APPENDIX A
PROOF OF LEMMA 1

According to the definition of $Q'_a(t) = \max\{Q_m(t), m \in \mathcal{M}_a\}$, it is can be derived that

$$Pr\left(Q_m(t) \le Q'_a(t)\right) = 1, \forall m \in \mathcal{M}_a, t, \tag{22}$$

which can also be denoted by $Q_m(t) \le Q'_a(t), \forall m \in \mathcal{M}_a, t$. We take expectations of both sides. According to the monotonicity of expectation, there is

$$\mathbb{E}\{Q_m(t)\} \le \mathbb{E}\{Q'_a(t)\}, \forall m \in \mathcal{M}_a, t. \tag{23}$$

Similarly, the inequality can also be extended to

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T} \mathbb{E}\{Q_m(t)\} \le \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\}, \forall m \in \mathcal{M}_a. \tag{24}$$

Thus, if the constraint $\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\} < \infty, \forall a \in \mathcal{A}$ is satisfied, there must be

$$\begin{aligned} &\lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{Q_m(t)\} \le \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\} \le \infty, \\ &\forall a \in \mathcal{A}, m \in \mathcal{M}_a. \end{aligned} \tag{25}$$

This proved that if the constraint $\overline{Q'_a(t)} \triangleq \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{Q'_a(t)\} < \infty, \forall a \in \mathcal{A}$ is satisfied, the constraint $\overline{Q_m(t)} \triangleq \lim_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{Q_m(t)\} < \infty, \forall m \in \mathcal{M}$ satisfies.

This completes the proof.

APPENDIX B
PROOF OF LEMMA 2

Recalling the equation (8), we can denote the $\Delta\left(\mathbf{\Theta}(\mathbf{t})\right)$ by

$$\begin{aligned} \Delta\left(\mathbf{\Theta}(\mathbf{t})\right) &= \mathbb{E}\left\{\frac{1}{2}\sum_{a=1}^{A} Q'_a(t+1)^2 - \frac{1}{2}\sum_{a=1}^{A}(Q'_a(t))^2\right\} \\ &= \mathbb{E}\left\{\frac{1}{2}\sum_{a=1}^{A}\left(y'_a(t) + Q'_a(t)\right)^2 - \frac{1}{2}\sum_{a=1}^{A}(Q'_a(t))^2\right\} \\ &= \mathbb{E}\left\{\frac{1}{2}\sum_{a=1}^{A}\left(y'_a(t)\right)^2 + \sum_{a=1}^{A} Q'_a(t)y'_a(t)\right\}. \end{aligned} \tag{26}$$

According to the definition of $y'_a(t)$ in (8), we have

$$\begin{aligned} y'_a(t) &= Q'_a(t+1) - Q'_a(t) \\ &= \max_{m\in\mathcal{M}_a} Q_m(t+1) - \max_{m\in\mathcal{M}_a} Q_m(t) \end{aligned} \tag{27}$$

According to (4), considering that each $m \in m \in \mathcal{M}_a$, there is $y_m(t) \le \max_{m\in\mathcal{M}_a} y_m(t)$, and

$$Q_m(t+1) = Q_m(t) + y_m(t) \le Q_m(t) + \max_{m\in\mathcal{M}_a} y_m(t), \forall m \in \mathcal{M}_a. \tag{28}$$

Based on (28), we have

$$\begin{aligned} \max_{m\in\mathcal{M}_a} Q_m(t+1) &\le \max_{m\in\mathcal{M}_a}\left(Q_m(t) + \max_{m\in\mathcal{M}_a} y_m(t)\right) \\ &\le \max_{m\in\mathcal{M}_a} Q_m(t) + \max_{m\in\mathcal{M}_a} y_m(t). \end{aligned} \tag{29}$$

Recalling (27), there is

$$\begin{aligned} y'_a(t) &= \max_{m\in\mathcal{M}_a} Q_m(t+1) - \max_{m\in\mathcal{M}_a} Q_m(t) \\ &\le \max_{m\in\mathcal{M}_a} y_m(t). \end{aligned} \tag{30}$$

According to (3), considering $x_m(t) \le Q_m(t)$ in (5), there is $Q_m(t) + A_m(t) - x_m(t) \ge 0$ and $Q_m(t+1) = Q_m(t) + A_m(t) - x_m(t)$. Thus, there is

$$y_m(t) = Q_m(t+1) - Q_m(t) = A_m(t) - x_m(t). \tag{31}$$

Associated with (1) and (5), we have

$$\begin{aligned} y_m(t) &= Q_m(t+1) - Q_m(t) = A_m(t) - x_m(t) \\ &\le D_m, \forall m \in \mathcal{M}. \end{aligned} \tag{32}$$

Therefore, the (30) can be rewritten as

$$y'_a(t) \le \max_{m\in\mathcal{M}_a} y_m(t) \le \max_{m\in\mathcal{M}_a} D_m, \forall a \in \mathcal{A}. \tag{33}$$

and also

$$\frac{1}{2}\sum_{a=1}^{A}\left(y'_a(t)\right)^2 \le \frac{1}{2}\sum_{a=1}^{A}\left(\max_{m\in\mathcal{M}_a} D_m\right)^2 = B. \tag{34}$$

Back to (26), we have

$$\Delta\left(\mathbf{\Theta}(\mathbf{t})\right) \le \mathbb{E}\left\{B + \sum_{a=1}^{A} Q'_a(t)y'_a(t)|\mathbf{\Theta}(\mathbf{t})\right\}. \tag{35}$$

and

$$\Delta_V\left(\mathbf{\Theta}(\mathbf{t})\right) \le B + \mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'_a(t)|\mathbf{\Theta}(\mathbf{t})\right\} + V\mathbb{E}\{P(t)|\mathbf{\Theta}(\mathbf{t})\}\}, \tag{36}$$

This completes the proof.

APPENDIX C
PROOF OF THE CONVERSION FROM PROBLEM 2 TO PROBLEM 3

Due to at the beginning of time slot t, we have obtained the values of $\mathbf{\Theta}(\mathbf{t})$, $\{Q_m(t)\}$, $\{A_m(t)\}$, $\mathbf{b}(\mathbf{t})$. We can re-express the objective function as

$$\begin{aligned} &\mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'_a(t)|\mathbf{\Theta}(\mathbf{t})\right\} + V\mathbb{E}\{P(t)|\mathbf{\Theta}(\mathbf{t})\} \\ &= \sum_{a=1}^{A} Q'_a(t)\mathbb{E}\left\{y'_a(t)\right\} + VP(t) \\ &\overset{(8)}{=} \sum_{a=1}^{A} Q'_a(t)\mathbb{E}\left\{Q'_a(t+1) - Q'_a(t)\right\} + V\frac{1}{A}\sum_{a\in\mathcal{A}} T'_a(t) \\ &\overset{(9)}{=} \sum_{a=1}^{A}\left(Q'_a(t)\mathbb{E}\left\{Q'_a(t+1)\right\} - (Q'_a(t))^2\right) \\ &\quad + V\frac{1}{A}\sum_{a\in\mathcal{A}} \max_{m\in\mathcal{M}_a} T_m(t) \end{aligned} \tag{37}$$

Since the $(Q'_a(t))^2$ is a constant item, we can ignore it and continue rewriting as

$$\begin{aligned}
&\sum_{a=1}^{A} Q'_a(t)\mathbb{E}\left\{Q'_a(t+1)\right\} + V\frac{1}{A}\sum_{a\in\mathcal{A}}\max_{m\in\mathcal{M}_a} T_m(t)\\
\overset{(6)(7)}{=}&\sum_{a=1}^{A} Q'_a(t)\mathbb{E}\left\{\max_{m\in\mathcal{M}_a} Q_m(t+1)\right\}\\
&+V\frac{1}{A}\sum_{a\in\mathcal{A}}\max_{m\in\mathcal{M}_a}\mathbf{1}(Q_m(t)+A_m(t)-x_m(t)>0)\\
\overset{5}{=}&\sum_{a=1}^{A} Q'_a(t)\mathbb{E}\left\{\max_{m\in\mathcal{M}_a}\left(Q_m(t)+A_m(t)-x_m(t)\right)\right\}\\
&+V\frac{1}{A}\sum_{a\in\mathcal{A}}\max_{m\in\mathcal{M}_a}\mathbf{1}(Q_m(t)+A_m(t)-x_m(t)>0)\\
=&\sum_{a=1}^{A} Q'_a(t)\max_{m\in\mathcal{M}_a}\left(Q_m(t)+A_m(t)-x_m(t)\right)\\
&+V\frac{1}{A}\sum_{a\in\mathcal{A}}\max_{m\in\mathcal{M}_a}\mathbf{1}(Q_m(t)+A_m(t)-x_m(t)>0),
\end{aligned}\tag{38}$$

which is the objective function in Problem 3

This completes the proof.

Appendix D
Proof of Theorem 1

For the (20), we first denote the optimal expected average communication fraction of Problem 1 by P^*_{av} and the corresponding optimal flow control solution by $\mathbf{x}^*(\mathbf{t})$ for each time slot. And we denote the OFCDCN's expected average communication fraction by P^{our}_{av}, the associated $P(t)$, and the corresponding OFCDCN's flow control solution by $\mathbf{x}(\mathbf{t})$. For each time slot, we denote the value of $P(t)$ based on solution $\mathbf{x}^*(\mathbf{t})$ by $P^*(t)$, and the value of $P(t)$ based on solution $\mathbf{x}(\mathbf{t})$ by $P(t)$. Thus, we have

$$P^*_{av} = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{P^*(t)|\boldsymbol{\Theta}(\mathbf{t})\},\tag{39}$$

and

$$P^{our}_{av} = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}.\tag{40}$$

According to (15) and (16), we have

$$\begin{aligned}
&\Delta\left(\boldsymbol{\Theta}(\mathbf{t})\right)+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}\\
\leq& B+\mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned}\tag{41}$$

[5]Here, due to our constraint $x_m(t)\leq Q_m(t)$ in (5), we have $Q_m(t)+A_m(t)-x_m(t)\geq 0$. And we can rewrite (3) as $Q_m(t+1)=Q_m(t)+A_m(t)-x_m(t)$.

Because the solution $x(t)$ is the optimal solution for the Problem 2, we have

$$\begin{aligned}
&B+\mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}\\
\leq& B+\mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'^*_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+V\mathbb{E}\{P^*(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned}\tag{42}$$

Joint inequalities (41) and (42) and take the time average for both two sides. We have

$$\begin{aligned}
&\frac{1}{T}\sum_{t=0}^{T-1}\left(\Delta\left(\boldsymbol{\Theta}(\mathbf{t})\right)+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}\right)\\
\leq&\frac{1}{T}\sum_{t=0}^{T-1}\left(B+\mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'^*_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+V\mathbb{E}\{P^*(t)|\boldsymbol{\Theta}(\mathbf{t})\}\right)\\
=&B+\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\left\{\sum_{a=1}^{A} Q'_a(t)y'^*_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+\frac{1}{T}\sum_{t=0}^{T-1}V\mathbb{E}\{P^*(t)|\boldsymbol{\Theta}(\mathbf{t})\}\\
=&B+\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\sum_{t=0}^{T-1} Q'_a(t)y'^*_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+\frac{1}{T}\sum_{t=0}^{T-1}V\mathbb{E}\{P^*(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned}\tag{43}$$

We focus on the second term, which is

$$\begin{aligned}
&\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\sum_{t=0}^{T-1} Q'_a(t)y'^*_a(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}\\
=&\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\sum_{t=0}^{T-1} Q'_a(t)(Q'_a(t+1)-Q'_a(t))|\boldsymbol{\Theta}(\mathbf{t})\right\}\\
=&\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\sum_{t=0}^{T-1} Q'_a(t)Q'_a(t+1)-(Q'_a(t))^2\right)|\boldsymbol{\Theta}(\mathbf{t})\right\}\\
\leq&\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\sum_{t=0}^{T-1}\frac{(Q'_a(t))^2+(Q'_a(t+1))^2}{2}-(Q'_a(t))^2\right)|\boldsymbol{\Theta}(\mathbf{t})\right\}\\
=&\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\sum_{t=0}^{T-1}\frac{(Q'_a(t+1))^2-(Q'_a(t))^2}{2}\right)|\boldsymbol{\Theta}(\mathbf{t})\right\}\\
=&\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\frac{(Q'_a(T))^2-(Q'_a(0))^2}{2}\right)|\boldsymbol{\Theta}(\mathbf{t})\right\}.
\end{aligned}\tag{44}$$

Due to $\sum_{t=0}^{T-1} \Delta\left(\boldsymbol{\Theta}(\mathbf{t})\right) \geq 0$, we have

$$
\begin{aligned}
&\frac{1}{T}\sum_{t=0}^{T-1} V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
\leq&\frac{1}{T}\sum_{t=0}^{T-1}\left(\Delta\left(\boldsymbol{\Theta}(\mathbf{t})\right)+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}\right) \\
\leq&B+\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\sum_{t=0}^{T-1} Q_a'(t)y_a'^{*}(t)|\boldsymbol{\Theta}(\mathbf{t})\right\} \\
+&\frac{1}{T}\sum_{t=0}^{T-1} V\mathbb{E}\{P^{*}(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
=&B+\frac{1}{T}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\frac{(Q_a'(T))^2-(Q_a'(0))^2}{2}\right)|\boldsymbol{\Theta}(\mathbf{t})\right\} \\
+&\frac{1}{T}\sum_{t=0}^{T-1} V\mathbb{E}\{P^{*}(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned} \tag{45}
$$

Both sides of the equation are divided by V, and we have

$$
\begin{aligned}
&\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
\leq&\frac{B}{V}+\frac{1}{TV}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\frac{(Q_a'(T))^2-(Q_a'(0))^2}{2}\right)|\boldsymbol{\Theta}(\mathbf{t})\right\} \\
+&\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{P^{*}(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned} \tag{46}
$$

We take the limit on both sides, and we have

$$
\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
\leq&\frac{B}{V}+\lim_{T\to\infty}\frac{1}{TV}\mathbb{E}\left\{\sum_{a=1}^{A}\left(\frac{(Q_a'(T))^2-(Q_a'(0))^2}{2}\right)|\boldsymbol{\Theta}(\mathbf{t})\right\} \\
+&\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{P^{*}(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
=&\frac{B}{V}+\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{P^{*}(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned} \tag{47}
$$

According to the (39) and (40), we have

$$
P_{av}^{our}=\lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}\leq P_{av}^{*}+\frac{B}{V}. \tag{48}
$$

For the (21), according to [13], we can assume that there exists a solution $\mathbf{x}^{\bullet}(\mathbf{t})$, which satisfies

$$
\exists\epsilon>0, \mathbb{E}\left\{y_a'^{\bullet}(t)\right\}\leq -\epsilon, \forall a\in\mathcal{A}. \tag{49}
$$

According to (41) and the solution $x(t)$ is the optimal solution for the Problem 2, we have

$$
\begin{aligned}
&\Delta\left(\boldsymbol{\Theta}(\mathbf{t})\right)+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
\leq&B+\mathbb{E}\left\{\sum_{a=1}^{A} Q_a'(t)y_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+V\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
\leq&B+\mathbb{E}\left\{\sum_{a=1}^{A} Q_a'(t)y_a'^{\bullet}(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+V\mathbb{E}\{P^{\bullet}(t)|\boldsymbol{\Theta}(\mathbf{t})\}.
\end{aligned} \tag{50}
$$

Thus, we have

$$
\begin{aligned}
&\Delta\left(\boldsymbol{\Theta}(\mathbf{t})\right)+Vp_{min} \\
\leq&B+\mathbb{E}\left\{\sum_{a=1}^{A} Q_a'(t)y_a'^{\bullet}(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+Vp_{max} \\
\leq&B+\sum_{a=1}^{A}\mathbb{E}\left\{Q_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}\mathbb{E}\left\{y_a'^{\bullet}(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+Vp_{max} \\
\leq&B-\epsilon\sum_{a=1}^{A}\mathbb{E}\left\{Q_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}+Vp_{max}
\end{aligned} \tag{51}
$$

where p_{min} is the minimum value of $\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}$, which is equal to 0. And the p_{max} is the maximum value of $\mathbb{E}\{P(t)|\boldsymbol{\Theta}(\mathbf{t})\}$, which is equal to A.

We take the sum of all time slots and have

$$
\begin{aligned}
&\mathbb{E}\{L\left(\boldsymbol{\Theta}(\mathbf{T})\right)\}-\mathbb{E}\{L\left(\boldsymbol{\Theta}(\mathbf{0})\right)\} \\
\leq&TB+TV(p_{max}-p_{min})-\epsilon\sum_{a=1}^{A}\sum_{t=0}^{T-1}\mathbb{E}\left\{Q_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}.
\end{aligned} \tag{52}
$$

Both sides of the equation are divided by T and taken the limit, and we have

$$
\begin{aligned}
&\lim_{T\to\infty}\frac{\mathbb{E}\{L\left(\boldsymbol{\Theta}(\mathbf{T})\right)\}-\mathbb{E}\{L\left(\boldsymbol{\Theta}(\mathbf{0})\right)\}}{T} \\
\leq&B+V(p_{max}-p_{min})-\epsilon\lim_{T\to\infty}\frac{1}{T}\sum_{a=1}^{A}\sum_{t=0}^{T-1}\mathbb{E}\left\{Q_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\right\}.
\end{aligned} \tag{53}
$$

Thus, we have

$$
\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\sum_{a=1}^{A}\sum_{t=0}^{T-1}\mathbb{E}\left\{Q_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\right\} \\
&\leq\frac{B+V(p_{max}-p_{min})}{\epsilon}=\frac{B+VA}{\epsilon},
\end{aligned} \tag{54}
$$

which can be rewritten as

$$
\begin{aligned}
Q_{av}^{our}&=\lim_{T\to\infty}\frac{1}{T}\sum_{a=1}^{A}\sum_{t=0}^{T-1}\mathbb{E}\{Q_a'(t)|\boldsymbol{\Theta}(\mathbf{t})\} \\
&\leq\frac{B+VA}{\epsilon}.
\end{aligned} \tag{55}
$$

This completes the proof.