

APPENDIX A  
PROOF OF LEMMA 1

According to the definition of  $Q'_a(t) = \max\{Q_m(t), m \in \mathcal{M}_a\}$ , it is can be derived that

$$Pr(Q_m(t) \leq Q'_a(t)) = 1, \forall m \in \mathcal{M}_a, t, \quad (22)$$

which can also be denoted by  $Q_m(t) \leq Q'_a(t), \forall m \in \mathcal{M}_a, t$ . We take expectations of both sides. According to the monotonicity of expectation, there is

$$\mathbb{E}\{Q_m(t)\} \leq \mathbb{E}\{Q'_a(t)\}, \forall m \in \mathcal{M}_a, t. \quad (23)$$

Similarly, the inequality can also be extended to

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T \mathbb{E}\{Q_m(t)\} \leq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\}, \forall m \in \mathcal{M}_a. \quad (24)$$

Thus, if the constraint  $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\} < \infty, \forall a \in \mathcal{A}$  is satisfied, there must be

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q_m(t)\} \leq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\} \leq \infty, \quad (25)$$

$\forall a \in \mathcal{A}, m \in \mathcal{M}_a.$

This proved that if the constraint  $\overline{Q'_a(t)} \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\} < \infty, \forall a \in \mathcal{A}$  is satisfied, the constraint  $\overline{Q_m(t)} \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{Q_m(t)\} < \infty, \forall m \in \mathcal{M}$  satisfies.

This completes the proof.

APPENDIX B  
PROOF OF LEMMA 2

Recalling the equation (8), we can denote the  $\Delta(\Theta(t))$  by

$$\begin{aligned} \Delta(\Theta(t)) &= \mathbb{E} \left\{ \frac{1}{2} \sum_{a=1}^A Q'_a(t+1)^2 - \frac{1}{2} \sum_{a=1}^A (Q'_a(t))^2 \right\} \\ &= \mathbb{E} \left\{ \frac{1}{2} \sum_{a=1}^A (y'_a(t) + Q'_a(t))^2 - \frac{1}{2} \sum_{a=1}^A (Q'_a(t))^2 \right\} \\ &= \mathbb{E} \left\{ \frac{1}{2} \sum_{a=1}^A (y'_a(t))^2 + \sum_{a=1}^A Q'_a(t) y'_a(t) \right\}. \end{aligned} \quad (26)$$

According to the definition of  $y'_a(t)$  in (8), we have

$$\begin{aligned} y'_a(t) &= Q'_a(t+1) - Q'_a(t) \\ &= \max_{m \in \mathcal{M}_a} Q_m(t+1) - \max_{m \in \mathcal{M}_a} Q_m(t) \end{aligned} \quad (27)$$

According to (4), considering that each  $m \in m \in \mathcal{M}_a$ , there is  $y_m(t) \leq \max_{m \in \mathcal{M}_a} y_m(t)$ , and

$$Q_m(t+1) = Q_m(t) + y_m(t) \leq Q_m(t) + \max_{m \in \mathcal{M}_a} y_m(t), \forall m \in \mathcal{M}_a. \quad (28)$$

Based on (28), we have

$$\begin{aligned} \max_{m \in \mathcal{M}_a} Q_m(t+1) &\leq \max_{m \in \mathcal{M}_a} \left( Q_m(t) + \max_{m \in \mathcal{M}_a} y_m(t) \right) \\ &\leq \max_{m \in \mathcal{M}_a} Q_m(t) + \max_{m \in \mathcal{M}_a} y_m(t). \end{aligned} \quad (29)$$

Recalling (27), there is

$$\begin{aligned} y'_a(t) &= \max_{m \in \mathcal{M}_a} Q_m(t+1) - \max_{m \in \mathcal{M}_a} Q_m(t) \\ &\leq \max_{m \in \mathcal{M}_a} y_m(t). \end{aligned} \quad (30)$$

According to (3), considering  $x_m(t) \leq Q_m(t)$  in (5), there is  $Q_m(t) + A_m(t) - x_m(t) \geq 0$  and  $Q_m(t+1) = Q_m(t) + A_m(t) - x_m(t)$ . Thus, there is

$$y_m(t) = Q_m(t+1) - Q_m(t) = A_m(t) - x_m(t). \quad (31)$$

Associated with (1) and (5), we have

$$\begin{aligned} y_m(t) &= Q_m(t+1) - Q_m(t) = A_m(t) - x_m(t) \\ &\leq D_m, \forall m \in \mathcal{M}. \end{aligned} \quad (32)$$

Therefore, the (30) can be rewritten as

$$y'_a(t) \leq \max_{m \in \mathcal{M}_a} y_m(t) \leq \max_{m \in \mathcal{M}_a} D_m, \forall a \in \mathcal{A}. \quad (33)$$

and also

$$\frac{1}{2} \sum_{a=1}^A (y'_a(t))^2 \leq \frac{1}{2} \sum_{a=1}^A \left( \max_{m \in \mathcal{M}_a} D_m \right)^2 = B. \quad (34)$$

Back to (26), we have

$$\Delta(\Theta(t)) \leq \mathbb{E} \left\{ B + \sum_{a=1}^A Q'_a(t) y'_a(t) | \Theta(t) \right\}. \quad (35)$$

and

$$\Delta_V(\Theta(t)) \leq B + \mathbb{E} \left\{ \sum_{a=1}^A Q'_a(t) y'_a(t) | \Theta(t) \right\} + V \mathbb{E}\{P(t) | \Theta(t)\}, \quad (36)$$

This completes the proof.

APPENDIX C  
PROOF OF THE CONVERSION FROM PROBLEM 2 TO PROBLEM 3

Due to at the beginning of time slot  $t$ , we have obtained the values of  $\Theta(t)$ ,  $\{Q_m(t)\}$ ,  $\{A_m(t)\}$ ,  $\mathbf{b}(t)$ . We can re-express the objective function as

$$\begin{aligned} &\mathbb{E} \left\{ \sum_{a=1}^A Q'_a(t) y'_a(t) | \Theta(t) \right\} + V \mathbb{E}\{P(t) | \Theta(t)\} \\ &= \sum_{a=1}^A Q'_a(t) \mathbb{E}\{y'_a(t)\} + V P(t) \\ &\stackrel{(8)}{=} \sum_{a=1}^A Q'_a(t) \mathbb{E}\{Q'_a(t+1) - Q'_a(t)\} + V \frac{1}{A} \sum_{a \in \mathcal{A}} T'_a(t) \quad (37) \\ &\stackrel{(9)}{=} \sum_{a=1}^A (Q'_a(t) \mathbb{E}\{Q'_a(t+1)\} - (Q'_a(t))^2) \\ &\quad + V \frac{1}{A} \sum_{a \in \mathcal{A}} \max_{m \in \mathcal{M}_a} T_m(t) \end{aligned}$$

Since the  $(Q'_a(t))^2$  is a constant item, we can ignore it and continue rewriting as

$$\begin{aligned}
& \sum_{a=1}^A Q'_a(t) \mathbb{E}\{Q'_a(t+1)\} + V \frac{1}{A} \sum_{a \in \mathcal{A}} \max_{m \in \mathcal{M}_a} T_m(t) \\
& \stackrel{(6)(7)}{=} \sum_{a=1}^A Q'_a(t) \mathbb{E}\left\{\max_{m \in \mathcal{M}_a} Q_m(t+1)\right\} \\
& \quad + V \frac{1}{A} \sum_{a \in \mathcal{A}} \max_{m \in \mathcal{M}_a} \mathbf{1}(Q_m(t) + A_m(t) - x_m(t) > 0) \\
& \stackrel{5}{=} \sum_{a=1}^A Q'_a(t) \mathbb{E}\left\{\max_{m \in \mathcal{M}_a} (Q_m(t) + A_m(t) - x_m(t))\right\} \\
& \quad + V \frac{1}{A} \sum_{a \in \mathcal{A}} \max_{m \in \mathcal{M}_a} \mathbf{1}(Q_m(t) + A_m(t) - x_m(t) > 0) \\
& = \sum_{a=1}^A Q'_a(t) \max_{m \in \mathcal{M}_a} (Q_m(t) + A_m(t) - x_m(t)) \\
& \quad + V \frac{1}{A} \sum_{a \in \mathcal{A}} \max_{m \in \mathcal{M}_a} \mathbf{1}(Q_m(t) + A_m(t) - x_m(t) > 0),
\end{aligned} \tag{38}$$

which is the objective function in Problem 3

This completes the proof.

#### APPENDIX D PROOF OF THEOREM 1

For the (20), we first denote the optimal expected average communication fraction of Problem 1 by  $P_{av}^*$  and the corresponding optimal flow control solution by  $\mathbf{x}^*(\mathbf{t})$  for each time slot. And we denote the OFCDCN's expected average communication fraction by  $P_{av}^{our}$ , the associated  $P(t)$ , and the corresponding OFCDCN's flow control solution by  $\mathbf{x}(\mathbf{t})$ . For each time slot, we denote the value of  $P(t)$  based on solution  $\mathbf{x}^*(\mathbf{t})$  by  $P^*(t)$ , and the value of  $P(t)$  based on solution  $\mathbf{x}(\mathbf{t})$  by  $P(t)$ . Thus, we have

$$P_{av}^* = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P^*(t) | \Theta(\mathbf{t})\}, \tag{39}$$

and

$$P_{av}^{our} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P(t) | \Theta(\mathbf{t})\}. \tag{40}$$

According to (15) and (16), we have

$$\begin{aligned}
& \Delta(\Theta(\mathbf{t})) + V \mathbb{E}\{P(t) | \Theta(\mathbf{t})\} \\
& \leq B + \mathbb{E}\left\{\sum_{a=1}^A Q'_a(t) y'_a(t) | \Theta(\mathbf{t})\right\} + V \mathbb{E}\{P(t) | \Theta(\mathbf{t})\}.
\end{aligned} \tag{41}$$

<sup>5</sup>Here, due to our constraint  $x_m(t) \leq Q_m(t)$  in (5), we have  $Q_m(t) + A_m(t) - x_m(t) \geq 0$ . And we can rewrite (3) as  $Q_m(t+1) = Q_m(t) + A_m(t) - x_m(t)$ .

Because the solution  $x(t)$  is the optimal solution for the Problem 2, we have

$$\begin{aligned}
& B + \mathbb{E}\left\{\sum_{a=1}^A Q'_a(t) y'_a(t) | \Theta(\mathbf{t})\right\} + V \mathbb{E}\{P(t) | \Theta(\mathbf{t})\} \\
& \leq B + \mathbb{E}\left\{\sum_{a=1}^A Q'_a(t) y_a^{*'}(t) | \Theta(\mathbf{t})\right\} + V \mathbb{E}\{P^*(t) | \Theta(\mathbf{t})\}.
\end{aligned} \tag{42}$$

Joint inequalities (41) and (42) and take the time average for both two sides. We have

$$\begin{aligned}
& \frac{1}{T} \sum_{t=0}^{T-1} (\Delta(\Theta(\mathbf{t})) + V \mathbb{E}\{P(t) | \Theta(\mathbf{t})\}) \\
& \leq \frac{1}{T} \sum_{t=0}^{T-1} \left( B + \mathbb{E}\left\{\sum_{a=1}^A Q'_a(t) y_a^{*'}(t) | \Theta(\mathbf{t})\right\} + V \mathbb{E}\{P^*(t) | \Theta(\mathbf{t})\} \right) \\
& = B + \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left\{\sum_{a=1}^A Q'_a(t) y_a^{*'}(t) | \Theta(\mathbf{t})\right\} + \frac{1}{T} \sum_{t=0}^{T-1} V \mathbb{E}\{P^*(t) | \Theta(\mathbf{t})\} \\
& = B + \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \sum_{t=0}^{T-1} Q'_a(t) y_a^{*'}(t) | \Theta(\mathbf{t})\right\} + \frac{1}{T} \sum_{t=0}^{T-1} V \mathbb{E}\{P^*(t) | \Theta(\mathbf{t})\}.
\end{aligned} \tag{43}$$

We focus on the second term, which is

$$\begin{aligned}
& \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \sum_{t=0}^{T-1} Q'_a(t) y_a^{*'}(t) | \Theta(\mathbf{t})\right\} \\
& = \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \sum_{t=0}^{T-1} Q'_a(t) (Q'_a(t+1) - Q'_a(t)) | \Theta(\mathbf{t})\right\} \\
& = \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \left(\sum_{t=0}^{T-1} Q'_a(t) Q'_a(t+1) - (Q'_a(t))^2\right) | \Theta(\mathbf{t})\right\} \\
& \leq \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \left(\sum_{t=0}^{T-1} \frac{(Q'_a(t))^2 + (Q'_a(t+1))^2}{2} - (Q'_a(t))^2\right) | \Theta(\mathbf{t})\right\} \\
& = \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \left(\sum_{t=0}^{T-1} \frac{(Q'_a(t+1))^2 - (Q'_a(t))^2}{2}\right) | \Theta(\mathbf{t})\right\} \\
& = \frac{1}{T} \mathbb{E}\left\{\sum_{a=1}^A \left(\frac{(Q'_a(T))^2 - (Q'_a(0))^2}{2}\right) | \Theta(\mathbf{t})\right\}.
\end{aligned} \tag{44}$$

Due to  $\sum_{t=0}^{T-1} \Delta(\Theta(t)) \geq 0$ , we have

$$\begin{aligned}
& \frac{1}{T} \sum_{t=0}^{T-1} V \mathbb{E}\{P(t)|\Theta(t)\} \\
& \leq \frac{1}{T} \sum_{t=0}^{T-1} (\Delta(\Theta(t)) + V \mathbb{E}\{P(t)|\Theta(t)\}) \\
& \leq B + \frac{1}{T} \mathbb{E} \left\{ \sum_{a=1}^A \sum_{t=0}^{T-1} Q'_a(t) y'_a(t) | \Theta(t) \right\} \\
& \quad + \frac{1}{T} \sum_{t=0}^{T-1} V \mathbb{E}\{P^*(t)|\Theta(t)\} \\
& = B + \frac{1}{T} \mathbb{E} \left\{ \sum_{a=1}^A \left( \frac{(Q'_a(T))^2 - (Q'_a(0))^2}{2} \right) | \Theta(t) \right\} \\
& \quad + \frac{1}{T} \sum_{t=0}^{T-1} V \mathbb{E}\{P^*(t)|\Theta(t)\}.
\end{aligned} \tag{45}$$

Both sides of the equation are divided by  $V$ , and we have

$$\begin{aligned}
& \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P(t)|\Theta(t)\} \\
& \leq \frac{B}{V} + \frac{1}{TV} \mathbb{E} \left\{ \sum_{a=1}^A \left( \frac{(Q'_a(T))^2 - (Q'_a(0))^2}{2} \right) | \Theta(t) \right\} \\
& \quad + \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P^*(t)|\Theta(t)\}.
\end{aligned} \tag{46}$$

We take the limit on both sides, and we have

$$\begin{aligned}
& \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P(t)|\Theta(t)\} \\
& \leq \frac{B}{V} + \lim_{T \rightarrow \infty} \frac{1}{TV} \mathbb{E} \left\{ \sum_{a=1}^A \left( \frac{(Q'_a(T))^2 - (Q'_a(0))^2}{2} \right) | \Theta(t) \right\} \\
& \quad + \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P^*(t)|\Theta(t)\} \\
& = \frac{B}{V} + \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P^*(t)|\Theta(t)\}.
\end{aligned} \tag{47}$$

According to the (39) and (40), we have

$$P_{av}^{our} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{P(t)|\Theta(t)\} \leq P_{av}^* + \frac{B}{V}. \tag{48}$$

For the (21), we assume that there exists a solution  $\mathbf{x}^\bullet(t)$ , which satisfies

$$\exists \epsilon > 0, \mathbb{E}\{y_a^\bullet(t)\} \leq -\epsilon, \forall a \in \mathcal{A}. \tag{49}$$

According to (41) and the solution  $x(t)$  is the optimal solution for the Problem 2, we have

$$\begin{aligned}
& \Delta(\Theta(t)) + V \mathbb{E}\{P(t)|\Theta(t)\} \\
& \leq B + \mathbb{E} \left\{ \sum_{a=1}^A Q'_a(t) y'_a(t) | \Theta(t) \right\} + V \mathbb{E}\{P(t)|\Theta(t)\} \\
& \leq B + \mathbb{E} \left\{ \sum_{a=1}^A Q'_a(t) y_a^\bullet(t) | \Theta(t) \right\} + V \mathbb{E}\{P^\bullet(t)|\Theta(t)\}.
\end{aligned} \tag{50}$$

Thus, we have

$$\begin{aligned}
& \Delta(\Theta(t)) + V p_{min} \\
& \leq B + \mathbb{E} \left\{ \sum_{a=1}^A Q'_a(t) y_a^\bullet(t) | \Theta(t) \right\} + V p_{max} \\
& \leq B + \sum_{a=1}^A \mathbb{E}\{Q'_a(t)|\Theta(t)\} \mathbb{E}\{y_a^\bullet(t)|\Theta(t)\} + V p_{max} \\
& \leq B - \epsilon \sum_{a=1}^A \mathbb{E}\{Q'_a(t)|\Theta(t)\} + V p_{max}
\end{aligned} \tag{51}$$

where  $p_{min}$  is the minimum value of  $\mathbb{E}\{P(t)|\Theta(t)\}$ , which is equal to 0. And the  $p_{max}$  is the maximum value of  $\mathbb{E}\{P(t)|\Theta(t)\}$ , which is equal to  $A$ .

We take the sum of all time slots and have

$$\begin{aligned}
& \mathbb{E}\{L(\Theta(T))\} - \mathbb{E}\{L(\Theta(0))\} \\
& \leq TB + TV(p_{max} - p_{min}) - \epsilon \sum_{a=1}^A \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)|\Theta(t)\}.
\end{aligned} \tag{52}$$

Both sides of the equation are divided by  $T$  and taken the limit, and we have

$$\begin{aligned}
& \lim_{T \rightarrow \infty} \frac{\mathbb{E}\{L(\Theta(T))\} - \mathbb{E}\{L(\Theta(0))\}}{T} \\
& \leq B + V(p_{max} - p_{min}) - \epsilon \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{a=1}^A \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)|\Theta(t)\}.
\end{aligned} \tag{53}$$

Thus, we have

$$\begin{aligned}
& \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{a=1}^A \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)|\Theta(t)\} \\
& \leq \frac{B + V(p_{max} - p_{min})}{\epsilon} = \frac{B + VA}{\epsilon},
\end{aligned} \tag{54}$$

which can be rewritten as

$$\begin{aligned}
Q_{av}^{our} &= \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{a=1}^A \sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)|\Theta(t)\} \\
&\leq \frac{B + VA}{\epsilon}.
\end{aligned} \tag{55}$$

This completes the proof.