Polar method

① $y' + ky = k\cos\omega t$

② $\tilde{y} + k\tilde{y} = k e^{i\omega t} = \frac{1}{1+\frac{\omega}{k}} e^{i\omega t}$

③ $$\tilde{y} = \underbrace{\frac{1}{\sqrt{1+(\frac{\omega}{k})^2}} e^{i(\omega t - \Phi)}}_{\text{real part}} \rightarrow \frac{1}{\sqrt{1+(\frac{\omega}{k})^2}} \cos(\omega t - \Phi)$$

the Cartesian method:

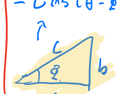
$$\tilde{y} = \frac{1}{1+i\cdot(\frac{\omega}{k})} \cdot e^{i\omega t}$$

$$= \frac{1 - i(\frac{\omega}{k})}{1+(\frac{\omega}{k})^2} (\cos\omega t + i\cdot\sin\omega t)$$

Real part:

$$\frac{1}{1+(\frac{\omega}{k})^2} (\cos\omega t + \frac{\omega}{k}\sin\omega t)$$

$a\cos\theta + b\sin\theta = C\cos(\theta - \Phi)$



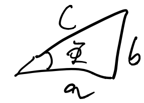
“三角恒等式”

trigonometric identity

“辅助角公式”

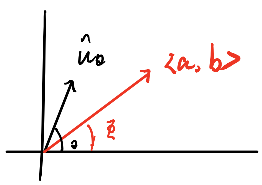
$$“\frac{1}{\sqrt{1+(\frac{\omega}{k})^2}} \cos(\omega t - \Phi)”$$

Proof of formal. $a\cos\theta + b\sin\theta = c\cdot\cos(\theta-\Phi)$



①. H·S 高中证法 反推: $c\cdot\cos(\theta-\Phi) = c\cdot\cos\theta\underbrace{\cos\Phi}_{\frac{a}{c}} + c\cdot\sin\theta\underbrace{\sin\Phi}_{\frac{b}{c}} = a\cos\theta + b\sin\theta$

②. 正推:



$\langle a, b\rangle \cdot \langle \cos\theta, \sin\theta\rangle$

$= |\langle a, b\rangle| \cdot 1 \cdot \cos(\theta-\Phi)$

③. Complex # Proof. → 两复数的乘积

$\boxed{(a-bi)(\cos\theta + i\sin\theta)} \xrightarrow{\text{取实部}} a\cos\theta + b\sin\theta$

$= \sqrt{a^2+b^2}\, e^{-i\Phi} \cdot 1\cdot e^{i\theta}$

$= \boxed{\sqrt{a^2+b^2}\, e^{i(\theta-\Phi)}} \xrightarrow{\text{取实部}} \sqrt{a^2+b^2}\cos(\theta-\Phi)$

总结!

Basic linear ODE

① $y' + ky = k\,q_e(t)$ "Temp - Conc. conduction-diffusion"

② $y' + ky = q(t)$ "$k>0$"

③ $y' + p(t)\,y = q(t)$

Basic → General

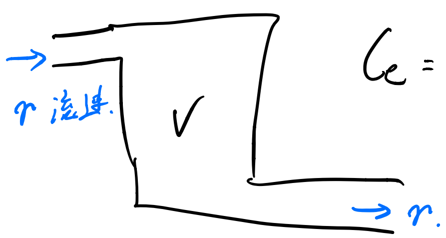
Mixing example.

"t时刻水箱的盐容量"

$X(t)$ = amt salt in tank at time t

C_e = Conc. of the incoming salt

"流体流入浓度"



Prob. what is $X(t)$?

解:

$x(t)$

$\frac{d\,x(t)}{dt}$ → t时刻的浓度.

$$\frac{d\,x(t)}{dt} = \text{rate salt inflow} - \text{rate salt outflow}$$

$$= r \cdot C_e - r \cdot \frac{x}{V}$$

$$\frac{dx}{dt} + \frac{rx}{V} = r \cdot C_e$$

/ $C(t) = \frac{x}{V} \Rightarrow \frac{dx}{dt} = V \cdot \frac{dc}{dt}$

$$V \cdot \frac{dc}{dt} + r \cdot c = r \cdot C_e$$

form.

$$\frac{dc}{dt} + \boxed{\frac{r}{V}} \cdot c = \boxed{\frac{r}{V}} \cdot C_e \rightarrow \text{" } y' + ky = k\,q_e(t) \text{ "}$$

"解率"

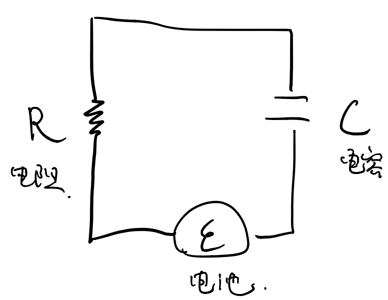
$k = \frac{r}{V}$ "basic param."

流出部分所占比例 "fractional" "$\frac{1}{t}$"

① C_e: sinusoidally "正弦变化" → $\cos \omega t$

"How closely does $C(t)$ follow C_e" → "池子浓度会多大程度上跟随 C_e 变化"

if k big → r big, → $V \downarrow$, 且 small. $A = 1$.



q = charge on the capacitance
电容上的电荷.

$\frac{dq}{dt} = i$

$$R \frac{dq}{dt} + \frac{q}{C} = \varepsilon(t)$$

$$q' + \frac{q}{RC} = \frac{\varepsilon}{R}$$

② 衰变

$$Ⓐ \xrightarrow[\text{decay}]{} Ⓑ \longrightarrow$$

$$\frac{dB}{dt} = k_1 A + k_2 B$$

$$= k_1 A_0 e^{-k_1 t} - k_2 B \Rightarrow B' + k_2 B = k_1 A_0 e^{-kt}$$

对 $k<0$，以下术语变得不适用，

- transient “暂态”
- steady-state “稳态”
- input 输入
- response 响应 ORZ
- applies 应用，

$$\frac{dy}{dt} - ay = q(t)$$

$k=-a$ $(a>0)$

LIFE.

★ 有生命特征的

$k<0$

无 - - -

$k \geq 0$

★ $$e^{at}\int q(t)e^{-at}\,dt + ce^{at}$$

FIXED

不是 transient

↳ ∞

INPUT