LINEAR FUNC

$$y' + \underset{\sim}{k} y = k\,q(t)$$

"constant"

Solve: $y' + ky = q(t)$ $\rightsquigarrow$ $\boxed{\frac{1}{k}y' + y = q_e(t)}$

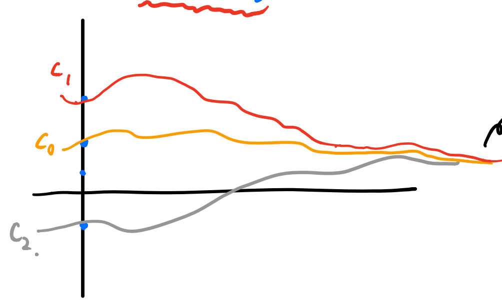
$$y = \underbrace{e^{-kt}\int q(t)e^{kt}\,dt}_{\text{"steady-state solution"}} + \underbrace{ce^{-kt}}_{0 \text{ as } t\to\infty}$$

当k>0时:

"steady-state solution"

"the SSS"

"transient"



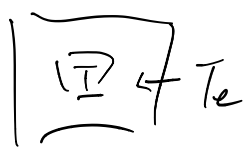
"选最简单的一个 fn"

the SSS

"Input response point of view" ⟶ "输入-响应的概念"

Input : $q(t)$ / $q_e(t)$ ⟶ physical input

Response : $y(t)$ soln to ODE



superposition of inputs

"输入的叠加"

$q_1(t) \to y_1(t)$

$q_2(t) \to y_2(t)$

$q_1 + q_2 \to y_1 + y_2$

$c q_1 \to c y_1$

LINEAR

线性才有的特性

$$y' + ky = k\, q_e(t)$$

"物理输入"

$q_e(t)$: $\cos(wt)$

w = 角频率 "angular frequency" = 全振动次数 (2π 内)

"the number of complete oscillations in 2π"

Prob: $q_e = \cos(wt)$ find the response

"Complexification of the problem":

复化问题.

步骤

① take differential equation "取微分方程"

② turn it into a ODE involving complex numbers. "带复数的微分方程" (转换为)

③ solve that "求解"

④ go back to the real domain to get "回到实数范围"

advantage: "easier to integrate exponentials" 指数函数积分更容易

try to introduce: "change the trigonometric functions into complex exponentials" "将三角函数转化为复化指数".

$$e^{iwt} = \cos wt + i \sin wt$$

$$\Rightarrow \text{①}\ \tilde{y}' + k\tilde{y} = k e^{iwt}$$

$(\tilde{y} = y_1 + i y_2)$ (complex soln)

② Find $\tilde{y}$, $\to$ ③ y_1 原方程的解 "real ODE"

解: $(\tilde{y}e^{kt})' = k e^{(k+iw)t}$

$$\tilde{y}e^{kt} = \frac{k}{k+iw} e^{(k+iw)t}$$

$$\tilde{y} = \frac{k}{k+iw} e^{iwt} = \underbrace{\frac{1}{1+i(\frac{w}{k})} e^{iwt}}$$

取实部 (两种方法)

Two methods:

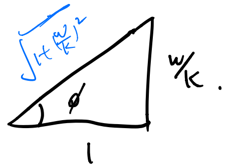
①. GO POLAR "极坐标" ☆☆☆

②. GO Cartesian "笛卡尔坐标系" ☆

①: **Polar form.**

$$\frac{1}{1+i(\frac{w}{k})} = \underset{\sim}{A} e^{-i\phi} = \frac{1}{\sqrt{1+(\frac{w}{k})^2}} e^{-i\phi}$$

$\left|\frac{1}{\text{complex}}\right|$ $=$ $\frac{1}{\sqrt{1+(\frac{w}{k})^2}}$



$$\text{ang}\left[1+i\left(\frac{w}{k}\right)\right] = \phi$$

$$\tilde{y} = A e^{iwt - i\phi}$$

$$= \frac{1}{\sqrt{1+(\frac{w}{k})^2}} e^{i(wt-\Phi)}$$

则 $y = \frac{1}{\sqrt{1+(\frac{w}{k})^2}} \cdot \cos(wt - \boxed{\Phi})$

real

Ans

what is Φ

相位滞后 = "phase lag" or "phase delay"

$\Phi = \arctan(\frac{w}{k})$

解:

$$y = \frac{1}{\sqrt{1+(\frac{w}{k})^2}} \cos(wt - \Phi)$$

$$\boxed{\begin{aligned} &y' + ky = k\cos(wt) \\ &\text{解} \\ &y = \frac{1}{\sqrt{1+(\frac{w}{k})^2}} \cdot \cos(wt - \underset{\sim}{\Phi}) \qquad \Phi = \arctan(\tfrac{w}{k}) \end{aligned}}$$

若 $k\uparrow$, $A\uparrow$, $\Phi\downarrow$.

$k\downarrow$, $A\downarrow$, $\Phi\uparrow$.

$x = we^t$

$x' + 2x = e^t$

$x' = we^t + w'e^t$

$w'e^t + we^t + 2we^t = e^t$

$w' + 3w = 1$

$w = \frac{1}{3}$

$x_p(t) = \frac{1}{3}e^t$

$$w'e^t + we^t + 2we^t = e^{2it}$$

V

$x = V \cdot e^{2it}$

$$\cancel{w'e^t + 3we^t = e^{2i}} \quad 2i - 1$$

$\cancel{x' = v'e^{2it}} + v'e^{2it}$

$$v'e^{2it} + ve^{2it} + 2ve^{2it} = e^{2it}$$

v' $\quad 3v$

$$e^{2it} = \cos 2t + \sin 2t\, i$$

$$x' + 2x = \cos(2t)$$

$$x' + 2x = e^{2it}$$

$$z = \frac{1}{2+2i} e^{2it} - C \cdot e^{-2t}$$

$z = ve^{2it}$

$z' = v'e^{2it} + v \cdot e^{2it} \cdot 2i$

$$z' + 2z = e^{2it}$$

$$v'\cancel{e^{2it}} + v \cdot \cancel{e^{2it}} \cdot 2i + 2v\cancel{e^{2it}} = \cancel{e^{2it}}$$

$$v' + (2+2i)v = 1$$

$$v = \frac{1}{2+2i}$$

$$z = \quad x_p(z) = \frac{1}{2+2i} e^{2it}$$

$$x' + 2x = e^{3t}$$

$$x = Be^{3t}$$

$$x' = B'e^{3t} + 3Be^{3t}$$

$$B'e^{3t} + 3Be^{3t} + 2Be^{3t} = e^{3t}$$

$$B' + 5B = 1$$

$$B = \frac{1}{5}$$

即 $x_p = \frac{1}{5} \cdot e^{3t}$

$$x' + 2x = 0$$

$$x = Ce^{-2t}$$

$$x = C \cdot e^{-2t} + \frac{e^{3t}}{5}$$

$$x' + 2x = e^{3it}$$

$$x = v \cdot e^{3it}$$

$$x' = v'e^{3it} + v \cdot e^{3it} \cdot 3i$$

$$v'e^{3it} + v \cdot e^{3it} \cdot 3i + 2ve^{3it} = e^{3it}$$

$$v' + (3i+2)v = 1$$

$$v = \frac{1}{2+3i}$$

$$x = C \cdot e^{-2t} + \frac{e^{3it}}{2+3i}$$

$$\frac{e^{3it}}{\sqrt{3}+i} \qquad e^{3it} = \cos(3t) + \sin(3t)\, i$$

$$\frac{1}{\sqrt{3}+i} = \frac{\sqrt{3}-i}{4} = \left(\frac{\sqrt{3}}{4} - \frac{i}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{4} - \frac{i}{4}\right) \cdot \left[\cos(3t) + \sin(3t)\, i\right]$$

$$= \frac{\sqrt{3}}{4}\cos(3t) + \frac{1}{4}\sin(3t)$$

$\angle 1$

$$= \frac{1}{2} \cos\left(3t - \frac{\pi}{6}\right)$$

$$\frac{e^{3it}}{2e^{i\cdot\frac{\pi}{6}}} = \frac{1}{2} e^{(3t-\frac{\pi}{6})t}$$

$\sqrt{3}+i$

b $\quad x = \frac{1}{2i+3} e^{2it} + C e^{-3t}$

Re