MSc in CSTE
High Performance Technical Computing

Assignment

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Hand in date: 16/1/17 (FT), 23/1/17 (PT), 09:30am

1 Introduction

In this assignment you are asked to apply distributed memory parallel programming techniques to the numerical solution of the linear advection equation:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} = 0$$

using the following schemes:

- Explicit Upwind
- Implicit Upwind
- Crank-Nicolson

The partial differential equation (PDE) is to be solved in a domain

$$x \in [-50, 50]$$

with

$$u = 1.75$$

and the following initial/boundary conditions:

$$\begin{aligned} f\left(x,0\right) &= \frac{1}{2}exp\left(-x^2\right) \\ f\left(-50,t\right) &= 0 \\ f\left(50,t\right) &= 0 \end{aligned}$$

where the analytical solution is given by:

$$f\left(x,t\right) = \frac{1}{2}exp\left(-\left(x-1.75t\right)^2\right)$$

2 Tasks

- Devise a way to parallelise the above numerical methods and analyse your algorithms.
- Implement your algorithms in a program written in C/C++/FORTRAN and coupled with MPI.
- For the purposes of this study you may assume a uniform grid containing $1,000$ and $10,000$ points in x and an appropriate number of points in t. Solve the PDE for $t = 5$.
- Compare the numerical solutions obtained by the serial programs and the parallel programs with the analytical solution.
- Measure the cost of communication in the boundary exchange and the cost of computing a time-step for each process.
- Measure the performance of your serial and parallel codes and discuss. How does the parallel program performance compare to the theoretical one? Is the performance of your parallel program expected?
- Replace your linear system solver with one from an external mathematical library. Study how the performance of a different implementation of the linear system solver affects the overall performance of your implicit scheme for the solution of the prescribed PDE.
- Based on your results, comment on the problem sizes deemed necessary in order for MPI parallelisation to become efficient for the above numerical algorithms.

3 Report

The source program will need to compile on Astral and your simulations for the above tasks should be performed on Astral using the scheduler. Write a report summarising your findings. The report should be no less than 1500 words and must not exceed 3000 words. The report can contain any number of figures. All figures and tables in the report should be numbered and discussed.

The report must include the source code as an Appendix and should be submitted electronically via the **TurnItInUK** link by 9:30am on the 16$^{\text{th}}$ of January (full-time students) or the 23$^{\text{rd}}$ of January (part-time students). The source code file must also be submitted directly to the instructor via **e-mail** at i.moulitsas@cranfield.ac.uk by the prescribed deadline for the assignment submission to be considered complete.