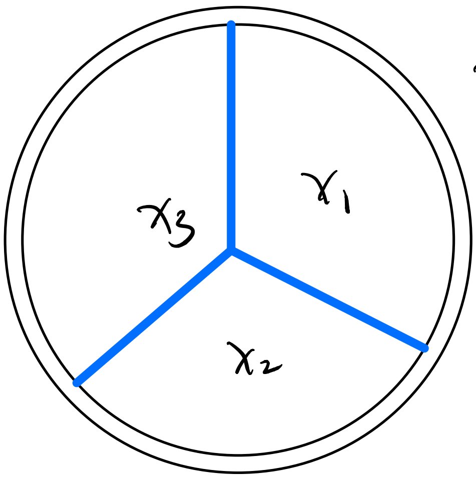


鱼缸 温度.

x_i = temp in tank i ，热量跑不出鱼缸

Problem:

$x_i(t) = ?$

什么决定了温度的变化？ 隔板传导率，温差，...

$$x_1' = a(x_3 - x_1) + a(x_2 - x_1)$$

即
$$x_1' = -2ax_1 + ax_2 + ax_3$$
$$x_2' = ax_1 - 2ax_2 + ax_3$$
$$x_3' = ax_1 + ax_2 - 2ax_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}' = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -2-\lambda & 1 & 1 \\ 1 & -2-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = -(\lambda+2)^3 + 2 + 3(2+\lambda) = 0$$

$\Downarrow$

$$\lambda^3 + 3\lambda^2 - 2 + 3\lambda \cdot 2^2 + 2^3 - 2 - 3(2+\lambda) = 0$$

$\Downarrow$

$$\lambda^3+6\lambda^2+9\lambda=0$$

$$\lambda(\lambda+3)^2=0$$

eigenvalues: $\lambda=0$, $\underline{\lambda=-3}$ 双重根

① $\lambda=0$, $\vec{\alpha}=?$

$$\begin{pmatrix}-2&1&1\\1&-2&1\\1&1&-2\end{pmatrix}\vec{\alpha}=0 \Rightarrow \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix}e^{0t}$$ → 常数解 → 稳定解

不随时间变化而变化的解

⇒ $x_1=x_2=x_3$.

② $\lambda=-3$, $\vec{\alpha}=?$

$$\underbrace{\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}}_{\text{相关方程组}}\vec{\alpha}=0$$ → 有很多解.

如 $\underbrace{\begin{pmatrix}1\\0\\-1\end{pmatrix}}_{\vec{\alpha}_1}e^{-3t}$ 解 / $\underbrace{\begin{pmatrix}1\\-1\\0\end{pmatrix}}_{\vec{\alpha}_2}e^{-3t}$ 另一个解.

相互独立

若 $\begin{pmatrix}0\\1\\-1\end{pmatrix}e^{-3t}$ ⊗. 不行

因为 $\begin{pmatrix}1\\0\\-1\end{pmatrix}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ 不独立!

$$\Rightarrow \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\underbrace{C_1\begin{pmatrix}1\\0\\-1\end{pmatrix}e^{-3t}+C_2\begin{pmatrix}-1\\1\\0\end{pmatrix}e^{-3t}}_{\text{当 } t\to\infty \text{ 时, 该部分趋于0.}}+\underbrace{C_3\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\text{稳定解.}}$$

③. 若 λ 是 重根 "repeated eigenvalues", 但可以找到足够的独立特征向量.

来构造所需数量的独立解.

完备特征值.

"Complete eigenvalue"

相反，

e.g. "defective eigenvalue"

不完备特征值 (incomplete)

$\boxed{\text{Prob. 2}}$

Ths.

A real $n \times n$ matrix

with is symmetric → 对称. ($A^T = A$)

$\Longrightarrow$ all it eigenvalues are complete

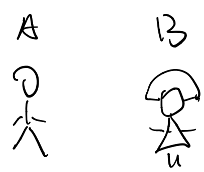
它们的所有特征值都完整.

Complex eigenvalues:

→ calc. cx. eigenvectors.

→ form soln. $\underset{\text{c.x}}{\underset{\sim}{\vec{\alpha}}}\, e^{(a+bi)t}$

→ take $\underbrace{\text{real}}_{\text{解1}}$ & $\underbrace{\text{imaginary}}_{\text{解2}}$ parts.



例:

$$\begin{cases} x' = x + 2y \\ y' = -x - y \end{cases}$$

x表示 A对B的 相对爱意

y表示 B对A ------

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 表示 平等的爱.

⊙ 当$y\uparrow$, $x'\uparrow$, → $x\uparrow$ 正常的爱.

当$x\uparrow$, $y'\downarrow$, → $y\downarrow$ 婊

$\rightarrow A = \begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$

$\rightarrow \quad \lambda^2 + 1 = 0$

$\lambda = \pm i$

$\rightarrow$ 令 $\lambda = i$

$$\begin{pmatrix} 1-i & 2 \\ -1 & -1-i \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = 0$$

$\Big({}^{2}$

$1-i+2(\cancel{?}-\frac{i}{2}+\frac{i}{2})$

$-(1-i)\cdot(1+i) = 2 \qquad \begin{pmatrix} 2 \\ i-1 \end{pmatrix}$

-2

$$\begin{cases} (1-i)\alpha_1 + 2\alpha_2 = 0 \\ -\alpha_1 - (1-i)\alpha_2 = 0 \end{cases} \Rightarrow \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} 2 \\ i-1 \end{pmatrix}$$

$\rightarrow$ Soln $\quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ i-1 \end{pmatrix} e^{it} =$

分离 Re & Im

$$= \left[\begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} i \right] (\cos t + i \sin t)$$

$$Re = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2\cos t \\ -\cos t - \sin t \end{pmatrix} \rightsquigarrow \text{椭圆}$$

$$Im = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t + \begin{pmatrix} 2 \\ -1 \end{pmatrix} \sin t$$