Research Log from 2023.11.29-2023.12.7

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Work: Go over SODA 2024 interested topics.

1 INTERESTED PAPERS IN SODA 2024

1. Fast and Accurate Approximations of the Optimal Transport in Semi-Continuous and Discrete Settings
2. On Approximability of Steiner Tree in ℓ_p-metrics
3. Optimal thresholds for Latin squares, Steiner Triple Systems, and edge colorings
4. Euclidean Bottleneck Steiner Tree is Fixed-Parameter Tractable
5. Induced-Minor-Free Graphs: Separator Theorem, Subexponential Algorithms, and Improved Hardness of Recognition
6. New SDP Roundings and Certifiable Approximation for Cubic Optimization
7. A Parameterized Family of Meta-Submodular Functions

2 PROOF OF BOUNDED DEGREE

Algorithm 4 Reduce3(G_2)

$\mathcal{X} \leftarrow \{FT_{G_2}(v) : v \in V(G_2)\}$
for $X \in \mathcal{X}$ **do**
  $d_X \leftarrow |N_{G_2}(X)|$
  $V_X \leftarrow$ an arbitrary subset of X of size $\min\{c(k,\epsilon) \cdot d_X, |X|\}$
end for
$G_3 \leftarrow G_2[\cup_{X \in \mathcal{X}} V_X]$

Last time finished proof of $(1+\epsilon)$-approximation preserving. This time prove bounded local radius.

Claim 2.1

For each $X \in \mathcal{X}$, $|V_X| = O_\epsilon(1)$.

Proof. To see this, first note that any two vertices $v, v' \in V(G_3)$ are false twins if.f. they are false twins in G_2. One simple fact can be shown that a disk graph H of ply p has at most $O(p \cdot |V(H)|)$ edges. Consider the induced subgraph $G_2[X \cup N_{G_2}(X)]$ contains at least $|X| \cdot d_X$. By the fact, this induced subgraph can contain at most $O_\epsilon(|X| + d_X)$

edges. Therefore, either $|X| = O_\epsilon(1)$ or $d_X = O_\epsilon(1)$. If $|X| = O_\epsilon(1)$, then trivially $|V_X| = O_{G_2}(1)$. If $d_X = O_\epsilon(1)$, then $|V_X| \leq (1 + \frac{1}{\epsilon}) \cdot d_X = O_\epsilon(1)$. $\square$

Now we only need a little extra effort to bound the local radius of G_3. Recall that for each $v \in V(G_3)$, the neighbor of v can be partitioned in to $S(v)$ and $I(v)$. Now we slightly modify the partition, then we can have local radius $O_\epsilon(1)$. Notice that in previous counterexample, the local radius is unbounded mainly because the disks in $I(v)$ is blocking the faces in $D(v)$. We can prevent this situation happens by creating a new partition $S^*(v)$ and $I^*(v)$ and guaranteeing for each $u \in I^*(v)$, $D(u) \not\subseteq \cup_{w \in \{v\} \cup S*(v)} D_w$. We create $(S^*(v), I^*(v))$ as follows: a vertex $u \in N_{G_3}(v)$ is included in S^* if $u \in S(v) \cup S^2(v)$ or $N_{G_3}(u) \subseteq \{v\} \cup S(v) \cup S^2(v)$, then $I^*(v)$ is simply $N_{G_3}(v) \setminus S^*(v)$. This justification should be simple. We can still have $|S^*(v)| = O_\epsilon(1)$ and $|I^*(v)| = O_\epsilon(1)$.

To see why this gives us bounded radius, let's consider $E_S = \cap_{w \in \{v\} \cup S} D_w$ for $S \subset S^*$. A geometric observation is that if a disk D is not contained in the union of a set of disks, then the boundary of D crosses the boundary of the intersection of the disks in the set at most twice. So the intersection pattern of E_S and the disks D_u for $u \in I^*$ should be a star. Therefore, within this induced arrangement subgraph, any two faces have a distance 3-path (cross the boundary of $I^*(v)$ twice).

Let $\mathcal{S} = \{S \subseteq S^* : E_S \neq \emptyset\}$, let $A[E_S]$ denote the induced subgraph of the arrangement graph of $\mathcal{D}_3$ consisting of the faces contained in E_S. Then we can apply induction on size of S to get bounded radius.

Inductive hypothesis: If the radius of $A[E_S]$ is at most r for any $S \in \mathcal{S}$ with $|S^*| - |S| = d$, then the radius of $A[E_S]$ is at most $f(\epsilon, r)$ for any $S \in \mathcal{S}$ with $|S^*| - |S| = d + 1$, where f is a fixed function. Note that, if $|S^*| - |S| = |S^*|$, then $|S| = 0$, so the arrangement graph $A[E_S] = A[E_\emptyset] = A[D_v]$. Thus by induction, we get bounded local radius.

Base case: $d = 0$, the arrangement graph $A[E_S]$ is a star, so local radius is 1.

Inductive step: Let $|S^*| - |S| = d$ If E_S does not intersect D_w for any $w \in S^* \setminus S$, then $A[E_S]$ is the star graph again. So local radius is 1. Suppose E_S intersect some $w \in S^* \setminus S$, then $A[E_{S \cup \{w\}}]$ has local radius at most r. We further observe that each vertex in $A[E_S]$ is within distance 3 from $A[E_{S \cup \{w\}}]$ for some $w \in S^* \setminus S$. Based on this observation, if we "expand" all $A[E_{S \cup \{w\}}]$ for $w \in S^* \setminus S$ with $D_w \cap A[E_S] \neq \emptyset$ a little bit, they will cover all vertices in $A[E_S]$. Specifically, let $A^+[E_{S \cup \{w\}}]$ be the induced subgraph of $A[E_S]$ consisting of vertices within distance at most 3 from $A[E_{S \cup \{w\}}]$. Then the radius of $A^+[E_{S \cup \{w\}}]$ is at most $r + 3$, and all $A^+[E_{S \cup \{w\}}]$ covers the vertices in $A[E_S]$. A simple argument shows that if a connected graph H can be covered by k induced subgraphs of radii at most $\rho \geq 1$, then the radius of H is at most $O(k\rho)$. Here $k = O_\epsilon(1)$ and $\rho = r + 3$. So the radius of $A[E_S]$ is at most $O_\epsilon(r + 3)$.

3 HANDLING BOUNDED RADIUS

Definition 3.1 SQGM Property

A graph class $\mathcal{G}$ has the subquadratic grid minor property if there exist constants $\alpha > 0$ and $1 \leq c \leq 2$ such that, for any $t > 0$, every graph $G \in \mathcal{G}$, excluding the $t \times t$-grid as a minor, has treewidth at most $\alpha \cdot t^c$.

Proposition 3.1

Let G be a planar graph with treewidth w, then G contains $\lfloor w/5 \rfloor \times \lfloor w/5 \rfloor$ minor.

Lemma 3.1

Given a disk graph G with local radius r. Let $\mathcal{D}$ be some realization of G, and let $t' \in \mathbb{N}$. If $A_{\mathcal{D}}$ contains the grid of size $t' \times t'$ as a minor, then G contains a grid of size $t \times t$ as a minor for $t = \Omega(t'/r)$

Proposition 3.2

Let G be a geometric graph that has a realization of ply p whose arrangement graph has treewidth w. Then, the treewidth of G is $O(w \cdot p)$.

Claim 3.1

Let $\mathcal{G}$ be the class of disk graphs with local radius at most r. Then, $\mathcal{G}$ has the SQGM property with parameters α and $c = 1$ where α depends on r.

Proof Sketch (Logic Chain). Let $t > 0$ *s.t.* G excludes as a minor, then by Lemma 3.1, $A_{\mathcal{D}}$ does not contain $t' \times t'$ as a minor for some t'. Since $A_{\mathcal{D}}$ is planar, by proposition 3.2, $A_{\mathcal{D}}$ has bounded treewidth. At last, By Proposition 3.3, G has bounded treewidth. □

Proposition 3.3

Let $\prod$ be an η-modulated and reducible graph optimization problem, then $\prod$ has an EPTAS on every induced-subgraph-closed graph class with the *SQGM* property.

Thus bounded local radius disk graph has the SQGM property and thus admits EPTAS on a bunch of graph optimization problem.

REFERENCES