COMP2012 Discrete Mathematics Assignment 2

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1 Question 1

1.1 Question 1 (a)

The min-cut of the graph is as follows:

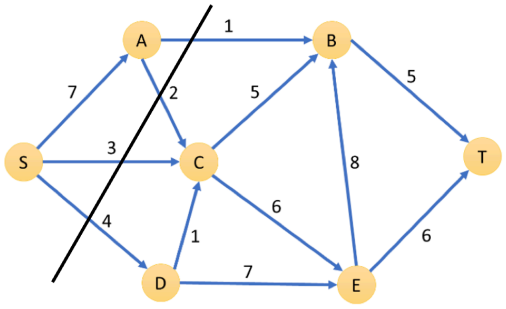
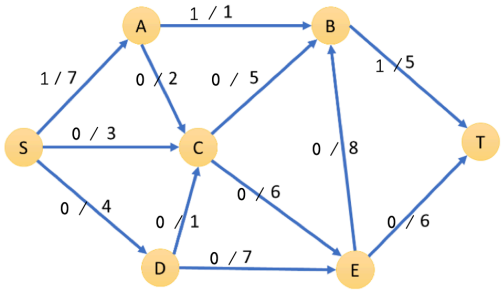


Figure 1: The min-cut of the graph

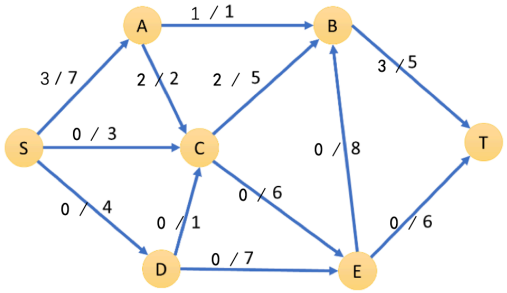
By adding the capacity of the cut edges, the maximum flow is $1 + 2 + 3 + 4 = 10$.

1.2 Question 1 (b)

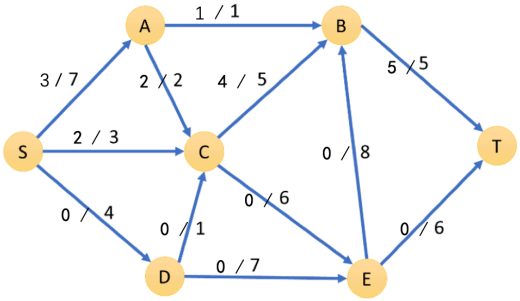
One of the possible procedures using Folk-Fulkerson method:



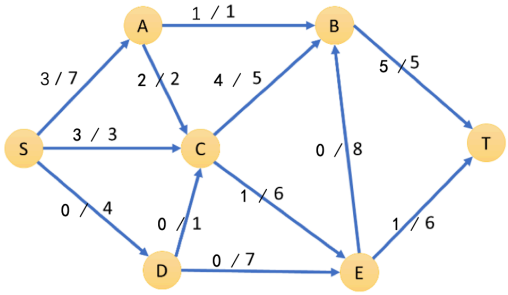
(a) Iteration 1: $S \to A \to B \to T$



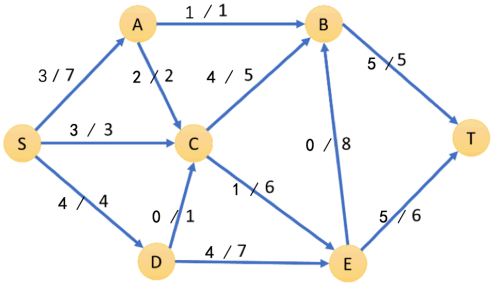
(b) Iteration 2: $S \to A \to C \to B \to T$



(c) Iteration 3 : $S \to C \to B \to T$



(d) Iteration 4: $S \to C \to E \to T$

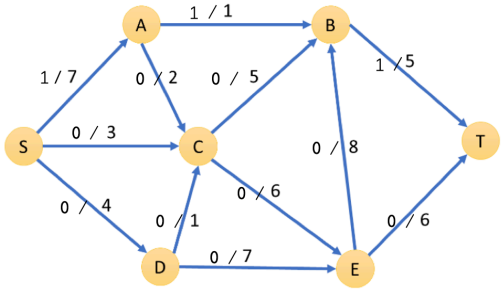


(e) Iteration 5: $S \to D \to E \to T$

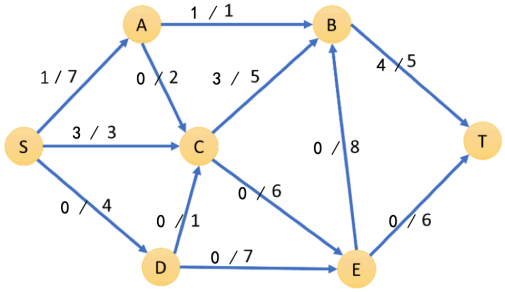
In conclusion, the maximum flow is 10.

1.3 Question 1 (c)

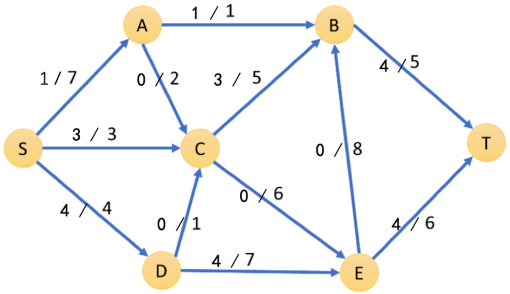
One of the possible procedures using Edmonds-Karp algorithm:



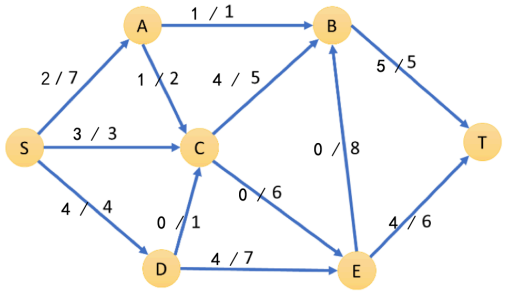
(f) Iteration 1: $S \to A \to B \to T$



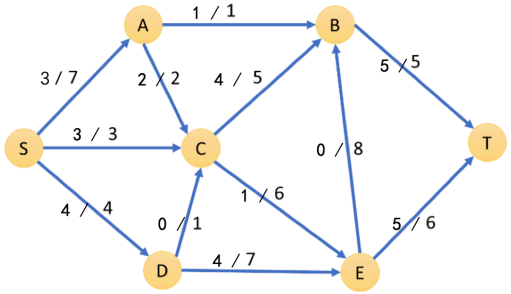
(g) Iteration 2: $S \to C \to B \to T$



(h) Iteration 3: $S \to D \to E \to T$



(i) Iteration 4: $S \to A \to C \to B \to T$



(j) Iteration 5: $S \to A \to C \to E \to T$

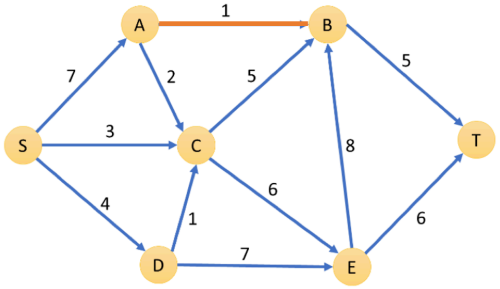
In conclusion, the maximum flow is 10.

1.4 Question 1 (d)

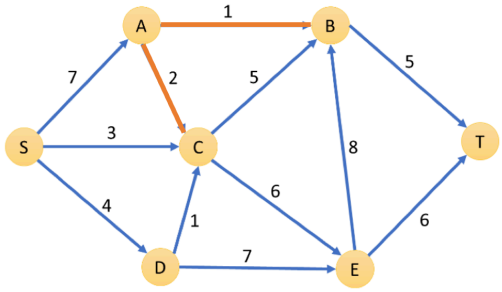
In the given graph G, the Edmonds-Karp algorithm does not necessarily outperform the Folk-Fulkerson algorithm, as in my answer, they both spend 5 iterations to find out the maximum flow. However, Edmonds-Karp usually has an smaller time completely than the Folk-Fulkerson method as it adapts the Breadth-First Search strategy, leading to improved performance compared to other path-finding strategies.

1.5 Question 1 (e)

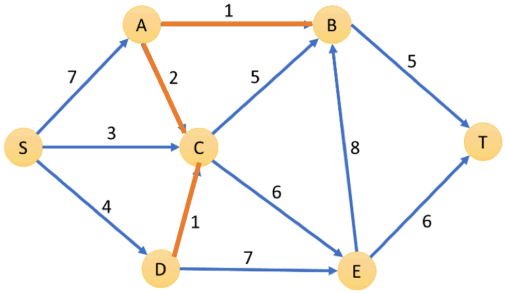
A possible minimum spanning tree based on Prim's algorithm:



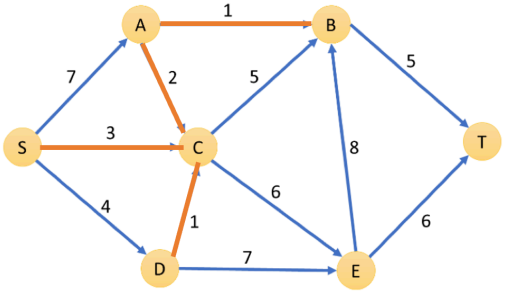
(k) Iteration 1: $c_f(A, B)$



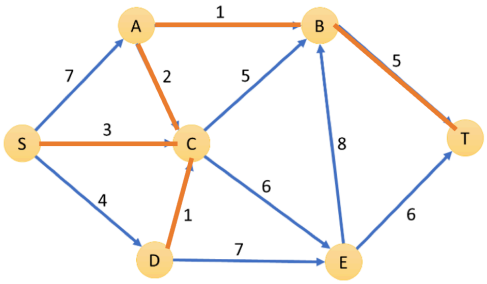
(l) Iteration 2: $c_f(A, C)$



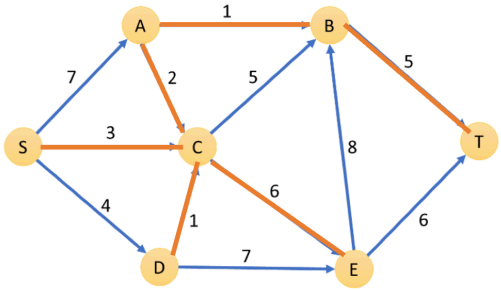
(m) Iteration 3: $c_f(C, D)$



(n) Iteration 4: $c_f(C, S)$



(o) Iteration 5: $c_f(B, T)$



(p) Iteration 6: $c_f(C, E)$

2 Question 2

2.1 Question 2 (a)

Since $A + \bar{A} = 1$, $(A + \bar{A}) \cdot (AB + AB\bar{C}) = AB + AB\bar{C}$. Then, according to the Absorption law, $AB + AB\bar{C} = AB$. Therefore, the logic of the given expression can be simplified as AB.

2.2 Question 2 (b)

The circuit of the given boolean expression is as follows:

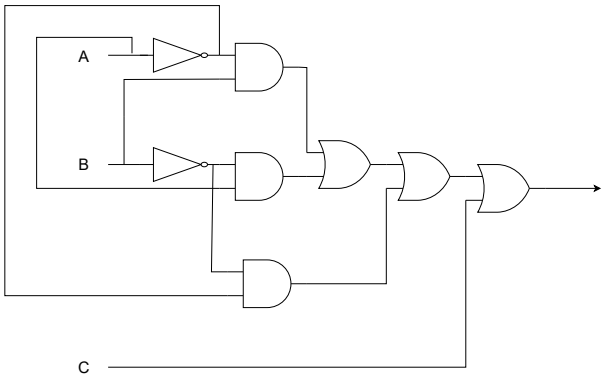


Figure 2: The logic circuit of the given expression

2.3 Question 2 (c)

The Karnaugh Map of the given expression is as follows:

C \ AB	00	01	11	10
0	0	0	0	0
1	1	1	0	1

Figure 3: The karnaugh map of the given expression

According to the map above, the simplified expression should be $\bar{A}C + \bar{B}C$. And it can be expressed by the following circuit:

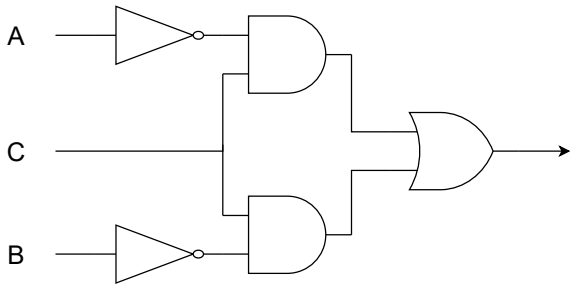


Figure 4: The logic circuit of the simplified expression

3 Question 3

A flow network can be established according to the given situation:

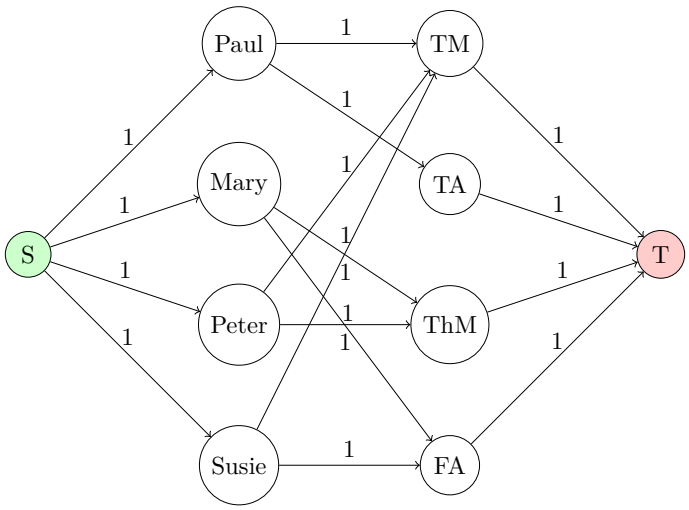


Figure 5: The flow network of the situation

Obviously, the maximum flow of this network is 4 along with the one of the possible schedules:

1. Paul attends the session on Tuesday Afternoon;
2. Mary attends the session on Thursday Morning;
3. Peter attends the session on Tuesday Morning;
4. And Susie attends the session on Friday Afternoon.

4 Question 4

4.1 Question 4 (a)

The binary search tree is as follows:

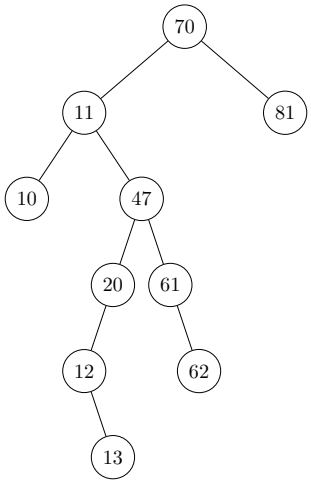


Figure 6: The binary search tree of the given array

4.2 Question 4 (b)

The binary search tree is not balanced. An obvious counterexample is that, the difference of the height of the two subtrees of the root node, 70, is 4, which is larger than 1.

4.3 Question 4 (c)

Preorder traversal: 70, 11, 10, 47, 20, 12, 13, 61, 62, 81.

4.4 Question 4 (d)

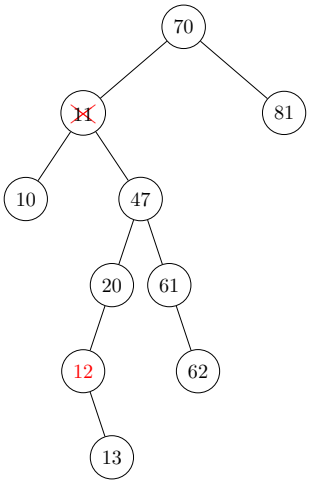
Postorder traversal: 10, 13, 12, 20, 62, 61, 47, 11, 81, 70.

4.5 Question 4 (e)

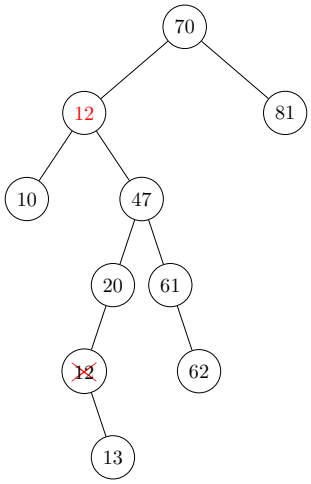
Inorder traversal: 10, 11, 12, 13, 20, 47, 61, 62, 70, 81.

4.6 Question 4 (f)

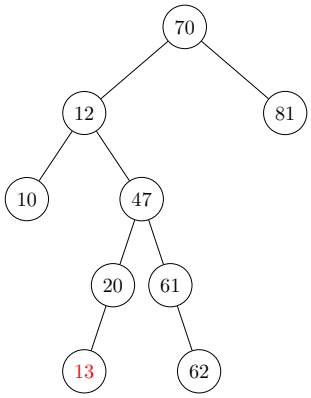
The procedure of deleting node 11 followed by deleting node 47 is as follows:



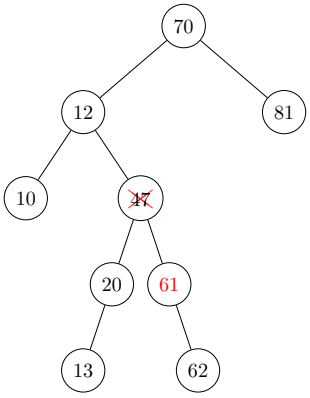
(a) Delete the node 11 and find its successor 12



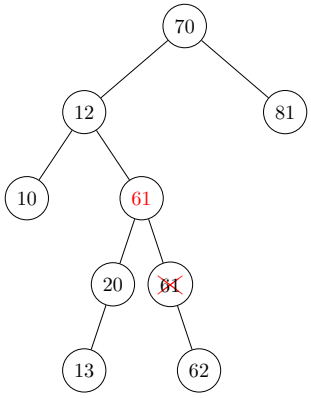
(b) Replace the node 11 by its successor 12 and delete 12



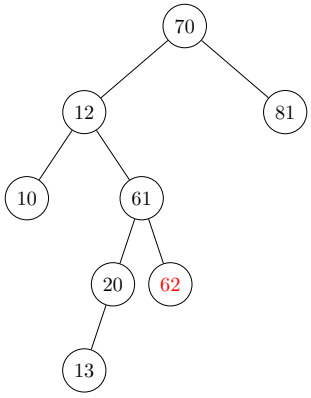
(c) Replace the node 12 by its only child 13



(d) Delete the node 47 and find its successor 61



(e) Replace the node 47 by its successor 61 and delete 61



(f) Replace the node 61 by its only child 62

5 Question 5

5.1 Question 5 (a)

(1) $Q \to M$: 185 meters.

(2) $Q \to T \to W \to Y$: 276 meters.

(3) $Q \to R \to VA$: 271 meters.

5.2 Question 5 (b)

$Q \to T \to W \to Y$: 276 meters.