# Financial Technology Report

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## Abstract

This report focuses on discussing how to look for arbitrage opportunities in changing market from the perspective of market makers. Besides, statistical relationships among stocks and options, which are traded in our virtual exchange (i.e. optibook), are determined mathematically. Inspired by these relationships, a novel trading algorithm is created, which subtly combines option-quoting strategy and cointegration strategy. Surprisingly, this algorithm achieves remarkable profits in the process of trading options and/or stocks in optibook.


## 1. Introduction

### 1.1 Market maker

Generally, financial market participants could be classified into two groups according to their business, namely directional group and non-directional group. Directional group refers to people who trade financial instruments and make profit from the directional changes of value of those financial instruments, such as asset managers, investment banks, retail investors, etc. On the contrary, non-directional participants usually are reluctant to be exposed to directional risk, such as market makers, brokers, intermediaries, etc. Unlike the former, non-directional participants try their best to be market neutral and avoid loss in directional changes. As a representative of non-directional market participants, market makers play an important role in increasing the liquidity of market through trading stocks and other derivatives in different exchanges. If market makers adopt proper strategies to trade financial instruments, there will be a win-win situation where not only the liquidity of market is improved, but also market makers make a tiny profit from each transaction. Therefore, a reasonable trading strategy is crucial for market makers.

### 1.2 Cointegration trading strategy

Cointegration (pairs trading) strategy, which aims to trade a pair of stocks in a spread, is commonly used by market makers. In financial market, traders like to look for the price difference (i.e. spread) of same stock listed in different exchanges. If they trade a pair of same stocks with a high spread then do a reversing trade when their spread converges to zero or lower value, traders will be able to make profits because a pair of same stocks are fungible. However, for two different stocks having a particular statistical relation between them, cointegration strategy could be employed as well. The solution is looking for a tradable and stationary quantity, which is similar to spread, from a pair of two different stocks. According to the Engle-Granger representation theorem, if two time series, $x_t$ & $y_t$, are cointegrated, they will have an Error Correction Model (ECM) representation as follows:

$$z_t = y_t - (c + \gamma x_t),$$

$$\Delta x_t = \alpha_x z_{t-1} + \varepsilon_t^x, (\Delta x_t = x_t - x_{t-1}),$$

$$\Delta y_t = \alpha_y z_{t-1} + \varepsilon_t^y, (\Delta y_t = y_t - y_{t-1}),$$

where $c$ & $\gamma$ are the parameters of the cointegration relationship between $x_t$ & $y_t$, and $\alpha_x$ & $\alpha_y$ are the error correction speeds. Then we can use Ordinary Least Squares (OLS) to estimate the values of these parameters, and $z_t$ is the residuals of the regression. Furthermore, the Dickey-Fuller (1979) test is the most common statistical test to check whether a given series is stationary or not. Generally, a low $p$-value for non-stationarity of the residuals means that processes are cointegrated. In reality, traders try to find a stationary $z_t$ from the long-run relationship between a pair of stock prices, $X_t$ & $Y_t$, then we have:

$$z_t = y_t - (c + \gamma\, x_t),$$
$$x_t = \ln X_t,$$
$$y_t = \ln Y_t.$$

Similarly, we sell $y$ and buy $x$ when the value of $z_t$ comes into a high position, then we can make profits when $z_t$ returns to zero or a lower position. Noticeably, as log prices are not tradable, we need to use small log returns approximation ($\ln \frac{x_t}{x_{t-1}} \approx \frac{x_t}{x_{t-1}} - 1$) to derive the relationship between actual returns:

$$\Delta Y_t \approx \gamma\, \frac{Y_{t-1}}{X_{t-1}}\, \Delta X_t,$$

which means that we have to buy $\gamma\, \frac{Y_{t-1}}{X_{t-1}}$ stocks of X for every sold unit of Y in order to be hedged. Therefore, cointegration strategy properly utilizes the statistical relationship between a pair of stocks to seek for arbitrage opportunities in market.

**1.3 Option-quoting trading strategy**

Option-quoting strategy is another common method adopted by market makers to look for arbitrage opportunities through trading options and hedging them with underlying stocks. Generally speaking, the value of options consists of two blocks, namely intrinsic value and time value. In terms of call option, its intrinsic value at expiration is:

$$\begin{cases} (S - X), & S > X \\ 0, & S \leq X \end{cases}$$

where $S$ is the price of underlying stock at expiration and $X$ is the strike price of call option. And the time value of call option depends on the volatility of the price of underlying stocks. Besides, there are some other factors contributing to the option value, such as option type, exercise style, strike, time to expiration, interest rate, dividends, etc. Fortunately, Black-Scholes model provide us a formula to calculate option value:

$$C = \begin{cases} max[0, (S - X)], & at\ expiration \\ S * N(d_1) - X * e^{-rt} * N(d_2), & before\ expiration \end{cases}$$

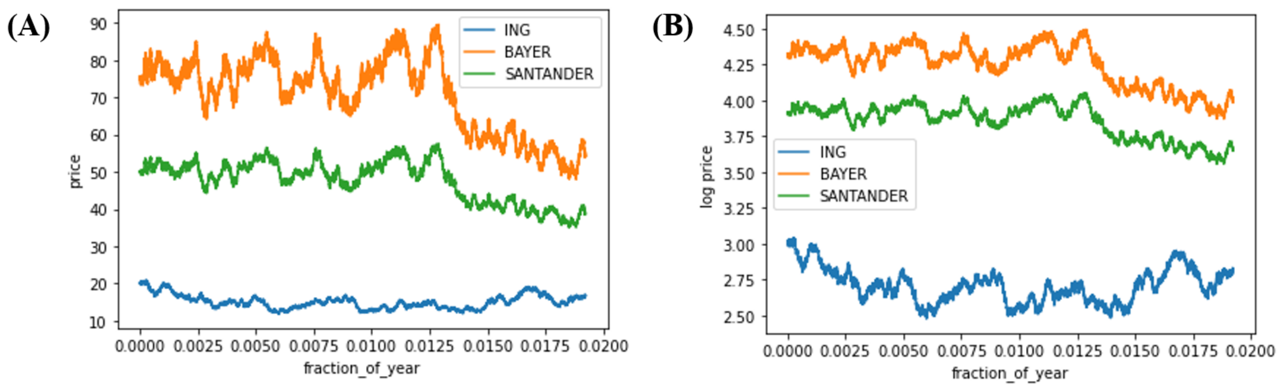
$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t},$$

where $C$ is the value of call option, $t$ is time to expiration, $r$ is interest rate, $\sigma$ is volatility. As market makers, what we can do is calculating the option value, subtracting or adding a fixed offset to this value, then showing bid & ask orders to the market. Once we make a deal with other traders in the market, we would calculate current delta position. In order to avoid directional risk, market makers need to trade the underlying asset until having neutralized the delta caused by options trading. Because both long call and long put options have positive gamma, which generates favorable delta when the underlying asset ($S$) changes in price, market makers are able to re-hedge their delta positions to be neutral and make a tiny profit from every hedging transaction. Therefore, option-quoting strategy is good at increasing the market liquidity and helping market markers make profits.

## 2. Data analysis

In this assignment, there are totally 3 stocks and 18 options tradable in optibook. In order to determine statistical relationships among these financial instruments and calculate parameters of their relationships, the historical data, containing the information of stock prices in the past, is analyzed in detail.




(C)

| | BAYER & SANTANDER | BAYER & ING | SANTANDER & ING |
|---|---|---|---|
| **DF-stat** | -128.2622 | -1.6888 | -0.6257 |
| **P-value** | 0.0 | 0.4369 | 0.8651 |

(D)

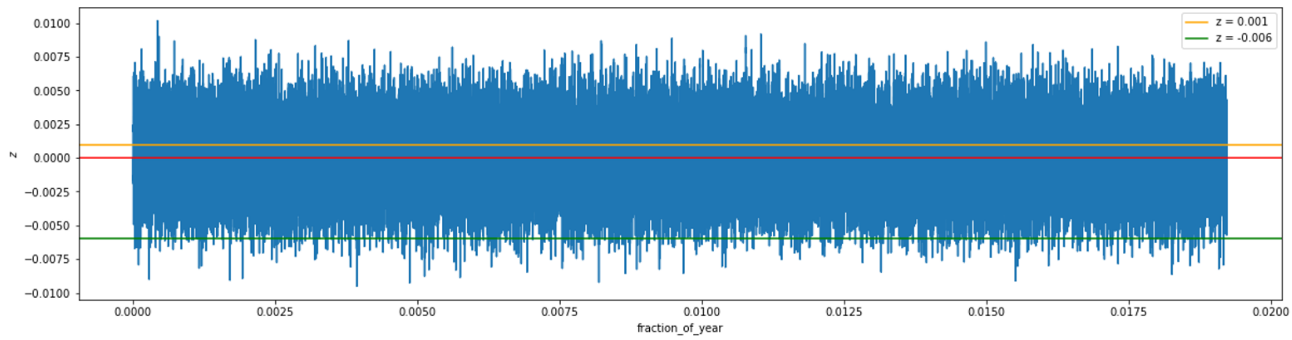



**Figure 1. (A, B)** Plots of stock price & log stock price as a function of time; **(C)** Outputs from Engle-Granger test; **(D)** Plot of z-series of BAYER &SANTANDER stock pair.

First of all, the cointegration analysis is implemented, and the results are shown in Fig. 1. Fig. 1(A) & (B) are the plots of stock prices & stock log prices as a function of time respectively. On the one hand, the stock price of ING seemingly is unrelated to those of the other two stocks. On the other hand, BAYER stock has a very similar tendency with SANTANDER stock in the aspect of price movements, so we can suppose that there probably exists a statistical relationship between the prices of these two stocks, which implies this pair of stocks has great potential to practice cointegration strategy. To confirm this hypothesis, choosing a stock pair randomly, I use Engle-Granger test to assess whether their log prices are cointegrated quantitively. As Fig. 1 (C) shows, there are two indicators returns by the testing function. Generally, the Dickey Fuller test-statistic (DF-stat) implies the existence of stronger cointegration if it has a more negative value, so the pair of BAYER & SANTANDER, which has the most negative DF-stat value (-128.2622), is likely to have a cointegrated relationship. Moreover, the $p$-value corresponding to the Dickey Fuller test-statistic further confirms this conclusion, because a lower p-value is stronger evidence of cointegration and the $p$-value of pair of BAYER & SANTANDER is 0. Therefore, we can claim that the log prices of BAYER & SANTANDER are cointegrated. The parameters of the "long-run relationship" between the log prices of BAYER & SANTANDER are calculated by calling the function of OLS method. According to the output from the function,

$$c = -0.5727, \ \gamma = 1.2500,$$
$$z_t = y_t + (-0.5727 + 1.25\, x_t),$$

where $y_t$, $x_t$ are the log prices of BAYER and SANTANDER respectively. Besides, the residual-series $z_t$ is plotted in Fig. 1(D), which indicates it is a stationary process. Although log price is not tradable, the small log returns approximate mentioned previously is able to solve this problem. Thus, $z_t$ is the quantity that we are looking for to implement cointegration strategy on the stock pair of BAYER & SANTANDER.

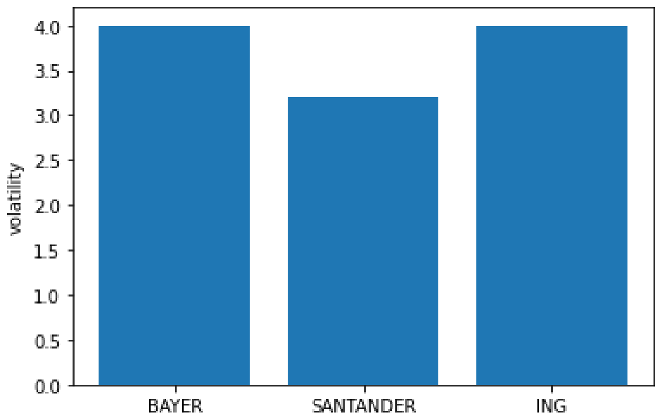



**Figure 2.** Volatilities of stock BAYER, SANTANDER and ING.

In addition, the volatilities of 3 stocks are calculated through the method of bisection. As Fig. 2 shows, the volatilities of BAYER, SANTANDER & ING are approximately floating at 4.00, 3.20 & 4.00 respectively. Because historical volatility can be assumed as implied volatility and keep constant in this assignment, for calculating the option values through Black-Scholes formula, the parameter $\sigma$ is set to these above values according to their underlying stocks in my algorithm.

## 3. Algorithm

After having obtained all parameters required for cointegration and option-quoting trading strategies, a novel algorithm is developed. The pseudocode of algorithm is shown in Alg. 1.

Firstly, two functions have to be defined before the program steps into the main loop. The function

**trade_would_breach_position_limit()** aims to avoid position-limit breach caused by new orders, and the function **print_positions_and_pnl()** is to obtain the current positions across all instruments. Besides, there are some important variables which would be frequently used in the main loop should be declared. Significantly, the type of variable **cointegration_positions** is dictionary, which stores the current positions of 3 stocks only for cointegration strategy (initial values of them are zero). And it will play an important role in coordinating two strategies in this algorithm.

**Algorithm 1** Pseudocode of trading algorithm

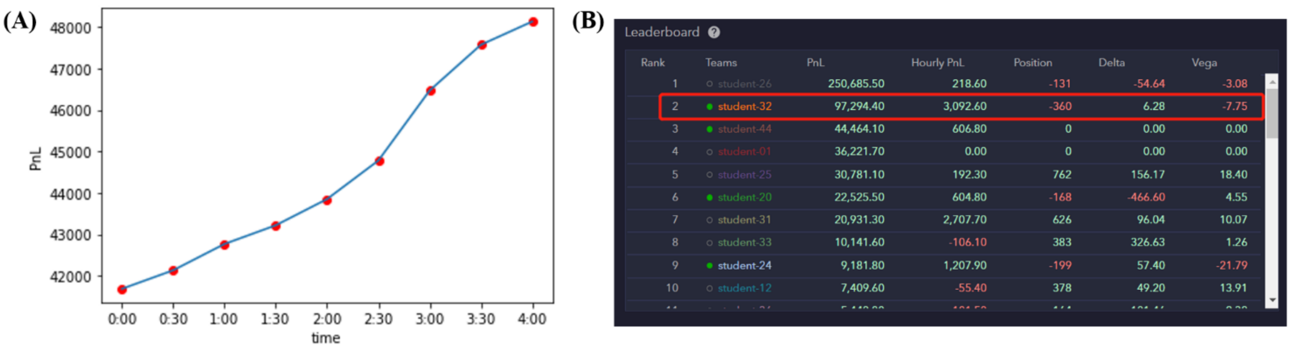
```
Define trade_would_breach_position_limit()
Define print_positions_and_pnl()
Declare some variables used frequently in the main loop:
    STOCK_IDS, volatility, OPTIONS, underlying, cointegration_postions
Main loop:
    Obtain information of all existing orders in the market
    Calculate the fair prices of all instruments based on the average of their current bid and ask prices

    ✦Implement option-quoting strategy:
    Delete all existing orders on options
    Calculate option value by calling the functions from black-scholes module
    Calculate desired bid and ask prices
    Insert limit orders on those prices for a desired volume
    Calculate current delta position across all instruments in option-quoting strategy
    Calculate stocks to buy / sell to become close to delta-neutral
    Perform the hedging stock trade by inserting IOC orders on the stock against the current top-of-book
    •option_quoter_positions ← exchange.get_positions() - cointegration_positions

    ✦Implement cointegration strategy:
    x ← the log stock price of SANTANDER
    y ← the log stock price of BAYER
    z ← y – ( -0.57 + 1.25 * x)
    Insert IOC ask orders on stock BAYER if z > 0.001
    Insert IOC bid orders on stock BAYER if z < -0.006
    Calculate the current positions of stocks BAYER & SANTANDERS in cointegration strategy
    Perform the hedging stock trade by inserting IOC orders on stock SANTANDER
    •cointegration_positions ← exchange.get_positions() - option_quoter_positions

    time.sleep(2)
```

In the main loop, the first task of algorithm is obtaining real-time information of all existing orders in the market, and calculating the fair prices of all instruments based on the average of their current bid and ask prices in order books.

The first strategy implemented in the main loop is option-quoting. In this strategy, the algorithm needs to delete all existing orders which were inserted but not traded in the previous iteration. Then the algorithm follows every procedure of option-quoting which has been described in introduction. After calculating values of all options by the method of Black-Scholes model, desired bid & ask prices are also calculated by subtracting or adding a fixed offset (the offset is set to 0.1 in this algorithm). Next, the algorithm inserts limit orders on those desired bid & ask prices for a volume of 30 lots. Then the current delta position across all instruments in option-quoting strategy is calculated according to the volume of all instruments in the inventory. After calculating the volume of stocks which are required to neutralize the current delta position, the algorithm will perform the hedging stock trade by inserting IOC orders on the stock against the current top-of-book. In this process, we are able to make profits from re-hedging. In addition, the last step aims to calculate the positions of all instruments only in the options quoting strategy through subtracting the positions in cointegration strategy from the current total positions. This step ensures that two strategies are able to work coordinately in my algorithm.

Next, the algorithm turns to implement cointegration strategy. The first task is calculating $z$ value based on "the long run relationship" between SANTANDER & BAYER, which has been mentioned previously. Then what algorithm needs to do is trading this pair of stocks at different positions of $z$. In this algorithm, I sell BAYER stock & buy SANTANDER stock if $z > 0.001$, and sell SANTANDER stock & buy BAYER stock if $z < -0.006$. Therefore, through this pairs trading strategy, we can also make profits from transactions. Similarly, the positions of instruments in cointegration strategy need to be updated in the final step. After 3 seconds sleep, my algorithm will step into next iteration, which means it will continue looking for arbitrage opportunities in the market.

## 4. Result & Conclusion




| Rank | Teams | PnL | Hourly PnL | Position | Delta | Vega |
|---|---|---|---|---|---|---|
| 1 | student-26 | 250,685.50 | 218.60 | -131 | -54.64 | -3.08 |
| 2 | student-32 | 97,294.40 | 3,092.60 | -360 | 6.28 | -7.75 |
| 3 | student-44 | 44,464.10 | 606.80 | 0 | 0.00 | 0.00 |
| 4 | student-01 | 36,221.70 | 0.00 | 0 | 0.00 | 0.00 |
| 5 | student-25 | 30,781.10 | 192.30 | 762 | 156.17 | 18.40 |
| 6 | student-20 | 22,525.50 | 604.80 | -168 | -466.60 | 4.55 |
| 7 | student-31 | 20,931.30 | 2,707.70 | 626 | 96.04 | 10.07 |
| 8 | student-33 | 10,141.60 | -106.10 | 383 | 326.63 | 1.26 |
| 9 | student-24 | 9,181.80 | 1,207.90 | -199 | 57.40 | -21.79 |
| 10 | student-12 | 7,409.60 | -55.40 | 378 | 49.20 | 13.91 |

**Figure 3. (A)** Plot of P&L as a function of time from 0:00 to 4:00 on Feb. 23rd; **(B)** Snapshot of optibook leaderboard at 8:00 on Feb. 24th.

In order to test the performance of this algorithm, I let it run without human intervention from 0:00 to 4:00 on Feb. 23rd GMT, and P&L it produced is recorded and shown in Fig. 3(A). During these 4 hours, the P&L rises from 41684.90 to 48127.40. Meanwhile, its hourly P&L keeps relatively stable during this period. After this test, this algorithm has been optimized again and again. Up to 8:00 Feb. 24th GMT, this algorithm has made 97294.40 profits in optibook (Fig. 3(B)), which implies this algorithm has an outstanding performance in this virtual exchange.

However, many hypotheses in this assignment rarely exist in the real world, so there remains a long distance from theory to practice. As financial market is usually full of uncertainty, participants have to keep humble all the time. That's the most important thing I learned from this Financial Technology module.