COMP4011 Theory of Computation
Assignment 2

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Problem 1

(1)

S is nullable since $S \to \epsilon$. Hence, the grammar without nullable productions is:

$$S \to ASB|AB$$
$$A \to aAS|aA|a$$
$$B \to SbS|Sb|bS|b|A|bb$$

(2)

The only unit production is $B \to A$. Then the grammar without unit productions is:

$$S \to ASB|AB$$
$$A \to aAS|aA|a$$
$$B \to SbS|Sb|bS|b|aAS|aA|a|bb$$

(3)

Since $S \to AB, A \to a, B \to b$, there is not any useless symbol.

(4)

The resulting grammar in Chomsky Normal Form is:

$$C \to a$$
$$D \to b$$
$$E \to AS$$
$$F \to DS$$
$$S \to EB|AB$$
$$A \to CE|CA|a$$
$$B \to SF|SD|DS|b|CE|CA|a|DD$$

Problem 2

Assume that the language is context-free, and the pumping length is n. Suppose that $z = a^n b^n c^n = uvwxy$ with $|vwx| \leq n$ and $|vx| > 0$. If vwx contains only a, b, or c, then uv^2wx^2y will not be in the language since the number of the terminal symbols that is contained by vwx is larger than others. If vwx contains only a and b, then uv^2wx^2y will not be in the language since the number of a's and c's, or the number of b's and c's will not be equal. If vwx contains only b and c, then uv^2wx^2y will not be in the language since the number of a's and b's, or the number of a's and c's will not be equal. Since $|vwx| \leq n$, it is impossible for vwx to contain only a and c, or contain a, b, and c at the same time. Therefore, the language is not context-free.

Problem 3

For the string *aabab*, we can perform the following inductions:

$$\begin{array}{llllll} X_{11} = \{A, C\} & X_{22} = \{A, C\} & X_{33} = \{B\} & X_{44} = \{A, C\} & X_{55} = \{B\} \\ X_{12} = \{B\} & X_{23} = \{S, C\} & X_{34} = \{A, S\} & X_{45} = \{S\} & \\ X_{13} = \{C\} & X_{24} = \{B\} & X_{35} = \{S\} & & \\ X_{14} = \{S, A, B\} & X_{25} = \{B\} & & & \\ X_{15} = \{S\} & & & & \end{array}$$

Since the start state S is in X_{15}, the string *aabab* is in the language.