FIRST-ORDER LINEAR

性质：可解

y与y'有线性关系.

若: $C=0$, eq. called "homogeneous" 齐次方程.

⊙ $a(x)y' + b(x)y = c(x)$

linear: $ay_1 + by_1 = c$.

STANDARD LINEAR form: 将y'的系数取作1.

$$y' + \underbrace{P(x)}_{\frac{b(x)}{a(x)}} y = \underbrace{q(x)}_{\frac{c(x)}{a(x)}}$$

MODELS:

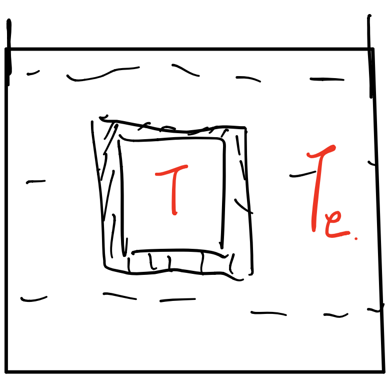
① 1. Temp - concentration. "温度-浓度" 模型 → "conduction-diffusion model" "传导-扩散" 模型.

$\boxed{\text{CONDUCTION}}$: "传导"

Newton cooling law:

$$\boxed{\frac{\partial T}{\partial t} = k(T_e - T),\ k>0. \quad T(0) = T_0}$$

→ conductivity 热传导



T: temperature

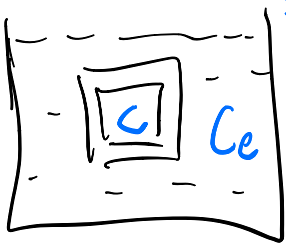
⇓ standard linear eq:

$$\frac{dT}{dt} + \underbrace{k}_{\text{usually constant, not necessarily}} T = \underbrace{kT_e}_{\text{generally}}$$

DIFFUSION : "扩散"

K可以随着时间改变.

← salt conc 盐水



$$\boxed{\frac{dC}{dt} = k_1 \cdot (C_e - C)}$$

C = concentration 浓度.

⊙ 解: solve me!

$$y' + p(x)\,y = q(x)$$

"Integrating factor" 积分因子 → $u(x)$.

$$\left.\begin{aligned} uy' + puy &= qu. \\ (uy)' &= qu \\ uy' + u'y &= qu. \end{aligned}\right\} \Rightarrow$$

works $u(x)$: 当 $u' = pu$. $\Rightarrow \frac{du}{dx} = p(x)\cdot u(x) \Rightarrow \ln u = \int p(x)\,dx$

$$\boxed{u = e^{\int p\,dx}}$$

⊙ Method:

$$y' + py = q.$$

1. standard. linear form.

2. calculate $e^{\int p\,dx}$: int. factor.

3. Mult both sides by $e^{\int p\,dx}$

4. Integrate

例子.

例A: $xy' - y = x^3$

1. $y' - \frac{1}{x}y = x^2$

2. $e^{\int -\frac{1}{x}dx} = e^{-\ln x} = (e^{\ln x})^{-1} = \frac{1}{x}$

3. $\frac{1}{x}\cdot y' - \frac{1}{x^2}y = x$

4. $(\frac{1}{x}y)' = x \Rightarrow \frac{1}{x}y = \frac{1}{2}x^2 \Rightarrow \boxed{y = \frac{1}{2}x^3 + C\cdot x}$

例B: $(1+\cos x)y' - (\sin x)y = 2x$

1. $y' - \frac{\sin x}{1+\cos x}\cdot y = \frac{2x}{1+\cos x}$

2. $e^{\int -\frac{\sin x}{1+\cos x}dx} = e^{\ln(1+\cos x)} = 1+\cos x$

3. $[(1+\cos x)\cdot y]' = 2x$

4. $(1+\cos x)\cdot y = x^2 + C \Rightarrow y = \frac{x^2}{1+\cos x} + \frac{C}{1+\cos x}$

⊙. Linear with k constant.

Temp. $\frac{dT}{dt} + k\cdot T = k\cdot T_e$

1. $e^{\int k\,dt} = e^{kt}$

2. $(e^{kt}T)' = k\cdot T_e\cdot e^{kt}$

3. $e^{kt}\cdot T = \int k\cdot T_e(t)\, e^{kt}\,dt + C$

4. $T = e^{-kt}\cdot \underbrace{\int k\cdot T_e(t)\cdot e^{kt}\,dt}_{\text{I}} + \underbrace{C\cdot e^{-kt}}_{\text{II}}$ ✓

$T(0) = T_0$

稳定解

T_0''.

当 $t \to \infty$, $\Rightarrow 0$.

暂定解.