GENERAL CERTIFICATE OF EDUCATION (GCE) BOARD

General Certificate of Education Examination

Pure Maths With Mech 2
0765/2

JUNE 2021

ADVANCED LEVEL

Subject Title	**Pure Mathematics With Mechanics**
Paper No.	Paper 2
Subject Code No.	**0765**

Three hours.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page

Turn Over

1. (i) The polynomial $f(x)$ is defined by $f(x) = 3 - ax + bx^2 - x^3$.
When $f(x)$ is divided by $(x + 2)$ the remainder is 45. Given that $(3 - x)$ is a factor of $f(x)$, find the values of the real constants a and b. **(5 marks)**

(ii) Express $\frac{5x}{(x+3)(x-2)}$ into partial fractions. **(3 marks)**

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2. (i) Given that the roots of the quadratic equation $x^2 + (2k + 2)x + 2k + 5 = 0$ are α and β,
(a) find the values of the constant k, for which $\alpha = \beta$. **(3 marks)**
(b) for $k > 0$, find the quadratic equation with integral coefficients, whose roots are $2\alpha + \alpha\beta$ and $2\beta + \alpha\beta$. **(5 marks)**

(ii) Find the range of values of x for which $\frac{2-x}{x+1} \geq 0$. **(3 marks)**

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3. (i) Show that $\frac{\sin 3A - \sin A}{\cos 3A + \cos A} = \tan 2A$. **(3 marks)**

(ii) Given that $f(\theta) = \sin\theta - \sqrt{3}\cos\theta$, express $f(\theta)$ in the form $R\sin(\theta - \lambda)$, where R is a positive constant and λ is an acute angle.
Hence, find the general solution of the equation $\sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$. **(7 marks)**

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4. (i) The real valued function f is defined as $f: x \mapsto \frac{4x}{x-1}, x \in \mathbb{R}, x \neq 1$.
(a) Show that f is injective. **(4 marks)**
(b) Find $(f \circ f)(x)$, stating its domain. **(3 marks)**

(ii) Two statements p and q are given by:
p: The principal goes to the ministry,
q: The principal will meet the minister.
Write out the following proposition in ordinary English:
(c) $p \wedge q$,
(d) $\sim p \vee \sim q$,
(e) $\sim p \Rightarrow \sim q$. **(3 marks)**

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5. The equations of two lines are given by
$$\mathbf{r}_1 = \mathbf{i} + a\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$
$$\mathbf{r}_2 = a\mathbf{i} + 2\mathbf{k} + \mu(-\mathbf{j} + 3\mathbf{k}),$$ where λ and μ are scalar parameters.
Given that $\mathbf{r}_1$ and $\mathbf{r}_2$ intersect,
find
(a) the value of the constant a, **(6 marks)**
(b) the position vector of the point of intersection, **(1 mark)**
(c) the cosine of the acute angle between $\mathbf{r}_1$ and $\mathbf{r}_2$. **(3 marks)**

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6. (a) Evaluate $\int_0^{\frac{\pi}{3}} \cos^4 3x \sin 3x \, dx$. **(5 marks)**

(b) Find the volume of the solid generated when the curve $y = x^3$ is rotated completely about the x-axis between the ordinates $x = 0$ and $x = 3$. **(5 marks)**

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7. (i) Show that the equation f(x) = 0, where f(x) = $3x^3 - x^2 + 3x - 1$ has a root between 0 and 1. **(3 marks)**

(ii) Given that the parametric equations of a curve are $x = \sin 2\theta$, $y = \cos 2\theta$, find $\frac{d^2y}{dx^2}$ in terms of the parameter θ. **(5 marks)**

(iii) Show that the function f: $\mathbb{R} \longrightarrow \mathbb{R}$, where $f(x) = (2x-5)^3 + x$, is an increasing function. **(3 marks)**

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8. (i) The sum of the first and second terms of a geometric progression is -5 while the sum of the fourth and fifth terms of the same progression is 40. Find the first term and common ratio of the progression. **(6 marks)**

(ii) Find the term indepentdent of x in the expansion of $\left(x + \frac{2}{x^2}\right)^{15}$. **(4 marks)**

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9. (i) Given the complex numbers $z_1 = 4 - 5i$ and $z_2 = p + 4i$, where $p \in \mathbb{R}$, find the value of p for which $z_1\bar{z}_2$ is purely imaginary, where $\bar{z}_2$ is the complex conjugate of z_2. Hence, find $|z_1|$ and $\arg(z_2)^2$. **(8 marks)**

(ii) Given that $z = x + iy$, $x, y \in \mathbb{R}$, find the locus of the point z such that $|z| = |z - 2 + 2i|$. **(3 marks)**

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10. (a) Find the value of the constant k for which the matrix **A** is singular,

where $\mathbf{A} = \begin{pmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 5 & 0 & k \end{pmatrix}$. **(2 marks)**

(b) Find the inverse of matrix **B**, if $\mathbf{B} = \begin{pmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 5 & 0 & 2 \end{pmatrix}$

Hence solve the system of equations

$$\begin{aligned} 2x + 3y + 4z &= 1, \\ x - y + 2z &= 2, \\ 5x + \quad\quad 2z &= 5. \end{aligned}$$

(9 marks)

(c) Find the image of the point $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ under the transformation matrix defined by **B**. **(2 marks)**
