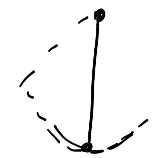
phenomenon of resonance.



共振.

input：/驱动项

输入频率 $\quad$ 自然频率

$(\omega_1 \neq \omega_0)$

$$y'' + \omega_0^2 y = \cos \omega_1 t$$

$$(D^2 + \omega_0^2) y = \cos \omega_1 t$$

复化 $\quad e^{i\theta} = \cos\theta + i\cdot\sin\theta$

$$(D^2 + \omega_0^2)\tilde{y} = e^{i\omega_1 t}$$

$$\tilde{y}_p = \frac{e^{i\omega_1 t}}{(i\omega_1)^2 + \omega_0^2} = \frac{e^{i\omega_1 t}}{\omega_0^2 - \omega_1^2}$$

$$\mathrm{Re}(\tilde{y}_p) = y_p = \frac{\cos \omega_1 t}{\omega_0^2 - \omega_1^2}$$

y_p的大小取决于 ω_0与ω_1的相对大小

↳ $\omega_1 \approx \omega_0$，振幅就会↑

↳ 相反

If $\omega_1 = \omega_0$

$$(D^2 + \omega_0^2) y = \cos \omega_0 t$$

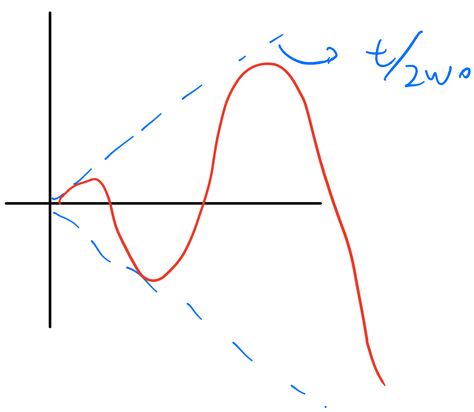
Complex.

$$(D^2 + \omega_0^2)\tilde{y} = e^{i\omega_0 t}$$

$i\omega_0$是$D^2 + \omega_0^2$的一个单根.

$$\tilde{y}_p = \frac{t e^{i\omega_0 t}}{D'(\omega_0 t)} = \frac{t e^{i\omega_0 t}}{2 i \omega_0}$$

$$\mathrm{Re}(\hat{y}_p) = y_p = \frac{t \sin w_0 t}{2w_0}$$



$$(D^2 + w_0^2)\, y = \cos w_1 t$$

y_c 补充解之一

$$y_p = \frac{\cos w_1 t}{w_0^2 - w_1^2} + \left(- \frac{\cos w_0 t}{w_0^2 - w_1^2} \right)$$

$$\lim_{w_1 \to w_0} \frac{\cos w_1 t - \cos w_0 t}{w_0^2 - w_1^2} \xrightarrow[\text{L'Hopital rule}]{\text{洛必达法则}} \lim_{w_1 \to w_0} \frac{-\sin(w_1 t)\, t}{-2w_1} = \boxed{\frac{t \sin w_0 t}{-2w_0}}$$

w_1 是变量.

几何意义?

$$\cos B - \cos A = 2\sin\frac{A-B}{2}\sin\frac{A+B}{2}$$

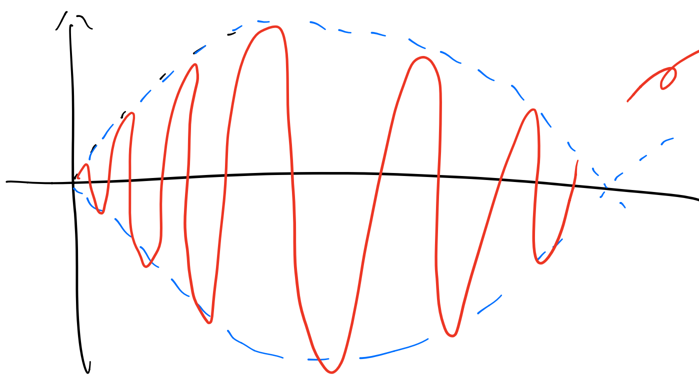
将sin用复数表出，反向旋转

频率较低，振幅

纯振荡

$$\frac{\cos w_1 t - \cos w_0 t}{w_0^2 - w_1^2} = \frac{\overbrace{2\sin\underbrace{\left(\frac{w_0-w_1}{2}\right)}_{\text{频率}}t}^{\approx 0}}{\underbrace{w_0^2 - w_1^2}_{\text{振幅}}} \cdot \overbrace{\sin\left(\frac{w_0+w_1}{2}\right)t}^{\approx w_0}$$

$\Rightarrow$



"beats"

拍是两个频率的组合.

⊙ Damped resonance 阻尼共振

Book: $mx'' + cx' + kx = f(t)$

$\Downarrow$

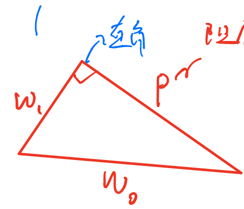
$x'' + 2px' + \underset{\downarrow}{w_0^2}\, x = f(t)$

natural freq.
无阻尼固定频率

伪频率
"pseudo freq."

w_1 阻尼固有频率

visual: $x'' + b x' + kx = f(x)$



$w_1^2 = w_0^2 - p^2$

$$y'' + \underbrace{2p}_{\text{存在阻尼}} y' + w_0^2 y = \cos wt$$

Q: 什么样的输入频率才能使响应的振幅最大

A: $w_r = \sqrt{w_0^2 - 2p^2}$

$$(D^2 + 2pD + w_0^2)\tilde{y} = e^{iwt}$$

$$\tilde{y}_p = \frac{e^{iw}}{(iw)^2 + 2p \cdot iw + w_0^2}$$

$$Re(\tilde{y}_p) = y_p = \frac{\cos wt}{-w^2 + w_0^2}$$