

6-4: 已知

$$J_0(\lambda) = \begin{bmatrix} \lambda_0 & 1 & & \\ & \lambda_0 & 1 & \\ & & \lambda_0 & \\ & & & \lambda_0 \end{bmatrix}$$

求 $J_0^2(\lambda_0), J_0^3(\lambda_0), J_0^4(\lambda_0), J_0^k(\lambda_0), k \geq 5$.

6-5: 求矩阵A的最小多项式.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

6-8: 已知矩阵

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 6 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

求 $e^A, e^{tA}, \sin A$.

6-11 设 A 为 n 阶矩阵, 证明:

$$(1) \quad e^{2\pi i I} = I, \quad e^{2\pi i I + A} = e^A$$

$$(2) \quad \sin 2\pi I = 0, \quad \cos 2\pi I = I;$$

$$(3) \quad \|e^A\| \leq e^{\|A\|}. \quad (\|\cdot\| \text{ 是算子范数})$$

6-16: 求矩阵幂级数 $\sum_{k=0}^{\infty} \frac{k+1}{10^k} \begin{bmatrix} 1 & 2 \\ 8 & 1 \end{bmatrix}^k$ 的和.