COMP2012 Discrete Mathematics Notes - Logic and Proof

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1 Logical Calculation

1. Negation $\neg$
2. Conjunction $\wedge$
3. Disjunction $\vee$

Their connections to Boolean algebra is that:

$$\neg p \equiv \bar{p},\ p \wedge q \equiv p \cdot q,\ p \vee q \equiv p + q$$

2 Propositions

Given two propositions, p and q, we say that $p \leftrightarrow q$ is a biconditional statement. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow q)$. $\neg q \rightarrow \neg p$ is called the contrapositive of $p \rightarrow q$ which has the same truth value as $p \rightarrow q$ in all cases. $\neg p \rightarrow \neg q$ is called the inversion of $p \rightarrow q$, and $\neg(p \rightarrow q)$ is the negation of $p \rightarrow q$. The latter two's truth values have no relationship with $p \rightarrow q$, and they are contrapositive for one another.

3 De Morgan's Law

For two propositions, p and q, we have:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q,\ \neg(p \vee q) \equiv \neg p \wedge \neg q$$

4 Quantifier

1. Universal quantifier $\forall$
2. Existential quantifier $\exists$

$$\forall x \in D,\ P(x) \equiv x_1 \cup x_2 \cup \cdots \cup x_n = x,\ P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n)$$
$$\exists x \in D,\ P(x) \equiv x_1 \cup x_2 \cup \cdots \cup x_n = x,\ P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$$