Inhomogenous equation. 非齐次方程.

(x is usually time)

$$y'' + p(x)y' + q(x)y = f(x)$$

input / signal / driving term.

soln: response / output

$y(x)$

齐次项的解 is essential part 对 非齐次项的解.

$$\boxed{y'' + p(x)y' + q(x)y = 0}$$

associated homogeneous equation. 相伴齐次方程. or reduced equation.

soln: $y_c = c_1 y_1 + c_2 y_2$.

"complementary solution." 补充解

Examples:

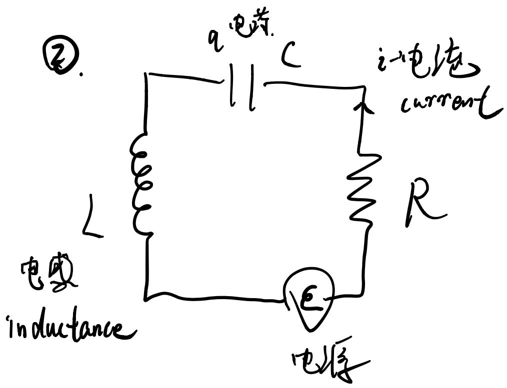
① $mx'' + bx' + kx = f(t)$ (mass, dashpot, spring) external force!

spring - mass - dashpot system: 弹簧 - 质量 - 阻尼.

"force system" $f(t) \neq 0$ 受迫系统.

passive system $f(t) = 0$ 被动系统.

②



sum of voltage drops = ?

电压下降之和

$$Li' + Ri + \frac{q}{C} = \mathcal{E}(t)$$

电池：常数

e交流电 fn,

$$\downarrow$$

$$\underline{Li'' + Ri' + \frac{i}{C} = \mathcal{E}'(t)}$$

$q' = i$

Thm: $Ly = f(x)$ (L is 2^{nd} linear operator)

soln: $y_p + y_c$

$Ly = f(x)$ 的特殊解

$\Leftrightarrow y = y_p + c_1 y_1 + c_2 y_2$

particular

① 找到补充解

②. find y_p

any one solution

任意一解

To proof: ① All the $y_p + c_1 y_1 + c_2 y_2$ 是解.

$$L(y_p + c_1 y_1 + c_2 y_2) = Ly_p + \underbrace{L(c_1 y_1 + c_2 y_2)}_{=0} = f(x)$$

② no other solution 不存在其它解

反证法：假设 $u(x)$ 是一个解

$$L(u) = f(x)$$

$$-\ L(y_p) = f(x)$$

$$L(u - y_p) = 0$$

means:

$\Rightarrow$ $u - y_p$ 一定是齐次方程的解！

即 $u - y_p = \tilde{c}_1 y_1 + \tilde{c}_2 y_2$

$\Rightarrow u = \tilde{c}_1 y_1 + \tilde{c}_2 y_2 + y_p \subseteq c_1 y_1 + c_2 y_2 + y_p$

则 u 属于 $c_1y_1+c_2y_2+y_p$，方程不存在其它解

☆☆☆ Find a particular solution! y_p ↓ 后两节课讲

★解的结构

一阶情况：

$$y'+ky=q(t)$$

soln: $y=\underbrace{e^{-kt}\int q(t)e^{kt}\,dt}_{y_p}+\underbrace{ce^{-kt}}_{y_c}$

当 $k>0$， $y=\underbrace{\text{steady-state}}_{y_p}+\underbrace{\text{transient}}_{y_c}$ ～ 暂态解 — "渐趋于0" $t\to\infty$, $y_c\to 0$.

$k<0$，无意义。

二阶：　　　　A,B是常数.

$$y'' + Ay' + By = f(t)$$

$$y = y_p + \underbrace{c_1y_1 + c_2y_2}_{\text{use init. cond.}}$$

for all c_1, c_2.

Q: A, B 满足什么条件，$c_1y_1+c_2y_2$ 在 t趋于无穷时趋于零？

如果满足这个特征，ODE called “stable”.

齐次方程

特征根	解	稳定态，
$r_1 \neq r_2$	$c_1e^{r_1t} + c_2e^{r_2t}$	$r_1<0$, $r_2<0$
$r_1 = r_2$	$(c_1+c_2t)e^{r_1t}$	$r_1<0$
$r = a\pm bi$	$e^{at}(c_1\cos bt + c_2\sin bt)$	$a<0$

ODE $y'' + Ay' + By = 0$

若特征根拥有负实部，ODE会趋向稳定。
negative real part

“下讲继续”