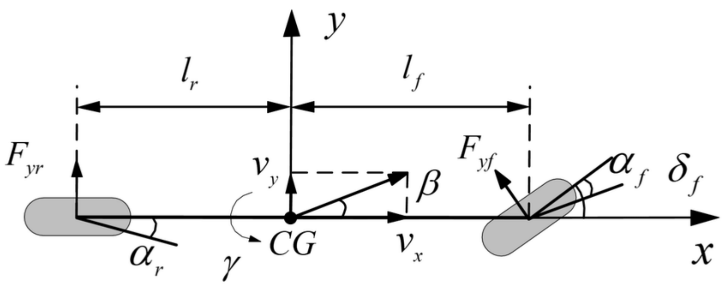
Linear vehicle lateral dynamics

$$\frac{d}{dt}\begin{Bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & 0 & -V_x-\dfrac{2C_{\alpha f}\ell_f-2C_{\alpha r}\ell_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{2\ell_f C_{\alpha f}-2\ell_r C_{\alpha r}}{I_z V_x} & 0 & -\dfrac{2\ell_f^{\,2} C_{\alpha f}+2\ell_r^{\,2} C_{\alpha r}}{I_z V_x} \end{bmatrix} + \begin{Bmatrix} 0 \\ \dfrac{2C_{\alpha f}}{m} \\ 0 \\ \dfrac{2\ell_f C_{\alpha f}}{I_z} \end{Bmatrix}\delta$$



- Constant longitudinal velocity (reasonable assumption)
- Nonlinearity in tire behavior ignored citing small steering angles (not reasonable in high speed evasive maneuvers)
- Cornering stiffness and Z-Inertia varying parameters over course of a maneuver.

Initial estimate of Cornering stiffness

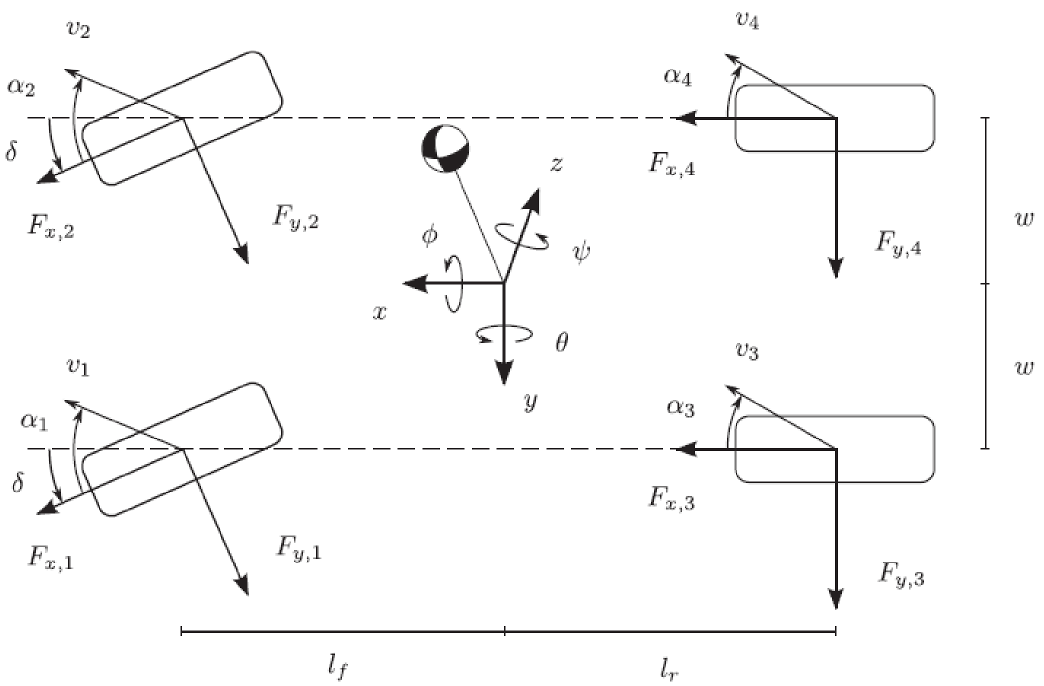
$$C_\alpha = \mu F_z BC$$

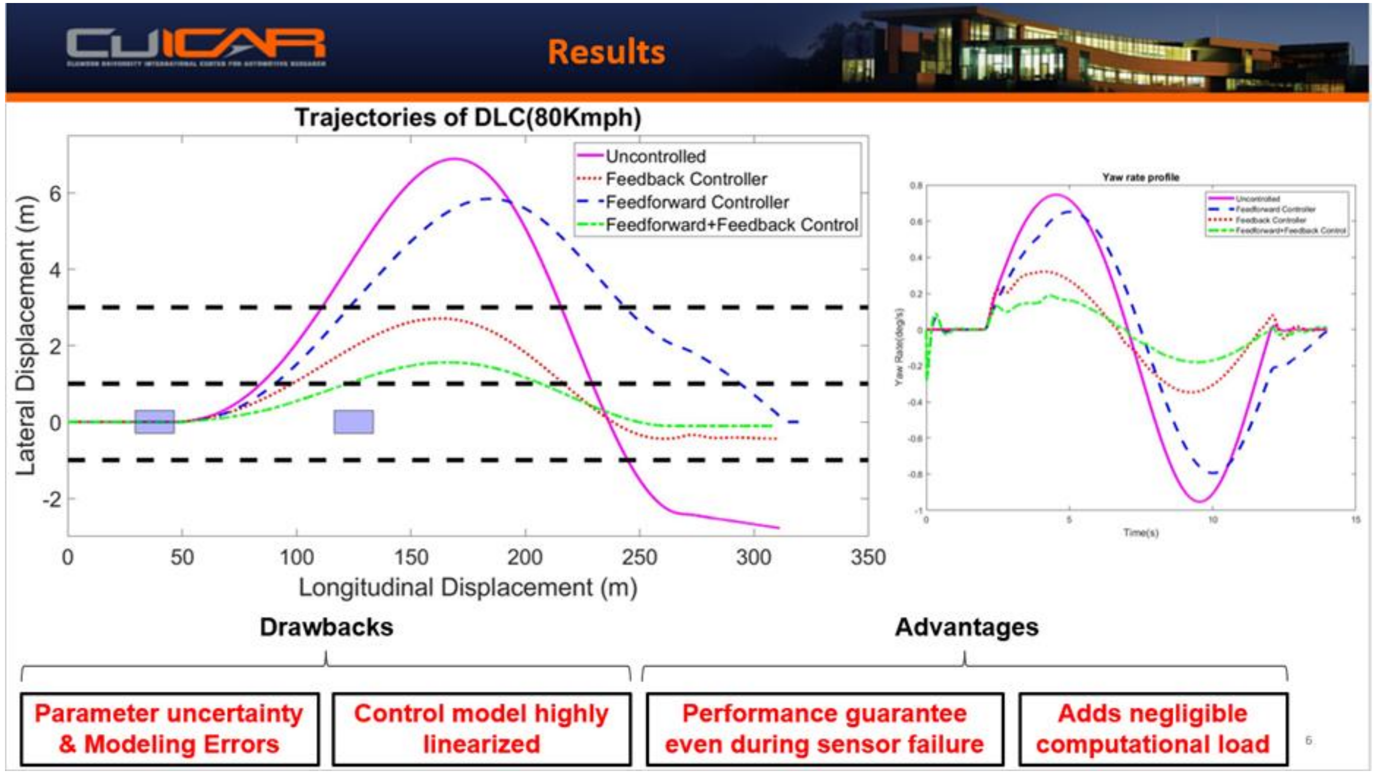
Where $F_{zf} = mgl_r/L$ and $F_{zf} = mgl_f/L \quad (L = l_f + l_r)$

Again not a reasonable assumption due to load shifts, roll, pitch, banking, etc.

IDEAL BEHAVIOR ALTHOUGH UNREALISTIC AFTER 55 Km/Hr

Nonlinear plant/Real vehicle

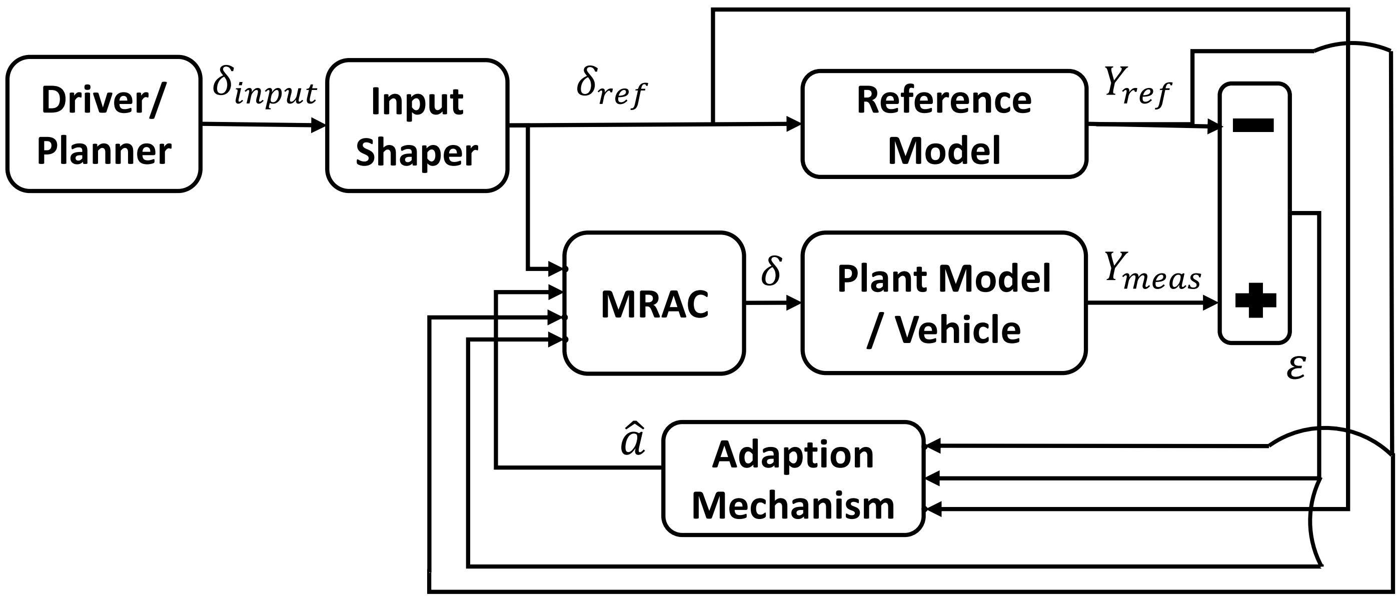




- Complex coupled dynamics.
- High nonlinearities.

- In previous paper we proved that Input shaper doesn't work standalone due to parameter variations in the nonlinear range (>55 KmpH).
- Needs to paired with a feedback controller in order to work as desired.

Steer-By-Wire Control Strategies To Enforce Linearity During High Speed Maneuvers



Nonlinear system dynamics (Unknown plant) $\dot{x} = f(x)$

Ideal linear system dynamics (Known model) $\dot{x} = A_{ref}x + B_{ref}u_{ref}$

States of interest: $\overline{\boldsymbol{x}} = \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix}$

Assuming smooth measurements/estimates are available for now, which is not an unreasonable assumption. IMU+Kalman would do.

$u_{ref} = IS * u_{raw}$

- Due to the nature of MRAC, the output of Nonlinear plant and Linear plant need to be the same.
- But the nonlinear plant is an 18 state system.
- Did not cause a problem however, the 2 states' output was good enough.
- Important to note, no velocity control here. 0 torque input and near constant velocity is assumed (Reasonable assumption for evasion/overtake during cruise control type maneuvers).

- When given an u_{ref} the linear vehicle will exactly provide the desired response.
- i.e. That is the best performance you can get in the given conditions.
- But as discussed earlier, the actual nonlinear plant will NOT.

We do not know the plant model or its dynamic parameters. We only get measurements.

Lets initially guess our plant to be of the form:

$$\dot{x} = Ax + Bu$$

Reference model is of the form:

$$\dot{x} = A_{ref}x + B_{ref}u$$

There exists a $\bar{u}$ which when applied causes model matching conditions and asymptotic tracking ($\lim_{t\to\infty} \varepsilon = 0$) such that:

$$\begin{matrix}\bar{u} \\ \bar{x}\end{matrix} = \begin{bmatrix} F11 & F12 \\ F21 & F22 \end{bmatrix} \begin{matrix} u_{ref} \\ x_{ref} \end{matrix}$$

So control law can just be:

$$\bar{u} = F11x_{ref} + F12u_{ref}$$

WONT WORK. Fallacy in assumption. Plant is not linear.
So the above control law will still cause errors.

So we introduce another state-feedback gain for the error dynamics, to drive down the error to 0.

So develop a control law that is of the form:

$$u_{MRAC} = \hat{a}_x^T * x_{ref} + \hat{a}_\varepsilon^T * \varepsilon + \hat{a}_u^T * u_{ref}$$

Feedback gains　　Feedforward gain

Learnt gains that have been adapted based on the errors Ɛ .

$$\varepsilon = x_p - x_r = \begin{bmatrix} \dot{y}_p \\ \dot{\psi}_p \end{bmatrix} - \begin{bmatrix} \dot{y}_r \\ \dot{\psi}_r \end{bmatrix}$$

Picked based on Lyapunov theory for stability. These gains result In boundedness and asymptotic tracking

The adaption laws are given by:

$$\dot{\hat{a}}_x = -\gamma_x x_{ref} \varepsilon^T P B$$

$$\dot{\hat{a}}_u = -\gamma_u u_{ref} \varepsilon^T P B$$

$$\dot{\hat{a}}_\varepsilon = -\gamma_\varepsilon \varepsilon \varepsilon^T P B$$

γ_x and γ_ε are matrices that represent a learning rate to adapt the feedback gain

γ_u is a similar matrix to adapt the feedforward gain

P is an unique symmetric positive definite solution of the algebraic Lyapunov equation $PA_{ref} + A_{ref}^T + Q = 0$

B is a coefficient matrix of the plant. Refer next slide

MRAC's structure here requires the system to be of the form:

$$\dot{x} = Ax + Bu + f(x)$$

Here 'A' can be unknown matrix but the 'B' is a known matrix.

We can consider B to be known for mild nonlinearities where in:

- The constant velocity and linear tire assumptions are kept true
- Small steering angles assumption is not.

Linear

$$m\ddot{y} = F_{yf} + F_{yr} - mV_x\dot{\psi}$$

$$I_z\ddot{\psi} = l_f F_{yf} - l_r F_{yr}$$

$$F_{yf} = 2C_{\alpha f}(\delta - \theta_{vf})$$

$$F_{yr} = 2C_{\alpha r}(-\theta_{vr})$$

$$\theta_{vf} = \frac{\dot{y} + l_f\dot{\psi}}{V_x} \qquad \theta_{vr} = \frac{\dot{y} - l_r\dot{\psi}}{V_x}$$

$$\longleftarrow \quad B_{ref} = 2 * \begin{bmatrix} \frac{C_{\alpha f}}{m} \\ \frac{C_{\alpha f} l_f}{I_z} \end{bmatrix}$$

$$B = 2\cos\delta * \begin{bmatrix} \frac{C_{\alpha f}}{m} \\ \frac{C_{\alpha f} l_f}{I_z} \end{bmatrix} \quad \longrightarrow$$

$$\cong \quad 2\Lambda * \begin{bmatrix} \frac{C_{\alpha f}}{m} \\ \frac{C_{\alpha f} l_f}{I_z} \end{bmatrix}$$

Λ is a tuning factor

Mild Nonlinearity

$$m\ddot{y} = F_{yf}\cos\delta + F_{yr} - mV_x\dot{\psi}$$

$$I_z\ddot{\psi} = l_f F_{yf}\cos\delta - l_r F_{yr}$$

$$F_{yf} = 2C_{\alpha f}(\delta - \theta_{vf})$$

$$F_{yr} = 2C_{\alpha r}(-\theta_{vr})$$

$$\theta_{vf} = \tan^{-1}\frac{\dot{y} + l_f\dot{\psi}}{V_x} \qquad \theta_{vr} = \tan^{-1}\frac{\dot{y} - l_r\dot{\psi}}{V_x}$$

Built in parameter adaption

MRAC can be built with inbuilt parameter adaption in the system. This is based on a the following assumptions:

- The system is of MRAC's general form i.e. $\dot{x} = Ax + Bu + f(x))$

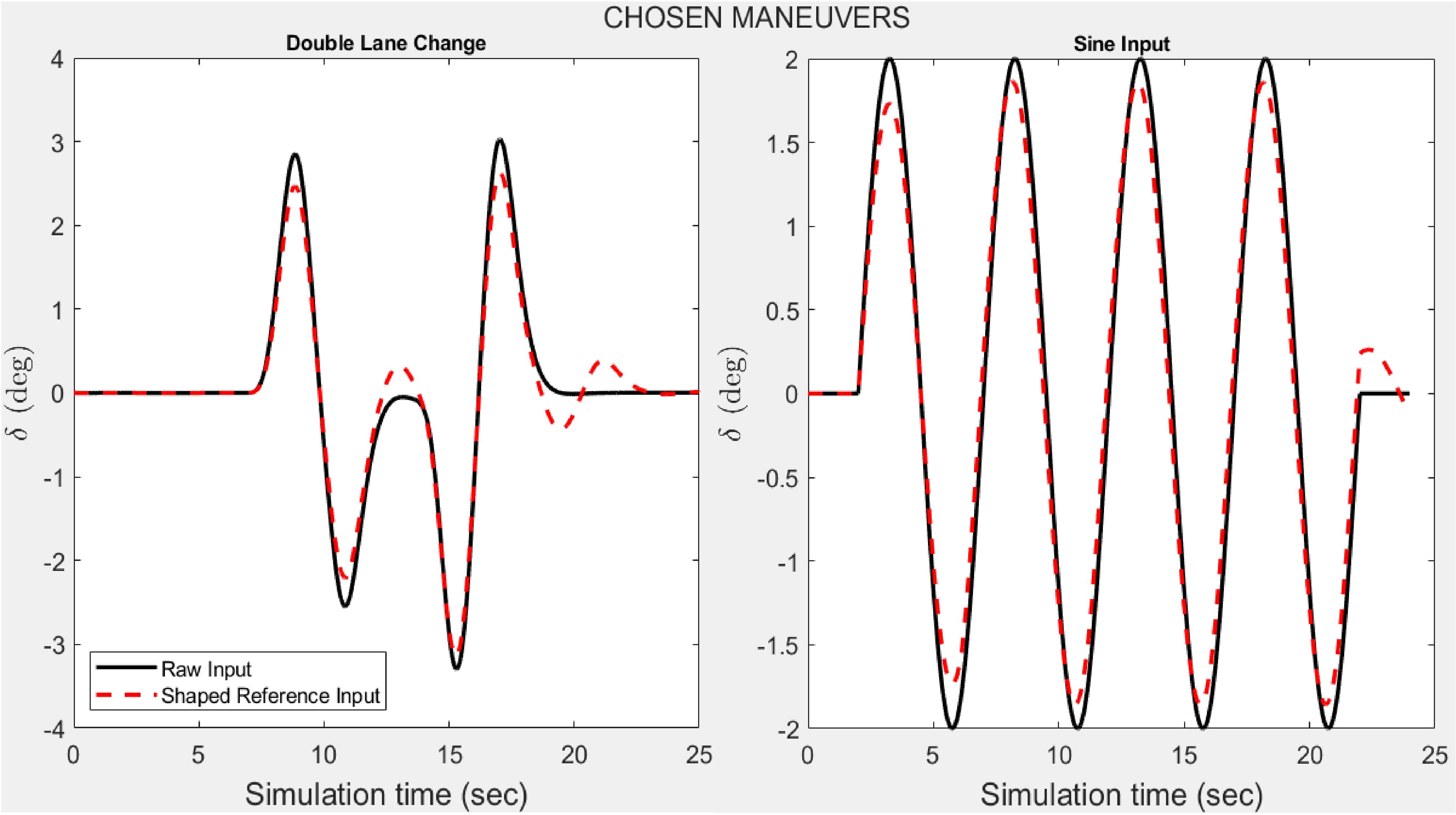
- $f(x)$ can be linearly parametrized as $\theta^T\phi$ or $\theta^T\phi + e(x)$ where $e(x)$ represents residual error.

- ϕ is a set of known basis functions known from our knowledge of the nonlinear plant.

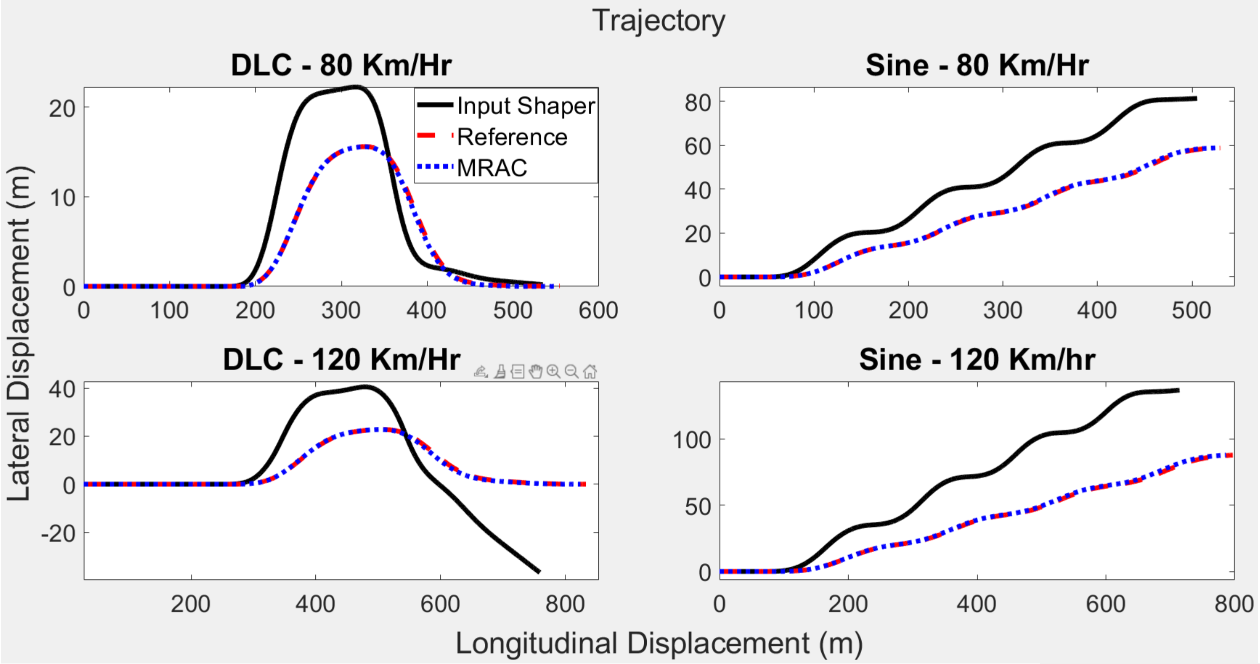
This assumption is not convenient in our case because the nonlinear vehicle system even for a case such as 3DoF nonlinear vehicle model cannot be parametrized in that manner.

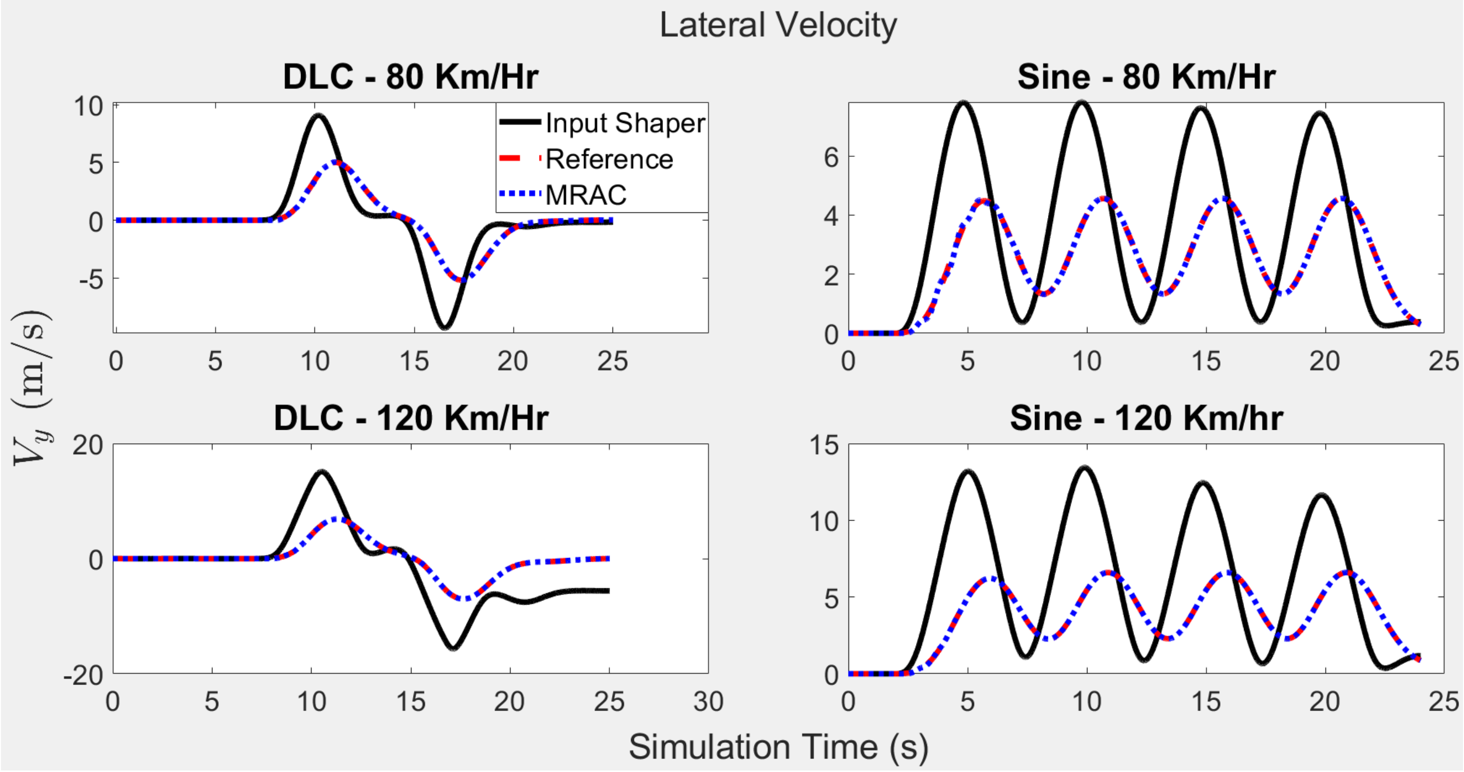
If we knew the basis functions control law would just be:

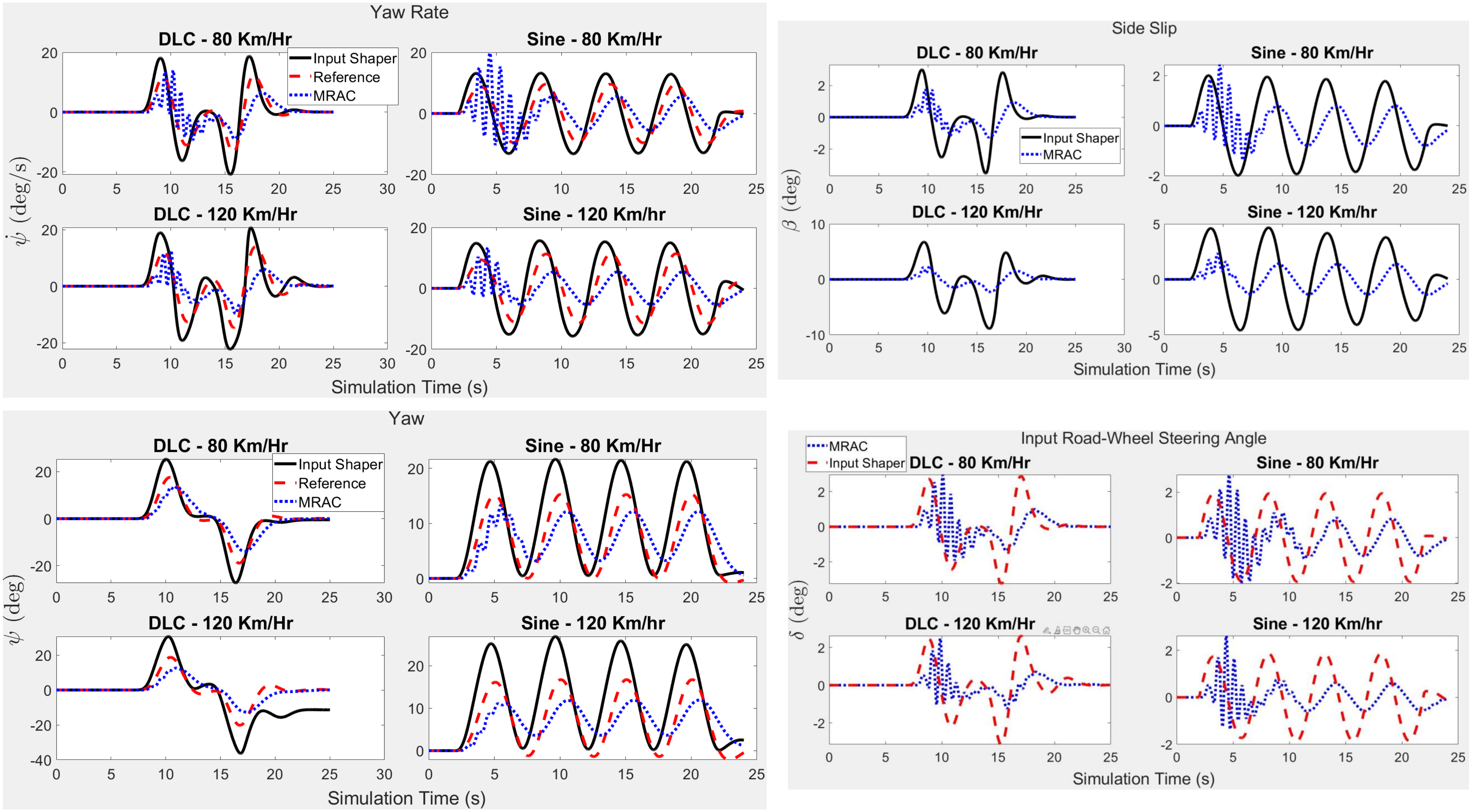
$$u_{MRAC} = \hat{a}_x^T * x_{ref} + \hat{a}_u^T * u_{ref} - \theta^T\phi$$



Test Velocities: 80 KmpH and 120 KmpH. Absolutely violent maneuvers for such speeds. Tend to excite all the nonlinearities in the system

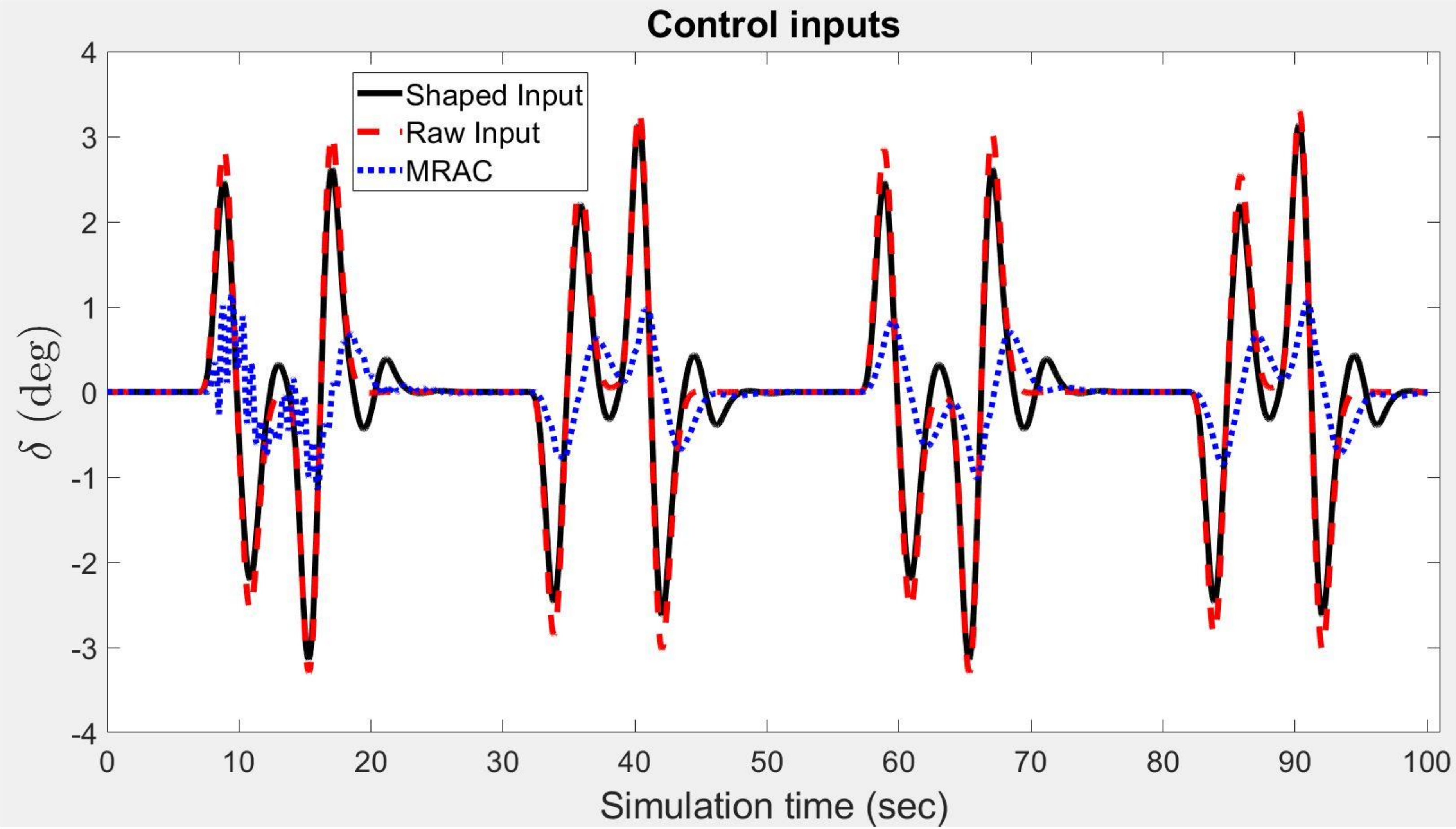

Trajectory
DLC - 80 Km/Hr
Sine - 80 Km/Hr
DLC - 120 Km/Hr
Sine - 120 Km/hr
Input Shaper
Reference
MRAC
Lateral Displacement (m)
Longitudinal Displacement (m)

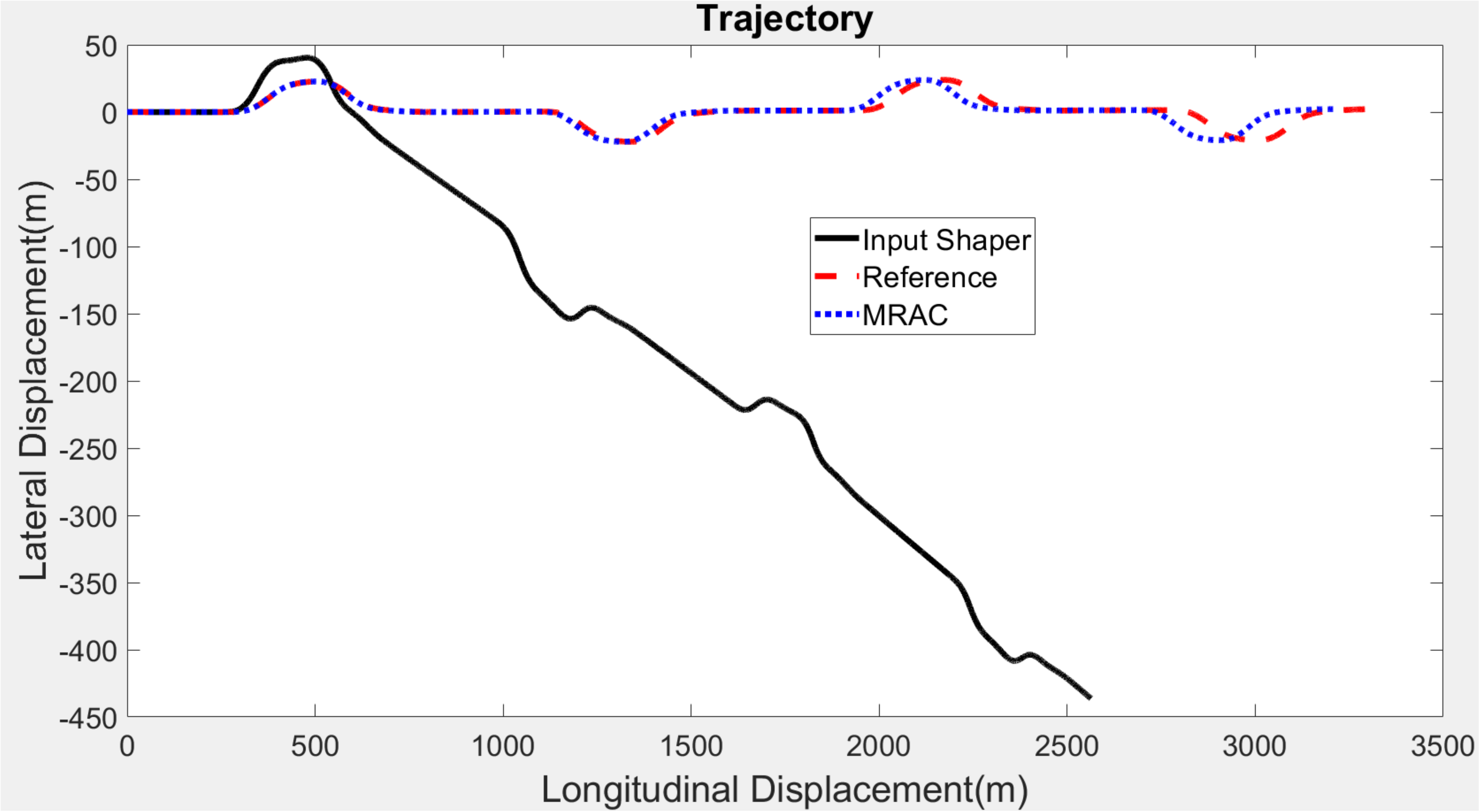

Lateral Velocity
DLC - 80 Km/Hr
Sine - 80 Km/Hr
DLC - 120 Km/Hr
Sine - 120 Km/hr
Input Shaper
Reference
MRAC
V_y (m/s)
Simulation Time (s)

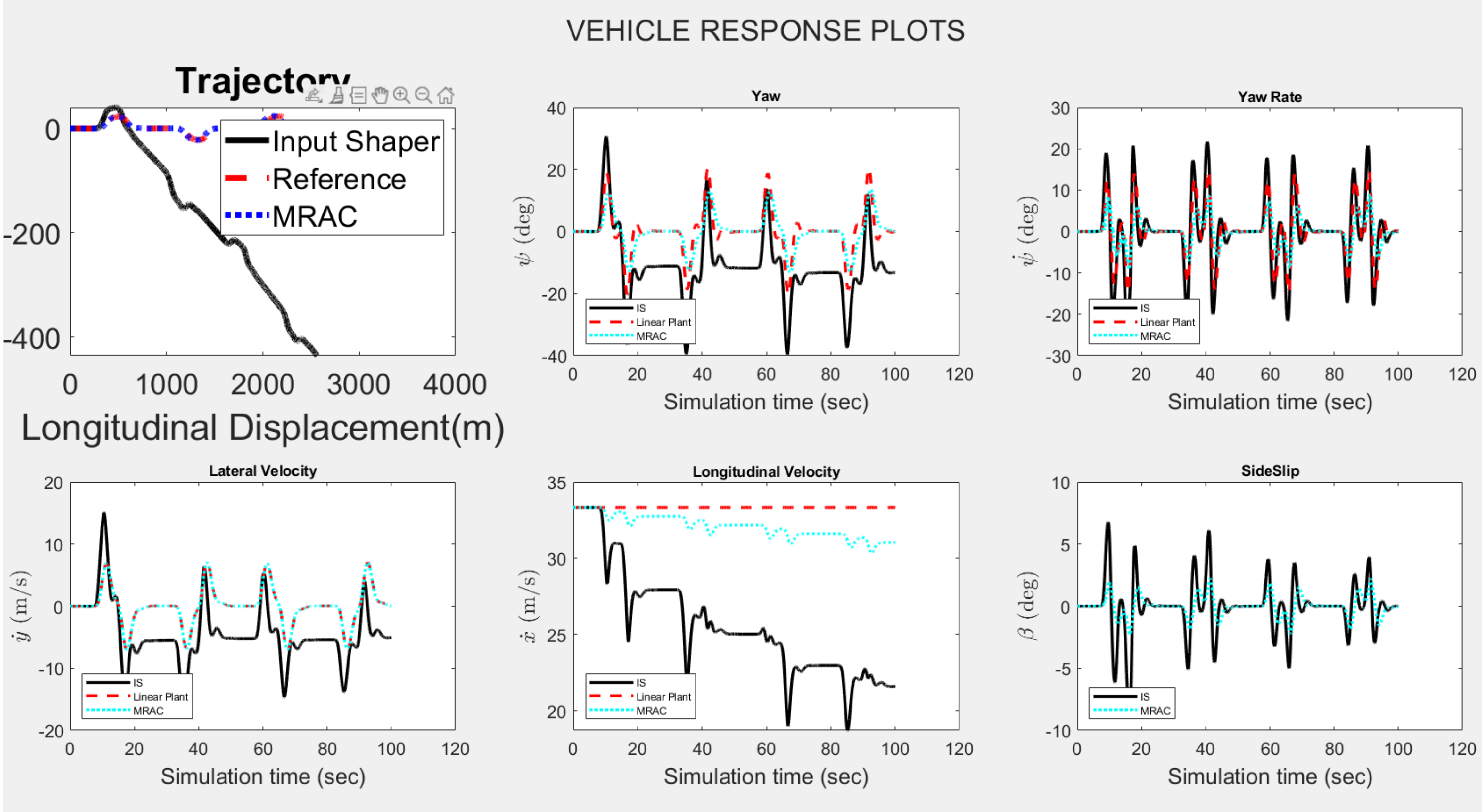


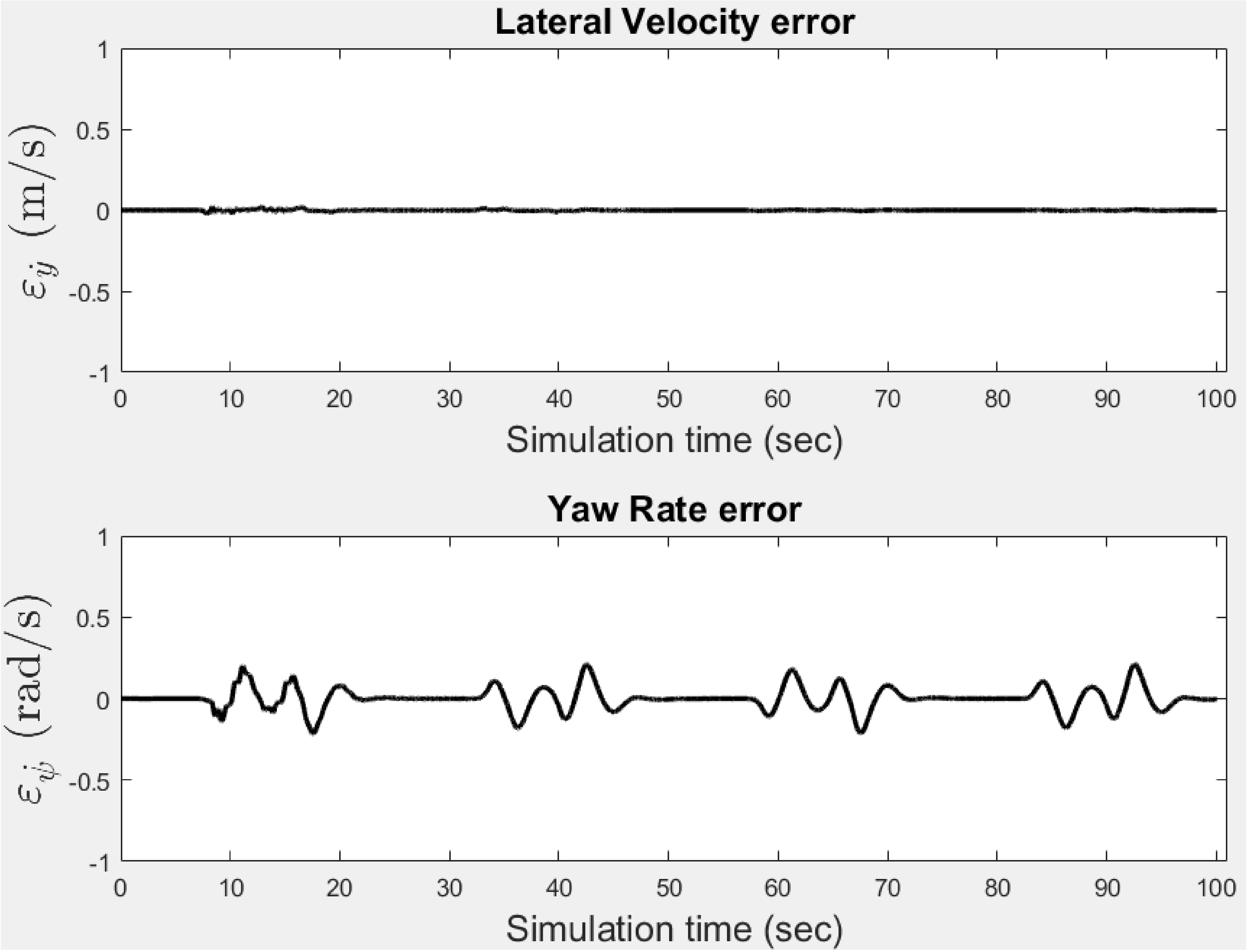
Yaw Rate
DLC - 80 Km/Hr
Sine - 80 Km/Hr
DLC - 120 Km/Hr
Sine - 120 Km/hr
Input Shaper
Reference
MRAC
$\dot{\psi}$ (deg/s)
Simulation Time (s)
Side Slip
DLC - 80 Km/Hr
Sine - 80 Km/Hr
DLC - 120 Km/Hr
Sine - 120 Km/hr
Input Shaper
MRAC
β (deg)
Simulation Time (s)
Yaw
DLC - 80 Km/Hr
Sine - 80 Km/Hr
DLC - 120 Km/Hr
Sine - 120 Km/hr
Input Shaper
Reference
MRAC
ψ (deg)
Simulation Time (s)
Input Road-Wheel Steering Angle
MRAC
Input Shaper
DLC - 80 Km/Hr
Sine - 80 Km/Hr
DLC - 120 Km/Hr
Sine - 120 Km/hr
δ (deg)
Simulation Time (s)

Testing boundedness and convergence : Repetitive DLC at 120 KmpH

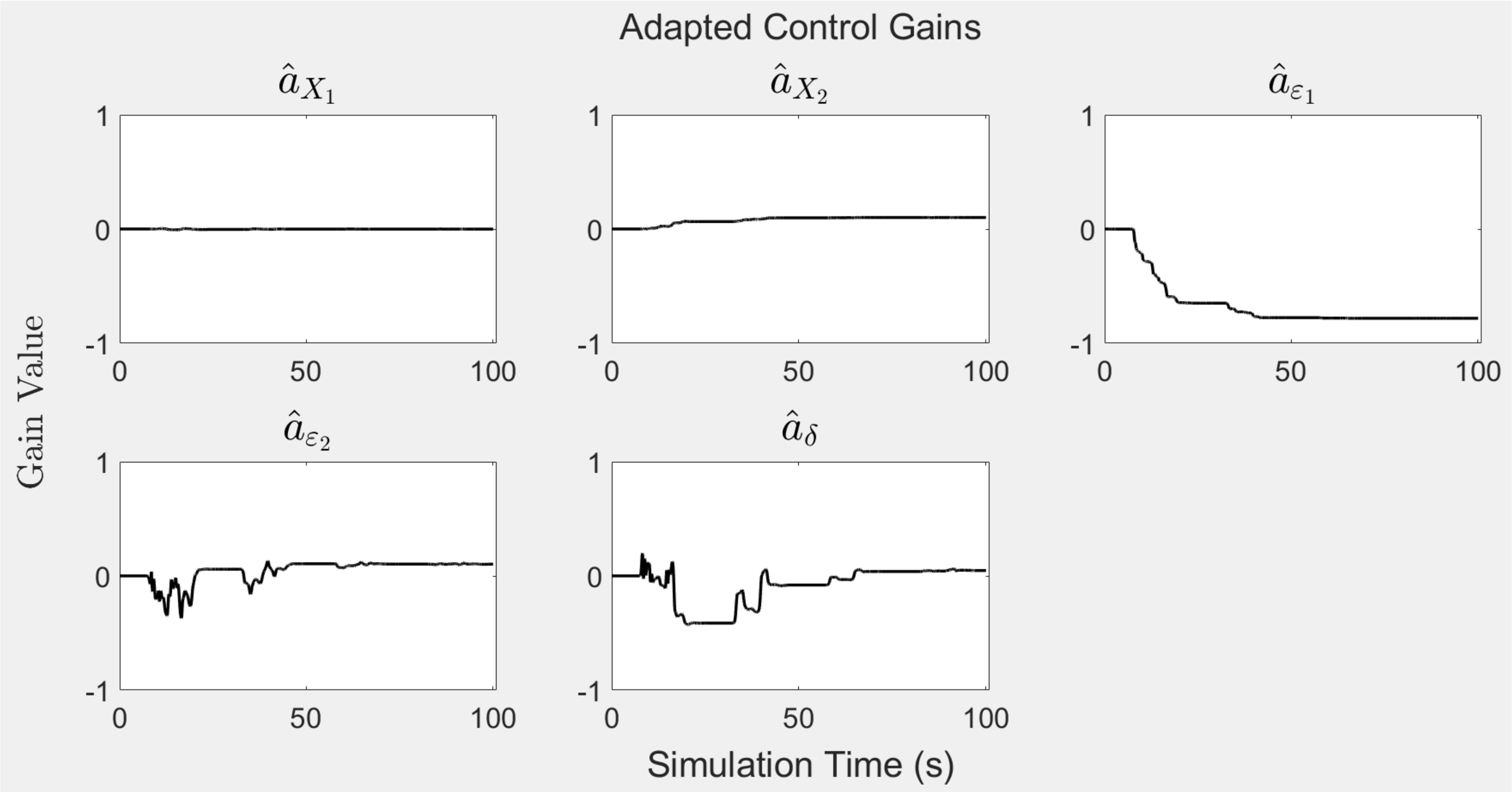



Trajectory
Lateral Displacement(m)
Longitudinal Displacement(m)
Input Shaper
Reference
MRAC
50
0
-50
-100
-150
-200
-250
-300
-350
-400
-450
0
500
1000
1500
2000
2500
3000
3500


VEHICLE RESPONSE PLOTS
Trajectory
Input Shaper
Reference
MRAC
Longitudinal Displacement(m)
Yaw
ψ (deg)
Yaw Rate
$\dot{\psi}$ (deg)
Lateral Velocity
$\dot{y}$ (m/s)
Longitudinal Velocity
$\dot{x}$ (m/s)
SideSlip
β (deg)
IS
Linear Plant
MRAC
Simulation time (sec)



Lateral Velocity error
$\varepsilon_{\dot{y}}$ (m/s)
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80
90
100
Simulation time (sec)
Yaw Rate error
$\varepsilon_{\dot{\psi}}$ (rad/s)
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80
90
100
Simulation time (sec)


Adapted Control Gains
$\hat{a}_{X_1}$
$\hat{a}_{X_2}$
$\hat{a}_{\varepsilon_1}$
$\hat{a}_{\varepsilon_2}$
$\hat{a}_{\delta}$
Gain Value
Simulation Time (s)
1
0
-1
0
50
100