COMP5411 Report: Laplacian Mesh Editing Implementation

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1 Introduction

As the one of the most representative mesh deformation methods, Laplacian Mesh Editing[2] helps us to find the new vertex coordinates for a plausible manipulation.

This report reproduces the method of Laplacian mesh editing and makes improvement on the local neighbouring differential weighting as well as computational complexity. We would demonstrate the effectiveness of our approach with examples, showing that the Laplacian mesh editing changes the shape while respecting the structural geometric detail.

At the discussion, we would analyze the results from Laplacian mesh editing and point out the crucial drawback of this model.

2 Methodology

2.1 Least-squares sense

Our goal of Laplacian mesh editing is to reconstruct the surface concerning the moved handles, at the same time, to preserve surface details as much as possible. Thus, we solve it in the least-squares sense, which is equivalent to minimizing the following quadratic energy with constraints in C:

$$\widetilde{V'} = \arg\min \left(\sum_{i=1}^{n} \left\| L\left(v_i'\right) - \delta_i \right\|^2 + \sum_{j \in C} \left\| v_j' - \mu_j \right\|^2 \right) \tag{1}$$

where the first term corresponds to the Regularization term and the second term corresponds to the Match term. With both terms, we can find a compromise in $\widetilde{V'}$ with δ_i and μ_i which denotes original Laplacian vectors and new vertex positions respectively.

2.2 Laplacian Matrix construction

Laplacian Matrix L is the nxn curvature-flow Laplace operator with elements, whose calculation formula is given by:

$$L_{i,j} = \begin{cases} -\frac{\omega_{i,j}}{\sum \omega_{i,j}} & if\ (i,j) \in E \\ 1 & if\ i = j \end{cases} \tag{2}$$

$$Otherwise\ L_{i,j} = 0$$

We use cotangent weighting to preserve angles and consequently areas, which are much better than umbrella weighting.[1] As a result:

$$\omega_{i,j} = \frac{1}{2}(\cot \alpha_{i,j} + \cot \beta_{i,j}) \tag{3}$$

$\alpha_{i,j}$ and $\beta_{i,j}$ are the opposite angles corresponding to the edge (i, j), here is an example in Figure 1:

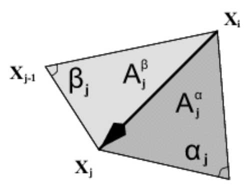
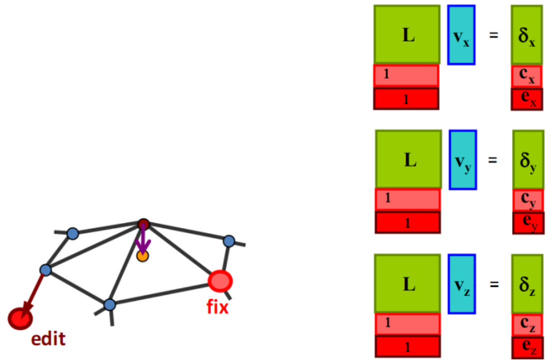


Figure 1: cotangent weighting example

2.3 A-matrix and b-matrix

After obtaining the Laplacian Matrix L, we can proceed to construct the A-matrix and b-matrix. The nxn matrix in the upper part of A-matrix is Laplacian Matrix L. The first n rows of b-matrix are δ_i, which is the product of Laplacian Matrix L and V. The remainings of A-matrix and b-matrix are determined according to the selection of constraints and handles.

An example is shown here 2a, this linear system assumes the simplest case: only one point as handle and only one point fixed as constraint.



(a) mesh form (b) matrix form

Figure 2: linear system example

For A-matrix, for each constraint or handle, the column entry of the corresponding row is one, and the rest entries in that row are zero. For b-matrix, the corresponding row of constraint is the initial position coordinate, and the corresponding row of handle is edited coordinate.Figure 2b shows the generation of A-matrix and b-matrix.

2.4 Reconstruction

We need to solve the reconstruction equation every time the modeling constraints are changed via moving the handles. Transform formulas by matrix multiplication:

$$\begin{aligned} AV &= B \\ A^T AV &= A^T\ B \\ V &= \left(A^T A\right)^{-1} A^T\ B \end{aligned} \tag{4}$$

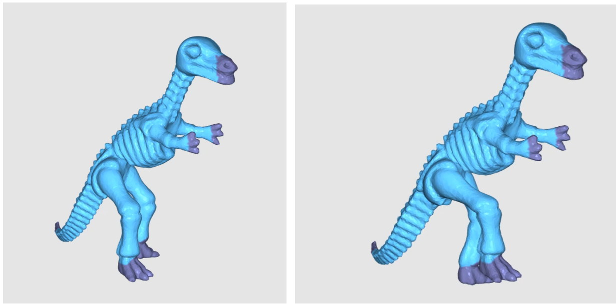
where $A^T A$ is sparse and positive definite.

Direct method is efficient up to over 100,000 vertices. However, for more vertices' situation, we would prefactorize use Cholesky factorization $A^T A = MM^T$ in Eigen library, where M is an upper-triangular sparse matrix. Then we only need to recompute the rhs formula when handles move.

3 Results

Figure 3a and Figure 3b show a mesh editing example of the dinosaur model. After implementing Laplacian mesh editing method, we could observe the smooth deformation from original meshes to the edited meshes.

Another observation is that the computation complexity of reconstruction mainly depends on the lhs factorization and matrix inversion. Solving the rhs formula is much faster.



(a) original (b) moving right foot

Figure 3: mesh editing example

4 Discussions

The experimental results fully demonstrate the excellent performance of Laplacian mesh editing in mesh deformation and mesh editing.

However, this method is based on differential coordinates. As a result, Laplacian mesh editing does not support large deformations as it is not rotation-invariant and attempts to preserve the original global orientation of the details.

5 Conclusion

In this assignment, we reproduce the method of Laplacian mesh editing and the algorithm was improved by replacing umbrella weighting with cotangent weighting. In the reconstruction process, we did not use the naive direct method, but applied Cholesky factorization to reduce the computational complexity.

Laplacian mesh editing could achieve smooth mesh deformation in small region. However, since the method itself uses differential coordinates, the mesh editing process is rotation-invariant. More researches in the future could be made on how to solve the shearing artifact because of this.

References

[1] Mathieu Desbrun, Mark Meyer, Peter Schröder, and Alan Barr. Implicit fairing of irregular meshes using diffusion and curvature flow. *SIGGRAPH*, 99, 04 2001. 2.2

[2] Olga Sorkine, Daniel Cohen-Or, Yaron Lipman, Marc Alexa, Christian Rossl, and Hans-Peter Seidel. Laplacian surface editing. In *Proceedings of the EUROGRAPHICS/ACM SIGGRAPH Symposium on Geometry Processing*, pages 179–188. ACM Press, 2004. 1