

Homework 4 计算题

姓名：王宝琪 学号：22210980075

1：极大似然估计：

通过计算，可以求得以下先验概率和条件概率

$$P(Y = 1) = \frac{5}{15}, P(Y = -1) = \frac{10}{15}$$

$$P(x^{(1)} = 1|Y = 1) = \frac{1}{5}, P(x^{(1)} = 2|Y = 1) = \frac{3}{5}, P(x^{(1)} = 3|Y = 1) = \frac{1}{5}$$

$$P(x^{(2)} = S|Y = 1) = \frac{1}{5}, P(x^{(2)} = M|Y = 1) = \frac{2}{5}, P(x^{(2)} = L|Y = 1) = \frac{2}{5}$$

$$P(x^{(1)} = 1|Y = -1) = \frac{2}{10}, P(x^{(1)} = 2|Y = -1) = \frac{2}{10}, P(x^{(1)} = 3|Y = -1) = \frac{6}{10}$$

$$P(x^{(2)} = S|Y = -1) = \frac{3}{10}, P(x^{(2)} = M|Y = -1) = \frac{4}{10}, P(x^{(2)} = L|Y = -1) = \frac{3}{10}$$

对给定的 $x = (2, M)^T$ ，计算后验概率得：

$$P(Y = 1)P(x^{(1)} = 2|Y = 1)P(x^{(2)} = M|Y = 1) = \frac{1}{3} \times \frac{3}{5} \times \frac{2}{5} = 0.08$$

$$P(Y = -1)P(x^{(1)} = 2|Y = -1)P(x^{(2)} = M|Y = -1) = \frac{2}{3} \times \frac{1}{5} \times \frac{2}{5}$$

$$= 0.053$$

所以经 MLE 预测得到的类标记为 $Y = +1$

2：贝叶斯估计：

通过计算，可以求得以下先验概率和条件概率：

$$P(Y = 1) = \frac{6}{17}, P(Y = -1) = \frac{11}{17}$$

$$P(x^{(1)} = 1|Y = 1) = \frac{2}{8}, P(x^{(1)} = 2|Y = 1) = \frac{4}{8}, P(x^{(1)} = 3|Y = 1) = \frac{2}{8}$$

$$P(x^{(2)} = S|Y = 1) = \frac{2}{8}, P(x^{(2)} = M|Y = 1) = \frac{3}{8}, P(x^{(2)} = L|Y = 1) = \frac{3}{8}$$

$$P(x^{(1)} = 1|Y = -1) = \frac{3}{13}, P(x^{(1)} = 2|Y = -1) = \frac{3}{13}, P(x^{(1)} = 3|Y = -1) = \frac{7}{13}$$

$$P(x^{(2)} = S|Y = -1) = \frac{4}{13}, P(x^{(2)} = M|Y = -1) = \frac{5}{13}, P(x^{(2)} = L|Y = -1) = \frac{4}{13}$$

对给定的 $x = (2, M)^T$, 计算后验概率得:

$$P(Y = 1)P(x^{(1)} = 2|Y = 1)P(x^{(2)} = M|Y = 1) = \frac{6}{17} \times \frac{4}{8} \times \frac{3}{8} = 0.066$$

$$P(Y = -1)P(x^{(1)} = 2|Y = -1)P(x^{(2)} = M|Y = -1) = \frac{11}{17} \times \frac{3}{13} \times \frac{5}{13}$$

$$= 0.057$$

所以经贝叶斯估计得到的类别标记为 $Y = +1$