

BIOMEDICAL SIGNAL ANALYSIS

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BIOMEDICAL SIGNAL ANALYSIS

Second Edition

RANGARAJ M. RANGAYYAN



IEEE Engineering in Medicine
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IEEE Press Series in Biomedical Engineering
Metin Akay, *Series Editor*



IEEE Press

WILEY

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Library of Congress Cataloging-in-Publication is available.

ISBN 978-0-470-91139-6

Printed in the United States of America.

DEDICATION

Mátr dévô bhava

Pitr dévô bhava

Áchárya dévô bhava

Look upon your mother as your God

Look upon your father as your God

Look upon your teacher as your God

— from the sacred Vedic hymns of the *Taittireeya Upanishad* of India.

*This book is dedicated to the fond memory of
my mother Srimati Padma Srinivasan Rangayyan
and my father Sri Srinivasan Mandayam Rangayyan,
and to all of my teachers,
in particular, Professor Ivaturi Surya Narayana Murthy.*

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PREFACE

The first edition of this book has been received very well around the world. Professors at several universities across North America, Europe, Asia, and other regions of the world are using the book as a textbook. A low-cost paperback edition for selected regions of the world and a Russian edition have been published. I have received several messages and comments from many students, professors, and researchers via mail and at conferences with positive feedback about the book. I am grateful to IEEE and Wiley for publishing and promoting the book and to the many users of the book for their support and feedback.

I have myself used the book to teach my course ENEL 563 Biomedical Signal Analysis at the University of Calgary. In addition to positive responses, I have received suggestions from students and professors on revising the book to provide additional examples and including advanced topics and discussions on recent developments in the book. I also made notes identifying parts of the book that could be improved for clarity, augmented with details for improved comprehension, and expanded with additional examples for better illustrations of application. I have also identified a few new developments, novel applications, and advanced techniques for inclusion in the second edition to make the book more interesting and appealing to a wider readership.

New Material in the Second Edition

In view of the success of the first edition, I have not made any major change in the organization and style of the book. Notwithstanding a tighter format to reduce white space and control the total number of pages, the second edition of the book remains similar to the first edition in terms of organization and style of presentation. New material has been inserted into the same chapters as before, thereby expanding the book. The new topics have been chosen with care not only to fit with the structure and organization of the book but also to provide additional support material and advanced topics that can be assimilated and appreciated in a first course or an advanced study of the subject area.

Some of the substantial and important additions made to the book deal with the following topics:

- analysis of the variation of parameters of the electromyogram with force;
- illustrations of the electroencephalogram with application to sleep analysis and prediction of epileptic seizures;
- details on the theory of linear systems and numerical examples related to convolution;
- details on the z -transform and the Fourier transform along with additional examples of Fourier spectra and spectral analysis of biomedical signals;
- details on linear filters and their characteristics, such as the impulse response, transfer function, and pole–zero diagrams;
- description and demonstration of nonlinear order-statistic filters;
- derivation of the matched filter;
- derivations related to the complex cepstrum;
- details on random processes and their properties;
- wavelets and the wavelet transform with biomedical applications;
- fractal analysis with biomedical applications;
- time-frequency distributions and analysis of nonstationary signals with biomedical applications;
- principal component analysis, independent component analysis, and blind source separation with biomedical applications;
- monitoring of sleep apnea;
- analysis of various types of bioacoustic signals that could bear diagnostic information; and

- methods for pattern analysis and classification with illustrations of application to biomedical signals.

Discussions related to the topics listed above are spread throughout the book with several new references added to assist the reader in further studies. Many more problems and projects have been added at the ends of the chapters.

The first edition of the book (2002) has 516 pages (plus xxxv pages of front matter) with nine chapters, 538 numbered equations (with many more equations not numbered but as parts of procedures), 232 numbered figures (many with multiple subfigures), and 265 references. The second edition (2015) has 672 pages (plus xliii pages of front matter) in a more compact layout than the first edition, with nine chapters, 814 numbered equations (and many more equations not numbered but as parts of procedures), 370 numbered figures (many with multiple subfigures), and 505 references. The discussions on some of the new topics added were kept brief in order to control the size of the book; regardless, the second edition is approximately 50% larger than the first edition in several aspects.

Intended Audience

As with the first edition, the second edition is directed at engineering students in their final (senior) year of undergraduate studies or in the first year of their graduate studies. Electrical Engineering students with a good background in signals and systems will be well prepared for the material in the book. Students in other engineering disciplines, or in computer science, physics, mathematics, or geosciences, should also be able to appreciate the material in the book. A course on digital signal processing or digital filters would form a useful link to the material in the present book, but a capable student without this background should be able to gain a basic understanding of the subject matter. The introductory materials on systems, filters, and transforms added in the second edition should assist the reader without formal training on the same topics. Practicing engineers, computer scientists, information technologists, medical physicists, and data-processing specialists working in diverse areas such as telecommunications, seismic and geophysical applications, biomedical applications, and hospital information systems may find the book useful in their quest to learn advanced techniques for signal analysis. They could draw inspiration from other applications of signal processing or analysis, and satisfy their curiosity regarding computer applications in medicine and computer-aided medical diagnosis.

Teaching and Learning Plan

The book starts with an illustrated introduction to biomedical signals in Chapter 1. Chapter 2 continues the introduction, with emphasis on the analysis of multiple channels of correlated signals.

Chapter 3 deals exclusively with filtering of signals for removal of artifacts as an important step before signal analysis. The basic properties of systems and transforms

as well as signal processing techniques are reviewed and described where required. The chapter is written as a mix of theory and application so as to facilitate easy comprehension of the basics of signals, systems, and transforms [1–4]. The emphasis is on the application of filters to particular problems in biomedical signal analysis. A large number of illustrations are included to provide a visual impression of the problem and the effectiveness of the various filtering methods described.

Chapter 4 presents techniques that are particularly useful in the detection of events in biomedical signals. Analysis of waveshape and waveform complexity of events and components of signals is the focus of Chapter 5. Techniques for frequency-domain characterization of biomedical signals and systems are presented in Chapter 6. A number of diverse examples are provided in all of the chapters. Attention is directed to the characteristics of the problems that are encountered when analyzing and interpreting biomedical signals, rather than to any specific diagnostic application with particular signals.

The material in the book up to and including Chapter 6 provides more than adequate material for a one-semester (13-week) course at the senior (fourth-year) engineering level. My own teaching experience indicates that this material will require about 38 hours of lectures. It would be desirable to augment the lectures with about 12 hours of tutorials (problem-solving sessions) and 10 laboratory sessions.

Modeling biomedical signal-generating processes and systems for parametric representation and analysis is the subject of Chapter 7. Chapter 8 deals with the analysis of nonstationary and multicomponent signals. The topics in these chapters are of higher mathematical complexity than suitable for undergraduate courses. Some sections may be selected and included in a first course on biomedical signal analysis if there is particular interest in these topics. Otherwise, the two chapters could be left for self-study by those in need of the techniques, or included in an advanced course.

Chapter 9 presents the final aspect of biomedical signal analysis, and provides an introduction to pattern classification and diagnostic decision making. Although this topic is advanced in nature and could form a graduate-level course on its own, the material is introduced so as to draw the entire exercise of biomedical signal analysis to its concluding stage of diagnostic decision. It is recommended that a few sections from this chapter be included even in a first course on biomedical signal analysis so as to give the students a flavor of the end result.

The topic of data compression has been deliberately left out of the book. Advanced topics such as time-frequency distributions, wavelet-based analysis, independent component analysis, and fractal analysis have been introduced briefly in the second edition, but require further detailed study for completeness. Adaptive filters and nonstationary signal analysis techniques are introduced in the book, but deserve more attention, depth, and breadth. The references provided should assist the interested reader in obtaining further advanced material.

Each chapter includes a number of study questions and problems to facilitate preparation for tests and examinations. A number of laboratory exercises are also provided at the end of each chapter, which could be used to formulate hands-on exercises with real-life signals. Data files related to the problems and exercises at the end of each chapter are available at the site

<http://people.ucalgary.ca/~ranga/enel563>

MATLAB[®] programs to read the data are also provided in some cases.

It is strongly recommended that the first one or two laboratory sessions in the course include visits to a local hospital, health sciences center, or clinical laboratory to view and experience procedures related to biomedical signal acquisition and analysis in a practical (clinical) setting. Signals acquired from fellow students and professors could form interesting and motivating material for laboratory exercises, and may be used to supplement the data files provided. A few workshops by physiologists, neuroscientists, and cardiologists should also be included in the course so as to provide the students with a nonengineering perspective on the subject.

Practical experience with real-life signals is a key element in understanding and appreciating biomedical signal analysis. This aspect could be difficult and frustrating at times, but provides professional satisfaction and educational fun!

RANGARAJ MANDAYAM RANGAYYAN

Calgary, Alberta, Canada

February, 2015

ACKNOWLEDGMENTS

In addition to the various people and sources acknowledged in the first edition and in captions of figures, I thank the following for their assistance with and contributions to the second edition: Faraz Oloumi, Alexander Kalinichenko, Douglas Bell, Tingting Mu, Fábio José Ayres, Shantanu Banik, Thanh Minh Cabral, Sridhar Krishnan, Yunfeng Wu, Suxian Cai, Adrian D.C. Chan, Anca Lazăr, Karthikeyan Umapathy, Ronald Platt, April Khademi, Markad V. Kamath, Rajeev Agarwal, Maarten De Vos, Hisham Alshaer, Jayasree Chakraborty, Ashis Kumar Dhara, Ivan Cruz Aceves, Behnaz Ghoraani, Paola Casti, Mehrnaz Shokrollahi, Zahra Moussavi, and T. Douglas Bradley.

I am grateful to the University of Calgary for facilitating and supporting my academic activities. I am grateful to IEEE Press and Wiley for publishing and promoting this book.

As always, I am grateful to my family — my wife Mayura, my daughter Vidya, and my son Adarsh — for their love and support.

I hope that this book will assist those who seek to enrich their lives and those of others with the exciting field of biomedical signal analysis.

RANGARAJ MANDAYAM RANGAYYAN

*Calgary, Alberta, Canada
February, 2015*

PREFACE: FIRST EDITION

Background and Motivation

The establishment of the clinical electrocardiograph (ECG) by the Dutch physician Willem Einthoven in 1903 marked the beginning of a new era in medical diagnostic techniques, including the entry of electronics into health care. Since then, electronics, and subsequently computers, have become integral components of biomedical signal analysis systems, performing a variety of tasks from data acquisition and pre-processing for removal of artifacts to feature extraction and interpretation. Electronic instrumentation and computers have been applied to investigate a host of biological and physiological systems and phenomena, such as the electrical activity of the cardiovascular system, the brain, the neuromuscular system, and the gastric system; pressure variations in the cardiovascular system; sound and vibration signals from the cardiovascular, the musculoskeletal, and the respiratory systems; and magnetic fields of the brain, to name a few.

The primary step in investigations of physiological systems requires the development of appropriate sensors and instrumentation to transduce the phenomenon of interest into a measurable electrical signal. The next step of analysis of the signals, however, is not always an easy task for a physician or life-sciences specialist. The clinically relevant information in the signal is often masked by noise and interference, and the signal features may not be readily comprehensible by the visual or

auditory systems of a human observer. Heart sounds, for example, have most of their energy at or below the threshold of auditory perception of most humans; the interference patterns of a surface electromyographic signal are too complex to permit visual analysis. Some repetitious or attention-demanding tasks, such as on-line monitoring of the ECG of a critically ill patient with cardiac rhythm problems, could be uninteresting and tiring for a human observer. Furthermore, the variability present in a given type of signal from one subject to another, and the interobserver variability inherent in subjective analysis performed by physicians or analysts make consistent understanding or evaluation of any phenomenon difficult, if not impossible. These factors created the need not only for improved instrumentation, but also for the development of methods for objective analysis via signal processing algorithms implemented in electronic hardware or on computers.

Processing of biomedical signals, until a few years ago, was mainly directed toward filtering for removal of noise and power-line interference; spectral analysis to understand the frequency characteristics of signals; and modeling for feature representation and parameterization. Recent trends have been toward quantitative or objective analysis of physiological systems and phenomena via signal analysis. The field of biomedical signal analysis has advanced to the stage of practical application of signal processing and pattern analysis techniques for efficient and improved non-invasive diagnosis, on-line monitoring of critically ill patients, and rehabilitation and sensory aids for the handicapped. Techniques developed by engineers are gaining wider acceptance by practicing clinicians, and the role of engineering in diagnosis and treatment is gaining much-deserved respect.

The major strength in the application of computers in biomedical signal analysis lies in the potential use of signal processing and modeling techniques for quantitative or objective analysis. Analysis of signals by human observers is almost always accompanied by perceptual limitations, interpersonal variations, errors caused by fatigue, errors caused by the very low rate of incidence of a certain sign of abnormality, environmental distractions, and so on. The interpretation of a signal by an expert bears the weight of the experience and expertise of the analyst; however, such analysis is almost always subjective. Computer analysis of biomedical signals, if performed with the appropriate logic, has the potential to add objective strength to the interpretation of the expert. It thus becomes possible to improve the diagnostic confidence or accuracy of even an expert with many years of experience. This approach to improved health care could be labeled as *computer-aided diagnosis*.

Developing an algorithm for biomedical signal analysis, however, is not an easy task; quite often, it might not even be a straightforward process. The engineer or computer analyst is often bewildered by the variability of features in biomedical signals and systems, which is far higher than that encountered in physical systems or observations. Benign diseases often mimic the features of malignant diseases; malignancies may exhibit a characteristic pattern, which, however, is not always guaranteed to appear. Handling all of the possibilities and degrees of freedom in a biomedical system is a major challenge in most applications. Techniques proven to work well with a certain system or set of signals may not work in another seemingly similar situation.

The Problem solving Approach

The approach I have taken in presenting material in this book is primarily that of development of algorithms for problem solving. Engineers are often said to be (with admiration, I believe) problem solvers. However, the development of a problem statement and gaining of a good understanding of the problem could require a significant amount of preparatory work. I have selected a logical series of problems, from the many case studies I have encountered in my research work, for presentation in the book. Each chapter deals with a certain type of a problem with biomedical signals. Each chapter begins with a statement of the problem, followed immediately with a few illustrations of the problem with real-life case studies and the associated signals. Signal processing, modeling, or analysis techniques are then presented, starting with relatively simple “textbook” methods, followed by more sophisticated research approaches directed at the specific problem. Each chapter concludes with one or more applications to significant and practical problems. The book is illustrated copiously with real-life biomedical signals and their derivatives.

The methods presented in the book are at a fairly high level of technical sophistication. A good background in signal and system analysis [1,3,4] as well as probability, random variables, and stochastic processes [5–10] is required in order to follow the procedures and analysis. Familiarity with systems theory and transforms such as the Laplace and Fourier, the latter in both continuous and discrete versions, will be assumed. We will not be getting into details of the transducers and instrumentation techniques essential for biomedical signal acquisition [11–14]; instead, we will be studying the problems present in the signals after they have been acquired, concentrating on how to solve the problems. Concurrent or prior study of the physiological phenomena associated with the signals of specific interest, with a clinical textbook, is strongly recommended.

Intended Readership

The book is directed at engineering students in their final year of undergraduate studies or in their graduate studies. Electrical Engineering students with a rich background in signals and systems [1,3,4] will be well prepared for the material in the book. Students in other engineering disciplines, or in computer science, physics, mathematics, or geophysics should also be able to appreciate the material in the book. A course on digital signal processing or digital filters [15] would form a useful link, but a capable student without this topic may not face much difficulty.

Practicing engineers, computer scientists, information technologists, medical physicists, and data-processing specialists working in diverse areas such as telecommunications, seismic and geophysical applications, biomedical applications, and hospital information systems may find the book useful in their quest to learn advanced techniques for signal analysis. They could draw inspiration from other applications of signal processing or analysis, and satisfy their curiosity regarding computer applications in medicine and computer-aided medical diagnosis.

Teaching and Learning Plan

The book starts with an illustrated introduction to biomedical signals in Chapter 1. Chapter 2 continues the introduction, but with emphasis on the analysis of multiple channels of related signals. This part of the book may be skipped in the teaching plan for a course if the students have had a previous course on biomedical signals and instrumentation. In such a case, the chapters should be studied as review material in order to get oriented toward the examples to follow in the book.

Chapter 3 deals exclusively with filtering for removal of artifacts as an important precursive step before signal analysis. Basic properties of systems and transforms as well as signal processing techniques are reviewed and described as and when required. The chapter is written so as to facilitate easy comprehension by those who have had a basic course on signals, systems, and transforms [1, 3, 4]. The emphasis is on the application to particular problems in biomedical signal analysis, and not on the techniques themselves. A large number of illustrations are included to provide a visual impression of the problem and the effectiveness of the various filtering methods described.

Chapter 4 presents techniques particularly useful in the detection of events in biomedical signals. Analysis of waveshape and waveform complexity of events and components of signals is the focus of Chapter 5. Techniques for frequency-domain characterization of biomedical signals and systems are presented in Chapter 6. A number of diverse examples are provided in these chapters. Attention is directed to the characteristics of the problems one faces in analyzing and interpreting biomedical signals, rather than to any specific diagnostic application with particular signals.

The material in the book up to and including Chapter 6 will provide more than adequate material for a one-semester (13-week) course at the senior (fourth-year) engineering level. My own teaching experience indicates that this material will require about 36 hours of lectures, augmented with about 12 hours of tutorials (problem-solving sessions) and 10 laboratory sessions.

Modeling biomedical signal-generating processes and systems for parametric representation and analysis is the subject of Chapter 7. Chapter 8 deals with the analysis of nonstationary signals. The topics in these chapters are of higher mathematical complexity than suitable for undergraduate courses. Some sections may be selected and included in a first course on biomedical signal analysis if there is particular interest in these topics. Otherwise, the two chapters could be left for self-study by those in need of the techniques, or included in an advanced course.

Chapter 9 presents the final aspect of biomedical signal analysis, and provides an introduction to pattern classification and diagnostic decision. Although this topic is advanced in nature and could form a graduate-level course on its own, the material is introduced so as to draw the entire exercise of biomedical signal analysis to its concluding stage of diagnostic decision. It is recommended that a few sections from this chapter be included even in a first course on biomedical signal analysis so as to give the students a flavor of the end result.

The topic of data compression has deliberately been left out of the book. Advanced topics such as nonlinear dynamics, time-frequency distributions, wavelet-

based analysis, chaos, and fractals are not covered in the book. Adaptive filters and nonstationary signal analysis techniques are introduced in the book, but deserve more attention, depth, and breadth. These topics will form the subjects of a follow-up book that I intend to write.

Each chapter includes a number of study questions and problems to facilitate preparation for tests and examinations. A number of laboratory exercises are also provided at the end of each chapter, which could be used to formulate hands-on exercises with real-life signals. Data files related to the problems and exercises at the end of each chapter are available at the site (revised for 2015)

<http://people.ucalgary.ca/~ranga/enel563>

MATLAB[®] programs to read the data are also provided where required.

It is strongly recommended that the first one or two laboratory sessions in the course be visits to a local hospital, health sciences center, or clinical laboratory to view biomedical signal acquisition and analysis in a practical (clinical) setting. Signals acquired from fellow students and professors could form interesting and motivating material for laboratory exercises, and should be used to supplement the data files provided. A few workshops by physiologists, neuroscientists, and cardiologists should also be included in the course so as to provide the students with a nonengineering perspective on the subject.

Practical experience with real-life signals is a key element in understanding and appreciating biomedical signal analysis. This aspect could be difficult and frustrating at times, but provides professional satisfaction and educational fun!

RANGARAJ MANDAYAM RANGAYYAN

*Calgary, Alberta, Canada
September, 2001*

ACKNOWLEDGMENTS: FIRST EDITION

To write a book on my favorite subject of biomedical signal analysis has been a long-cherished ambition of mine. Writing this book has been a major task with many facets: challenging, yet yielding more knowledge; tiring, yet stimulating the thirst to understand and appreciate more; difficult, yet satisfying when a part was brought to a certain stage of completion.

A number of very important personalities have shaped me and my educational background. My mother, Srimati Padma Srinivasan Rangayyan, and my father, Sri Srinivasan Mandayam Rangayyan, encouraged me to keep striving to gain higher levels of education and to set and achieve higher goals all the time. I have been very fortunate to have been taught and guided by a number of dedicated teachers, the most important of them being Professor Ivaturi Surya Narayana Murthy, my Ph.D. supervisor, who introduced me to the topic of this book at the Indian Institute of Science, Bangalore, Karnataka, India. It is with great respect and admiration that I dedicate this book as a humble offering to their spirits.

My basic education was imparted by many influential teachers at Saint Joseph's Convent, Saint Joseph's Indian High School, and Saint Joseph's College in Mandya and Bangalore, Karnataka, India. My engineering education was provided by the People's Education Society College of Engineering, Mandya, affiliated with the University of Mysore. I express my gratitude to all of my teachers.

My association with clinical researchers at the University of Calgary and the University of Manitoba has been invaluable in furthering my understanding of the subject matter of this book. I express my deep gratitude to Cyril Basil Frank, Gordon Douglas Bell, Joseph Edward Leo Desautels, Leszek Hahn, and Reinhard Kloiber of the University of Calgary, and Richard Gordon and George Collins of the University of Manitoba, Winnipeg, Manitoba, Canada.

My understanding and appreciation of the subject of biomedical signal analysis has been boosted by the collaborative research and studies performed with my many graduate students, postdoctoral fellows, research associates, and colleagues. I would like to place on record my gratitude to Sridhar Krishnan, Naga Ravindra Mudigonda, Margaret Hilary Alto, Ricardo José Ferrari, Liang Shen, Roseli de Deus Lopes, Antonio César Germano Martins, Marcelo Knörich Zuffo, Begoña Acha Piñero, Carmen Serrano Gotarredona, Sílvia Delgado Olabarriaga, Christian Roux, Basel Solaiman, Olivier Menut, Denise Guliato, Mihai Ciuc, Vasile Buzuloiu, Titus Zaharia, Constantin Vertan, Sarah Rose, Salahuddin Elkadiki, Kevin Eng, Nema Mohamed El-Faramawy, Arup Das, Farshad Faghih, William Alexander Rolston, Yiping Shen, Zahra Marjan Kazem Moussavi, Joseph Provine, Hieu Ngoc Nguyen, Djamel Boulfelfel, Tamer Farouk Rabie, Katherine Olivia Ladly, Yuanting Zhang, Zhi-Qiang Liu, Raman Bhalachandra Paranjape, Joseph André Rodrigue Blais, Robert Charles Bray, Gopinath Ramaswamaiah Kuduvalli, Sanjeev Tavathia, William Mark Morrow, Timothy Chi Hung Hon, Subhasis Chaudhuri, Paul Soble, Kirby Jaman, Atam Prakash Dhawan, and Richard Joseph Lehner. In particular, I thank Sridhar and Naga for assisting me in preparing illustrations and examples; Sridhar for permitting me to use sections of his M.Sc. and Ph.D. theses; and Sridhar, Naga, Hilary, and Ricardo for careful proofreading of the drafts of the book. Sections of the book were reviewed by Robert Clark, Martin Paul Mintchev, Sanjay Srinivasan, and Abu Bakarr Sesay, University of Calgary; and Ioan Tăbuș, Tampere Technical University, Tampere, Finland; I express my gratitude to them for their comments and advice.

The book has benefited significantly from illustrations and text provided by a number of researchers worldwide, as identified in the references and permissions cited. I thank them all for enriching the book with their gifts of knowledge and kindness. I thank Bert Unterberger for drafting some of the illustrations in the book.

The research projects that have provided me with the background and experience essential in order to write the material in this book have been supported by many agencies. I thank the Natural Sciences and Engineering Research Council of Canada, the Alberta Heritage Foundation for Medical Research, the Alberta Breast Cancer Foundation, the Arthritis Society of Canada, the Nickle Family Foundation of Calgary, Control Data Corporation, the University of Calgary, the University of Manitoba, and the Indian Institute of Science for supporting my research projects.

I thank the Killam Foundation for awarding me a Resident Fellowship to facilitate work on this book. I gratefully acknowledge support from the Alberta Provincial Biomedical Engineering Graduate Programme, funded by a grant from the Whitaker Foundation, toward student assistantship for preparation of exercises and illustrations for this book and the related course ENEL 563 Biomedical Signal Analysis at the University of Calgary. I am pleased to place on record my gratitude for the gen-

erous support from the Department of Electrical and Computer Engineering and the Faculty of Engineering at the University of Calgary in terms of supplies, services, and relief from other duties.

My association with the IEEE Engineering in Medicine and Biology Society (EMBS) in many positions has benefited me considerably in numerous ways. In particular, the period as an Associate Editor of the *IEEE Transactions on Biomedical Engineering* was very rewarding, as it provided me with a wonderful opportunity to work with many leading researchers and authors of scientific articles. I thank IEEE EMBS for lending professional support to my career on many fronts. I am grateful to the IEEE Press, in particular, Metin Akay, Series Editor, IEEE Press Series in Biomedical Engineering, for inviting me to write this book.

Writing this book has been a monumental task, often draining me of all of my energy. The infinite source of inspiration and recharging of my energy has been my family — my wife Mayura, my daughter Vidya, and my son Adarsh. While supporting me with their love and affection, they have had to bear the loss of my time and effort at home. I express my sincere gratitude to my family for their love and support, and record their contribution toward the preparation of this book.

It is my humble hope that this book will assist those who seek to enrich their lives and those of others with the wonderful powers of biomedical signal analysis. Electrical and Computer Engineering is indeed a great field in the service of humanity!

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September, 2001

ABOUT THE AUTHOR

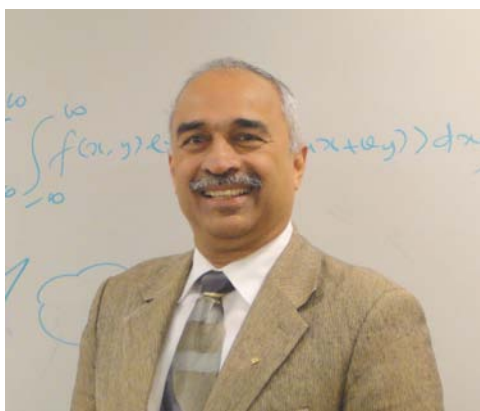


Photo by Faraz Oloumi.

Rangaraj (Raj) Mandayam Rangayyan was born in Mysore, Karnataka, India, on 21 July 1955. He received the Bachelor of Engineering degree in Electronics and Communication in 1976 from the University of Mysore at the People's Education Society College of Engineering, Mandya, Karnataka, India, and the Ph.D.

degree in Electrical Engineering from the Indian Institute of Science, Bangalore, Karnataka, India, in 1980. He was with the University of Manitoba, Winnipeg, Manitoba, Canada, from 1981 to 1984.

He is, at present, a Professor with the Department of Electrical and Computer Engineering as well as an Adjunct Professor of Surgery and Radiology at the University of Calgary, Calgary, Alberta, Canada. His research interests are in the areas of digital signal and image processing, biomedical signal analysis, medical imaging and image analysis, and computer vision. His research projects have been focused on mammographic image enhancement and analysis for computer-aided diagnosis of breast cancer; region-based image processing; knee-joint vibration signal analysis for noninvasive diagnosis of articular cartilage pathology; and analysis of textured images by cepstral filtering and sonification. He has published more than 150 papers in journals and 250 papers in proceedings of conferences. He has lectured extensively in many countries, including India, Canada, United States, Brazil, Argentina, Uruguay, Chile, United Kingdom, The Netherlands, France, Spain, Italy, Finland, Russia, Romania, Croatia, Egypt, Malaysia, Thailand, China, and Japan. He has collaborated with many research groups in India, Brazil, Italy, Spain, France, Romania, and China.

He was an Associate Editor of the *IEEE Transactions on Biomedical Engineering* from 1989 to 1996; the Program Chair and Editor of the Proceedings of the IEEE Western Canada Exhibition and Conference on “Telecommunication for Health Care: Telemetry, Teleradiology, and Telemedicine,” July 1990, Calgary, Alberta, Canada; the Canadian Regional Representative to the Administrative Committee of the IEEE Engineering in Medicine and Biology Society (EMBS), 1990–1993; a Member of the Scientific Program Committee and Editorial Board, International Symposium on Computerized Tomography, Novosibirsk, Siberia, Russia, August 1993; the Program Chair and Coeditor of the *Proceedings of the 15th Annual International Conference of the IEEE EMBS*, October 1993, San Diego, CA; and Program Cochair, 20th Annual International Conference of the IEEE EMBS, Hong Kong, October 1998.

He has been awarded the Killam Resident Fellowship thrice in support of his book-writing projects. He has been recognized with the 1997 and 2001 Research Excellence Awards of the Department of Electrical and Computer Engineering, the 1997 Research Award of the Faculty of Engineering, and by appointment as “University Professor” (2003–2013), at the University of Calgary. He is the author of two textbooks: *Biomedical Signal Analysis* (IEEE/ Wiley, 2002/2015) and *Biomedical Image Analysis* (CRC, 2005). He has coauthored and coedited several other books, including *Color Image Processing with Biomedical Applications* (SPIE, 2011). He has been elected as Fellow, IEEE (2001); Fellow, Engineering Institute of Canada (2002); Fellow, American Institute for Medical and Biological Engineering (2003); Fellow, SPIE (2003); Fellow, Society for Imaging Informatics in Medicine (2007); Fellow, Canadian Medical and Biological Engineering Society (2007); and Fellow, Canadian Academy of Engineering (2009). He has been recognized with the IEEE Third Millennium Medal (2000), the Distinguished Alumni award of the PES College of Engineering (2012), and the Outstanding Engineer Medal by IEEE Canada (2013).

SYMBOLS AND ABBREVIATIONS

Note: Boldfaced letters represent the vectorial or matrix form of the variable indicated by the corresponding italicized letters. Variables or symbols used within limited contexts are not listed: they are described within their contexts. The mathematical symbols listed may stand for other entities or variables in different applications; only the common associations are listed for ready reference.

a_k	autoregressive model or filter coefficients
au	arbitrary units
$aV\{F, L, R\}$	augmented ECG leads
Ag	silver
$AgCl$	silver chloride
A_z	area under the ROC curve
ACF	autocorrelation function
ADC	analog-to-digital converter
AHI	apnea–hypopnea index
AI	aortic insufficiency
AM	amplitude modulation
ANC	adaptive noise cancellation
ANN	artificial neural network
ANS	autonomic nervous system
AO	aorta, aortic (valve or pressure)

AP	action potential
AR	interval between atrial activity and the corresponding QRS
AR	autoregressive (model or filter)
ARMA	autoregressive, moving-average (model or filter)
AS	aortic stenosis
ASD	atrial septal defect
AV	atrioventricular
A2	aortic component of the second heart sound
b	bit
b_l	moving-average model or filter coefficients
bpm	beats per minute
BCG	ballistocardiogram
BP	blood pressure
BSS	blind source separation
C	covariance matrix
Ca	calcium
C_i	the i^{th} class in a pattern classification problem
Cl	chlorine
C_{xy}	covariance between x and y
CA	constant area
CAD	computer-aided diagnosis
CCF	cross-correlation function
CD	compact disk
CNS	central nervous system
CO	carbon monoxide
CP	carotid pulse
CPAP	continuous positive airway pressure
CSA	central sleep apnea
CSD	cross-spectral density, cross-spectrum
CV	coefficient of variation
CWT	continuous wavelet transform
$diag$	diagonal of a matrix
D	dicrotic notch in the carotid pulse
DAC	digital-to-analog converter
DC	direct current; zero frequency
DFT	discrete Fourier transform
DM	diastolic murmur
DW	dicrotic wave in the carotid pulse
DWT	discrete wavelet transform
$e(n), E(\omega)$	model or estimation error
ECG	electrocardiogram, electrocardiography
ECoG	electrocorticogram
EEG	electroencephalogram
EGG	electrogastrogram
EM	electromagnetic

EMD	empirical mode decomposition
EMG	electromyogram
ENG	electroneurogram
EOG	electrooculogram
EP	energy parameter
ERP	event-related potential
ESP	energy spread parameter
E_x	total energy of the signal x
$E[\]$	statistical expectation operator
f	frequency variable, usually in Hertz
fBm	fractional Brownian motion
f_c	cutoff frequency (usually at -3 dB) of a filter in Hertz
f_s	sampling frequency in Hertz
FD	fractal dimension
FDI	first dorsal interosseus
FF	form factor
FFT	fast Fourier transform
FIR	finite impulse response (filter)
FLDA	Fisher linear discriminant analysis
FM	frequency modulation
FN	false negative
FNF	false-negative fraction
FP	false positive
FP	frequency parameter
FPF	false-positive fraction
FSP	frequency spread parameter
FT	Fourier transform
GLR	generalized likelihood ratio
GTFD	generalized time-frequency distribution
h	hour
$h(t), h(n)$	impulse response of a filter
H	entropy
H	Hurst coefficient
H	Hermitian (complex-conjugate) matrix transposition
Hg	mercury
$H(s), H(z)$	transfer function of a filter
$H(s)$	Laplace transform of $h(t)$
$H(z)$	z -transform of $h(n)$
$H(\omega)$	frequency response of a filter
$H(\omega)$	Fourier transform of $h(t)$
HR	heart rate
HRV	heart rate variability
HSS	hypertrophic subaortic stenosis
Hz	Hertz
i	index of a series or discrete-time signal

ICA	independent component analysis
IFT	inverse Fourier transform
IIR	infinite impulse response (filter)
IMF	instantaneous mean frequency
IMF	intrinsic mode function
IPI	interpulse interval
j	index of a series or discrete-time signal
j	$\sqrt{-1}$
JM	Jeffries–Matusita
k -NN	k nearest neighbors
K	kurtosis
K	potassium
KLD	Kullback–Leibler distance or divergence
KLT	Karhunen–Loève transform
\ln	natural logarithm (base e)
L_{ij}	loss function in pattern classification
LA	left atrium
LED	light emitting diode
LHS	left-hand side
LMS	least mean squares
LOO	leave one out
LP	linear prediction (model)
LSI	linear shift-invariant
LTI	linear time-invariant
LV	left ventricle
m	mean
m	mean vector of a pattern class
mA	milliamperes
min	minute
mm	millimeter
ms	millisecond
mV	millivolt
M	number of samples
MA	moving average
MCI	muscle-contraction interference
MI	mitral insufficiency
MMSE	minimum mean-squared error
MP	Matching Pursuit
MPC	minimum-phase correspondent
MPTFD	Matching Pursuit time-frequency distribution
MR	mitral regurgitation
MS	mitral stenosis
MS	mean-squared (value)
MSE	mean-squared error
MU	motor unit

MUAP	motor-unit action potential
MVC	maximal voluntary contraction
nA	nanoamperes
N	number of samples
N	filter order
Na	sodium
NPV	negative predictive value
p_k	pole of a model
$p(x)$	probability density function of the random variable x
$p(x C_i)$	likelihood function of class C_i or state-conditional PDF of x
ppm	pulses per minute
pps	pulses per second
Pa	Pascal
OAE	otoacoustic emission
OLS	orthogonal least-squares
OMPTFD	optimized Matching Pursuit time-frequency distribution
OSA	obstructive sleep apnea
P	atrial contraction wave in the ECG
P	percussion wave in the carotid pulse
P	model order or number of poles
$P(x)$	probability of the event x
$P(C_i x)$	posterior probability that the observation x is from class C_i
PA	predictive area
PCA	principal component analysis
PCG	phonocardiogram
PDA	patent ductus arteriosus
PDF	probability density function
PFP	patellofemoral pulse trains or signals
PI	pulmonary insufficiency
PLP	posterior leaflet prolapse
PNS	parasympathetic nervous system
PPC	physiological patellofemoral crepitus
PPV	positive predictive value
PQ	isoelectric segment in the ECG before ventricular contraction
PS	pulmonary stenosis
PSA	power spectral analysis
PSD	power spectral density, power spectrum
PSG	polysomnography
PVC	premature ventricular contraction
P2	pulmonary component of the second heart sound
Q	model order or number of zeros
QRS	ventricular contraction wave in the ECG
QRSTA	area under the QRS and T waves
r, \mathbf{r}	reference input to an adaptive filter
$r_j(\mathbf{x})$	average risk or loss in pattern classification

RA	right atrium
RBF	radial basis function
RBFN	radial basis function network
REM	rapid eye movement
RHS	right-hand side
RF	radio-frequency
RLS	recursive least-squares
RLSL	recursive least-squares lattice
RMS	root mean squared (value)
ROC	receiver operating characteristics
ROC	region of convergence
RR	interval between two successive QRS waves in an ECG
RSPWVD	reassigned smoothed pseudo Wigner–Ville distribution
RV	right ventricle
s	second
s	Laplace-domain variable
sec	second (in figures from other sources)
sgn	signum (sign)
S	skewness
$S(\omega), S(k)$	auto- or cross-spectral density; power spectral density
SA	sinoatrial
SpO_2	level of oxyhemoglobin in blood
SD	standard deviation
SEM	spectral error measure
SEP	somatosensory evoked potential
SL	signal length
SM	systolic murmur
SMUAP	single-motor-unit action potential
SNR	signal-to-noise ratio
SNS	sympathetic nervous system
SpO_2	level of oxyhemoglobin in blood
SPWVD	smoothed pseudo Wigner–Ville distribution
ST	isoelectric segment in the ECG during ventricular contraction
STFT	short-time Fourier transform
SWVD	smoothed Wigner–Ville distribution
S1	first heart sound
S2	second heart sound
S3	third heart sound
S4	fourth heart sound
S^+	sensitivity of a test
S^-	specificity of a test
t	time variable
T	ventricular relaxation wave in the ECG
T	tidal wave in the carotid pulse
T	sampling interval

T	as a superscript: vector or matrix transposition
T^+	positive test result
T^-	negative test result
TCR	turns count rate
TF	time-frequency
TFD	time-frequency distribution
Th	threshold
TI	tricuspid insufficiency
TN	true negative
TNF	true-negative fraction
TP	true positive
TPF	true-positive fraction
Tr	trace of a matrix (sum of the diagonal entries)
TS	tricuspid stenosis
TSE	total squared error
TV	television
V	Volt
V1 – V6	chest leads for ECG
VAG	vibroarthrogram
VCG	vectorcardiography
VLf	very low frequency
VMG	vibromyogram
VSD	ventricular septal defect
w	filter tap weight; weighting function
\mathbf{w}	filter weight vector
WP	wavelet packet
WT	wavelet transform
WVD	Wigner–Ville distribution
$x(t), x(n)$	a signal in the time domain; usually denotes input
\mathbf{x}	vectorial representation of the signal $x(n)$
\mathbf{x}	a feature vector in pattern classification
$X(f), X(\omega)$	Fourier transform of $x(t)$
$X(k)$	discrete Fourier transform of $x(n)$
$X(z)$	z -transform of $x(n)$
$X(\tau, \omega)$	short-time Fourier transform or time-frequency distribution of $x(t)$
$y(t), y(n)$	a signal in the time domain; usually denotes output
\mathbf{y}	vectorial representation of the signal $y(n)$
$Y(f), Y(\omega)$	Fourier transform of $y(t)$
$Y(k)$	discrete Fourier transform of $y(n)$
$Y(z)$	z -transform of $y(n)$
z	the z -transform variable
z^{-1}	unit delay operator in discrete-time systems
z_l	zeros of a system
\mathbf{z}	a prototype feature vector in pattern classification
ZCR	zero-crossing rate

ZT	the z -transform
1D	one-dimensional
2D	two-dimensional
3D	three-dimensional
I, II, III	limb leads for ECG
α	an EEG wave
β	an EEG wave
β	spectral component
γ	an EEG wave
γ_i	reflection coefficient
γ_{xy}	correlation coefficient between x and y
Γ_{xy}	coherence between x and y
δ	an EEG wave
δ	Dirac delta (impulse) function
ε	total squared error
η	a random variable or noise process
θ	an angle
θ	a threshold
θ	an EEG wave
θ, Θ	cross-correlation function
λ	forgetting factor in the RLS filter
μ	the mean (average) of a random variable
μ	a rhythmic wave in the EEG
μ	step size in the LMS filter
μV	microvolt
μm	micrometer
μs	microsecond
ρ	correlation coefficient
σ	the real part of the Laplace variable s (Neper frequency)
σ	the standard deviation of a random variable
σ^2	the variance of a random variable
τ	a time interval, delay, or shift
ϕ, Φ	autocorrelation
ω	frequency variable in radians per second
Ω	frequency variable in radians per second
*	when in-line: convolution
*	as a superscript: complex conjugation
$-$	average or normalized version of the variable
\wedge	complex cepstrum of the signal, if a function of time
\wedge	complex logarithm of the signal, if a function of frequency
\sim	estimate of the variable under the symbol
$'$, $''$	first and second derivatives of the preceding function
\forall	for all
\in	belongs to or is in (the set)
$ $	absolute value or magnitude of

\angle	argument of, angle of
$\lceil \rceil$	ceiling
$\lfloor \rfloor$	floor
\langle, \rangle or \cdot	dot (inner) product