The Hong Kong Polytechnic University

COMP2012 Discrete Mathematics

Assignment 2 Suggested solutions

Questions:

Question 1 **[20 marks]**

Determine the maximum flow of the network *G* in Figure 1-1,

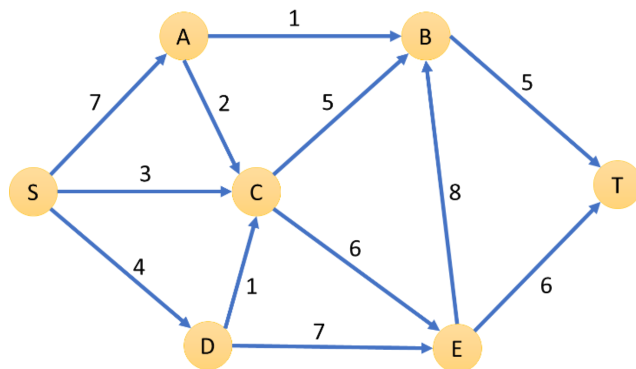


Figure 1-1

1(a) Using the *max-flow min-cut* theorem. (3 marks)

1(b) Using the *Folk-Fulkerson* algorithm. (5 marks)

1(c) Using the *Edmonds-Karp* algorithm. (5 marks)

1(d) Discuss whether the 1(c)'s algorithm outperforms 1(b)'s algorithm. (2 marks)

1(e) Suppose a network *G'* is an undirected graph with the same vertices and edges (without directions) as in *G*. Find the minimum spanning tree (MST) from the network *G'* with any method you have learned in the lesson. (5 marks)

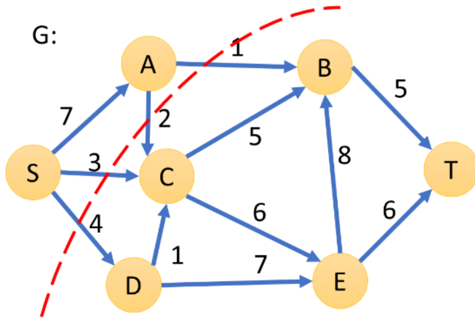
Solution:

(a) Max-flow min-cut theorem (3 marks for correct min-cut at max flow=10 by drawing)

To solve this problem, you may need to try different ways of cutting the graph *G*, making it become two disconnected sub-graphs, where one contains S while the other contains T.

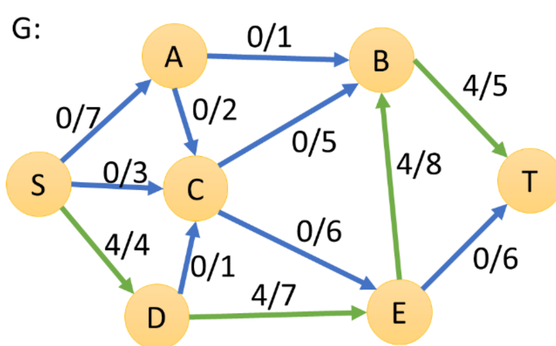
It is not allowed if a subgraph contains both S and T!!

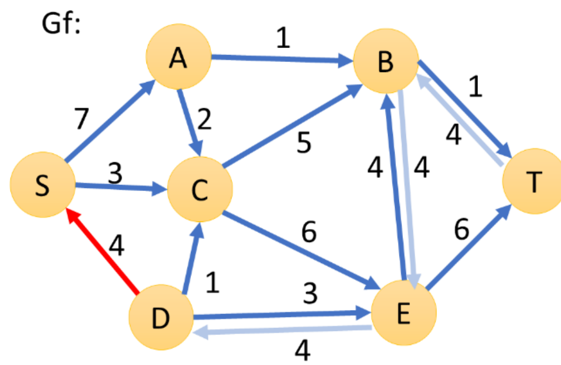
In the answer, you may need to show a few different cuttings and their total capacity (S→T direction) along the cut edges. The max flow found by min-cut theorem is 10.



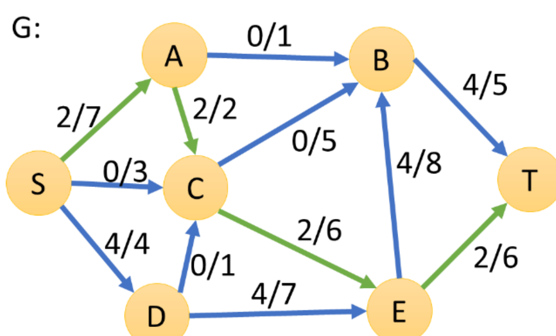
(b) Ford-Fulkerson algorithm (4 marks for steps, and 1 mark for correct max flow=10)

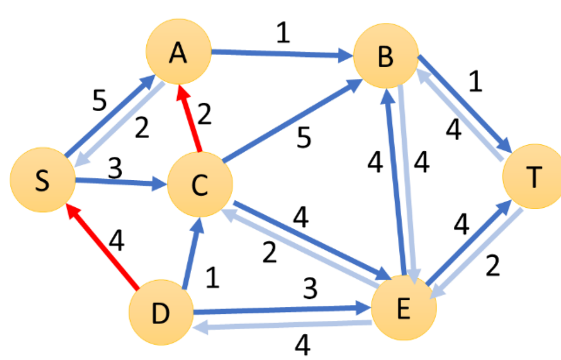
1st iteration: Path=S→D→E→B→T Flow=0+4=4



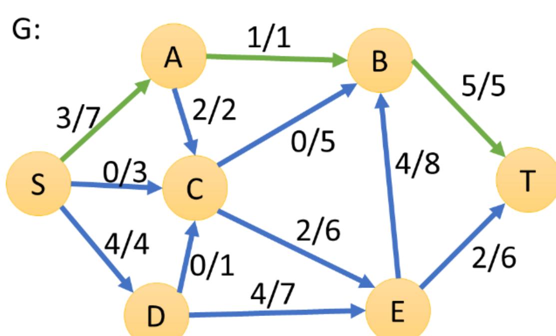


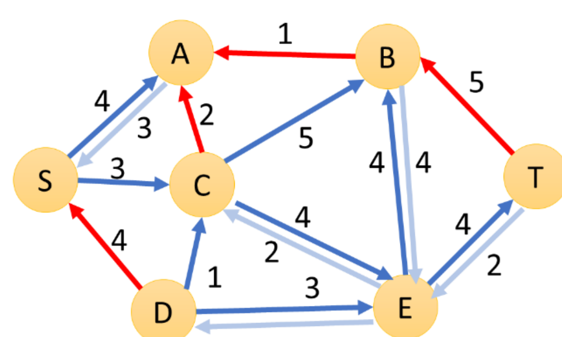
2nd iteration: Path=S→A→C→E→T Flow=4+2=6





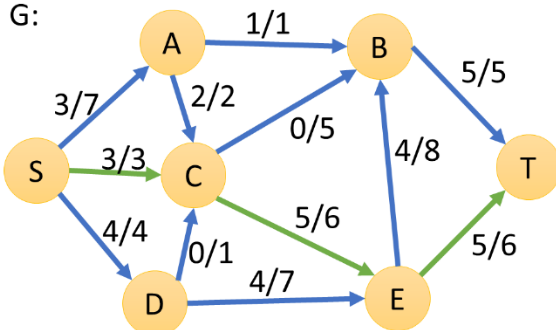
3rd iteration: Path=S→A→B→T Flow=6+1=7

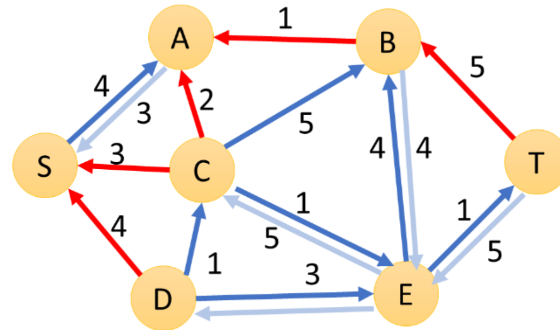




4th iteration: Path=S→C→E→T   Flow=7+3=10 (max. flow)

G:

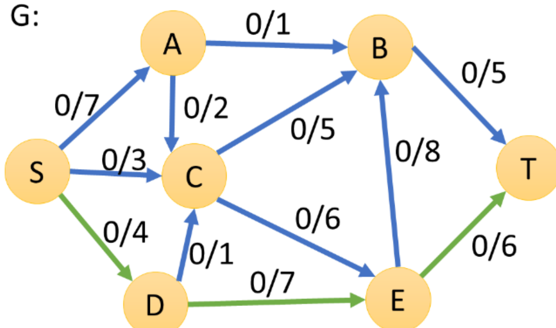




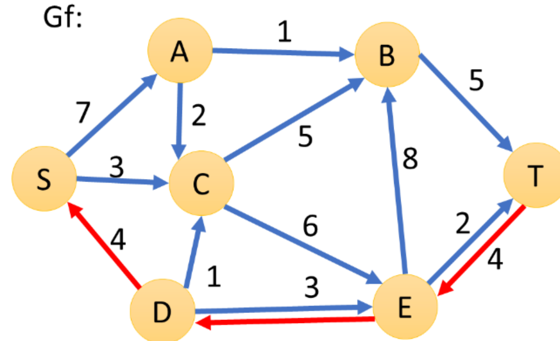
(c) Edmonds-Karp algorithm (4 marks for steps, and 1 mark for correct max flow=10)

1st iteration: Path=S→D→E→T   Flow=0+4=4

G:

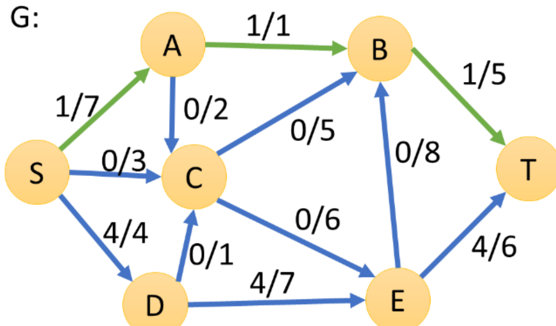


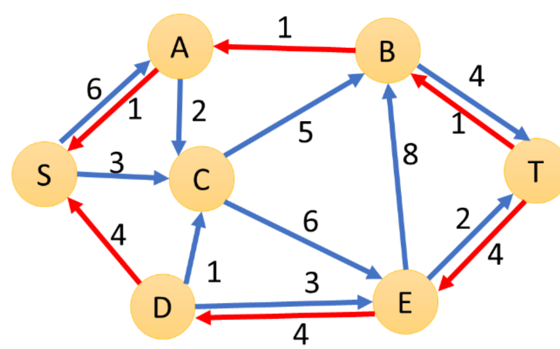
Gf:



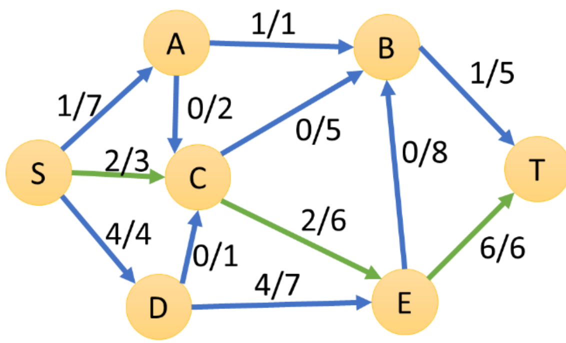
2nd iteration: Path=S→A→B→T   Flow=4+1=5

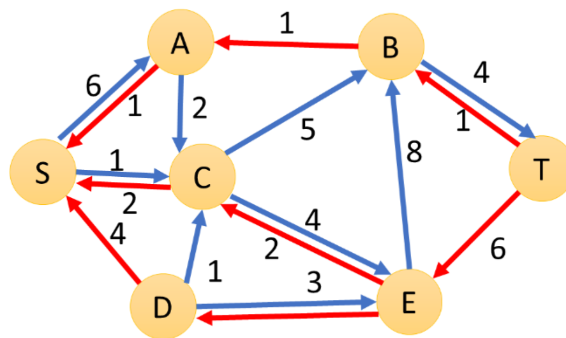
G:



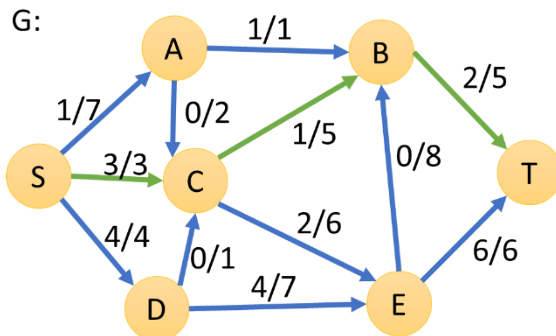


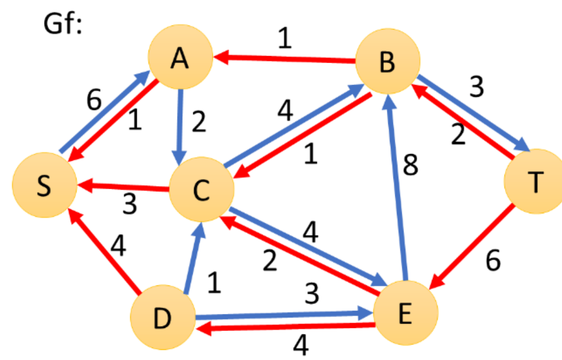
3rd iteration: Path=S→C→E→T   Flow=5+2=7



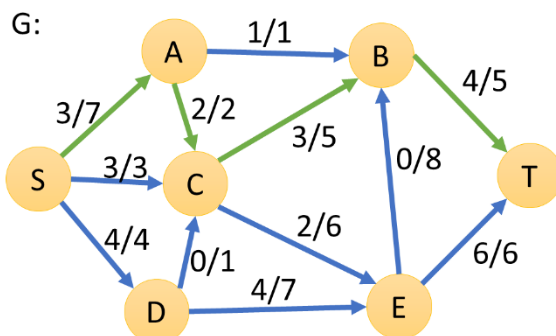


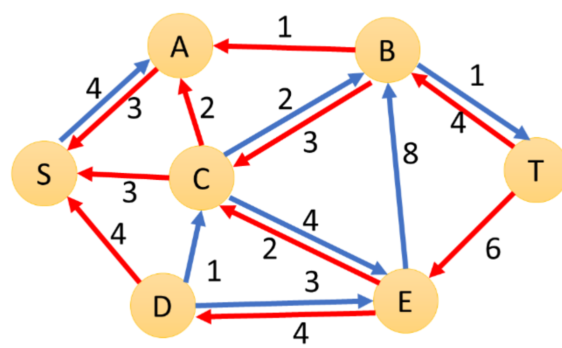
4th iteration: S→C→B→T Flow=7+1=8





5th iteration: Path=S→C→B→T Flow=8+2=10 (max. flow)

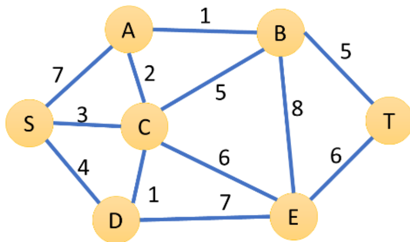




(d) In this case,
- E-K algorithm does not run fewer iterations than the F-F algorithm. (1 mark)
- Because the paths of fewer edges (e.g. S→A→B→T) have a bottleneck of capacity=1, using BFS wouldn't guarantee a benefit. (1 mark)

(e) MST can be found by the following steps

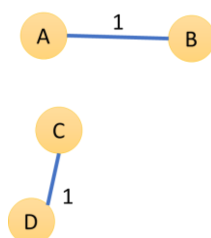
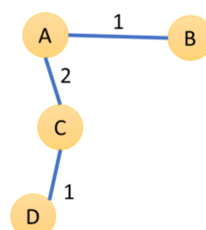
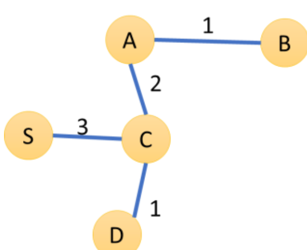
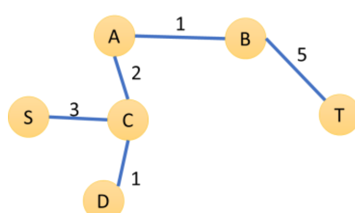
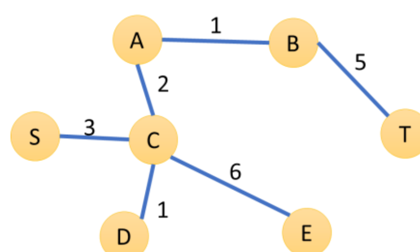
Consider the graph G (undirected version):



- You may use either Prim's algorithm or Kruskal's algorithm.
- Here, I illustrate Kruskal's algorithm: (1 mark)

Note: this is only one of the answer (other correct MST also accepted as correct)

(3 marks, 0.5 mark per step)

Step1: 	Step2: 	Step3: 
Step4: 	Step5: 	Step6: 

Total weight = 1+1+2+3+5+6 = 18 (1 mark for MST information is given, e.g. total weight)

Hint: a number of |V-1| = 7-1 = 6 edges are inserted into MST, so total we need 6 steps.

Question 2 **[20 marks]**

2(a) Simplify the logic of $(A+\bar{A})(AB+AB\bar{C})$ using Boolean Rules and Laws. (5 marks)

2(b) Express $F(A,B,C)=A\bar{B}C+\bar{A}BC+\bar{A}\bar{B}C$ using a combinational circuit (you can only use two-input logic gates). (5 marks)

Hint: in this question type, you are required to draw a logic circuit diagram

2(c) Simplify $F(A,B,C)=A\bar{B}C+\bar{A}BC+\bar{A}\bar{B}C$ using Karnaugh map. (5 marks) And then, express $F(A,B,C)$ using combinational circuit. (5 marks)

<u>Solution</u>

(a)

$(A+\bar{A})(AB+AB\bar{C})$	1 mark
$=1.(AB+AB\bar{C})$	Unit property (2 marks)
$=AB$	Law of absorption (2 marks)

In this question only, the name of rules/laws is not necessary.

(b)

The given statement is a sum of products with 3-input AND gates.

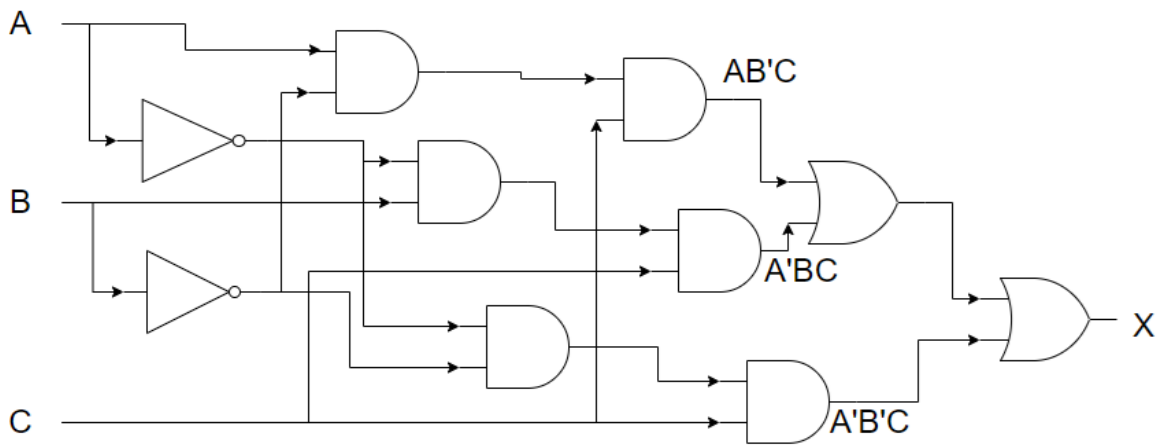
If only 1-input or 2-input gates are allowed, we need to convert 3-input ANDs to 2-input ones. Consider the associative laws (AB)C=A(BC)=ABC.

Hint: see the truth tables as proof (This part no need to include in your answer)

A	**B**	**C**	**ABC**	**A(BC)**	**(AB)C**
0	**0**	**0**	**0**	**0**	**0**
0	**0**	**1**	**0**	**0**	**0**
0	**1**	**0**	**0**	**0**	**0**
0	**1**	**1**	**0**	**0**	**0**
1	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**0**	**0**	**0**
1	**1**	**0**	**0**	**0**	**0**
1	**1**	**1**	**1**	**1**	**1**

A	**B**	**C**	**A+B+C**	**A+(B+C)**	**(A+B)+C**
0	**0**	**0**	**0**	**0**	**0**
0	**0**	**1**	**1**	**1**	**1**
0	**1**	**0**	**1**	**1**	**1**
0	**1**	**1**	**1**	**1**	**1**
1	**0**	**0**	**1**	**1**	**1**
1	**0**	**1**	**1**	**1**	**1**
1	**1**	**0**	**1**	**1**	**1**
1	**1**	**1**	**1**	**1**	**1**

The combinational circuit is hence: (5 marks)



(c)

We let X be the output of the function F(A,B,C): (1 mark)

A	**B**	**C**	**X**
0	**0**	**0**	**0**
0	**0**	**1**	**1**
0	**1**	**0**	**0**
0	**1**	**1**	**1**
1	**0**	**0**	**0**
1	**0**	**1**	**1**
1	**1**	**0**	**0**
1	**1**	**1**	**0**

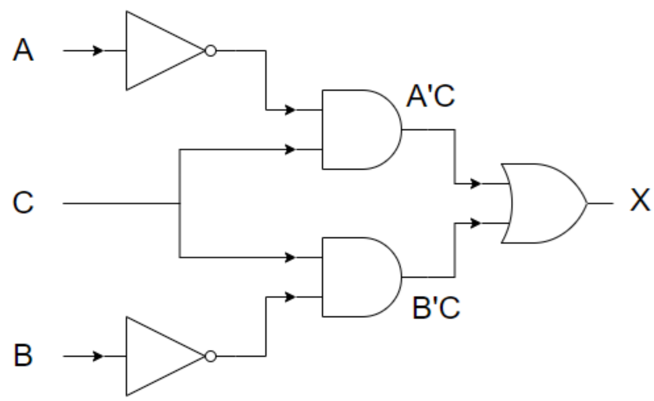
The truth table can be presented by the following K-map: (2 marks)

C \AB	**00**	**01**	**11**	**10**
0	**0**	**0**	**0**	**0**
1	**1**	**1**	**0**	**1**

Correct groupings (2 x 0.5 = 1 mark)

The simplified statement is hence X = $\bar{A}C + \bar{B}C$ (1 mark)

The combinational circuit is hence: (5 marks)



Question 3 **[10 marks]**

Computing students are enrolling for different class sessions of a course. Given:

Paul: available for Tuesday morning & Tuesday afternoon

Mary: available for Thursday morning & Friday afternoon

Peter: available for Tuesday morning & Thursday morning

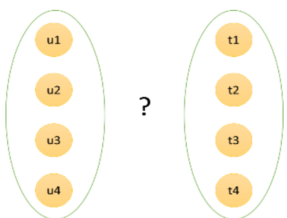
Susie: available for Tuesday morning & Friday afternoon

Use maximum flow method to solve this class assignment problem. (10 marks)

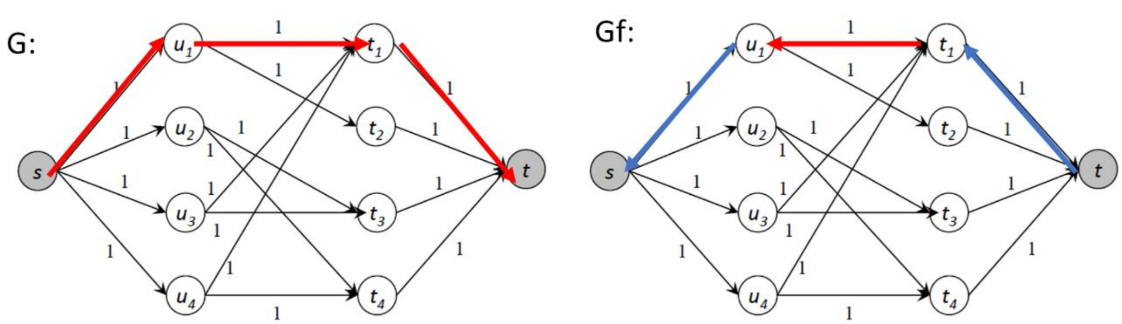
<u>Solution:</u>

(8 marks for at least 4 correct matching shown in steps, 2 marks each)

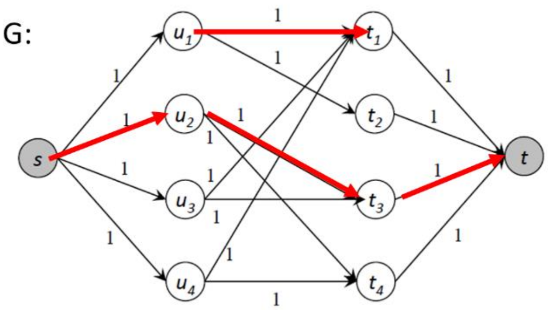
This is a very common problem for assigning n tasks (t) to n people, while compromise to their preferences.



It could be solved by flow networks with residual graph:



G:



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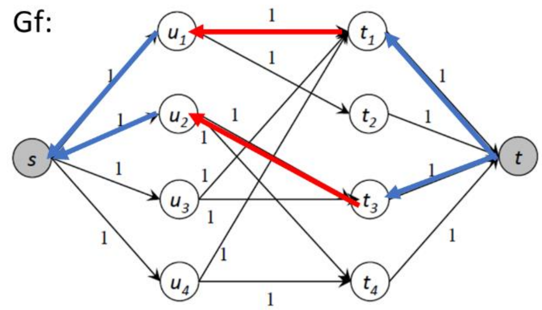

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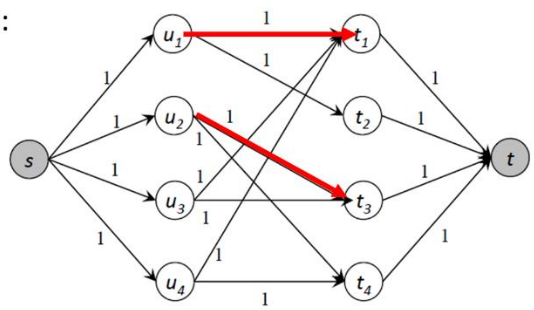

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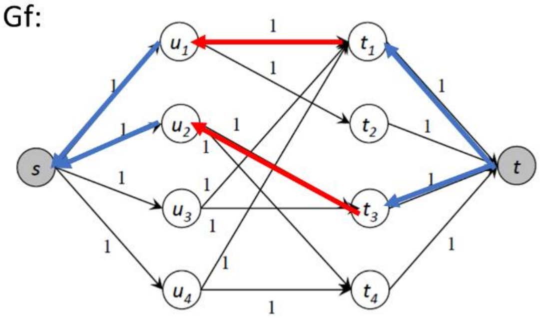
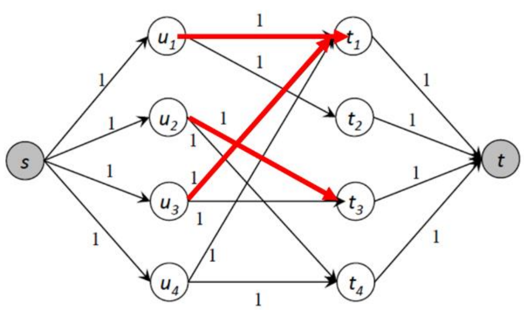


In the step below, u3 can produce two match to resolve the contradiction of (u1,t1) and (u3,t1) pairs. As compromise, form (u1,t2) and (u3,t1). This makes use of benefit of residual graph.

G:



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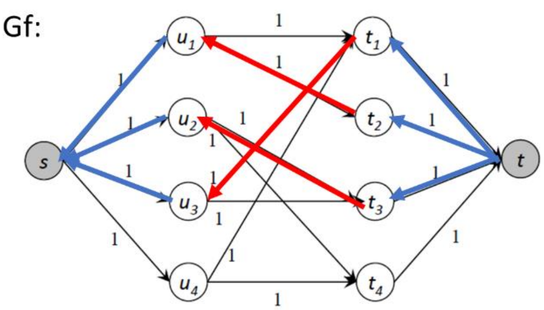

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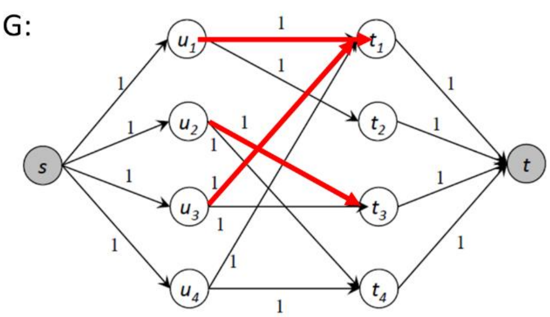

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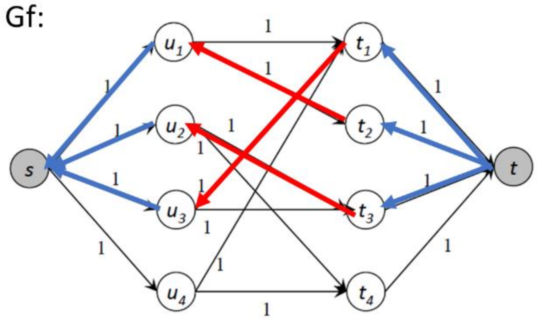
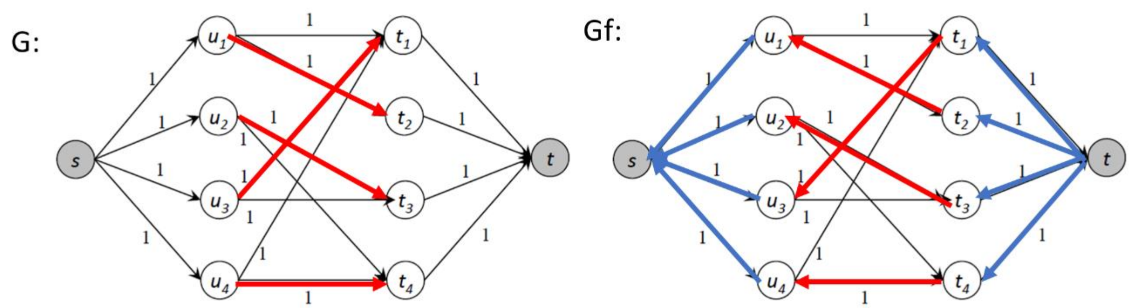




Hence, the answer is: (2 marks, for final result)

Paul (u1) will be assigned to Tuesday afternoon (t2)

Mary (u2) will be assigned to Thursday (t3)

Peter (u3) will be assigned to Tuesday morning (t1)

Susie (u4) will be assigned Friday afternoon (t4)

Question 4. **[30 marks]**

4(a) Given the array of integers below, draw a Binary Search Tree (BST). (5 marks)

70 11 47 81 20 61 10 12 13 62

4(b) Is this BST a balanced tree? (1 mark) Give your justification (2 marks)

4(c) List nodes in a *pre-order traversal*. (5 marks)

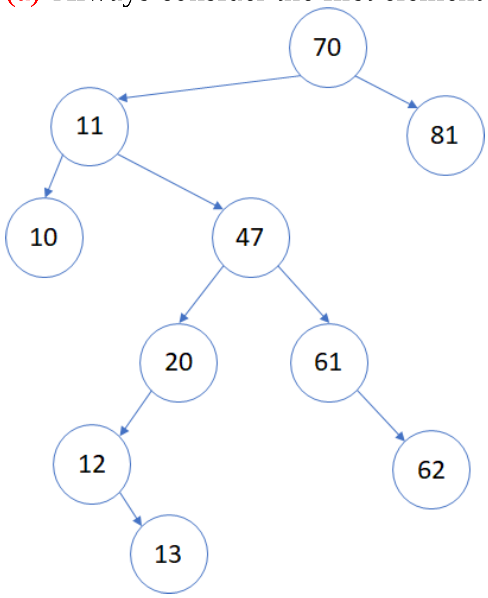
4(d) List nodes in a *post-order traversal*. (5 marks)

4(e) List nodes in an *in-order traversal*. (5 marks)

4(f) On the BST, show the steps to delete node 11 followed by deleting node 47? (7 marks)

Solution:

(a) Always consider the first element as root node. (5 marks)



(b) (3 marks)

No, not a balanced tree (1 mark)

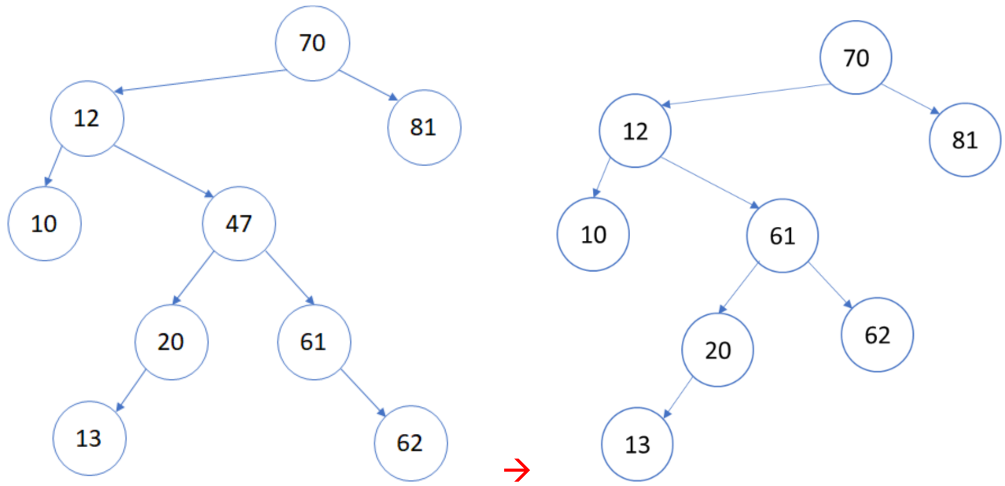
It is because there exist some leaves (or, leaf nodes) i.e. 10, 81 at levels < h-1 (2 marks)

(c) Pre-order: 70, 11, 10, 47, 20, 12, 13, 61, 62, 81 (5 marks)

(d) Post-order: 10, 13, 12, 20, 62, 61, 47, 11, 81, 70 (5 marks)

(e) In-order: 10, 11, 12, 13, 20, 47, 61, 62, 70, 81 (5 marks)

(f) Let z be node 11, which has two children (two subtrees). Take the smallest value of the right sub-tree of z to replace z.
As we also need to delete 47 from the original position, similarly 61 will take over the parent position. (7 marks)



Question 5. **[20 marks]**

Figure 5-1 shows the campus map of the Hong Kong Polytechnic University:

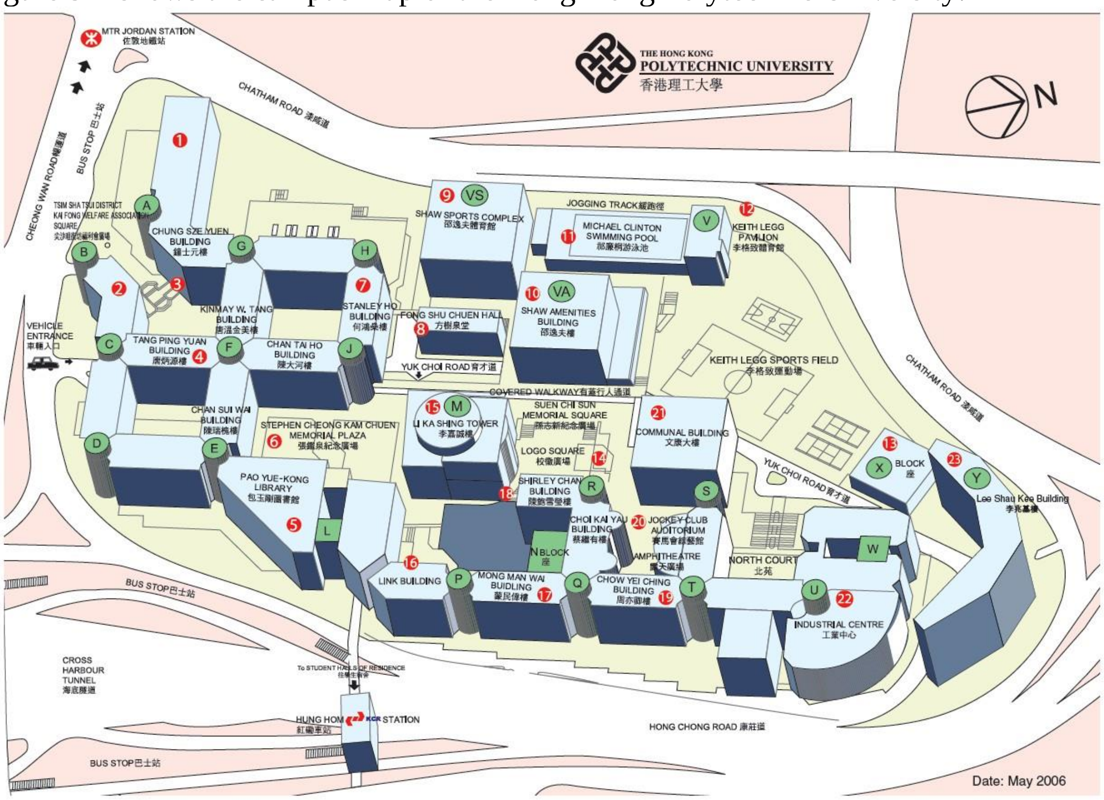


Figure 5-1

We define the distances between Cores/Blocks/Towers:

M to R: 150 metres, Q to R: 100 metres, Q to M: 185 metres, Q to T: 100 metres
R to S: 100 metres, S to T: 100 metres, T to W: 115 metres, S to W: 122 metres
T to U: 100 metres, U to W: 55 metres, W to Y: 61 metres. U to Y: 83 metres
M to VA: 112 metres, R to VA: 171 metres, S to VA: 202 metres

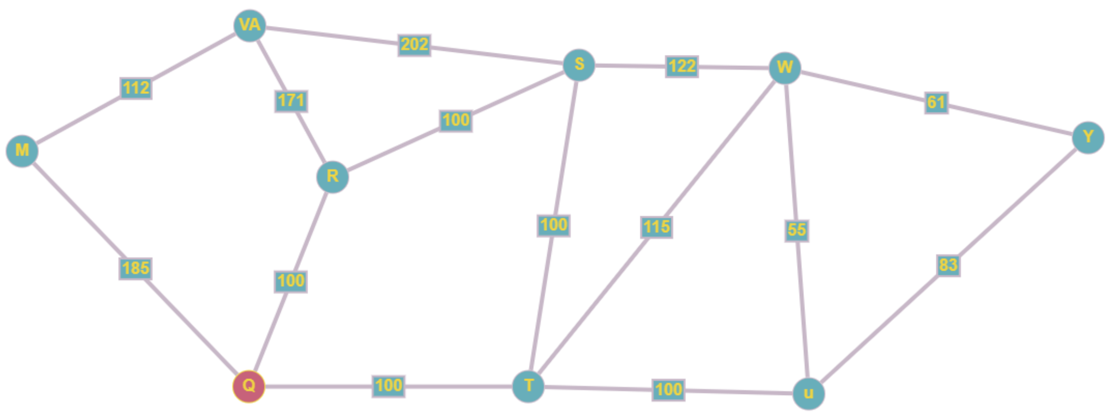
5(a) Start from Core Q, find the lowest cost distances to the building/tower of the following landmarks:

(i) Tower M (5 marks)
(ii) Classroom Y302 (5 marks)
(iii) 7-Eleven (5 marks)

5(b) Write down the shortest path from Core Q to Classroom Y302 in order to attend the COMP2012 lecture. (5 marks)

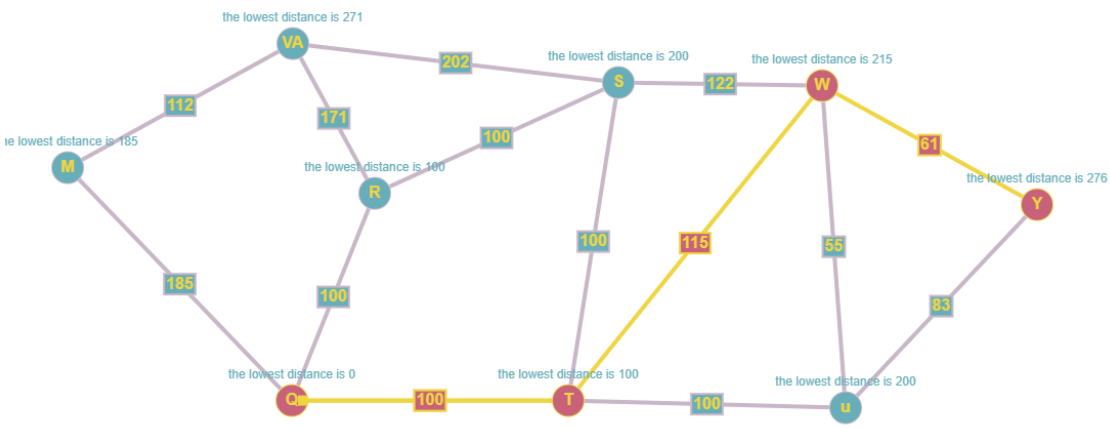
Solution:

Note: In this question, if you just write down the answer without steps/explanation (i.e. Dijkstra's Algorithm), even your answer is correct, this question will give 0 mark as a suspect case of copy-cat or plagiarism.



Suppose the **Dijkstra's algorithm** is run correctly. (max. 8 marks)
Distances from Q to followings are:
M: 185 → for part (i) (3 marks. This does not require step, as you can read directly)
R: 100
VA: 271 → for part (iii) 7-Eleven (2 marks, 0 if no proper steps)
S: 200
T: 100
U: 200
W: 215
Y: 276 → for part (ii) Classroom Y302 (2 marks, 0 if no proper steps)

5(c)
We assume the distance between the distance form Y's entrance to Y302 is negligible;
From Core Q to Tower Y.
The shortest weight/cost path would be Q→T→W→Y
(5 marks if provided explanations of shown the Dijkstra's algorithm steps in previous parts)
(0 mark if only write down the answer, no steps without explanations)



End of Assignment 2 solution