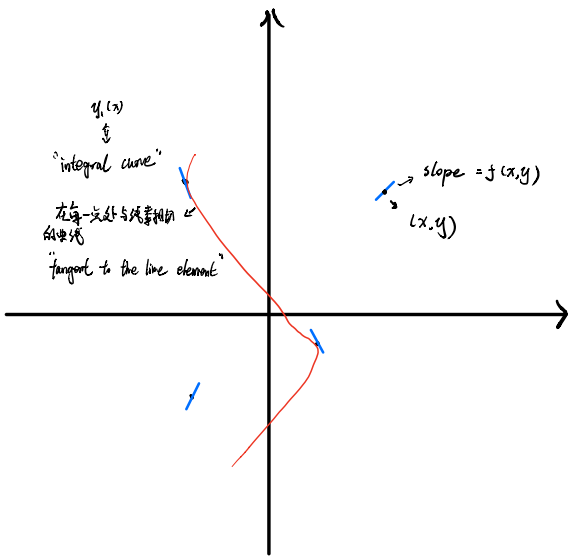
GEOM VIEW OF ODE's

1. analytic ideas.

$y' = f(x,y)$ <——> "direction field" ↷ 方向场/斜率场

$y_1(x)$ → solution. <——> "integral curve" ↷ 积分曲线

"斜率的斜率" - curve



DRAWing Dir. FLD.

Computer. Human.

1. Pick (x,y) [equally space]

2. $f(x,y)$ → find slope.

3. Draws / slope $f(x,y)$

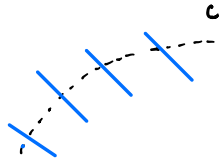
1. Pick slope C.

★ 2. find equation: $f(x,y) = C$.

(plot this) → isocline

∵ 等斜線

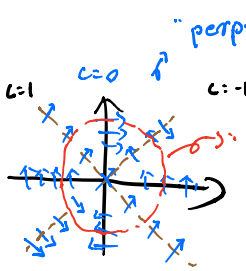
3.



例子 : example

① $y' = \frac{-x}{y}$

$\frac{-x}{y} = C \Rightarrow y = -\frac{1}{C}x.$

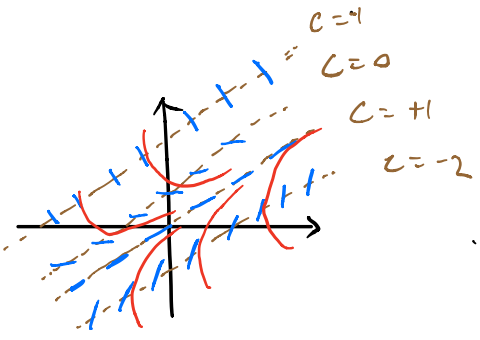


∴ integral curve = circles

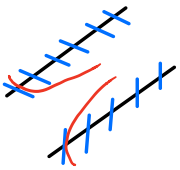
solution of ODE by separation of variables.

$\frac{dy}{dx} = \frac{-x}{y} \Rightarrow y\cdot dy = -x\,dx \Rightarrow y^2 + x^2 = C$

Corridor between.



∴ no escape



curve.

solution can't escape.

② $y' = 1 + x - y.$

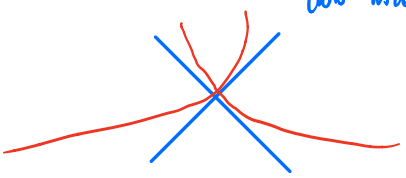
$y = 1 + x - C.$

$y = x + Ce^{(-x)}$

$x = \infty$ ∫ solutions.

✱ Principle I : 两条积分曲线不会相交.

"two integral curves can't cross at an angle."



方向场只允许这里存在一个斜率.

can't have two slope

✱ Principe II : 两条积分曲线不能相切. (touch)

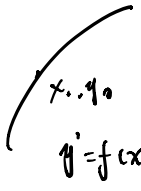
two integral curves can't touch/ be tangent

No!

Why?

↓

"Existence and Uniqueness Theorem." 存在与唯一性定理



有且仅有唯一解

Existence one and only one Uniqueness.

x_0, y_0

$y' = f(x,y)$ solution.

假设 HYP:

① f(x,y) 必须是连续函数.

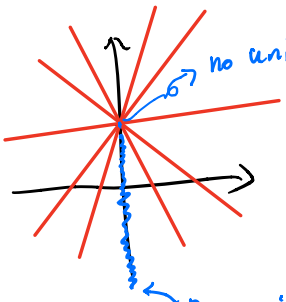
hypotheses continuous near, in the vicinity of that point.

② y 的偏导必须连续.

partial derivative with respect to y should be continuous near (x_0, y_0)

$xy' = y-1 \to \frac{dy}{y-1} = \frac{dx}{x} \to \ln|y-1| = \ln|x| + C \to y = Cx+1$

$y-1 = Cx$



平面内任意点都有解.

$y' = \frac{y-1}{x}$ $x=0$, 无意义.

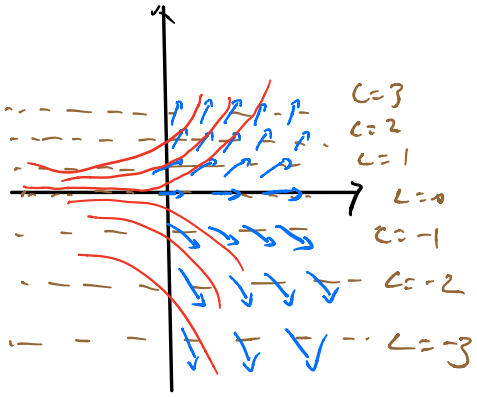
原因: ① $f(x,y)$ 上存在一些无定义的点.

$y' = y$

$\frac{dy}{dx} = y \Rightarrow \frac{1}{y}dy = 1 \cdot dx \to \ln y = x + C \quad y = Ce^x$

$y' = y$

$y = C$



Python.

$$\frac{dT(t)}{dt} = -k(T(t) - T_a)$$

$$y' = -k(y - c)$$

$$y' = -ky + a.$$

$$\frac{dy}{dx}$$

$$f(x, y) =$$

$$\frac{dy(x)}{dx} = f(x, y(x))$$

f_up: f(x, y(x))

$$\left(xx - \frac{Dx}{2},\ xx + \frac{Dx}{2}\right)$$

$$\left[yy - \frac{Dy}{2},\ yy + \frac{Dy}{2}\right]$$

$$x_0$$

$$0.8(dx)^2$$

$$\sqrt{dx^2 + dy^2}$$

f(dx, dy)

dy_x

y(x).

$$dx = Dy = slop \cdot dx$$