Fourier seires.

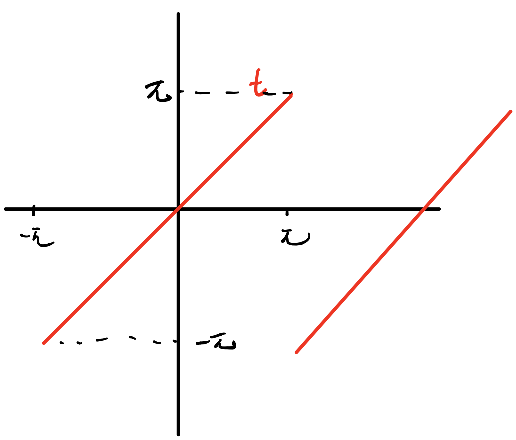
$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\, dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\, dt$$

(其中 $f(t)$ period, 2π is a period.)

若 $f(t) = g(t)$ $\Rightarrow$ F.S for $f(t)$ = F.S for $g(t)$ (因为 a_n, b_n 的计算公式相同)



[1] shorten the calculations

[2] 去掉函数的不同约束, 扩展傅里叶级数的范围.
Extend F.S

[1] method: 通过奇偶性. evenness and oddness

偶函数: $f(-t) = f(t)$

claim: If $f(t)$ is an even function

⊙ L $f(t) = \frac{a_0}{2} + \sum a_n \cos nt$ ($b_n = 0$)

Proof:

$f(-t) = f(t)$ ← F_s: $\frac{a_n}{2} + \sum a_n \cos nt + b_n \sin nt$

↑

F.S: $\frac{a_n}{2} + \sum a_n \cos nt - b_n \sin nt$

↳ $b_n = -b_n \Rightarrow b_n = 0$

⊙ 当 $f(t)$ is an odd function　奇函数：$f(-t) = -f(t)$

↳ $f(t) = \frac{a_0}{2} + \sum b_n \sin nt$　$(a_n = 0)$

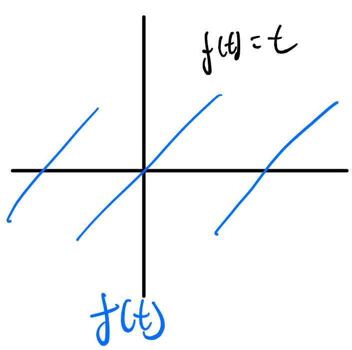
⇓ 进一步 simplify the integral 对 a_n / b_n.

⊙ 当 $f(t)$ even. $\Rightarrow f(t)\cos nt$ 偶

则 $\boxed{a_n = \frac{2}{\pi}\int_0^{\pi} f(t)\cos nt \, dt}$

⊙ 当 $f(t)$ odd $\Rightarrow f(t)\sin nt$ 偶　奇·奇=偶

则 $\boxed{b_n = \frac{2}{\pi}\int_0^{\pi} f(t)\sin nt \, dt}$



odd.

$\Rightarrow a_n = 0$

$$b_n = \frac{2}{\pi}\int_0^{\pi} t \sin nt \, dt$$

$$= \frac{2}{\pi}\left[-t\frac{\cos nt}{n}\Big|_0^{\pi} - \int_0^{\pi} -\frac{\cos nt}{n} dt \right]$$

$$= \frac{2}{\pi}\left[\frac{-\pi}{n}(-1)^{n+1} + \frac{\sin nt}{n^2}\Big|_0^{\pi} \right]$$

则 $b_n = \frac{2}{n}(-1)^{n+1}$

则 F.S for $f(t) = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nt$

$= 2(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t)$

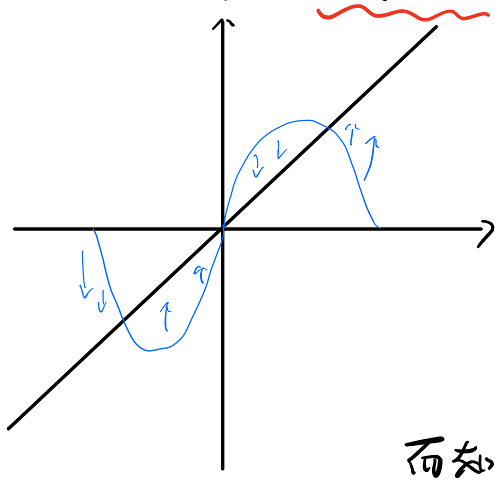
☆☆☆ 重点.

零点 zero

傅里叶级数不是在(中心点)附近近似函数,

而是尝试照顾整个区间. 在整个区间上近似函数

如[-π,π]



Thm: If $f(t)$ at t_0 是连续函数,

↳ Then. $f(t_0)$ = Sum of F.S at t_0

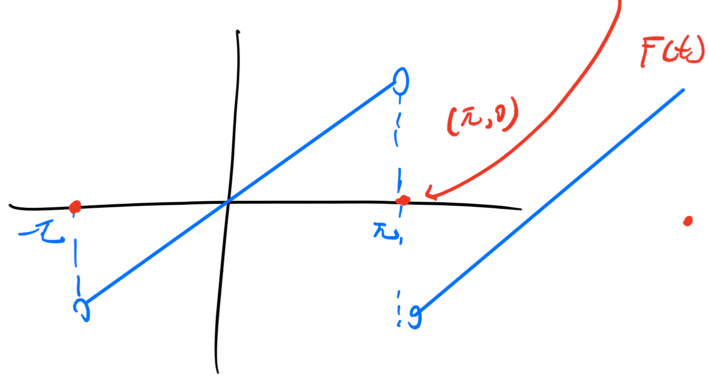
其中 F.S 是收敛的.

而如果 t_0 是不连续的跳跃点,

↳ Sum of F.S at t_0 收敛于 → 跳跃的中点

$$\sum \frac{(-1)^{n+1}}{n} \sin nt = F(t)$$

图



[2] Extend F.S

(not just 2π)

extension #1 : period is $2L$

t: 0, L $\quad t = \frac{L}{\pi}u$

u: 0, π $\quad u = \frac{\pi}{L}t$

即 $\to \int_{-L}^{L} \cos n\frac{\pi}{L}t$

$\to \int_{-L}^{L} \sin n\frac{\pi}{L}t$

即: $f(t) = \frac{a_0}{2} + \sum a_n \cos\frac{n\pi}{L}t + b_n \sin\frac{n\pi}{L}t$

$a_n = \frac{1}{L}\int_{-L}^{L} f(t) \cos\frac{n\pi}{L}t\, dt$

$b_n = \frac{1}{L}\int_{-L}^{L} f(t) \sin\frac{n\pi}{L}t\, dt$

$\to f(t)$ even:

$a_n = \frac{2}{L}\int_0^{L} f(t) \cos\frac{n\pi}{L}t\, dt.$

$b_n = 0$

$\to f(t)$ odd

. . . . .

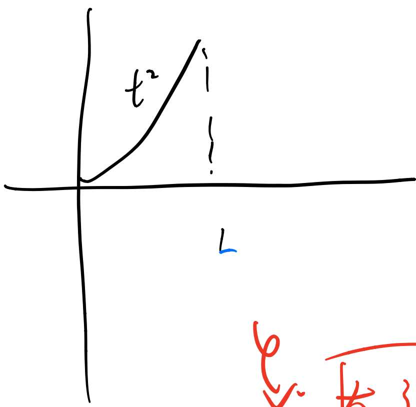
Extension #2:

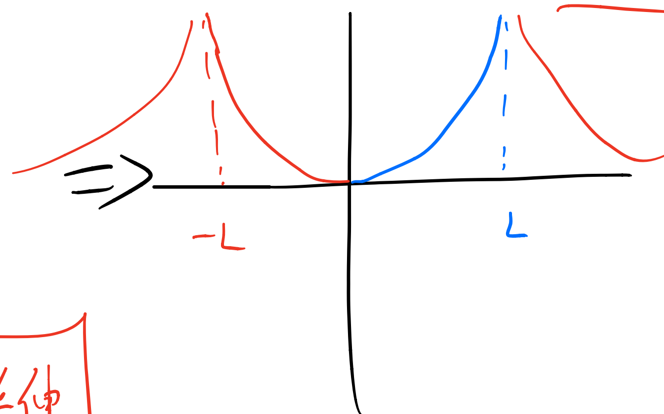
F.s 是针对 有限区间

若 $f(t)$ period at $(0, L]$,

$\leadsto$ periodic extend

对它做一个周期性延伸





偶延伸

奇延伸