OSCILLATIONS. 振荡.

回顾：

$$y'' + by' + ky = 0$$

try $y = e^{rt}$

must satisfy. $r^2 + br + k = 0$

"特征根复根时才能得到振荡"

complex root

$$r = a \pm bi \Rightarrow e^{(a+ib)t} \xrightarrow{\text{Re}} e^{at}\cos bt = y_1 \; ① \qquad \searrow e^{at}\sin bt = y_2 \; ②$$

通解 $y = c_1 y_1 + c_2 \cdot y_2$.

$$y = c_1 e^{(a+bi)t} + c_2 e^{(a-bi)t}$$

① chge $i \to -i$

② give condi's on the $c_1 + c_2$.

Which are real soln's?

Ans → by hack 蠢 : 相乘，让虚部=0 ?

Ans → $u + iv$ → $v = 0$

→ change "$i = -i$". 看等式是否有变。

$$C_1 e^{(a+bi)t} + C_2 e^{(a-bi)t}$$

$$\Big\downarrow \; i = -i$$

Complex

$$\bar{C_1} e^{(a-bi)t} + \bar{C_2} e^{(a+bi)t} \qquad \begin{cases} C_2 = \bar{C_1} \\ \bar{C_2} = C_1 \end{cases}$$

$$(c+id)e^{(a+bi)t} + (c-id)e^{(a-bi)t}$$

$$\begin{cases} \cos a = \dfrac{e^{ia} + e^{-ia}}{2} \\[2ex] \sin a = \dfrac{e^{ia} - e^{-ia}}{2i} \end{cases}$$

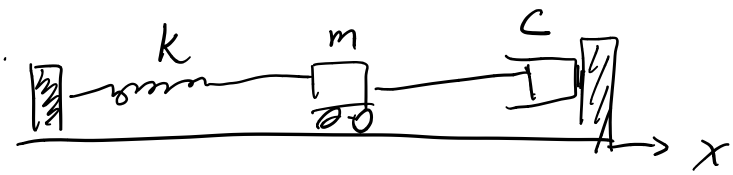
"逆向 Euler's form"

$$= e^{at}\left[c\,(e^{ibt} + e^{-ibt}) + id\,(e^{ibt} - e^{-ibt})\right]$$

$$2\cos bt \qquad + \qquad 2i\sin bt$$

$$= e^{at}\left(\underbrace{2c}_{c_1}\cos bt - \underbrace{2d}_{c_2}\sin bt\right)$$

②.



$$mx'' + cx' + kx = 0$$

$$\hookrightarrow x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$$

★ $\boxed{y'' + 2p\,y' + \omega_0^2\, y = 0}$

oscillations : $\rightarrow$ complex roots.

$$r^2 + 2p\,r + \omega_0^2 = 0$$

$$r = \frac{-2p \pm \sqrt{4p^2 - 4\omega_0^2}}{2} = -p \pm \sqrt{p^2 - \omega_0^2}$$

case ①, $p = 0$. $\Rightarrow$ 无阻尼. (undamp)

$$y'' + \underbrace{\omega_0^2}_{\text{circular frequency}} y = 0$$

圆频.

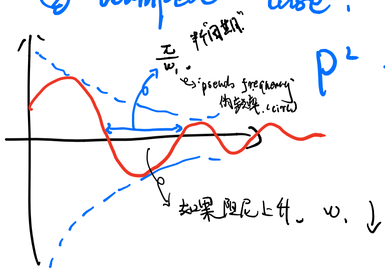
Soln's

$r = \pm i\omega_0$

$$y = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

$$= A \cos(\omega_0 t - \phi)$$

② damped case :



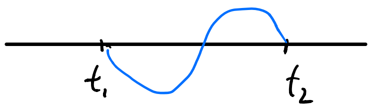
$p^2 - \omega_0^2 < 0 \Rightarrow p < \omega_0 \rightarrow$ 阻尼小于圆频率

$$r = -p \pm \sqrt{-(\omega_0^2 - p^2)} = -p \pm \sqrt{-\omega_1^2}$$

$$y = e^{-pt}(c_1 \cos \omega_1 t + c_2 \sin \omega_1 t)$$

$= e^{-pt} A \cos(\omega_1 t - \Phi)$

$\omega_1 \to$ 伪频率



$t_2 = t_1 + \frac{2\pi}{\omega_1}$

$\omega_1 t_1 - \Phi = \frac{\pi}{2}$.

next time $\Rightarrow$ $\omega_1\left(t_1 + \frac{2\pi}{\omega_1}\right) - \Phi = \frac{\pi}{2} + 2\pi$

$p \to$ only on ODE

$\left.\begin{array}{l}\Phi \\ A\end{array}\right\}$ depends on init. cond.

$\omega_1 \to$ only on ODE

$\omega_1^2 = \omega_0^2 - p^2$ $\to$ damp
$\to$ 回.