

SUBSTITUTIONS. "变量代换法"

1. Scaling "尺度变换"

$$y' = f(x, y) \xrightarrow{\begin{matrix} x_1 = \frac{x}{a} \\ y_1 = \frac{y}{b} \end{matrix}}$$

①. change units "改变单位"

②. make the variables dimensionless. "无量纲"
 不带单位的纯数字.

③ reduce or simplify the constants. "减少常数"

example:

Bio
Temperature

$$\frac{dT}{dt} = k(M^4 - T^4)$$

T : internal temperature

M : external temperature
constant

① $T_1 = \frac{T}{M}$ \rightarrow dimensionless

$\Rightarrow T = T_1 M$

$$M \frac{dT_1}{dt} = k \cdot M^4 (1 - T_1^4) \Rightarrow \frac{dT_1}{dt} = \underbrace{k_1}_{\substack{\text{km}^3 \approx \\ \text{..lumping..} \\ \text{constants}}} (1 - T_1^4)$$

1.1 DIRECT \rightarrow New Variable = old Var.s = $T_i = \frac{T}{m}$.

example: $y' = p(x) \cdot y + q(x) \cdot y^n$ ($n \neq 0$) \rightarrow "Bernoulli equation" 伯努利方程.

"Everything multiplied by y or a power of y , two terms"
这两项必须包含 y 或者 y 的幂.

\Downarrow

$$\frac{y'}{y^n} = p(x) \frac{1}{y^{n-1}} + q(x)$$

$$\Downarrow v = \frac{1}{y^{n-1}} = y^{1-n}, \quad v' = (1-n) \frac{1}{y^n} \cdot y'$$

$$\boxed{\frac{v'}{1-n} = p(x) \cdot v + q(x)} \rightarrow \text{"linear"}$$

★: "Learn methods, not final formulas"

$$y' = \frac{y}{x} - y^2$$

① $\frac{y'}{y^2} = \frac{1}{x} \cdot \frac{1}{y} - 1$

$$\Downarrow v = \frac{1}{y}, \quad v' = -\frac{1}{y^2} \cdot y'$$

② $-v' = \frac{1}{x} \cdot v - 1 \Rightarrow \boxed{v' + \frac{v}{x} = 1} \rightarrow \text{"linear"}$

$$\Downarrow \int \frac{1}{x^2} dx = -\frac{1}{x}$$

③ $xv' + v = x \Rightarrow (xv)' = x \Rightarrow xv = \frac{1}{2}x^2 + C \Rightarrow v = \frac{1}{2}x + \frac{C}{x}$

$$\Downarrow v = \frac{1}{y}$$

④ $\frac{1}{y} = \frac{x}{2} + \frac{C}{x} \Rightarrow \boxed{y = \frac{2}{x} \cdot \frac{2x}{x^2 + 2C + C_1}} \rightarrow \text{solution.}$

1.2 INVERSE

$$\text{old Var} = \begin{array}{c} \text{New} \\ \text{Var.} \end{array} = T = M \cdot T_1$$

"Homogeneous" \rightarrow 齐次式

$$y' = F\left(\frac{y}{x}\right)$$

$$\begin{cases} y' = \frac{x^2 y}{x^3 + y^3} = \frac{y/x}{1 + (\frac{y}{x})^3} \\ xy' = \sqrt{x^2 + y^2} \Rightarrow y' = \sqrt{1 + (\frac{y}{x})^2} \end{cases}$$

\rightarrow 具有不变性: "invariant under the operation zoom"
Invariant 缩放变换不变.

$$\hookrightarrow x = ax_1, y = ay_1, \quad \frac{dy_1}{dx_1} = F\left(\frac{y_1}{x_1}\right)$$

solve it:

$$y' = F\left(\frac{y}{x}\right)$$

$$\downarrow z = \frac{y}{x} \rightarrow y = zx \rightarrow y' = z'x + z$$

$$z'x + z = F(z) \rightarrow x \cdot \frac{dz}{dx} = F(z) - z \rightarrow \frac{dz}{F(z) - z} = \frac{dx}{x}$$

\downarrow
solve it \checkmark .

★ : differential equations model.

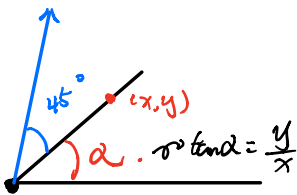
the escape strategy.

The diagram shows two scenarios of wave propagation. In the top scenario, a source (black dot) emits waves (red arcs) while moving to the right with velocity v . An observer (red dot) is moving upwards with velocity v . The text "the drug boat" is written in red, and "goes" is written in blue. In the bottom scenario, the source is stationary, and the observer is moving upwards at a 45° angle to the wave beam. The text "dope constants 45° to beam." is written in blue.

Q: what's the
boats path?

* PATH \Rightarrow Curve
a function.

解:



y' = slope

Let $y = y(x)$ is unknown graph.

$$y' = \tan(\alpha + 45^\circ) = \frac{\tan \alpha + \tan 45^\circ}{1 - \tan \alpha \cdot \tan 45^\circ}$$

$$\Rightarrow y' = \frac{y/x + 1}{1 - y/x} \quad \rightarrow \text{"homogeneous" ODE}$$

$$\downarrow z = y/x \rightarrow y' = z'x + z$$

$$z'x+2 = \frac{z+1}{1-z} \rightarrow (zx)' = \frac{z+1}{1-z}$$

$$\frac{dz}{dx} \cdot x = \frac{z+1}{1-z} - z = \frac{1+z^2}{1-z}$$

$$dx \cdot \frac{1-z}{1+z^2} = \frac{dx}{x}$$

~~$$2x = \int \frac{2+1}{1-z} dz =$$~~

$$\tan^{-1} z - \frac{\ln(Hz^2)}{2} = \ln \chi + C.$$

$$\tan^{-1} z = \ln \sqrt{1+z^2} + \ln x + \angle$$

$$\tan^{-1} \left(\frac{y}{x} \right) = \ln x \sqrt{1 + \left(\frac{y}{x} \right)^2} + \angle = \ln \sqrt{x^2 + y^2} + \angle.$$

↓. polar coordinates. "极坐标"

$$\theta = \ln r + \angle \rightarrow e^\theta = r \cdot \angle$$

↓.

$$r = C_1 \cdot e^\theta$$