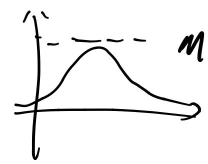
1. 保证某个函数的拉氏变换存在的条件

$f(t)$ of "exponential type"

$|f(t)| \leq C \cdot e^{kt}$ some $C>0$, $k>0$. all $t>0$

$|\sin t| \leq 1 \cdot e^{0t}$

$t^n \leq Me^t$ L'Hop $\frac{t^n}{e^t} \xrightarrow{t\to\infty} 0$.

$\frac{t^n}{e^t} \leq M \rightarrow$ 

反例: e^{t^2} not of exp type.

$e^{t^2} > e^{kt}$ 不能被 e^{kt} 所收敛.

②.

$$y'' + Ay' + By = h(t), \quad y(0)=y_0,\ y'(0)=y_0'$$

Laplace 变换必须有一个初值问题 → y必须要满足的初始条件

$$\boxed{y(t)\ \text{soln}} \rightsquigarrow Y(s)$$

① ② 相对简单.

代数

algebraic

$$\boxed{y'' + Ay' + By = h(t), \quad y(0)=y_0,\ y'(0)=y_0'} \xrightarrow[\text{①}]{\mathcal{L}} \text{alg. equ in } Y(s)$$

②

$$Y = \frac{p(s)}{q(s)} \xrightarrow[\text{③}]{\mathcal{L}^{-1}} \boxed{y = y(t)}$$

计算量大

比较难

$\mathcal{L}(f'(t))$

Integration of parts.

分部积分

$$\begin{cases} \int u\,dv = uv - \int v\,du \\ \int u v'\,dx = uv - \int u' v\,dx \end{cases}$$

$$= \int_0^{\infty} \underbrace{e^{-st}}_{\text{Diff}} \underbrace{f'(t)}_{\text{Int}}\, dt$$

$$= e^{-st} f(t)\Big|_0^{\infty} - (-s)\int_0^{\infty} e^{-st} f(t)\,dt$$

$$0 - f(0) + s\int_0^{\infty} e^{-st} f(t)\,dt$$

$\lim_{t\to\infty} \frac{f(t)}{e^{st}} \longrightarrow 0 \quad (s>k)$

$\Rightarrow |f(t)| < C e^{kt}$

$$= -f(0) + s \cdot F(s)$$

即 $\boxed{\mathcal{L}(f'(t)) = -f(0) + s \cdot F(s)}$

☆ $f'(t) \rightsquigarrow sF(s) - f(0)$

☆ $\boxed{\begin{aligned} f''(t) &\rightsquigarrow s \cdot [sF(s) - f(0)] - f'(0) \\ &= s^2F(s) - sf(0) - f'(0) \end{aligned}}$

$[f'(t)]' \rightsquigarrow s\mathcal{L}(f'(t)) - f'(0)$

例 $y'' - y = e^{-t}$, $y(0)=1$, $y'(0)=0$

Laplace → transform.

① $s^2 Y - s - Y = \dfrac{1}{s+1}$ → Alg. eql in $Y(s)$.

② $(s^2-1)Y = \dfrac{1}{s+1} + s \Rightarrow Y = \dfrac{s^2+s+1}{(s+1)^2(s-1)}$ → Soln of $Y(s)$

↓ 部分分式分解

→ 待定系数法 $s=0$. $-1 = -\frac{1}{2} + C - \frac{3}{4} \Rightarrow C = \frac{1}{4}$

$$\frac{s^2+s+1}{(s+1)^2(s-1)} = \frac{-\frac{1}{2}}{(s+1)^2} + \frac{\frac{1}{4}}{(s+1)} + \frac{\frac{3}{4}}{s-1}$$

cover-up method → 分子将均为常数.

捂住.

③ 逆变换. Laplace transform. inverse

$$\frac{-\frac{1}{2}}{(s+1)^2} + \frac{\frac{1}{4}}{s+1} + \frac{\frac{3}{4}}{s-1} \xrightarrow{-1} \underbrace{-\frac{1}{2} t e^{-t}}_{\text{特解 } y_p} \underbrace{+ \frac{1}{4} e^{-t} + \frac{3}{4} e^{t}}_{\text{补充解 } y_c}$$

$t \to \dfrac{1}{s^2}$

$te^{-t} \to \dfrac{1}{(s+1)^2}$ → 拉氏位移.