1.Solution:

The Lagrange function of this problem could be denoted as follows:



Then, we could obtain the KKT conditions of this problem as blow:



So, when taking , we have  and if , then we find a solution to the KKT conditions.

When , we have . Then the value of  could be derived from the following equation:



And after getting the value of , if it is greater or equal to 0, then this is a solution to KKT conditions. And the corresponding optimal  is .

2.Solution:

The Lagrange function of this problem is denoted with:



Then we could get the KKT conditions of this problem as follows:



When we take , we have  and . And if , then this a solution to the KKT conditions, and thus an optimal solution.

When , we have . And we have , if this value is greater or equal to 0, then this is a solution to the KKT conditions, and thus an optimal solution to the problem.

3.Solution:

a) The problem could be reformulated as the following form:



The Lagrange function of this version of the original problem is as follows:



And the KKT conditions of this problem is denoted as below:



By solving this system of equations, we get that:

If , the optimal solution is .

If , it is easy to see that  and . And . Thus, from the given information, we know that  and that if then  and that if , then .

Thus, if , then the optimal solution to the problem is .

b) First, we will show for , there is . Assume there is optimal solution  to this problem, which doesn’t satisfy . It is easy to see that  must be positive. Otherwise, change the position of  with some other positive component which must exist since the condition  and . Then we also get a feasible solution but is greater than the previous one. Similarly, we could prove that .

If , then there must exist some other negative number in other components, let say .Then the new solution , the new solution is also feasible. And the objective value of the new solution is greater than the previous one. This contradicts the assumptions. Similarly, when we take , this also could establish contradictions. Thus, we have .

Assume . Then similarly, we could get . Thus, .

By using the above properties of the problem, we could easily reformulate the problem as the following:



As you see, it is almost the same as the first problem. So, if , the optimal solution is .

And if  then the optimal solution to the problem is  and other components are zeros.

c) The problem could be reformulated as the following problem:



The Lagrange function of this version of the original problem is as follows:



And the KKT conditions of this problem is as follows:



If , the optimal solution is .

If , it is easy to see that  and . And . Thus, from the given information, we know that  and that if then  and that if , then .

Thus, if , then the optimal solution to the problem is  where  .