

Abduction Prover for Induction Problems at WAIT2024

Yutaka Nagashima x/twitter: YutakangE





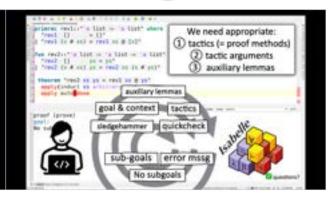














tactic as conjecturing

Back to DEMO





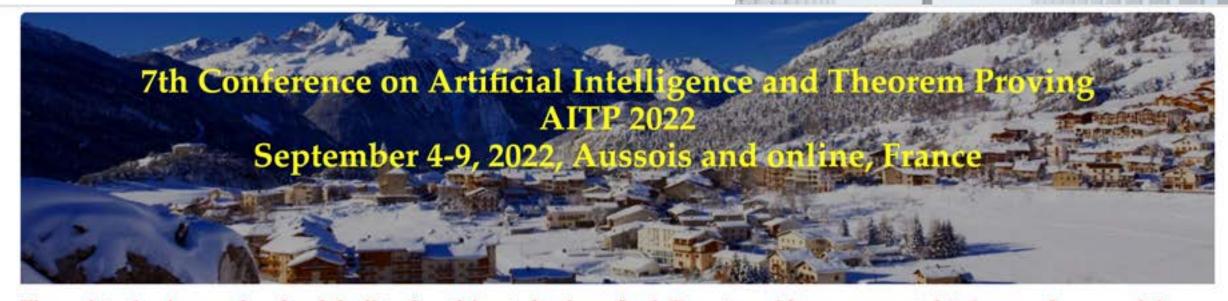
Who am 1?







Who am 1?



The registration is now closed and the list of participants has been fixed. Do not provide any payment data in any other way - it is very likely a scam!

Background

Large-scale semantic processing and strong computer assistance of mathematics and science is our inevitable future. New combinations of AI and reasoning methods and tools deployed over large mathematical and scientific corpora will be instrumental to this task. The AITP conference is the forum for discussing how to get there as soon as possible, and the force driving the progress towards that.

This year AITP will be co-located with a meeting of WG5 of Cost Action European Research Network on Formal Proofs.

Topics

- Al and big-data methods in theorem proving and mathematics
- · Collaboration between automated and interactive theorem proving, in particular their AI/ML aspects
- · Common-sense reasoning and reasoning in science
- · Alignment and joint processing of formal, semi-formal, and informal libraries, Formal Abstracts

3 Secretes of AITP

more of a working hypothesis than a scientific statement



3 things that are not *that* important.



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Engineering • September 2018 • Pages 362-372 • https://doi.org/10.1145/3238147.3238210

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(Machine)Learning.

compared to representation & search.

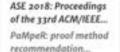








Tactic Selection.



■ ABSTRACT

Deciding which sub-tool to use for a given proof state requires expertise specific to each interactive theorem prover (ITP). To mitigate this problem, we present epaMpeR/

compared to tactic argument selection.

hod <U>r</U>ecommendation system for Isabelle/HOL. O e> recommends proof methods to discharge the proof go

e> recommends proof methods to discharge the proof god dons as to why it suggests these methods. PaMpeR/p

Useful only for special tactics.

Just use Sledgehammer.

References

Particularly for proof by induction itten proof corpora, thus

transferring experienced users' expertise to new users. Our evaluation shows that

pre> correctly predicts experienced users' proof methods invocation

it comes to special purpose proof methods.

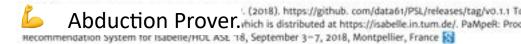


Conjecturing (Auxiliary Lemmas)

compared to filtering of good conjectures.



Human creativity?





Abduction Prover.

Automatic Conjecturing of Auxiliary lemmas

Automatic Filtering of Conjectures

Seamless Integration into Isabelle/HOL

Sledgehammer & Quickcheck & Nitpick Integration

PSL & Smart Induction Integration

Native Isabelle Proof Script Generation

Human-Friendly Proof Script Generation

(Mostly) Parallel Proof Search

Best-First Expansion rather than Depth-First Search

Simultaneous Abduction of Multiple Conjectures

Multi-Step Abduction of Auxiliary Lemmas

Template-Based Conjecturing



Used OCaml/Haskell for more than 10 hours?

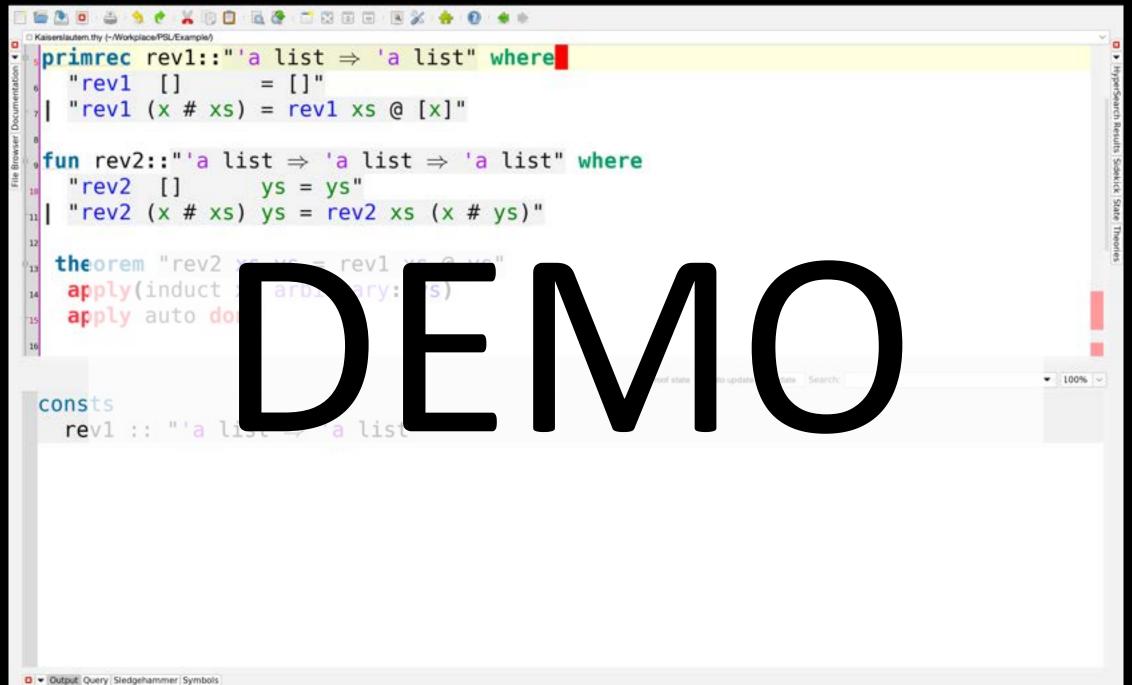
Used an Isabelle/Coq/LEAN for more than 10 hours?

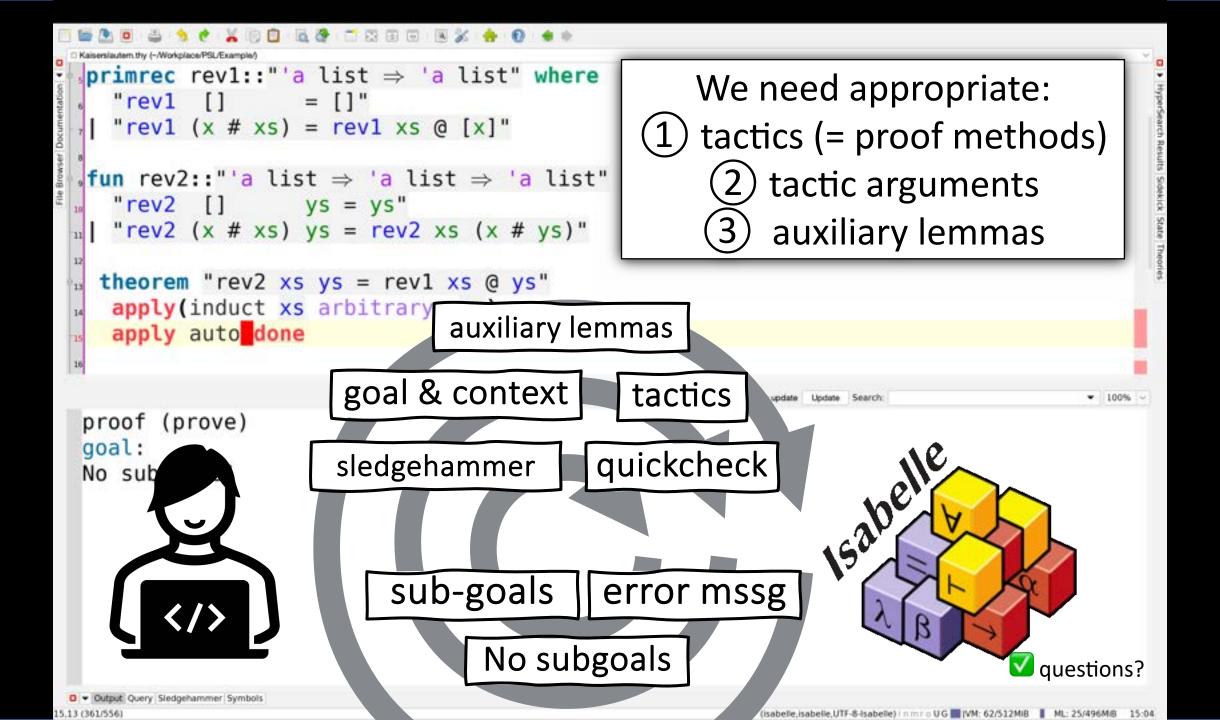
Invoked Sledgehammer at least once?

90 % ?

70 % ?

7/0 % ?





- 1) tactics (= proof methods)
 - 2 tactic arguments
 - 3 auxiliary lemmas

Conjecturing is not that important if we can ...



Good lemma?

- 1. Not obviously false.
- 2. Useful to prove the goal.
 - 3. Easy to prove.



- 1) tactics (= proof methods)
 - 2 tactic arguments
 - 3 auxiliary lemmas





Fail early.



1. Not obviously false.

2. Useful to prove the goal.

3. Easy to prove.



- 1 tactics (= proof methods)
 - 2 tactic arguments
 - 3 auxiliary lemmas

Conjecturing is not that important if we can ...



Fail early.

Q



- 1. Not obviously false.
- 2. Useful to prove the goal.
 - 3. Easy to prove.



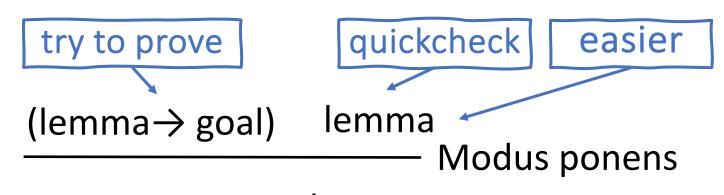


- 1 tactics (= proof methods)
 - 2 tactic arguments
 - 3 auxiliary lemmas

Conjecturing is not that important if we can ...



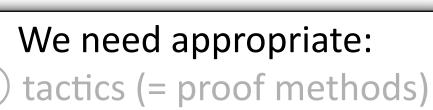
⊌ Fail early.





- 1. Not obviously false.
- 2. Useful to prove the goal.3. Easy to prove.





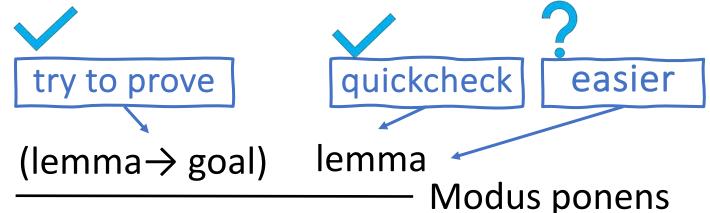
2 tactic arguments

3 auxiliary lemmas

Conjecturing is not that important if we can ...



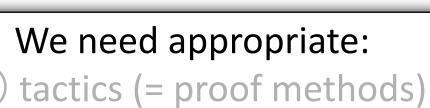
Second Fail early.



difficult
Good
lemma?

1. Not obviously false.
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3. Easy to prove.





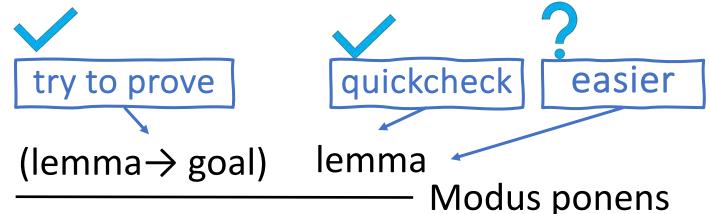
2 tactic arguments

3 auxiliary lemmas

Conjecturing is not that important if we can ...



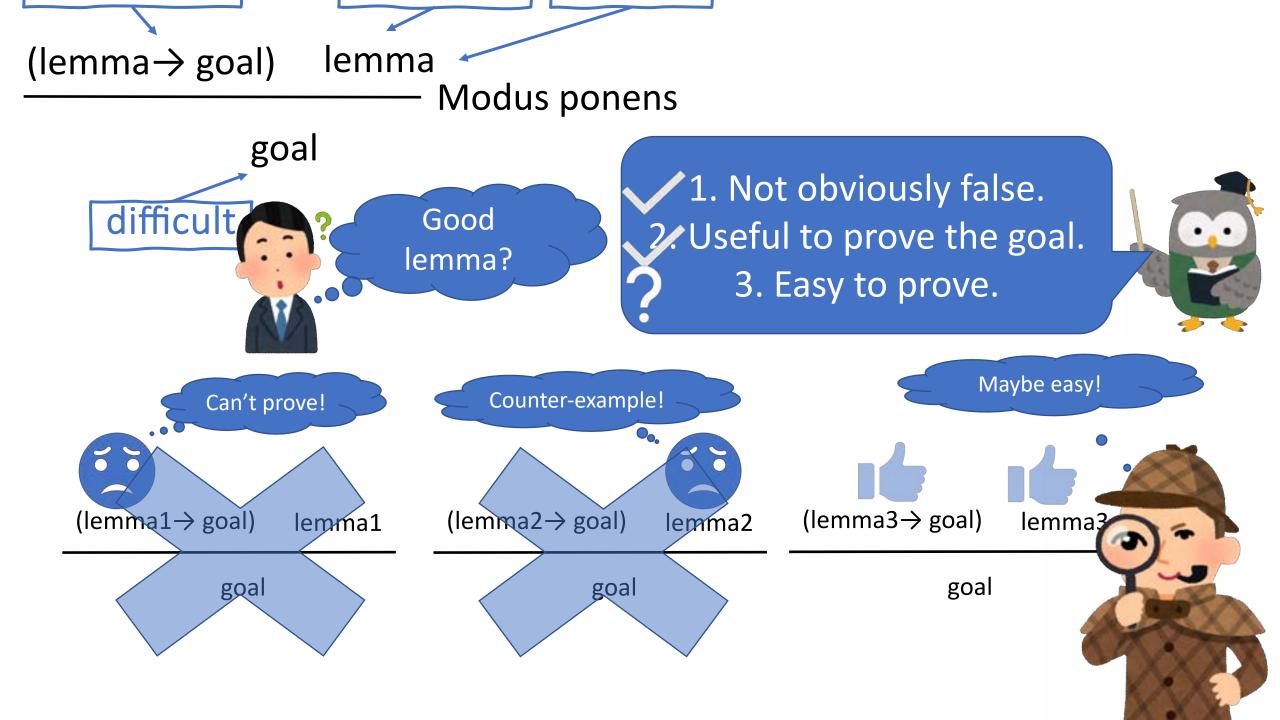
🤞 Fail early.



difficult Good lemma?

Not obviously false.
 Useful to prove the goal.
 3. Easy to prove.

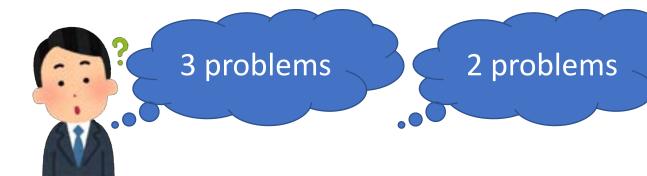




1 tactics (= proof methods)

2 tactic arguments

(3) auxiliary lemmas



1 problem





3 problems

2 problems

1 problem

Human-friendly proof script.

lemma abduction

(lemma → goal) lemma

goal

tactic application

(new-goal → goal) new-goal

goal

proof by induction

(base-case & step-case \rightarrow goal)

base-case step-case

goal

Back to

Reasoning

```
the Edit Switch Markets Fighting your Utilizes Magnic Ellipsis bely-
                  X II O A O CHIEF RX . O . .
 # TIP progr (6.8by ) Workplace/Prod/Prod/
                                                                                                                                                                                                                                                                   Proof some " Auto update: Luphan Search: a
 mi "42 (5 27) ± + 5 (42 22 2)"
                                                                                                                                                                                                                                    theorem
 Hareve property8 :
                                                                                                                                                                                                                                      priginal goal 7579816:
  '((length (rev ix y 2))) = (t2 (length y) (length 2)))'
                                                                                                                                                                                                                                        length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                                        t2 (length ?var 0.0) (length ?var 1.0)
  splemma abduced lemma tactic 14045356; "war 6 = 12 war 8 2"
  am apply ( induct "war 0" )
     opply (simp all ) done
 wilemma abduced temma tactic 14045358: "(As. t2 var 0 s = t2 s var 0) - 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
 as septy ( induct "war 1" "war 0" role : TEP prop 06.t2.induct )
     apply (nimp all ) does
 sulemma abduced lemma commutativity 7642214: "t2 var 8 var 1 - t2 var 1 var 8"
    apply ( induct "war 6" arhitrary : war 1 )
     opply (simp all )
     epoly ( simp will : abduced Lemma tactic 14045356 )
     using abduced lemma tactic 14045358 apply force done
 sileema abduced lemma tactic 13498028; "length var 0 = length (x var 0 nil2)"
     apply { induct "war 0" }
     apply (simp all ) dune
 nu[lemma abduced lemma remove assumption 47001480: "5 (length (x war 0 var 1)) = length (x war 0 (cons2 var 2 var 1))"
     spoty ( induct "yer 0" arbitrary a yer 1 1
     apply (simp all ) done
  selemma abduced terms factic 13498034:
  ma"(As. length (x vor 8 a) = length (x a vor 8)) -- 5 (length (x vor I vor 8)) = length (s vor 1 (coss vor 2 vor 8))"
     apply ( simp mid : abduced Lemma remove assumption 47001480 ) done
 aglesma abduced lemma composite commutativity 7642270: "length (x var 0 var 1) = length (x var 1 var 0)"
    apply ( induct "var 0" arbitrary a var 1 )
     apply (simp att )
     uning abduced lemma tactic 13498828 apply blast
     apply ( simp into a abduced lemma factic 13498034 ) does
 swlemma abduced Lemna tactic 13293202: "length (rev var 0) = Length var 0 --- Length (x (rev var 0) (cons2 var 1 mil2)) = 5 (Length var 0)"
     apply ( metis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
              abduced Lemma composite commutativity 7642270 ) done
 relemma abduced lemma generalisation them extension 7642332: "length (rev var 0) = length var 0"
     apply ( induct "war 0" )
      apply (simp all )
      apply ( simp min : abduced Lemma tactic 13293202 ) done
 malemma abduced terms identity 7579838: "x var 0 mil2 = var 0"
     apply [ induct "war 0" )
     monty (simp all ) done
 externs abduced temma generalisation them extension 17745958: "Length (rev (a var 0 mil2)) - Length var 5"
  as apply ( simp and a abduced Lemma generalisation them extension 7642332 abduced Lemma identity 7579838 )
  malemma abduced temma tactic 33224946; "(An. length (rev (x a var 0)) = 12 (length var 0) (length a)) ---
  arlength (rev (x var 1 (cons2 var 2 var 0))) = 5 (t2 (length var 0) (length var 1))*
  ma suply ( metis TIP prop 86.length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation then extension 7642332 )
 uplessa abduced Lemma generalise by renaming 7642286: "Length (rev (x var 8 var 1)) = 12 (length var 1) (length var 0)"
 as epply ( induct "war 1" arbitrary ; war 0 )
     apply (simp all )
     epoly ( simp old : abduced lemma generalisation them extension 17745958 )
     epply ( simp mid : abduced lemma tactic 33224946 )
 milemma original goal 7579816: "length (rev (x var 0 var 1)) = t2 (length var 0) (length var 1)"
 me spoly ( simp min : abduced lemma commutativity 7642214 abduced lemma generalise by renaming 7642286 )
```

8 * Query Medgehammer Symbols

1 tactics (= proof methods)



2 tactic arguments

auxiliar

These 3 things are not *that* important.



compared to representation & search.

Tactic Selection.

compared to tactic argument selection.

Conjecturing (Auxiliary Lemmas).

compared to filtering of good conjectures.

Automatic Conjecturing of Auxiliary lemmas

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Template-Based Conjecturing

```
the Edit Seath Markets Fighting your Utilizes Magnes Ehapes tech-
                                                                                                                                                                                                                                                   Proof some "Auto update Update Search:"
  mil "42 (5 23) 2 = 5 (42 22 2)"
 sepreve property8 :
                                                                                                                                                                                                                       priginal goal 7579816:
   u "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                         Length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                         t2 (length ?var 0.0) (length ?var 1.0)
  splemma abduced lemma tactic 14045356; "war 6 = t2 war 0 2"
  se apply ( induct "war 6" )
  be coply (simp all ) done
  at lemma abduced terms tactic 14045358: "(As. t2 var 0 s = t2 s var 0) -- 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
 ap apply ( induct "war 1" "war 0" rale : TEP prop 86.12.induct )
 an apply (nimp all ) done
 solemns abduced lemma commutativity 7642214: "12 var 0 var 1 - 12 var 1 var 0"
  as apply ( induct "war 6" arhitrary : war 1 )
                                                                                                                       (lemma→ goal)
                                                                                                                                                                                    lemma
  ar opply (simp all )
     apply ( simp min : abduced Lemma factic 14045356 )
     using abduced lemma factic 14045358 apply force done
                                                                                                                                                                                                                     Modus ponens
  silemma abduced teems tactic 13498025; "Length yer 0 - Length (x var 0 nil2)"
     apply { indect 'war b' }
     apply [simp all ] done
   [lemma abdoced lemma remove assumption 47001480: "5 (length (x war 0 var 1)) = length (x war 0 (cons2 var 2 var 1))"
                                                                                                                                                                       goal
     apply ( induct feer of artifrary a yer 1 )
     apply (simp all ) done
  selemma abduced terms factic 13498034:
  ma (As. Length (a var 6 a) - Length (a a var 8)) - 5 (Length (a var I var 8)) - Length (a var 1 (coss2 var 2 var 8))
 as apply ( simp mid : abduced lemma remove assumption 47001480 ) done
 assessed terms composite commutativity 7642270: "length (s var 0 var 1) - length (s var 1 var 0)"
  as apply ( induct ?var 0" arbitrary a var 1 )
     uning abduced lemma tactic 13498628 apply blast
     apply ( simp into a abduced lemma tactic 13498034 ) down
  selemma abduced Lemma tactic 13293282: "Length (rev war 8) = Length war 8 --- Length (x (rev war 8) (coms2 war 1 mil2)) = 5 (Length war 8)"
     apply ( metis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
                                                                                                                        (sub-goal → goal) sub-goal
             abduced terms composite commutativity 7642270 ) done
  tilemma abduced temma generalisation them extension 7642332: "length (rew var 0) = length var 0"
     apply ( induct "war 0" )
                                                                                                                                                                                                                               tactic
     spoly (simp atl )
     spoly ( simp min : abduced lemma tactic 13293202 ) done
 malemma abduced terms identity 7579838: "x var 0 mil2 = var 0"
  TW spply [ induct "yer 0" )
                                                                                                                                                                       goal
  as soply (simp all ) done
 aglesma abduced lemma generalisation them extension 17745958: "Length (rev (a var 0 mil2)) - Length var 0"
 so apply ( simp will abduced lemma generalisation them extension 7642332 abduced lemma identity 7579638 )
  selemma abduced lemma tactic_33224946: "(As. length (rev (x a var 0)) = t2 (length var 0) (length a)) ---
 artength (rev (x var 1 (cons2 var 2 var 0))) = 5 (12 (length var 0) (length var 1))*
 am apply ( motis TIP prop 86.length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation them extension 7642332 )
 wiless abduced Lemma generalise by remaining 7642286: "Length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
 as apply ( induct "war 1" arbitrary ; war 8 )
     apply ( simp ood : abduced Lemma generalisation them extension 17745958 )
 no emply ( simp mit : abduced lemma tactic 33224946 )
 milemma original goal 7579816: "Length (rev (x var 0 var 1)) = 12 (Length var 0) (Length var 1)"
 mm apply ( simp min : abduced lemma commutativity 7642214 abduced lemma generalise by renaming 7642286 )
 8 * Query Nedgehammer Symbol
```

```
The Edit Switch Markets Fighting York Ullimes Magnic Physics Selp-
 # TIP_prop_06 My ( Workplace/Prod/Prod)
. ml "42 (5 27) 2 = 5 (42 22 2)"
                                                                                                                                                                                                                             theorem
supreve property8 :
  32 "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
  splemma abduced lemma tactic 14045356; "war 6 = 12 war 8 2"
  sm apply ( induct "war 0" )
  be opply (simp all ) done
 glemma abduced terms tactic 14045358: "(As. t2 var 0 s = t2 s var 0) == 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
 as soply ( induct "war 1" "war 0" rale : TEP prop 06.12.induct )
 was apply (nimp all ) done
  solemna abduced lemma commutativity 7642214: "12 var 0 var 1 = 12 var 1 var 0"
  as apply ( induct "war &" arbitrary t war 1 )
  ar opply (simp all )
  as spoly ( simp will : abduced lemma factic 14045356 )
  as axing abduced lemma factic 14045358 apply force done
  silemma abduced temma tactic 13498028; "tength yar 0 - length (x yar 0 nil2)"
  to apply ( induct "war 6" )
 an apply (simp all ) dune
  in[lemma abduced lemma remove assumption 47001400: "5 (length (x war 0 var 1)) = length (x war 0 (cons2 var 2 var 1))"
  me apply ( induct four of arbitrary ; ver 1 )
  ar apply (simp all ) done
  selemma abduced lemma tactic 13498034:
  sa"(Ao. length [x var 8 a) = length (x a var 8)) -- 5 (length (x var I var 8)) = length (x var I (cons var 2 var 8))*
  as apply ( simp mid ; abduced Lemma remove assumption 47001480 ) done
 aclemma abduced lemma composite commutativity 7642270: "length (a var 0 var 1) - length (a var 1 var 0)"
  to septy ( induct "yer 4" arbitrory a ver 1.1
 se apply (simp all )
  as uning abduced lemma tactic 13498828 apply blast
 or apply ( simp whi : abduced lemma tactic 13498034 ) down
 autenma abduced Lemna tactic 13293202: "length (rev var 0) = length var 0 --- length (x (rev var 0) (coms2 var 1 mil2)) = 5 (length var 0)"
  suply ( notis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
              abduced terms composite commutativity 7642270 ) done
  talesma abduced lemma generalisation them extension 7642332: "length (rew var 0) = length var 0"
  ya seply ( induct "war 0" )
      spoly (simp all )
      apply ( simp min : abduced Lemma tactic 13293202 ) done
  milemma abduced terms identity 7579838: "K var 0 mil2 - var 0"
  To spoly ( induct "yer 4" )
  as apply (simp all ) done
 splemma abduced terms generalisation then extension 17745958: "Length (rev (a var 0 mil2)) - Length var 0"
  apply ( simp min : abduced lemma generalisation them extension 7642332 abduced lemma identity 7579838 )
  as lemma abduced terms tactic 33224946: "(As. length (rev (x a var 0)) = t2 (length var 0) (length a)) ---
  aftength (rev (x var 1 (cons2 var 2 var 0))) = 5 (12 (length var 0) (length var 1))*
  am apply ( mutis TIP prop 86.length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation them extension 7642332 )
 THE DONE
 silemma abduced temma generalise by remaming 7642286: "Length (rev (a var 8 var 1)) = 12 (tength var 1) (tength var 0)"
 as epply ( induct "war 1" arbitrary ; war 0 )
 we apply (simp all )
 se apply ( simp wol : abduced lemma generalisation them extension 17745958 )
 no apply ( simp mid : abduced temma tactic 33224946 )
 SM DOOR
 milemma original goal 7579816: "(ength (rev (x war 0 var 1)) = t2 (length var 8) (length var 1)"
 se apply ( simp all : abduced lemma commutativity 7842214 abduced lemma generalise by renaming 7642286 )
```

8 Y Query Nedgehammer Symbols

Proof some Plaus update: Lucium Search: a

original goal 7579316: length (rev (x 7var 0.0 7var 1.0)) = 12 (length 7var 0.0) (length 7var 1.0)

```
The Edit Switch Markets Fighing York Ullimes Magnic Physics Selp-
 BEN G ST X II D GF CONT BY G O O
 # TIP_prop_06 My ( Workplace/Prod/Prod)
. III "42 (5 27) 2 = 5 (42 22 2)"
                                                                                                                                                                                                                                theorem
supreve property8 :
  34 "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
  splemma abduced lemma tactic 14045356; "war 6 = 12 war 8 2"
  to apply ( induct "war 0" )
  be opply (simp all ) done
 glemma abduced terms tactic 14045358: "(As. t2 var 0 s = t2 s var 0) == 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
 42 septy ( induct "war 1" "war 0" rule : TEP prop 06.12.induct )
 was smply (simp all ) done
 automos abduced terms commutativity 7642214: "12 var 8 var 1 - 12 var 1 var 8"
 se apply ( induct "war d" arbitrary : war 1 )
  on apply (simp all )
  as apply ( simp mid : abduced lemma factic 14045356 )
 as axing abduced lemma factic 14045358 apply force done
 silemma abduced leens tactic 13498028: "Length yar 0 - Length (x var 0 hil2)"
  so apply ( induct "war 0" )
  an apply (simp all ) done
  " lemma abduced lemma remove assumption 47001400: "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
  as apply ( induct four of artifrary a ver 1 )
  ar apply (simp all ) done
  selemma abduced terms tactic 13498034:
  mi"(As. length (x vor 8 a) = length (x a vor 8)) -- 5 (length (x vor I vor 8)) = length (x vor 1 (cosx vor 2 vor 8))"
  as emply ( simp mid : abduced lemma remove assumption 47001480 ) done
 aslemma abduced lemma composite commutativity 7642270: "length (a var 0 var 1) - length (a var 1 var 0)"
 as mosty ( induct Twar 07 arbitrary a ver 1 )
 se spoly (simp all )
  se uning abduced lemma tactic 13498828 apply blast
 ar apply ( simp whi : abduced lemma tactic 13498034 ) down
 autenma abduced Lemna tactic 13293202: "length (rev var 0) = length var 0 --- length (x (rev var 0) (coms2 var 1 mil2)) = 5 (length var 0)"
  suply ( metis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
             abduced Lemma composite commutativity 7642270 ) done
   ulemma abduced lemma generalisation them extension 7642332: "length (rev var 0) = length var 0"
  ne apply ( induct "war 0" )
      spoly (simp atl )
      apply ( simp min : abduced Lemma tactic 13293202 ) done
  malemma abduced temma identity 7579838: "k war 0 mit2 - war 0"
  De apply [ induct "yer 0" )
  as mosty (simp all ) done
  griems abduced temma generalisation then extension 17745958; "Length (nev (a var 0 mil2)) = Length var 0"
  so apply ( simp mir : abduced lemma generalisation them extension 7642332 abduced lemma identity 7579838 )
  as lemma abduced terms tactic 33224946: "(As. length (rev (x a ver 0)) = t2 (length ver 0) (length al) ---
  aftength (rev (x var 1 (cons2 var 2 var 0))) = 5 (12 (length var 0) (length var 1))*
  am apply ( mutis TIP prop 86.length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation them extension 7642332 )
  THE DONE
 afterms abduced terms generalise by remaning 7642286; "length (rev (a var 8 var 1)) = 12 (length var 1) (tength var 0)"
 as epoly ( induct (war 1) arbitrary ; war # )
 we apply (simp all )
 as apply ( simp old : abduced Lemma generalisation them extension 17745958 )
 no apply ( simp mit : abduced lemma tactic 33224946 )
 2000
 milemma original goat 7579816: "(ength (rev (x war 0 var 1)) = t2 (length var 8) (length var 1)"
 se apply I simp all I abduced lemma commutativity 7842214 abduced lemma generalise by renaming 7642286 )
 8 * Query Sledgehammer Symbols
```

Proof some IP has update - Update - Search - USDs - -

priginal goal 7579816: Length (rev (s 7var 0.0 7var 1.0)) = 12 (length 7var 0.0) (length 7var 1.0)

```
the Edit Seath Markets Fighting your Utilizes Magnes Ehapes tech-
 in TRP, prop. Oli Wy J.; Workplace/Prod/Prod/)
                                                                                                                                                                                                                                                         Proof some I Auto update: Update: Search:
 m1 "12 (5 22) 2 = 5 (12 22 2)"
 sepreve property® :
                                                                                                                                                                                                                             priginal goal 7579816:
  34 "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                              Length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                               t2 (length Tvar 0.0) (length Tvar 1.0)
  splemma abduced lemma tactic 14045356; "war 6 = t2 war 6 2"
  apply ( induct "war 0" )
  se opply (simp all ) done
 witeman abduced temms factic 14045358: "(As. t2 var 0 = +t2 = var 0) -> 5 (t2 var 1 var 0) - t2 var 1 (5 var 0)"
 as soply ( induct "war 1" "war 0" rule : TEP prop 06.t2.induct )
 as apply (nimp all ) done
 solemma abduced lemma commutativity 7642214: "12 var 8 var 1 - 12 var 1 var 6"
 as apply ( induct "war d" arbitrary : war 1 )
                                                                                                 (lemma1 → lmma2 → goal) lemma1 lemma2
  an opply (simp all )
  am apply ( simp wil : abduced Lemma tactic 14045356 )
 as axing abduced lemma tactic 14045358 apply force done
 silemma abduced lemma tactic 13498028: "length var 8 - length (x var 8 nil2)"
 to apply { induct "war 6" }
 Wi spoly (simp all ) done
  sillemma abduced lemma remove assumption 47001480: "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
     apply ( induct feer of artifrary a yer 1 )
 ar apply (simp all ) done
 selemma abduced lemma tactic 13498034:
 ma"(As. length (x var 8 a) = length (x a var 8)) -- 5 (length (x var 1 var 8)) = length (x var 1 (coss2 var 2 var 8))"
 as apply ( simp mid : abduced lemma remove assumption 47001480 ) done
 as lemma abduced lemma composite commutativity 7642270; "length (s var 0 var 1) - length (s var 1 var 0)"
 as mosty ( induct from 01 arbitrary a ver 1 )
  wa sming abduced lemma tactic 13498028 apply blast
 ar apply ( simp whi : abduced lemma tactic 13498034 ) down
 selemma abduced lemma tactic 13293202: "length (rev war 0) = length var 0 --> length (x (rev var 0) (coma2 var 1 mil2)) = 5 (length var 0)"
     apply ( metis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
             abduced terms composite commutativity 7642270 ) done
  filemma abduced temma generalisation them extension 7642332: "length (new var 0) = length var 0"
  ne septy ( induct "war 0" )
     spoly (simp atl )
     apply ( simp min : abduced Lemma tactic 13293202 ) done
 milemes abduced Lemma identity 7579838: "x var 0 mil2 = var 0"
 De apply [ induct "yer 0" )
 as mosty (simp all ) done
 externs abduced temms generalisation them extension 17745958: "Length Irev (a var 0 mil2)) - Length var 0"
 spoly ( simp mid : abduced lemma generalisation them extension 7642332 abduced lemma identity 7579638 )
 as terms abduced terms tactic 33224946: "(As. tength (rev (x a var 0)) = t2 (length var 0) (length a)) -
 astlength (rev (x var 1 (cons2 var 2 var 0))) = 5 (12 (length var 0) (length var 1))*
 mm apply ( metis TIP prop 86.length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation them extension 7642332 )
 wilessa abduced Lessa generalise by renaming 7642286: "Length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
 as epply ( induct "war 1" arbitrary ; war 0 )
 as apply ( simp old : abduced Lemma generalisation them extension 17745958 )
 no seply ( simp ald 1 abduced Lenna tactic 33224946 )
 milemma original goal 7579816: "length (rev (x var 0 var 1)) = t2 (length var 8) (length var 1)"
 se apply I simp all a abduced Lemma commutativity 7642214 abduced Lemma generalise by renaming 7642286 )
 8 * Query Nedgehammer Sym
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the Edit Seath Markets Folding your Utilizes Mayor Ellipsis belo-
  HI "17 (5 27) 2 = 5 (12 27 2)"
                                                                                                                                                                                                                              theorem
 sepreve property8 :
                                                                                                                                                                                                                                priginal goal 7579816:
  34 "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                                  length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                                  t2 (length ?var 0.0) (length ?var 1.0)
  splemma abduced lemma tactic 14045356; "war 6 = t2 war 0 2"
     apply [ induct "war 0" )
     opply (simp all ) done
  silemma abduced lemma factic 14845358: "(As. t2 var 0 s = t2 s var 0) - 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
     seply ( induct "war 1" "war 0" rule : TIP prop 06.t2.induct )
     apply (nimp all ) dose
           oduced lemma commutativity 7642214: "t2 war 8 war 1 = t2 war 1 war 6"
                                                                                                                            (lemma \rightarrow goal)
                                                                                                                                                                                              lemma
      apply (simp all )
      apply ( simp mil : abduted Lenna tactic 14045356 )
     axing abduced lemma tactic 14045358 apply force done
                                                                                                                                                                                                                       Modus ponens
  silemma abduced lemma tactic 13498028; "length yar 8 = length (x var 8 nil2)"
     apply { induct "war 6"
     spoty (simp all ) dune
  silemma abduced lemma remove assumption 47001480: "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
     apply ( induct "var 0" amitrary : var 1 1
     apply (simp all ) done
  selemma abduced lemma tactic 13498034:
  ma"(As. length (x var 8 a) = length (x a var 8)) -- 5 (length (x var I var 8)) = length (x var 1 (cos2 var 2 var 8))
     apply ( simp mid : abduced Lenna remove assumption 47001480 ) done
 asterna abduced terms composite commutativity 7642276: "tength (s var 0 var 1) = tength (s var 1 var 0)"
     septy ( induct "var 0" artificrary : ver 1 )
     spoly (simp att )
     using abduced lemma tactic 13498828 apply blast
     apply ( simp wit ; abduced lemma factic 13498034 ) door
  selemma abduced Lemma tactic 13283282: "length (rev war 8) = length var 8 --- length (x (rev war 8) (coms2 war 1 mil2)) = 5 (length war 8)"
     apply ( metis TIP prop 06.1 mgth.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
             abduced lemma composite commutativity 7642270 ) done
  filensa abduced temma generalisation them extension 7642332: "length (rew var 0) = length var 0"
     apply ( induct "war 0" )
      spoly (simp all )
      apply ( simp min : abduced Lemma tactic 13293202 ) done
  malemma abduced lemma identity 7579838: "x var 0 mit2 = var 0"
     apply [ induct "yer 0" )
     apply (simp all ) done
  externs abduced temms generalisation then extension 17745958: "Length (rev is var 0 mil2)) - Length var 0"
     apply ( simp mir : abduced lemma generalisation them extension 7642332 abduced lemma identity 7579838 )
  milemma abduced temma tactic 33224946: "(As. length (rev (x a var 0)) = t2 (length var 0) (length s)) --
  arlength (rev (x var 1 (cons2 var 2 or 0))) = 5 (t2 (length var 0) (length var 1))*
     mpty ( metis TIP prop 86. Length mimps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced Lemma composite commutativity 7642270 abduced Lemma generalisation them extension 7642332 )
          abduced temma generalise by remaming 7642286
                                                      "Length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
      spoly ( simp sod ; abduced Leema pereralization then extension 17745958 )
      apply ( simp mid : abduced lemma tactic 33224946 )
 malemas original goal PS70016: "Length
 me apply ( simp and
                       abduced lemma commutativity 7642214
                                                            abduced lemma generalise by renaming 7642286
8 * Query Medgehammer Symbols
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the Edit Seath Markets Folding your Utilizes Mayor Ellipsis belo-
  HI "12 (5 27) 2 = 5 (12 27 2)"
 sepreve property8 :
                                                                                                                                                                                                                                priginal goal 7579816:
     "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                                  length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                                  t2 (length ?var 0.0) (length ?var 1.0)
          abduced Lemna tactic 14045356
                                        "var 6 = 12 var 6 2"
      opply (simp all ) a
                              14045358: "(As. 12 var 0 = + 12 = var 0) -= 5 (12 var 1 var 0) - 12 var 1 (5 var 0)"
                                    Full : TEP prop 86.t2.induct )
      apply (nimp all ) do
           bduced lemna commutativity 7642214: "t2 var 8 var 1 = t2 var 1 var 8"
                                                                                                                            (lemma \rightarrow goal)
                                                                                                                                                                                              lemma
      opply (simp all )
                                                                                                                                                                                                                       Modus ponens
  silemma abduced lemma tactic 13498028: "Length yar 8 - Length (x var 8 hil2)"
     apply ( induct "war 6" )
     apply (simp all ) dune
  wilems abduced lemma remove assumption 47001480: "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
     apply ( induct "war 0" arbitrary ; wer 1 )
     apply (simp all ) done
  selemma abduced lemma tactic 13498034:
  ma"(As. length (x var 8 a) = length (x a var 8)) -- 5 (length (x var I var 8)) = length (x var 1 (cos2 var 2 var 8))
     apply ( simp mid : abduced Lemma remove assumption 47001480 ) done
 asterna abduced terms composite commutativity 7642276: "tength (s var 0 var 1) = tength (s var 1 var 0)"
     septy ( induct "var 0" arbitrary : ver 1 )
     spoly (simp att )
     uming abduced lemma tactic 13498828 apply blast
     apply ( simp win ; abduced lemma factic 13498034 ) door
  selemma abduced Lemma tactic 13293202: "length (rev var 0) = length var 0 -- length (x (rev var 0) (coms2 var 1 mil2)) = 5 (length var 0)"
     apply ( metis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
             abduced terms composite commutativity 7642276 ) done
  filemma abduced lemma generalisation them extension 7642332: "length (rev var 0) = length var 0"
     apply ( induct "war 0" )
     apply (simp att )
     spoly ( simp min : abduced Lemma tactic 13293202 ) done
  malemma abduced lemma identity 7579838: "k var 0 mil2 = var 0"
     apply [ induct "war 0" )
     monty (simp all ) done
            duced temma generalisation then extension 17745958
                                                              "Length (rev (a var 0 mil2)) = Length var 0"
                                terms conversionation them extension 7642332 abduced lemma identity 7579838 1
            luced Lemma tactic 33224946
                                        "Ma. length (rev (x a var 0)) = t2 (length var 0) (length a)) ---
  arlength (rev (x var 1 kcors2 var 2 var 0)) = 5 (t2 (length var 0) (length var 1))*
     mply ( metris TIP prop 86. length simps 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation them extension 7642332 )
          abduced temma generalise by renaming 7642286
                                                       "length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
                                      generalisation them extension 17745958
 material original goal 7579816: "Length (rev (x var 0 var 1)) = t2 (Length var 8) (Length var 1)"
     apply [ simp wif : abduced Lemma commutativity 7642214 abduced Lemma generalise by renaming 7642286 )
8 * Query Medgehammer Symbols
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the Edit Seath Markets Fighing your Littles Magnis Shapes bely
 HI "12 (5 27) 2 = 5 (12 27 2)"
 sepreve property8 :
                                                                                                                                                                                                                                priginal goal 7579816:
    "((length (rev (x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                                  length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                                  t2 (length Twar 0.0) (length Twar 1.0)
  sylemma abduced lemma tactic 14045356; "yar 6 = 52 war 8 2"
     apply [ induct "war 0" )
     opply (simp all ) done
  # lemma abduced lemma factic 14845358: "(As. t2 var 0 s = t2 s var 0) == 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
     seply ( induct "war 1" "war 0" rule : TEP prop 86.t2.induct )
     apply (nimp all ) done
 ou lemma abduced lemma commutativity 7642214: "12 var 8 var 1 - 12 var 1 var 8"
     apply ( induct "war 6" arhitrary : war 1 )
                                                                                                                            (lemma \rightarrow goal)
                                                                                                                                                                                             lemma
     opply (simp all )
     apply ( simp will : abduced Lemma tactic 14045356 )
     using abduced lemma factic 14045358 apply force doors
                                                                                                                                                                                                                       Modus ponens
  silemma abduced lemma tactic 13498028: "length yar 8 = length (x var 8 nil2)"
     apply ( induct "war 0" )
     apply (simp all ) done
  wilems abduced lemma remove assumption 47001480: "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
     apply ( induct "war 0" arbitrary ; wer 1 )
     apply (simp all ) done
  selemma abduced lemma tactic 13498034:
  ma"(As. length (x var 8 a) = length (x a ver 8)) -- 5 (length (x var I var 8)) = length (x var 1 (coex2 var 2 var 8))*
     apply ( simp mid : abduced Lemma remove assumption 47001480 ) done
           duced lemma composite commutativity 7642270; "length (x var 0 var 1) - length (x var 1 var 0)"
     apply (simp att )
     uning abduced lemma tactic 13498028 apply blast
     apply ( simp min : abduced leams twotic 13498034 ) does
 selemma abduced Lemma tactic 13293282: "length (rev war 8) = length var 8 --- length (x (rev war 8) (coms2 war 1 mil2)) = 5 (length war 8)"
     apply ( metis TIP prop 06.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
             abduced temma composite commutativity 7642270 ) dome
                                                             bangth (rev var 0) = Length var 0"
     apply (simp att )
      apply [ simp mid : abduced Lemma Dactic 13293202 ] done
     mostly (simp all ) don
                                                                                  New identity 7579838
            duced Lemma tactic 33224946
                                       *(As. length (rev (x a var 0)) = t2 (length var 0) (length s)) ---
 arlength (rev (x var 1 (cons2 var 2 var 0))) = 5 (t2 (length var 0) (length var 1))*
     apply ( metis TIP prop 06. length simps ( 2 ) TIP prop 06.x.simps 4 2 ) abduced lemma composite commutativity 764227
                                                                                                                        bduced temma generalisation them extension 7642333
 uplemma abduced Lamma generalise by remaining 7642286: "Length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
     apply ( induct "war 1" arhitrary ; war 8 )
     apply ( simp sol : abduced Lemma generalisation then extension 17745958 )
     epply ( simp mid : abduced lemma tactic 33224946 )
 material original goal 7579816: "Length (rev (x var 0 var 1)) = t2 (Length var 8) (Length var 1)"
 me epoly ( timp wif : abduced temma commutativity 7842214 abduced temma generalise by renaming 7642286 )
8 * Query Medgehammer Symbols
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the Edit Seath Markets Folding your Utilizes Mayor Ellipsis belo-
  HI "17 (5 27) 2 = 5 (12 27 2)"
 sepreve property8 :
                                                                                                                                                                                                                                priginal goal 7579816:
  34 "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                                  length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                                  t2 (length ?var 0.0) (length ?var 1.0)
  sylemma abduced lemma tactic 14045356: "var 6 = t2 var 8 2"
     apply [ induct "war 0" )
     opply (simp all ) done
  silemma abduced lemma factic 14845358: "(As. t2 var 0 s = t2 s var 0) - 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
     seply ( induct "war 1" "war 0" rule : TIP prop 06.t2.induct )
     apply (nimp all ) done
  ou lemma abduced lemma commutativity 7642214: "12 var 8 var 1 - 12 var 1 var 8"
     apply ( induct "war 6" arbitrary : war 1 )
                                                                                                                            (lemma \rightarrow goal)
                                                                                                                                                                                              lemma
     opply (simp all )
     epoly ( simp will : abduced Lemma tactic 14045356 )
     swing abduced lemma tactic 14045358 apply force done
                                                                                                                                                                                                                       Modus ponens
            duced lemma tactic 13498028; "Length var 8 - Length (x var 8 mil2)"
     spoty (simp all ) do
  silemma abduced lemma remove assumption 47001480: "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
     apply ( induct "war 0" andtrary ; wer 1 )
     apply (simp all ) done
  ma"(As. length (x var & a) = length (x a var 8)) -- 5 (length (x var I var 8)) = length (x var I (cos2 var 2 var 8))
     apply ( simp mid : abduced Lemma remove assumption 47001480 ) done
                                        tivity 7642270; "length (x var 0 var 1) = length (x var 1 var 0)"
           bduced lemma tactic 13203202: "tength (rev war 0) = length war 0 == length (x (rev war 0) (cons2 war 1 mil2)) = 5 (length war 0)"
      apply ( metis TIP prop 86.length.simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 06.x.simps ( 2 )
              abduced terms composite commutativity 7642270 ) done
                                                             "length (rev var 0) = length var 0"
      apply (simp all )
      apply ( simp mid : abduced Lenna tactic 13293205 ) done
  mlamma abduced lemma identity 7579838: "k var 0 nit2 = var 0"
     apply [ induct "yer 0" )
     spoty (simp all ) done
  externs abduced temms generalisation them extension 17745958: "Length (rev (a var 0 mil2)) = Length var 0"
     apply ( simp min's abduced lemma generalisation them extension 7642332 abduced lemma identity 7579838 }
  milemma abduced lemma tactic 33224946: "(Am. length (rev (x a var 0)) = t2 (length var 0) (length m)) ---
  arlength (rev (x var 1 (cons2 var 2 var 0))) = 5 (t2 (length var 0) (length var 1))*
  as apply ( metis TIP prop 86. length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation then extension 7642332 )
 uplesse abduced Lamma generalise by remaining 7642286: "Length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
     apply ( induct "war 1" arhitrary ; war 8 )
     apply ( simp sol : abduced Lemma generalisation then extension 17745958 )
     epply ( simp mid : abduced lemma tactic 33224946 )
 in terms original goal 7579816; "(ength (rev (x var 0 var 1)) = t2 (length var 8) (length var 1)"
 me epoly ( timp wif : abduced temma commutativity 7842214 abduced temma generalise by renaming 7642286 )
8 * Query Medgehammer Symbols
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the Edit Seath Markets Folding your Utilizes Mayor Ellipsis belo-
  HI "17 (5 27) 2 = 5 (12 27 2)"
                                                                                                                                                                                                                             theorem
 sepreve property8 :
                                                                                                                                                                                                                               priginal goal 7579816:
  34 "((length (rev [x y 2))) = (t2 (length y) (length 2)))"
                                                                                                                                                                                                                                 length (rev (x 7var 0.0 7var 1.0)) =
                                                                                                                                                                                                                                 t2 (length ?var 0.0) (length ?var 1.0)
  sylemma abduced lemma tactic 14045356: "var 6 = t2 var 8 2"
     apply [ induct "war 0" )
     opply (simp all ) done
  silemma abduced lemma factic 14845358: "(As. t2 var 0 s = t2 s var 0) - 5 (t2 var 1 var 0) = t2 var 1 (5 var 0)"
     seply ( induct "war 1" "war 0" rule : TIP prop 06.t2.induct )
     apply (nimp all ) done
  ou lemma abduced lemma commutativity 7642214: "12 var 8 var 1 - 12 var 1 var 8"
     apply ( induct "war 6" arhitrary : war 1 )
                                                                                                                            (lemma \rightarrow goal)
                                                                                                                                                                                             lemma
     opply (simp all )
     apply ( simp will ; abduced Lemma tactic 14045356 )
     using abduced lemma tactic 14045358 apply force done
                                                                                                                                                                                                                      Modus ponens
  silemma abduced lemma tactic 13498028: "length yar 8 = length (x var 8 nil2)"
     apply ( induct "war 0" )
     spoty (simp all ) dune
                                                  "5 (length (x var 0 var 1)) = length (x var 0 (cons2 var 2 var 1))"
     apply (simp all ) done
  velenna abduced terms factic 1349803
  ma (As. length (x var 8 a) = length (x a var 8)) - 3 length (x var I var 8)) = length (x var I (coms2 var 2 var 8))
           oduced lemma composite commutativity 7642270: "length (x var 0 var 1) - length (x var 1 var 0)"
                                                                                                                              already proved
     spoly (simp att )
     uning abduced lemma tactic 13498838 apply blast
     apply ( simp wid : abduced lemma tactic 13498034 ) down
           bduced lumna tactic 13293202: "tength (rec car 0) = length var 0 --- length (x (rev var 0) (cons2 var 1 mil2)) = 5 (length var 0)"
            notis TIP prop 06. Length. simps ( 2 ) TIP prop 06.x.simps ( 1 ) TIP prop 66.x.simps ( 2 )
                fuced terms composite commutativity 7642275
  tilemma abduced lemma generalisation them extension 7642332: "length (rev var 0) = length var 0"
     apply ( induct "war 0" )
      spoly (simp all )
      spoly ( simp min : abduced Lemma tactic 13293202 ) done
  malemma abduced lemma identity 7579838: "k var 0 mil2 = var 0"
     apply [ induct "yer 0" )
     spoty (simp all ) done
  externs abduced temms generalisation them extension 17745958: "Length (rev (a var 0 mil2)) = Length var 0"
     apply ( simp min's abduced lemma generalisation them extension 7642332 abduced lemma identity 7579838 )
  milemma abduced temma tactic 33224946: "(As. length (rev (x a var 0)) = t2 (length var 0) (length s)) --
  arlength (rev (x var 1 (cons2 var 2 var 0))) = 5 (t2 (length var 0) (length var 1))*
  as apply ( metis TIP prop 86. length.simps ( 2 ) TIP prop 86.x.simps ( 2 ) abduced lemma composite commutativity 7642270 abduced lemma generalisation then extension 7642332 )
  uplemma abduced Lamma generalise by remaining 7642286: "Length (rev (a var 8 var 1)) = 12 (length var 1) (length var 0)"
     apply ( induct "war 1" arhitrary ; war 8 )
     apply ( simp sol : abduced Lemma generalisation then extension 17745958 )
     epply ( simp mid : abduced lemma tactic 33224946 )
 in terms original goal 7579816; "(ength (rev (x var 0 var 1)) = t2 (length var 8) (length var 1)"
 se epoly ( timp wif : abduced temma commutativity 7842214 abduced temma generalise by renaming 7642286 )
8 * Query Medgehammer Symbols
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