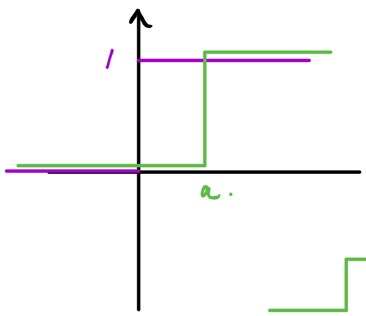
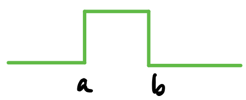
"跳跃不连续点" → Jump discontinuities



$u(t)$ "unit step"

$u(0)$ undefined.

$u_a(t) = u(t-a)$

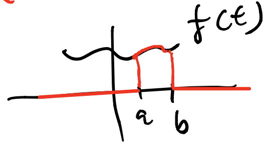


→ $U_{ab}(t)$ → "unit box"

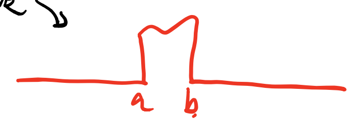
a处跳跃　　b处下降.

$$U_{ab}(t) = \underbrace{U_a(t)}_{} + [-\underbrace{U_b(t)}_{}]$$
$$= u(t-a) - u(t-b)$$

这些转换有什么好处? → 当在乘法中使用它们时，它们会"变换"为其它的函数　筛出



$\xrightarrow{U_{ab}(t)f(t)}$ "滤波"



$$\mathcal{L}(u(t)) = \int_0^\infty e^{-st} \underbrace{u(t)}_{=1} dt = \frac{1}{s}, \ (s>0)$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\right) = ?$$

确定 $\mathcal{L}^{-1}$ 存在的问题.

↓

→ 与过去相关：傅里叶变换

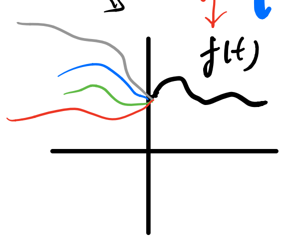
→ 与未来相关：Laplace 变换

(a, t)

$$f(t) \overset{\mathcal{L}}{\rightsquigarrow} F(s)$$

当 t<0 时
f(t)与任意一个小尾巴都会
同一个 Laplace 变换.

$$F(s) \overset{\mathcal{L}^{-1}}{\rightsquigarrow} u(t)\cdot f(t)$$



强制要求让函数在 t<0 时变为 0 →

⊗ 使得 $\mathcal{L}^{-1}$ 变为唯一

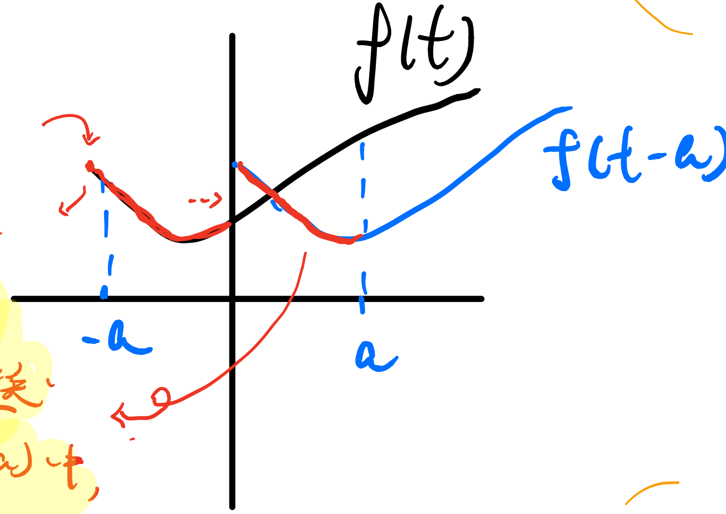
want formula: $(a>0)$

$\mathcal{L}(f(t-a))$ in trans of $\mathcal{L}(f(t))$

but 不存在

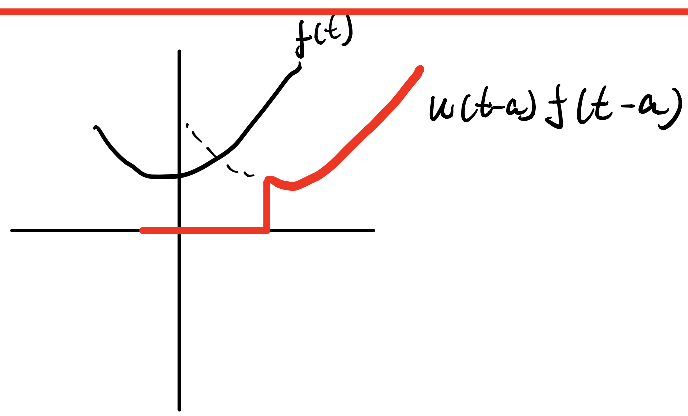
but，不存在

因为：
$f(t<0)$ 这部分没有被 $f(t)$ 作 Laplace 用到
但是，在变换 $f(t) \to f(t-a)$ 时，需要被用到



$$\boxed{\text{Right formula}}$$

$$\boxed{u(t-a)f(t-a) \rightsquigarrow e^{-as}F(s)}$$



$$\boxed{u(t-a)f(t) \rightsquigarrow e^{-as}\mathcal{L}(f(t+a))}$$

"t轴平移公式"

Proof: ①

$$\int_0^\infty e^{-st} u(t-a) f(t-a)\, dt \qquad \text{令 } t_1 = t-a$$

$$= \int_{-a}^\infty e^{-s(t_1+a)} u(t_1) f(t_1)\, dt_1$$

$(-a,0),\ u(t)=0$

$(0,+\infty),\ u(t)=1$

$$= e^{-as}\int_{-a}^\infty e^{-st_1} \underbrace{\cancel{u(t_1)}}_{=1} f(t_1)\, dt_1$$

$$= e^{-as}\int_0^\infty e^{-st_1} f(t_1)\, dt, \qquad \because u(t_1)=0 \text{ for } t_1<0$$

哑变量

$$= e^{-as}F(s)$$

② $u(t-a)f(t-a) \rightsquigarrow e^{-as}\mathcal{L}(f(t))$

$u(t-a)f(t) \rightsquigarrow ?$

$\|$

$u(t-a)f(t-a+a)$ $\xrightarrow[t \to t+a]{\text{replace}}$ to RHS.

$\|$ $(t+a)-a$

$u(t-a)f(\underline{(t+a)-a}) \rightsquigarrow e^{-as}\mathcal{L}(f(t+a))$

例: $\mathcal{L}(u(t)) = \frac{1}{s}$.

① $u_{ab}(t) = u(t-a) - u(t-b)$

$\overset{\mathcal{L}}{\rightsquigarrow} (e^{-as}\cdot\frac{1}{s}) - (e^{-bs}\frac{1}{s})$

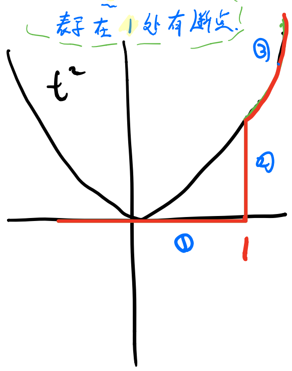
②

$u(t-1)\,t^2$

$$\xrightarrow[\mathcal{L}]{} e^{-s}\mathcal{L}[(t+1)^2] = e^{-s}\mathcal{L}(t^2+2t+1)$$

$$= e^{-s}\left(\boxed{\frac{2!}{s^3}+\frac{2}{s^2}+\frac{1}{s}}\right) \rightsquigarrow t^2.$$

$-1\cdot s$
表示在 1 处有断点



$\to u(t-1)\,t^2$.

有 2 段，有 3 部分

所以 Laplace 就有三项.

例 2. $\mathcal{L}^{-1}\left(\dfrac{1+e^{-\pi s}}{s^2+1}\right)$

逆变换

必须将其 $\boxed{\text{分成}}$ 伴随不同指数的项

$$= \mathcal{L}^{-1}\left(\frac{1}{s^2+1}+\frac{e^{-\pi s}}{s^2+1}\right),$$

$\dfrac{1}{s^2+1} \rightsquigarrow u(t)\sin t.$

$\dfrac{e^{-\pi s}}{s^2+1} \rightsquigarrow u(t-\pi)\cdot\sin t$

$$= \underbrace{u(t)}_{} \overset{t>0.}{} \sin t + \underbrace{u(t-\pi)}_{} \overset{t>\pi.}{} \sin t$$

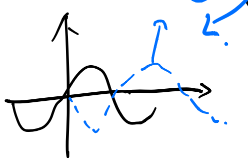
1

即:

$$f(t) = \begin{cases} u(t) \sin t, & 0<t<\pi \\ \sin(t-\pi) + u(t-\pi)\cdot \sin t, & t>\pi \end{cases}$$

$-\sin t$

$\Rightarrow -\sin t + \sin t$



即

$$f(t) = \begin{cases} \sin t, & 0 \le t \le \pi \\ 0, & t \geq 0 \end{cases}$$