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2018
MCM/ICM
Summary Sheet

Evaluation of the Attenuation of the Multi-hop HF Radio Propagation

Summary

In this paper, we build a model to simulate the propagation of the HF radio. In order to figure out the loss of signal strength, we divide the model into three parts: the attenuation in the ionosphere, the attenuation of reflection off the ocean & ground, the energy loss in the travel of waves. Among them, the key is to solve the problem of reflection on the ocean & ground. We modify the currently used model and add some assumptions to adapt to the environment proposed by the problem.

First, to solve the attenuation in the ionosphere, we collect the data of the ionosphere's electronic density and simulate the path of the radio. Then we obtain the relationship between the launching angel and the height of reflection. Based on the fact that the main form of energy loss in the ionosphere is through the forced vibration of electron, we can calculate the radiated energy of the radio to represent the attenuation.

Second, to analyse the situation of reflection in different environment, which is the key point of the problem and the breakthrough point of our model, we divide the task into two parts. For the reflection off the ocean, we use the reflection coefficient to simulate the attenuation. Considering the existence of turbulence, we introduce a modifying factor to suit the situation. For the reflection off the ground ,we make the assumption that the area and height of the terrain's cross section satisfy the normal distribution and obtain the reflection coefficient by Rayleigh criterion.

Third, we use the empirical formulas to figure out the attenuation in the process of travel and convert the unit to dB. The distance between the source of the signal and the receiver can be calculated by the height of the ionosphere and the launching angle of elevation.

At last, we draw the figures to show the influence of different elements, from which we can clearly see the attenuation. We analyze the sensitivity and evaluate the model. The result produced by our model is up to our expectations and correspond to the reality.

Keywords: ionosphere simulation; reflection coefficient; modifying factor

Evaluation of the Attenuation of the Multi-hop HF Radio Propagation

February 12, 2018

Contents

1 Introduction

1.1 Problem Background

High frequency radio(defined to be 3-30 MHz, also called shortwave) plays a significate role in data communication and broadcast. It has the advantage of high adaptation and safety, its devices have a smaller size compared with those of the long waves, also, it is hard to destroy and uneasy to detect. All of the superiorities indicate that it will have a bright future in both civilian and military field.

Shortwave can travel a long distance by multiple reflections. For frequencies under the *maximum usable frequency*(MUF), shortwave signal can be reflected by the ionosphere to the ground, then it may be reflected back to the ionosphere, and again back to the earth. . . through repeated reflections it can travel a long way from one point on earth to another, which is shown in Figure 1 [1]. For frequencies above the MUF, shortwave signal will not be reflected and will make its way through the ionosphere into the space. Besides, the MUF changes with the season, time of the day and solar conditions, thus affecting the travel of the HF radio.

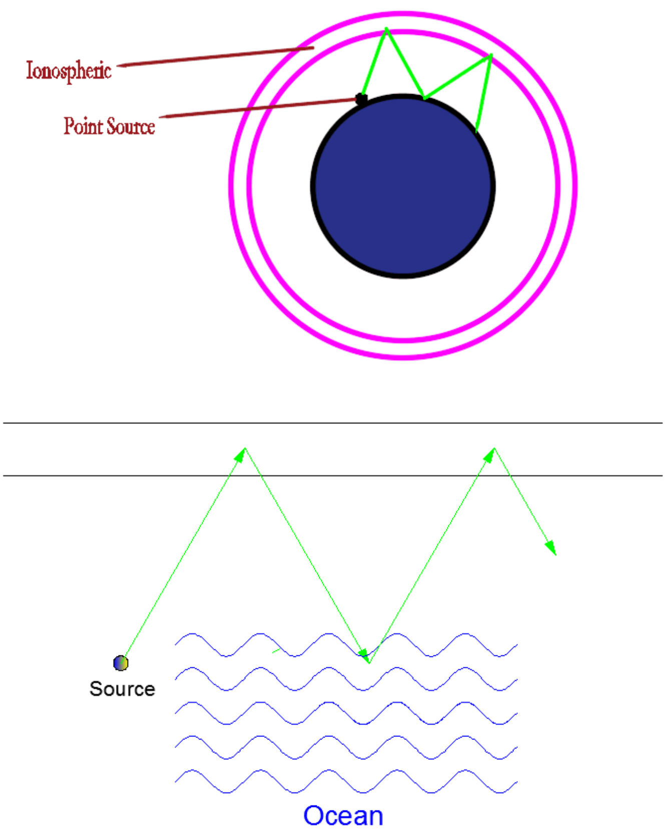

Figure 1: The Propagation of the HF Radio

The features of the reflecting surface largely decides how strong the reflecting wave is and how far it can travel while maintaining the *signal-to-noise ratio*(SNR) at a reasonable level. Among all the reflecting surface, the ocean surface is especially paid attention on not only because it makes up for about 70 percent of our planet surface, but also because it is affected by many elements such as ocean turbulence, which will change the local permittivity and permeability of the ocean. From our daily experience we find that the reflections off a calm ocean are attenuated less than reflections off a turbulent ocean, where wave heights, shapes, direction of the wave and frequencies change rapidly.

1.2 The Task at Hand

we are required to build a realistic, sensible and useful model to simulate the reflection of the HF radio propagation. By our analysis, we proposed to decompose the problem into three sub-problems as follows:

- Build a model that can simulate the first reflection of the HF radio off the ocean on a calm & turbulent condition.
- Compare the model on the ocean with the reflection off mountainous or flat ground.
- Modify our model to solve the problem of HF communication with the ship on a turbulent ocean.

1.3 Previous Research

Radio wave has a lower cost compared with other means of communication, but it can cover a long distance with low cost. So, it is widely used in Maritime Communications. Usually, radio broadcasting and prediction are based on empirical model. The empirical models are based on statistical analysis and digital maps with universality. But a specific environment has a unique geomorphic feature. So, we need analyze a specific environment in order to make sure that our models are suitable in different conditions. A relatively accurate propagation model can help us accurately estimate the coverage of the base station and predict the coverage of the network, it has an important strategic significance [2]. So far, with the help of extensive research and practice on the propagation environment, scientists' prediction about the spread of radio waves on land has been more accurate. There are about 10 commonly used models now. One of the most famous model among these models is called Okumura-Hata model, this model has been widely used in rural areas [3]. Also, this model has been improved by some scientists. Lida Akhoondzadeh-Asl and Narges Noori published some modification and tuning of the model for radio propagation predictions [4]. However, the model is Not suitable for the sea environment. The data collected does not fit the model. So, consider the particular environment, we need more targeted model.

1.4 Paper Organization

The structure of the paper is as follows: Section 3 includes the model for shortwave signal reflection off the ocean, Section 4 contains the model for reflection off mountainous & rugged terrain and makes comparison with the model in Section 3, Section 5 includes the model for signal communication with ships on the ocean, and in Section 6, we further evaluate our model and list its strengths and weaknesses. Finally, Section 7 make a summary of our findings in the form of a short synopsis suitable for publication as a short note in *IEEE Communications Magazine*.

2 Our Design of the Assessment Model

2.1 Modeling Objectives

We aim to develop a model that can reflect the relationship between the strength of signal after several reflection and the initial value. We suggest that the initial value mainly consists of the frequency of the signal and the launching angle of elevation. Also, we take the change of the environment into consideration to optimize our model. We hope our model can reflect the attenuation of HF radio under different conditions as comprehensive as possible and make comparison according to requirement of the problem.

In order to reveal the process of the multi-hop radio propagation, we decide to build several sub-models as follows:

- We aim to build a model to show the electronic density of ionosphere according to the data collected. Based on this model, we can find the relationship between the launching angle and the height where the reflection takes place
- The second model is aimed to simulate the attenuation in the process of propagation. In this model, we will not consider the attenuation of the reflection.
- The last model is aimed to simulate the attenuation of reflection. The reflection in the ionosphere mainly consults the electrodynamics, considering the electronic influence. To simulate the reflection on the ocean, we decide to calculate the reflection coefficient. Also, taking the turbulence into consideration, we further correct the reflection coefficient.

2.2 Notations

Here we list the symbols and notations used in this paper, as shown in Table 1. Some of them will be defined later in the following sections.

2.3 Assumptions and Justifications

- The attenuation of electromagnetic wave in the ionosphere is mainly caused by the forced vibration of the electric charge in the electromagnetic field.
- The law of reflection can be applied to our model because the scale of the fluctuation on a turbulent ocean is much smaller than the scale of the meter-wave.
- Since the Rayleigh scattering is proportional to the fourth power of frequency and the frequency of the HF radio is much lower than that of the light wave, we assume that the attenuation of the electromagnetic wave in the atmosphere can be ignored.

Table 1: Notation

Symbol	Description
H	The height where the reflection takes place
N_H	The density of electronics
θ	The launching angle of elevation
ϕ	The elevation of incidence
R_0	The radius of earth
r_e	Classical ionic radius
r	The reflection coefficient
rh	The reflection coefficient of horizontally polarized wave
rv	The reflection coefficient of vertical polarized wave
η	The logarithm of the proportion of energy loss due to the electric charge's forced vibration in the ionosphere
s	The integration of electronic density with respect to space
d	The distance that the signal travels
E	The electric field of the usable frequencies
y	The decibel of the attenuation after one reflection in the ionosphere and on the ocean
$f()$	The function relationship between the electronic density and height
$g_1()$	The difference compared to the electronic density of the height where reflection takes place suggested by the paper [5]

- In the process of the radio propagation, the locations near the source receive signal through ground wave while the locations far away receive the signal through sky wave.
- Our model is based on the *International Reference Ionosphere*(IRI-2007), we assume that the IRI-2007 can correctly reflect the ionosphere data.
- In order to simplify the problem, we assume that the shape of the mountainous terrain is similar to the shape of a normal distribution.

3 Model for Signal Reflection off the Ocean

3.1 Sub-model of the Ionosphere and Travel Distance

Ionosphere is the atmosphere around 60 to 450km away from the ground, it is full of free electron and ion, thus has electrical conductivity. Also, due to the existence of the free electron and ion, HF radio can be reflected, refracted or absorbed by the ionosphere. The characteristics of reflection and refraction are related to the permittivity and conductivity of the ionosphere. The elements that affect the reflection of HF radio in the ionosphere are the density of electronics, the frequency of the wave, and the angle of incidence of the radio, they satisfy the relationship as follows [5]:

$$N_H = \frac{f^2}{80.8}\left[1 - \left(\frac{R_0}{R_0 + H}\right)^2 \sin^2\phi\right] \tag{1}$$

NASA's measured the density of electronic in different height all around the world [6]. we take London for example and use a smooth curve to fit the data given by NASA. The result is shown in Figure 2, Figure 3, Figure 4,and Figure 5.

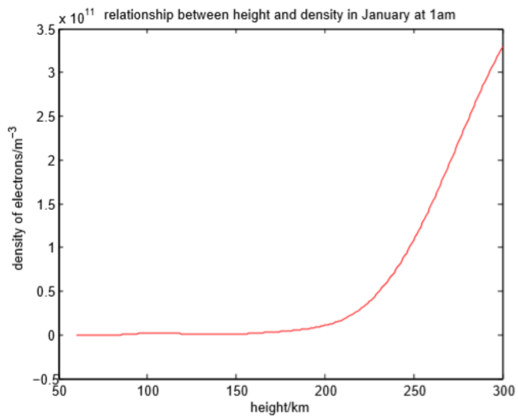

Figure 2: Relationship between Height and Density in January at 1am

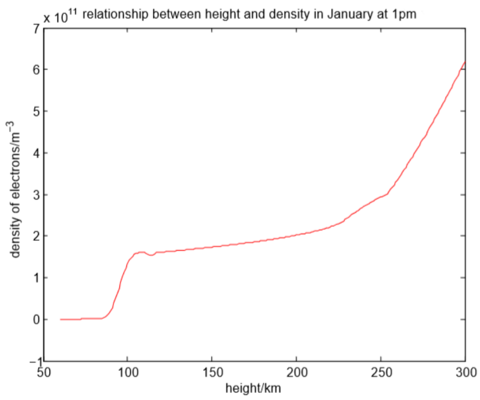

Figure 3: Relationship between Height and Density in January at 1pm

According to the figures we find that the climate change can influence the *maximum usable frequency*(MUF) and they have the following relationship [5]:

$$MUF = \frac{\sqrt{80.8N_H}}{\cos\phi} \tag{2}$$

Through all these relationships, for a given launching angle of elevation, we can calculate the height in the ionosphere where the signal is reflected, and as far as the height and the launching angle is known, we can easily obtain the propagation distance.

3.2 Sub-model of the Attenuation of Reflection in the Ionosphere

The atmosphere is very thin in the ionosphere, the electronics in motion are forced to vibrate under electromagnetic field and emit electromagnetic radiation, which is the

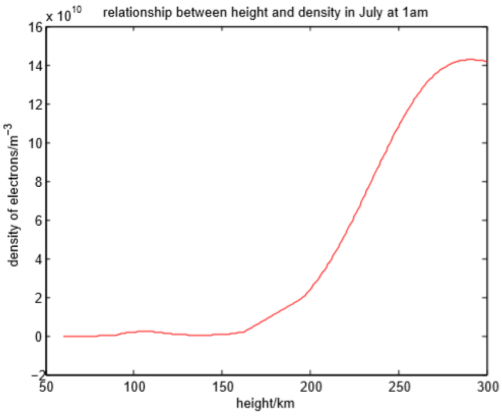

Figure 4: Relationship between Height and Density in July at 1am

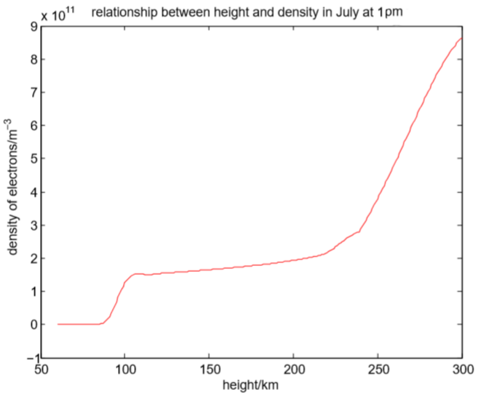

Figure 5: Relationship between Height and Density in July at 1pm

main form of the energy loss of the HF radio. In our model, we ignore other factors that may cause the attenuation of the wave.

David J.Griffiths and Reed College [7] introduced the method of calculating the power of energy loss, after combining several equations we rewrite one that fits this model:

$$\eta = \frac{8\pi r_e^2 W}{3} \int_0^{h_{max}} N_h dh \tag{3}$$

3.3 Sub-model of the Attenuation of Reflection on the Ocean

The attenuation on the ocean [8] is unpredictable, in order to simulate its features we divide the model into two parts. First, we simulate the reflection of radio on a calm ocean, on which condition we are able to use the Fresnel reflection coefficient formula. Then, in order to optimize our model to adapt the situation of a turbulent ocean, we decide to add a modifying factor [9].

3.3.1 Simulation of Reflection on a Calm Ocean

According to the Snell Law, the calm ocean is assumed as an ideal interface so we can obtain the Fresnel reflection coefficient formula of horizontally polarized wave & vertical polarization wave as follow:

$$rh = \frac{\sin\theta - \sqrt{\varepsilon - \cos^2\theta}}{\sin\theta + \sqrt{\varepsilon - \cos^2\theta}} \tag{4}$$

$$rv = \frac{\varepsilon\sin\theta - \sqrt{\varepsilon - \cos^2\theta}}{\varepsilon\sin\theta + \sqrt{\varepsilon - \cos^2\theta}} \tag{5}$$

Using the Equation 4 and the Equation 5, we can get the reflection coefficient of the horizontally polarized wave & vertical polarization wave, then we can use the reflection coefficient to simulate the attenuation of the HF radio.

3.3.2 Simulation of Reflection on a Turbulent Ocean

The law of the turbulence is hard to calculate and taken into account in such a short period time given by the MCM, so we adopt the way of adding modifying factor which not only simplify the problem but also can achieve outstanding results. The *International Radio Consultative Committee*(CCIR) offers a rough expression of the modifying factor ρ:

$$\rho = \frac{1}{\sqrt{3.2g - 2 + \sqrt{(3.2g)^2 - 7g + 9}}} \tag{6}$$

Among them:

$$g = 0.5\left(\frac{4\pi h^2 f\sin\theta}{c}\right)^2 \tag{7}$$

Among them, c represents the speed of light, f represents the frequency of the radio, h^2 represents the root-mean-square height of the sea level.

We can achieve the reflection coefficient of turbulence through multiplying the reflection coefficient of the flat ocean by the modifying factor ρ, then calculate the loss of energy in the same way as the model of reflection on the even ocean.

Thus, the decibel of attenuation after one reflection in the ionosphere and one on the ocean can be described as follows:

$$y = \left[\eta + \ln(\rho^2 \cdot \frac{rh^2 + rv^2}{2})\right]\frac{10}{\ln(10)} \tag{8}$$

3.4 The Empirical Formula of the Signal Strength

The attenuation during the process of the travel is related to the launching frequency and the distance that the signal travels, in order to determine the relationship between them and finish the unit conversion from w to dB, we introduce the empirical formulas [10] to model the energy change of the radio.

First, we set the launching frequency as $1kw$, then we have the relationship in Equation 9

$$E_{1kw} = 106.9 - 20\lg(d)dB(\mu V/m) \tag{9}$$

Based on Equation 9, we can expand the equation to all power of the source:

$$E = E_{1kw} + 10\lg(P)dB(\mu V/m) \tag{10}$$

And the final SNR can be calculated by adding E to$y$. Taking advantage of the two equations, we can measure and maintain the signal-to-noise ratio at a reasonable level according to the requirement of the task.

3.5 Evaluation and Calculation of the Model

We suggest the frequency of the HF radio is 6 MHz and the root-mean-square is 5 m^2 and draw the relationship between the strength of the signal and the launching angle of the elevation after the first reflection in the Figure 6.

Then we change the value of the root-mean-square height of the sea level to model different turbulence and obtain the results in Figure 7.

In Figure 8, we fix the elevation angle and the root-mean-square height of the sea level to find the influence of frequencies on the signal strength.

Figure 6, Figure 7,and Figure 8 imply the major three elements that affect the propagation of the HF radio and make comparison between the calm ocean and the turbulent ocean. As can be seen from the figures, the result of our model can fit the reality of radio communication well and basically reaches our expectations.

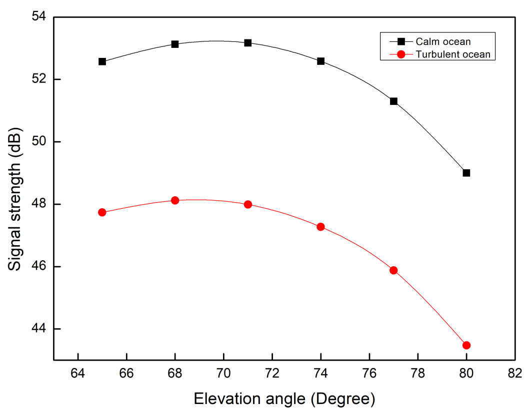

Figure 6: The Relationship between the Signal Strength and Elevation Angle

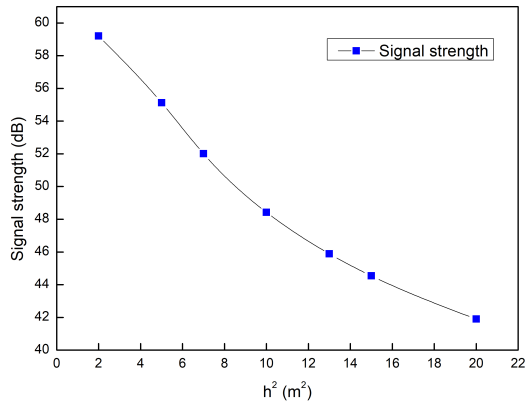

Figure 7: The Relationship between the Signal Strength and the Root-mean-square Height of the Sea Level

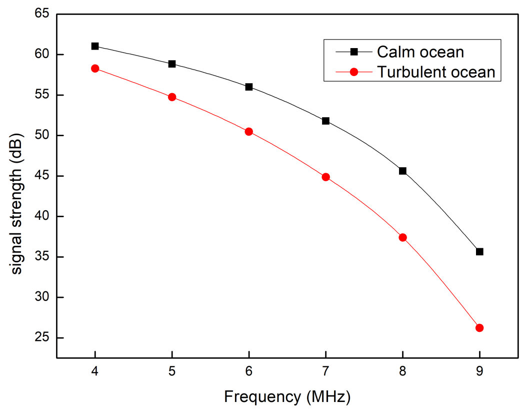

Figure 8: The Relationship between the Signal Strength and the Launching Frequencies

Then, we use our model to analyse the situation of multi-hop transmission. As can be seen in Figure 9, the signal strength falls below the useable signal-to-noise ratio threshold of 10 dB after the fourth reflection.

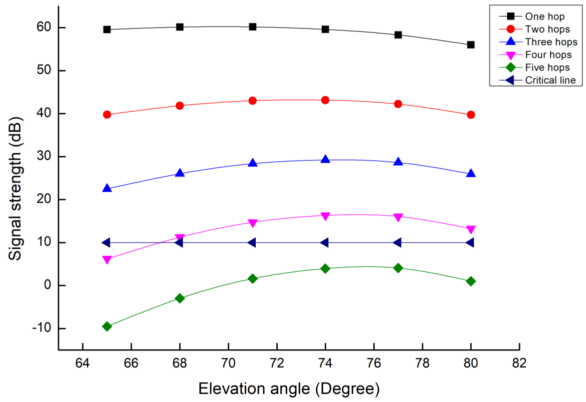

Figure 9: LEO Distribution of Space Debris

4 Model for Signal Reflection off Mountainous & Smooth terrain

4.1 Simulation of Reflection on the Smooth Ground

We assume that the reflection on the smooth ground is similar to the specular reflection. Thus the reflectivity is close to 1. On which condition, the ratio of the remaining signal's power to the initial power is e^{η}, thus the decibel of attenuation after the first reflection can be figured out as follows:

$$y = \eta \cdot \frac{10}{\ln 10} \tag{11}$$

4.2 Simulation of Reflection on the Rugged Ground

The area and height of the vertical cross-section of any terrain must satisfy certain distributions. Generally speaking, the normal distribution satisfies most of the occasion where the terrain is irregular. For mountainous & rugged areas, the increase of the relative height is a slow progress, thus, we suggest that the normal distribution can fit the situation of the rough terrain. So the relationship between the height and the area of the cross-section is

$$F(h) = \frac{\sigma}{\sqrt{2\pi}} e^{-\frac{h^2}{\sigma^2}} \tag{12}$$

According to the Rayleigh criterion, when the phase difference is smaller than $\frac{\lambda}{8}/2\pi$, we can roughly view the reflection as a specular reflection. Therefore, we calculate the ratio of specular reflection based on the assumption that the terrain satisfies the normal distribution. The ratio is

$$p = \frac{\sigma}{\sqrt{2\pi}} \int_0^{h_{max}} e^{-\frac{h^2}{\sigma^2}} h \tag{13}$$

Finally,we can use the equation $r = p \cdot \rho$ to obtain the reflectivity. The modifying factor rho because we take the irregular distribution as normal distribution. However, in actual practice, we suggest the value of ρ is 1 because the difference caused by the modifying factor can be ignored compared with the error caused by other factors. Due to the law of 3Sigma, the value of σ is around one third of the height of the mountain. Besides, we can also use the normal distribution to determine the value of σ, on which condition the value is $\sqrt{2\pi}h^2$.

4.3 Evaluation and Calculation of the Model

Similar to the model in Section 3, we set the frequency of the HF radio as 6 MHz and the root-mean-square height as 5 m^2 to obtain the relationship between the signal strength and the launching angle in Figure 10.

In Figure 11, we fix the launching angle of the signal to see the relationship between the frequency and the signal strength.

In Figure 12, we change the value of height to simulate the level of rise and fall of the mountain to study its influence on the strength of the signal.

And we compare the model of the ocean with the model of the ground in Figure 13 and Figure 14. As can be seen in the figure, the trend of the signal strength is similar on the ocean and ground.

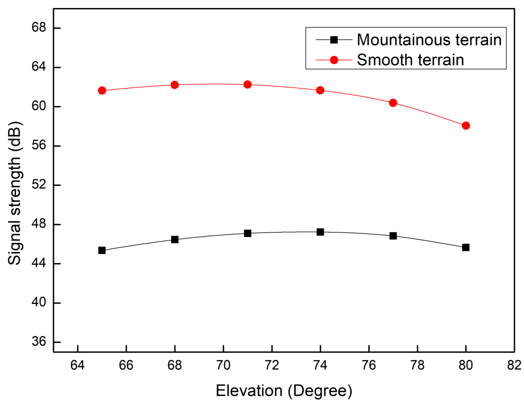

Figure 10: The Relationship between the Signal Strength and Elevation Angle

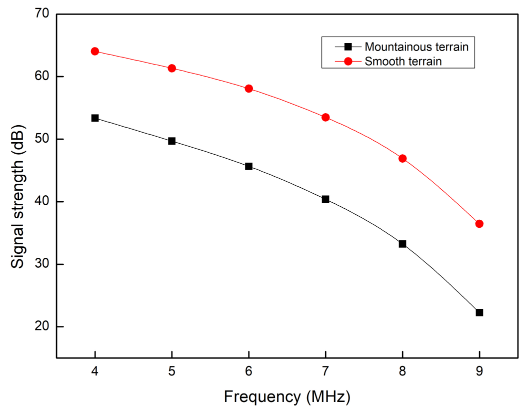

Figure 11: The Relationship between the Signal Strength and the Launching Frequencies

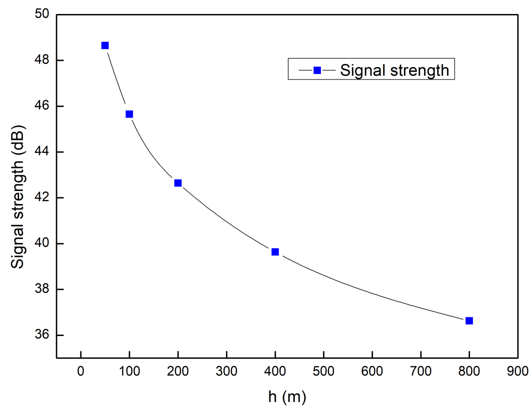

Figure 12: The Relationship between the Signal Strength and the Height

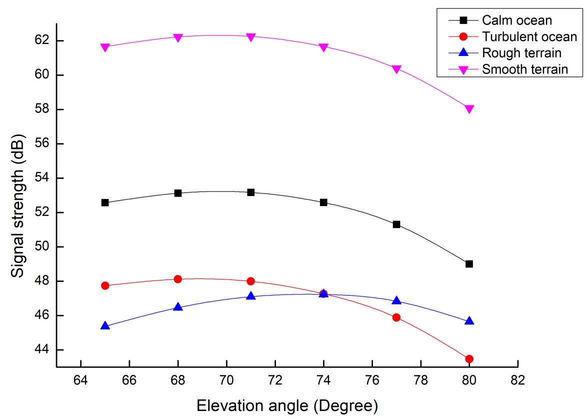

Figure 13: The Relationship between the Signal Strength and the Height

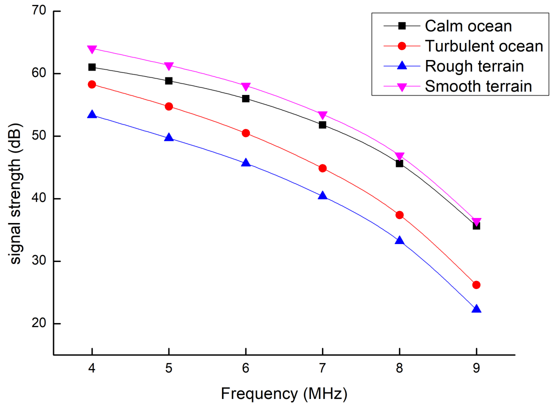

Figure 14: The Relationship between the Signal Strength and the Height

5 Model for HF Communications with Ships on the Ocean

The two models above mainly solve the task of attenuation. For task 3, we decide to transform our model on the basement of the two models above. This time we will take the upper and lower bounds of the elevation angle into consideration and decide the range of areas that the HF radio wave can reach.

The model in Section 3 reveals that for a fixed launching angel, the signal will travel a certain distance during each hop. However, due to the limitation of the ionosphere, some radio signal will not be reflected.Thus there exists an upper bound and a lower boundary. The launching angle must be between the two bounds so that the electromagnetic wave can be reflected by the ionosphere. We can figure out the range of areas that the radio wave can reach as soon as we work out the maximum and the minimum launching angle.

Let's take the radio of $6MHz$ for instance, with the help of our model, we obtain the result that the upper angle of the radio is 81° while the lower angle is 59°. In addition, we can figure out the area that the signal can reach after the first reflection. In our case, the ship that is between $1066km$ and $2663km$ away from the launching source can receive the signal of one hop. In the same method, we can calculate the range of area of the following reflection.

6 Model Evaluation & Sensitivity Analysis

6.1 Sensitivity Analysis

In our model, we mainly study the influence of different elements on the strength. And by our analysis, the launching angle of elevation, the height of the wave and the frequency of the signal are the three most important factors. As can be seen in the figures, the radio signal is quite sensitive to the three elements, which means the small change of these parameters can be reflected clearly on the signal strength. After observing the figures, we can come to some more specific conclusions:

- The signal reflection off the ground is more sensitive to the change of launching angle than the reflection off the ocean, which can be seen in Figure 6 and Figure 10. Thus, we can conclude that the reflection coefficient on the ocean is depend more on the elevation angle than that on the ground.
- Compare Figure 7 with other figures we can find that the change caused by root-mean-square height of the sea level is more obvious than the two other factors, which also reveal the importance of the ocean wave model in the propagation of radio wave. The simulation of turbulence is of great importance.
- Besides the elements mentioned above, the climate and weather can also change the ionospheric structure and the permittivity of the ocean. However, compared with the elements above, their influence can be ignored.

6.2 Strengths and Weaknesses

6.2.1 Strengths

- By plotting various figures, our model achieves a good visualization effect.
- On the condition that the analytic solutions are hard to obtain, we achieved the numerical solutions based on the simulative result of the MATLAB program.
- After consulting a large number of papers, we build an accurate, realistic and simplified model that can reflect the reflection on the ocean and in the ionosphere.
- We deduce the radiation loss in the ionosphere which is reasonable according to our result.
- Our model is innovative in that we establish a simulation of the mountainous & rugged area.
- We build a model that can calculate the attenuation during the propagation of the sky wave.

6.2.2 Weaknesses

- Our model applies many empirical formulas, which makes our deduction not so strict.
- The chaos phenomena make the turbulence hard to predict. Thus, it is hard to obtain the vibration of ocean during the reflection of the sky wave, which lead to the fact that our model is not of much value in practice.
- Due to the limitation of time, we do not have the chance to further analyze the actual terrain, such as mountains, valleys and cities.
- We fail to take the influence of climate on the ionosphere into consideration and we also ignore the sunspots and other solar activities which affects the ionosphere.
- We assume that the flat ground satisfies the specular reflections but many elements of the ground, such as the soil water content and the vegetation plant cover rate, makes a big difference to the reflection coefficient.

7 Conclusion

Our model clearly shows the three major parts of the radio's attenuation(the attenuation in the ionosphere, the attenuation of the reflection off the ocean, and the energy loss in the process of propagation). We adopt the modifying factor and many assumptions to get a complete model. The result of our model is shown in variable-controlling approach, which is good in visualization and offers an answer to the tasks given by the problem.

8 A Synopsis of Our Model for publication in *IEEE Communication Magazine*

Abstract: The study of the communication of the HF radio(defined to be 3-30MHz) is of great civilian and military importance. In this paper , we try to build a model to research on the attenuation of the HF radio in the process of the propagation.

The HF radio can travel a long distance by multiple reflections. For frequencies under the *maximum usable frequency*(MUF), radio signal can be reflected by the ionosphere to the ground, then it may be reflected back to the ionosphere, and again back to the earth... through repeated reflections it can travel a long way from one point on earth to another. For frequencies above the MUF, shortwave signal will not be reflected.

The prediction and the simulation of the radio transmission are mainly based on empirical formulas. Many factors contribute to the propagation of radio. For example, the geographic conditions are unique in certain area and there is turbulence on the ocean. Also, as the nature environment changes, the structure of the ionosphere changes. Thus, the current model of the radio propagation needs further modifying.

First, we correct the model of the ionosphere. With the help of the real-time monitored dates collected by NASA, we can predict the path of the HF radio in the ionosphere. In addition, we build a model of the radio's propagation based on the theory of electronic collision in electrodynamics. Through a series of deduction and calculations, we obtain the model as follows:

$$\eta = \frac{8\pi r_e^2 W}{3} \int_0^{h_{max}} N_h dh \tag{14}$$

Then, we analyze the reflection coefficient on the ocean. The attenuation on the ocean is unpredictable, in order to simulate its features we divide the model into two parts. First, we simulate the reflection of radio on a calm ocean, on which condition we are able to use the Fresnel reflection coefficient formula. Then, in order to optimize our model to adapt the situation of a turbulent ocean, we decide to add a modifying factor. And finally we obtain the result as follows:

$$y = \left[\eta + \ln(\rho^2 \cdot \frac{rh^2 + rv^2}{2})\right] \frac{10}{\ln(10)} \tag{15}$$

Finally, after using several unit conversion formula and adding some necessary modifications, we obtain the model of the signal strength:

$$E = E_{1kw} + 10\lg(P) dB(\mu V/m) \tag{16}$$

Also,we build a model of the reflection on the ground. Instead of using empirical formulas, we make an assumption of the distribution of the terrain. In our task, the angle of the ground generally fit the normal distribution. Based on the assumption, we make some calculations and obtain the results that suit the reality, which confirms the validity of our model.

At last, we analyze the range of area that the HF radio can reach. Due to the limit of the ionosphere and the signal strength, the travel distance of each hop is finite. The launching angle of the signal source is limited as well. These limitations can be figured out with the help of our model, which is the threshold value of the model.

The model we establish is complete and accurate, which offers explanation for the two most important elements(the signal strength and the outreach of the signal) in the propagation.

9 Appendix

9.1 Simulation Code

```
function y=h_theta1_1(x1,y1)
f=input('\Omega=');
%the frequency of the electromagnetic wave
phi=input('\Phi=');
%the elevation of incidence
R0=6371;
%the radius of earth
h=0;
g1=@(t) spline(x1,y1,t)-f^2/80.8*(1-(R0/(R0+t))^2*sin(phi)^2);
%the difference compared to the electronic density of the height where ...
    reflection
t1=Newtons(g1,200);
t2=Newtons(g1,100);
t3=Newtons(g1,150);
t4=Newtons(g1,300);
t5=Newtons(g1,250);
t=[t1(1),t2(1),t3(1),t4(1),t5(1)];
t=t(t>0)
h=min(t);
%find the height in the ionosphere where the reflection takes place
if or(or(t1(2)≠20,t2(2)≠20 ),or(t3(2)≠20,or(t4(2)≠20,t5(2)≠20)))
    y=h
    theta=h*tan(phi)/R0+phi
else
    y=300
end
end
```

```
function y=dB_theta(i,theta,x1,y1)
hh=h_theta1_1(x1,y1)
%the height in the ionosphere where the reflection take place
f=@(t) spline(x1,y1,t);
%using spline interpolation fitting to decide the relationship between ...
    the electronic density and the height
if hh<400
    s=integral(f,70,hh);
end
%the integral of electronic density in the ionosphere
eta=-32/3*pi^2*s*tan(theta)*2*1.668776045500509e-15
phi=theta+hh*tan(theta)/6371;
%the launching angle of elevation
rh=abs((sin(phi)-sqrt(70-cos(phi)^2))/(sin(phi)+sqrt(70-cos(phi)^2)));
rv=abs((70*sin(phi)-sqrt(70-cos(phi)^2))/(70*sin(phi)+sqrt(70-cos(phi)^2)));
r=(rv^2+rh^2)/2
%reflection coefficient
y=(i*eta/log(10)+(i)*log(r)/log(10))*10;
%the decibels of the energy loss
end
```

```
function y=dB_theta2(i,theta,x1,y1,dh)
hh=h_theta1_1(x1,y1)
```

```
f=@(t) spline(x1,y1,t);
omega=input('omega=')
if hh<400
    s=integral(f,70,hh);
end
eta=-32/3*pi^2*s*tan(theta)*2*1.668776045500509e-15
phi=theta+hh*tan(theta)/6371;
g=0.5*(4*pi*dh*omega*sin(theta)/299792458)^2;
rho=1/sqrt(3.2*g-2+sqrt((3.2*g)^2-7*g+9))
rh=abs((sin(phi)-sqrt(70-cos(phi)^2))/(sin(phi)+sqrt(70-cos(phi)^2)));
rv=abs((70*sin(phi)-sqrt(70-cos(phi)^2))/(70*sin(phi)+sqrt(70-cos(phi)^2)));
r=(rv^2+rh^2)/2*rho^2
y=(i*eta/log(10)+(i)*log(r)/log(10))*10;
end
```

```
function y=dB_theta3(i,theta,x1,y1,dh)
hh=h_theta1_1(x1,y1)
%the height in the ionosphere where the reflection take place
f=@(t) spline(x1,y1,t);
%using spline interpolation fitting to decide the relationship between ...
    the electronic density and the height
omega=input('omega=');%the frequency of the radio
if hh<400
    s=integral(f,70,hh);
%the integral of electronic density in the ionosphere
end
eta=-32/3*pi^2*s*tan(theta)*2*1.668776045500509e-15
phi=theta+hh*tan(theta)/6371;
%the launching angle of elevation
y=(i*eta/log(10))*10;
end
```

```
function y=s_theta(theta,height)
y=2*height*tan(theta);
end
```

```
function y=dB_theta4(i,theta,x1,y1,h)
hh=h_theta1_1(x1,y1)
%the height in the ionosphere where the reflection take place
f=@(t) spline(x1,y1,t);
%using spline interpolation fitting to decide the relationship between ...
    the electronic density and the height
omega=input('omega=');%the frequency of the radio
if hh<400
    s=integral(f,70,hh);
end
eta=-32/3*pi^2*s*tan(theta)*2*1.668776045500509e-15
%The logarithm of the proportion of energy loss due to the electric ...
    charge's forced vibration in the ionosphere
phi=theta+hh*tan(theta)/6371;%the launching angle
sigma=sqrt(2*pi)*h;
%root mean square height under the normal distribution
gg=@(tt) 1/(sqrt(2*pi)*sigma)*exp(-tt.^2/(2*sigma^2));
%the distribution of the area of the mountainous areas under the normal ...
    distribution
R=integral(gg,0,299792458/(16*omega*cos(theta)))*2
```

```
%reflection coefficient of the mountainous area
y=(i*eta/log(10)+(i)*log(R)/log(10))*10;
end
```

```
function t=Newtons(f,x)
x1=x;eps=abs(1);flag=1;
while eps>10^-14
    delt=weifen(f,x1);
    delt1=delt(1);
    eps1=f(x1)/delt1;
    eps=abs(eps1);
    x1=x1-eps1;
    if flag==20
        break
    else
        flag=flag+1;
    end
end
t=[x1,flag];
```

```
function t1=weifen(f,x)
delt=1;
delty=abs(f(x+delt)-f(x));
i=1;
flag=1;
while delty>10^-6
    if delt>10^-5
    delt=delt/10;
    else delt=delt/2;
    end
    delty=abs(f(x+delt)-f(x));
    if i==5;
        flag=5;
        break
    else
    flag=i;
    i=i+1;
    end
end
t=(f(x+delt)-f(x-delt))/(2*delt);
eps=abs((f(x+2*delt)-f(x-2*delt))/(4*delt)-t);
t1=[t,eps,flag];
end
```

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