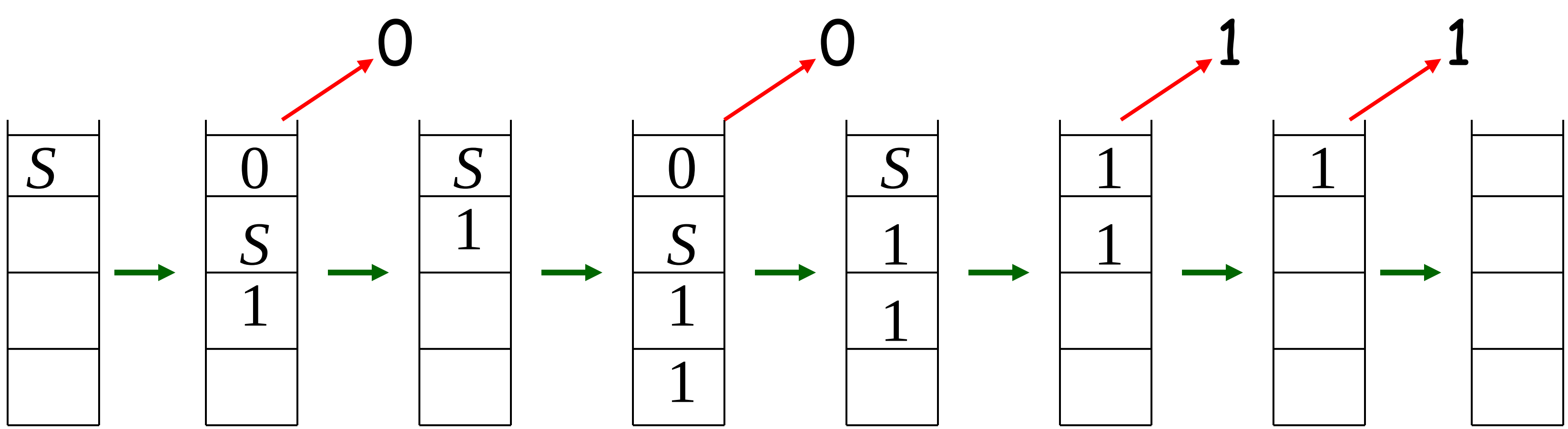
$\varepsilon, S / SS \leftarrow S\rightarrow SS$

$\varepsilon, S / 0S1 \leftarrow S\rightarrow 0S1$

$0,0 / \varepsilon$

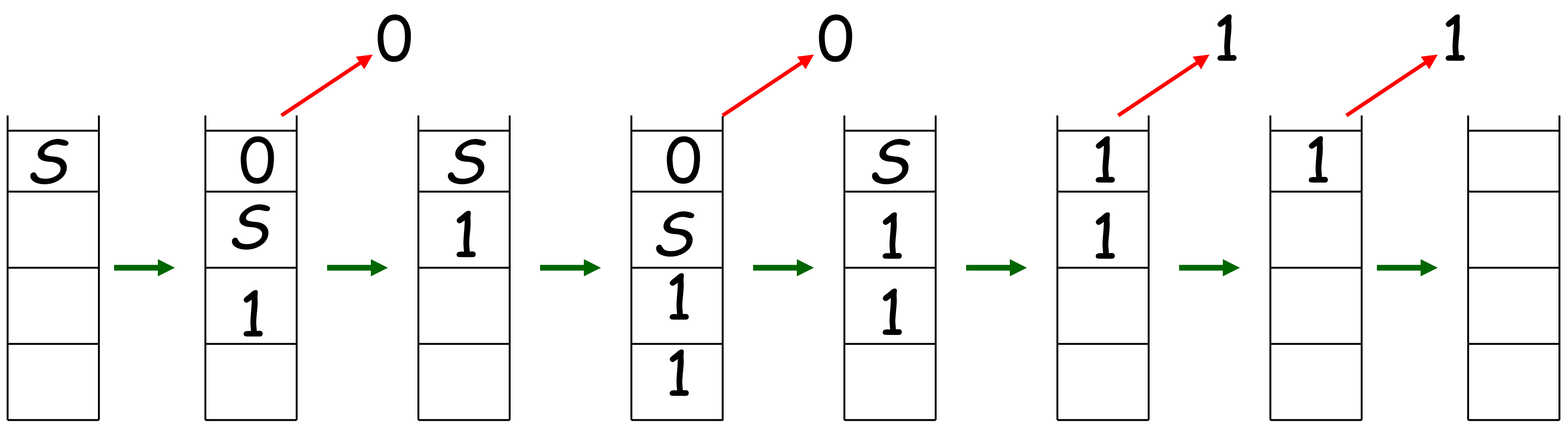
$1, 1 / \varepsilon$

Let w = 0 0 1 1



Example 1 CFG $\Rightarrow$ PDA

Let w = 0 0 1 1



$(q,0011,S) \vdash (q,0011,0S1) \vdash (q,011,S1) \vdash (q,011,0S11)$

$\vdash (q,11,S11) \vdash (q,11,11) \vdash (q,1,1) \vdash (q, \varepsilon, \varepsilon)$

$S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 0011$

$CFG \Rightarrow PDA$

Let GNF $G = (V, T, S, R)$

$R:\ A \to a\alpha \quad (A \in V,\ a \in T,\ \alpha \in V^*)$

$\Rightarrow$ PDA $P = (\{q\}, T, V \cup T, \delta, q, S, \{\ \})$

- $\delta(q, \varepsilon, A) = \{(q, a\alpha) \mid A \to a\alpha \in R\}$
- $\delta(q, a, a) = (q, \varepsilon)$

CFG $\Rightarrow$ PDA

For $w \in L(G)$, let $w = a_1 a_2 \ldots, a_n$

$$S \Rightarrow a_1 \alpha_1$$
$$\Rightarrow a_1 a_2 \alpha_2$$
$$\Rightarrow a_1 a_2 a_3 \alpha_3$$
$$\Rightarrow \ldots\ldots$$
$$\Rightarrow a_1 a_2 \ldots a_{n-1} \alpha_{n-1}$$
$$\Rightarrow a_1 a_2 \ldots\ a_{n-1}\ a_n$$

$\alpha_1, \ldots, \alpha_{n-1} \in V^*$

$\alpha_i \Rightarrow a_{i+1} \alpha_{i+1}$

$\alpha_{n-1} \rightarrow a_n$

CFG $\Rightarrow$ PDA

$$
\begin{aligned}
(q,w,S) &\vdash (q,\ a_1a_2\ldots a_n,\ a_1\alpha_1)\\
&\vdash (q,\ a_2\ldots a_n,\ \alpha_1)\\
&\vdash \ldots\ldots\\
&\vdash (q,\ a_{n-1}a_n,\ a_{n-1}\alpha_{n-1})\\
&\vdash (q,\ a_n,\ \alpha_{n-1})\\
&\vdash (q,\ a_n,\ a_n)\\
&\vdash (q,\ \varepsilon,\ \varepsilon)
\end{aligned}
$$

- $\delta(q,\varepsilon,S) = (q,\ a_1\alpha_1)$
- $\delta(q,\ a_1,\ a_1) = (q,\ \varepsilon)$
- $\delta(q,\ a_{n-1},\ a_{n-1}) = (q,\ \varepsilon)$
- $\delta(q,\ \varepsilon,\ \alpha_{n-1}) = (q,\ a_n)$
- $\delta(q,\ a_n,\ a_n) = (q,\ \varepsilon)$

CFG $\Rightarrow$ PDA

$(q, w, S) \vdash (q, a_1 a_2 \ldots a_n, a_1\alpha_1)$ $\quad S \Rightarrow a_1\alpha_1$

$\vdash (q, a_2 \ldots a_n, \alpha_1)$

$\vdash (q, a_2 \ldots a_n, a_2\alpha_2)$ $\quad \Rightarrow a_1 a_2 \alpha_2$

$\vdash \ldots\ldots$ $\quad \Rightarrow \ldots\ldots$

$\vdash (q, a_{n-1} a_n, a_{n-1}\alpha_{n-1})$ $\quad \Rightarrow a_1 a_2 \ldots a_{n-1}\alpha_{n-1}$

$\vdash (q, a_n, \alpha_{n-1})$

$\vdash (q, a_n, a_n)$ $\quad \Rightarrow a_1 a_2 \ldots a_{n-1} a_n$

$\vdash (q, \varepsilon, \varepsilon)$

PDA $\Rightarrow$ CFG

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F) \Rightarrow G = (V, \Sigma, S, R)$$

- V :

  start symbol S

  all symbols like $[qXp]$, $\forall q, p \in Q, X \in \Gamma$

- R :

  $S \rightarrow [q_0 z_0 p]$ for all $p \in Q$

  $[q X r_k] \rightarrow a[r Y_1 r_1][r_1 Y_2 r_2] \ldots [r_{k-1} Y_k r_k]$

  for $(r, Y_1 Y_2 \ldots Y_k) \in \delta(q, a, X)$

PDA $\Rightarrow$ CFG

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F) \Rightarrow G = (V, \Sigma, S, R)$$

- V :

start symbol S

all symbols like $[qXp]$

1. pop X from stack
2.transition from q to p

$$\delta(q, ?, X) = (p, \varepsilon)$$

$S \rightarrow [q_0 z_0 p]$ for all $p \in Q$

$$\delta(q_0, \varepsilon, z_0) = (p, \varepsilon) \Rightarrow [q_0 z_0 p] \rightarrow \varepsilon \Rightarrow \varepsilon \in L(P)$$

$$(q, w, z_0) \vdash^{*} (q, \varepsilon, \varepsilon) \Rightarrow (S \Rightarrow [q_0 z_0 p] \stackrel{*}{\Rightarrow} w \ )$$

PDA $\Rightarrow$ CFG

$$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F) \Rightarrow G = (V, \Sigma, S, R)$$

- R :

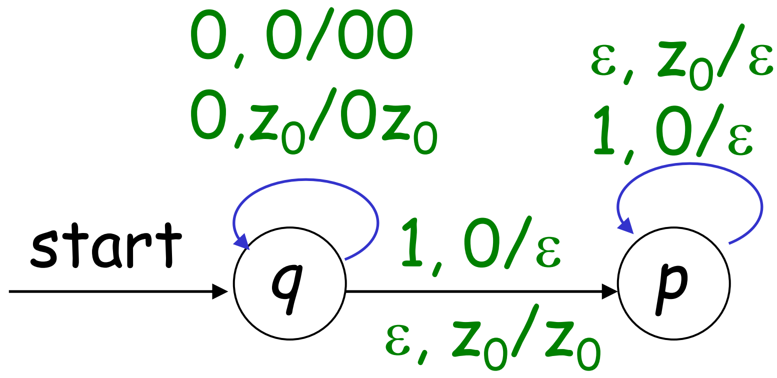
$$r, r_i \in Q,\ y_i \in \Gamma$$

$$[q X r_k] \rightarrow a[r Y_1 r_1][r_1 Y_2 r_2] \ldots [r_{k-1} Y_k r_k]$$

$$\text{for } (r, Y_1 Y_2 \ldots Y_k) \in \delta(q, a, X)$$

$$\delta(q, a, X) = (r, Y_1 Y_2 \ldots Y_k)$$

Example 2



$L = \{\, 0^n 1^n \mid n \geq 0 \,\}$

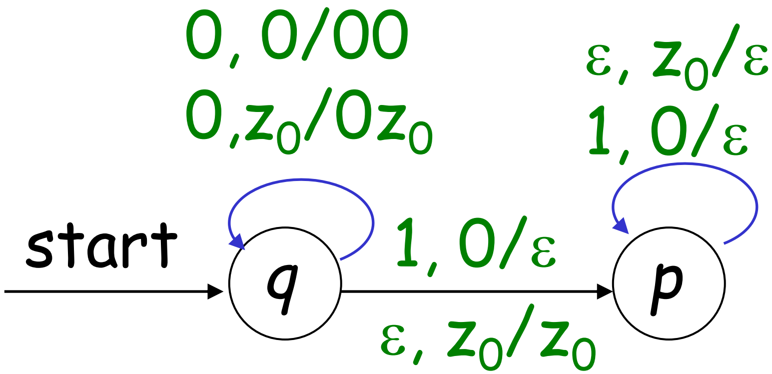
$P = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F) \Rightarrow G = (V, \Sigma, S, R)$

$V = \{$ S, $[qz_0q]$, $[qz_0p]$, [q0q], [q0p], [q1q], [q1p],

$[pz_0q]$, $[pz_0p]$, [p0q], [p0p], [p1q], [p1p] $\}$

Example 2

$L=\{ 0^n1^n \mid n \geq 0 \}$



$$\delta(q, 0, z_0) = (q, 0z_0) \Rightarrow$$

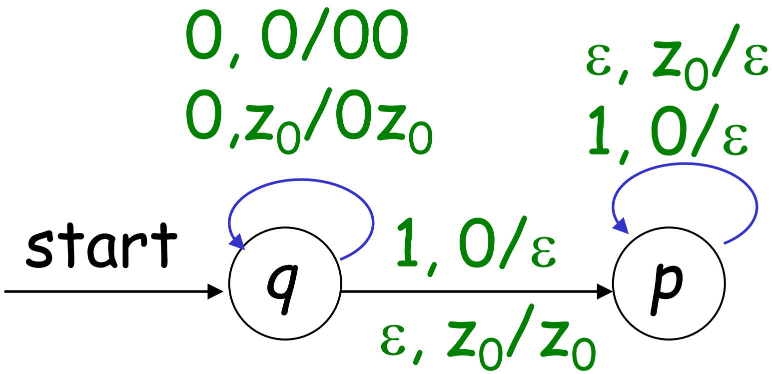
$$[qz_0r_2] \rightarrow 0[q0r_1][r_1z_0r_2],\ \forall r_1, r_2 \in Q \Rightarrow$$

$$[qz_0q] \rightarrow 0[q0q][qz_0q] \mid 0[q0p][pz_0q]$$

$$[qz_0p] \rightarrow 0[q0q][qz_0p] \mid 0[q0p][pz_0p]$$

Example 2

$L = \{ 0^n 1^n \mid n \geq 0 \}$



$$\delta(q, 0, 0) = (q, 00) \Rightarrow$$

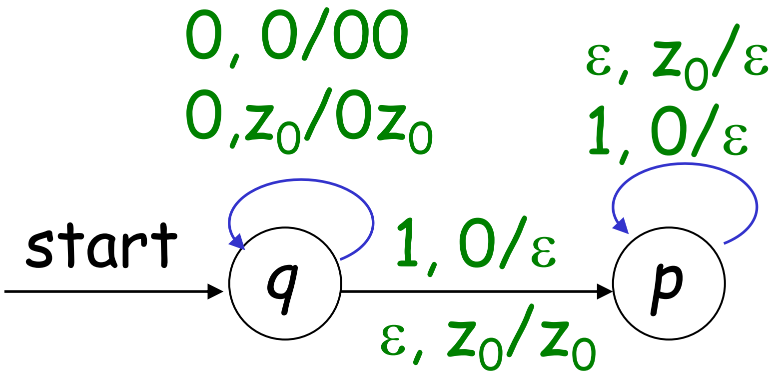
$$[q0r_2] \rightarrow 0[q0r_1][r_10r_2], \ \forall r_1, r_2 \in Q \Rightarrow$$

$$[q0q] \rightarrow 0[q0q][q0q] \mid 0[q0p][p0q]$$

$$[q0p] \rightarrow 0[q0q][q0p] \mid 0[q0p][p0p]$$

Example 2

$L=\{\ 0^n1^n \mid n \geq 0\ \}$



$$\delta(q, \varepsilon, z_0) = (p, z_0) \ \Rightarrow$$

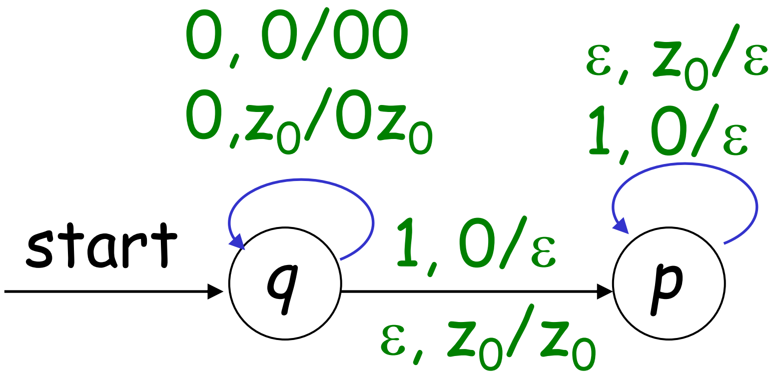
$$[qz_0r_1] \rightarrow [pz_0r_1],\ \ \forall r_1 \in Q \ \Rightarrow$$

$$[qz_0q] \rightarrow [pz_0q]$$

$$[qz_0p] \rightarrow [pz_0p]$$

Example 2

$L = \{ 0^n 1^n \mid n \geq 0 \}$

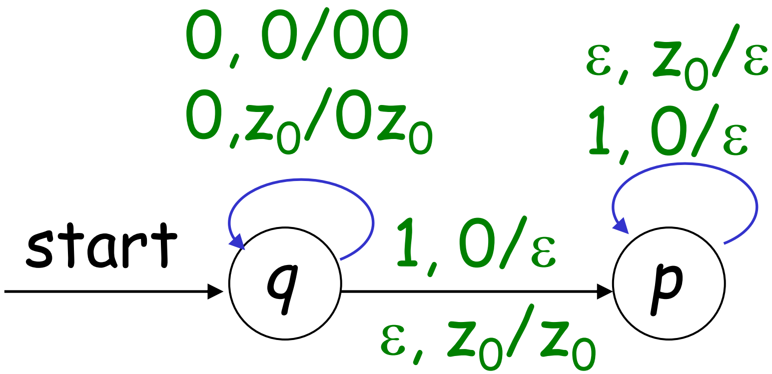


$\delta(q, 1, 0) = (p, \varepsilon) \Rightarrow$

$[q0p] \rightarrow 1$

Example 2

$L = \{ 0^n 1^n \mid n \geq 0 \}$



$\delta(p, 1, 0) = (p, \varepsilon) \Rightarrow$

$[p0p] \rightarrow 1$

$\delta(p, \varepsilon, z_0) = (p, \varepsilon) \Rightarrow$

$[pz_0p] \rightarrow \varepsilon$

Example 2

$$R = \{\ S \to [qz_0q] \mid [qz_0p],$$

$$[qz_0q] \to 0[q0q][qz_0q] \mid 0[q0p][pz_0q]$$

$$[qz_0p] \to 0[q0q][qz_0p] \mid 0[q0p][pz_0p]$$

$$[q0q] \to 0[q0q][q0q] \mid 0[q0p][p0q]$$

$$[q0p] \to 0[q0q][q0p] \mid 0[q0p][p0p]$$

$$[qz_0q] \to [pz_0q],\quad [qz_0p] \to [pz_0p]$$

$$[q0p] \to 1,\ [p0p] \to 1,\ [pz_0p] \to \varepsilon \qquad \}$$

Example 2

Not generating

$R = \{ S \to [qz_0q] \mid [qz_0p],$

$[qz_0q] \to 0[q0q][qz_0q] \mid 0[q0p][pz_0q]$

$[qz_0p] \to 0[q0q][qz_0p] \mid 0[q0p][pz_0p]$

$[q0q] \to 0[q0q][q0q] \mid 0[q0p][p0q]$

$[q0p] \to 0[q0q][q0p] \mid 0[q0p][p0p]$

Useless

$[qz_0q] \to [pz_0q], \quad [qz_0p] \to [pz_0p]$

$[q0p] \to 1, [p0p] \to 1, [pz_0p] \to \varepsilon \quad \}$

Example 2

$R = \{\ S \to [qz_0p]$ $\qquad L=\{\ 0^n1^n \mid n\geq 0\ \}$

$[qz_0p] \to 0[q0p][pz_0p]$

$[q0p] \to 0[q0p][p0p]$

$[qz_0p] \to [pz_0p]$

$[q0p] \to 1,\ [p0p] \to 1,\ [pz_0p] \to \varepsilon\ \}$

for $w = 0011 \in L$

$S \Rightarrow [qz_0p] \Rightarrow [qz_0p] \Rightarrow 0[q0p][pz_0p] \Rightarrow 0[q0p]$

$\Rightarrow 00[q0p][p0p] \Rightarrow 001[p0p] \Rightarrow 0011$

Example 2

$R = \{ S \rightarrow [qz_0p]$ $\qquad L=\{ 0^n1^n \mid n \geq 0 \}$

$[qz_0p] \rightarrow 0[q0p][pz_0p]$

$[q0p] \rightarrow 0[q0p][p0p]$

$[qz_0p] \rightarrow [pz_0p]$

$[q0p] \rightarrow 1,\ [p0p] \rightarrow 1,\ [pz_0p] \rightarrow \varepsilon \ \}$

Let $A=[qz_0p]$, $B=[q0p]$, $C=[p0p]$, $D=[pz_0p]$

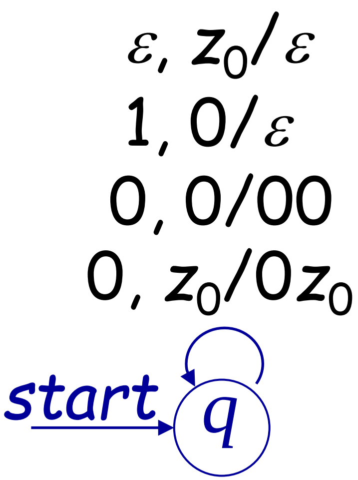
$R = \{ S \rightarrow A,\ A \rightarrow 0BD \mid D,\ B \rightarrow 1 \mid 0BC,\ \ C \rightarrow 1,\ D \rightarrow \varepsilon \}$

$R = \{ S \rightarrow 0B,\ B \rightarrow 1 \mid 0BC,\ \ C \rightarrow 1 \}$

Example 3 PDA $\Rightarrow$ CFG

L={ w | w contains equal number of 0's and 1's, and no prefix has more 1's than 0's }

PDA



CFG

$$[qz_0q] \to \varepsilon$$

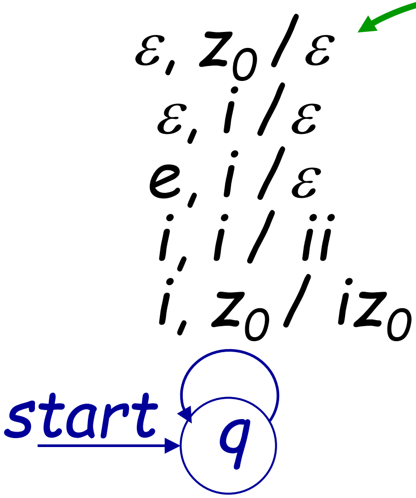
$$[q0q] \to 1$$

$$[q0q] \to 0[q0q][q0q]$$

$$[qz_0q] \to 0[q0q][qz_0q]$$

Example 4 " if-else"

PDA



CFG

$G = (V, \Sigma, S, P)$

$V = \{S, [qz_0q], [qiq]\}$

P :

$S \rightarrow [qz_0q]$

$\delta(q, \varepsilon, z_0) = (q, \varepsilon) \longrightarrow [qz_0q] \rightarrow \varepsilon$

$\delta(q, i, z_0) = (q, iz_0) \longrightarrow [qz_0q] \rightarrow i\,[qiq][qz_0q]$

$\delta(q, \varepsilon, i) = (q, \varepsilon) \longrightarrow [qiq] \rightarrow \varepsilon$

$\delta(q, i, i) = (q, ii) \longrightarrow [qiq] \rightarrow i\,[qiq][qiq]$

$\delta(q, e, i) = (q, \varepsilon) \longrightarrow [qiq] \rightarrow e$

Good good study

day day up !