AUTONOMOUS. 自治

no independ variable on RHS.

$$\frac{dy}{dt} = f(y) \leftarrow \text{no } t \text{ on RHS}$$

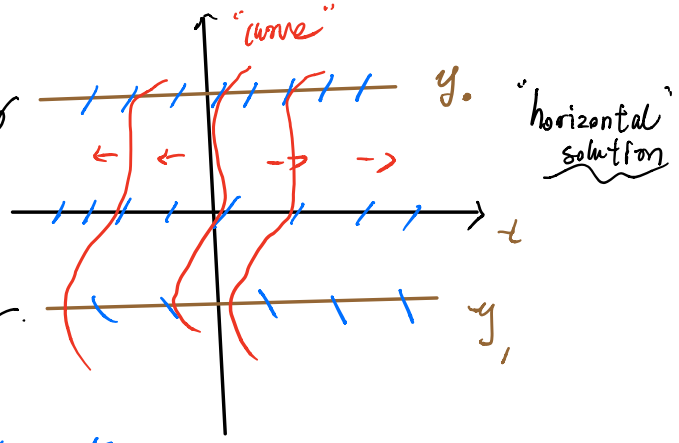
不解方程

"Get qualitative information about the solutions without actually solving"

DIR'N FIELD:

slope $\frac{dy}{dt} = f(y_0)$ isocline:

slope $\frac{dy}{dt} = f(y_1)$



① 自治方程的积分曲线对平移不变 "get them all by taking one"

CRITICAL POINT "临界点"

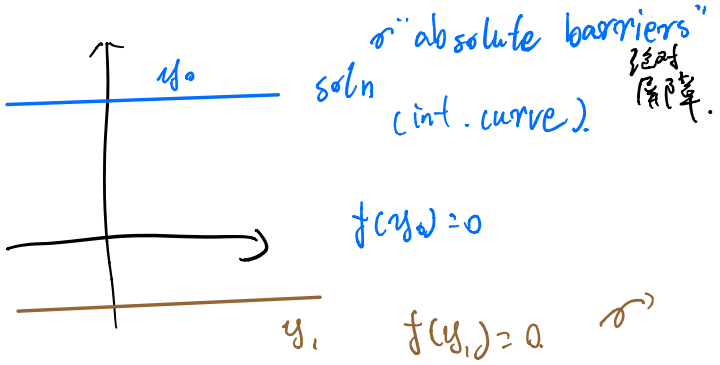
① $f(y_0) = 0$ ⇒

②

③

① $f(y_0) = 0$

$y = y_0$ is a soln. ⇒ $\frac{dy_0}{dt} = 0$ ⇒ a constant function.



① Find the critical points "$=0$"

② Graph. $f(y)$. >0 ? <0 ?

$\frac{dy}{dt} = f(y) > 0 \Rightarrow y \uparrow$

$< 0 \Rightarrow y \downarrow$

example:

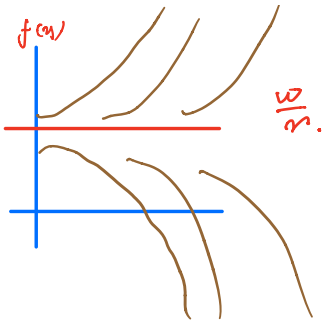
y: money in the bank account

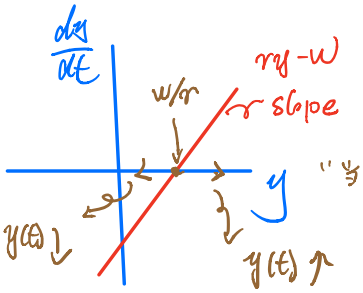
r = interest rate → "continuously"

eq: $\frac{dy}{dt} = r \cdot y - w$

w = rate of embezzlement "盜用率" → "cont."

解: ① crit. pts: $ry - w = 0 \Rightarrow y = \frac{w}{r}$



② 

"当 y(t) 远离 $\frac{w}{r}$ 时, $\frac{dy}{dt}$ ↑, y ↑. 而且越跳越快"

example 2:

logistic equation → "describes how population increases"

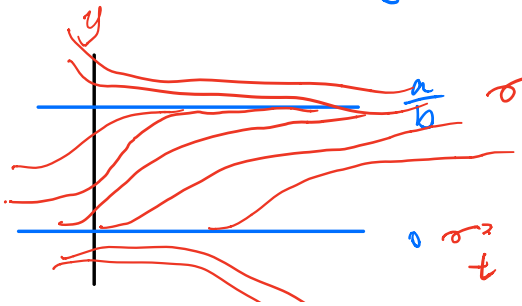
$y(t)$: 某地. population.

$\frac{dy}{dt} = ky$ k: growth rate. ("生-死")

Logistic growth: k is unrealistic. { 当 y ↑, k ↓. y ↓, k ↑.

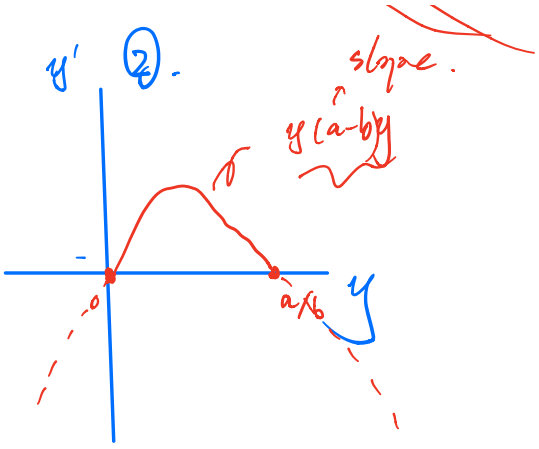
simplest choice: "$k = a - by$" → $\frac{dy}{dt} = (a - by)y$.

解: ① Crit. pts. $y(a - by) = 0 \Rightarrow y = 0, y = \frac{a}{b}$.



→ 吸引的 → "stable soln"

→ "unstable soln" 排斥力



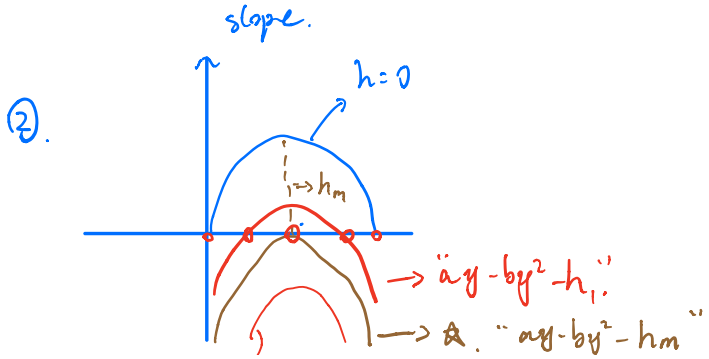
②. Logistic EQN.

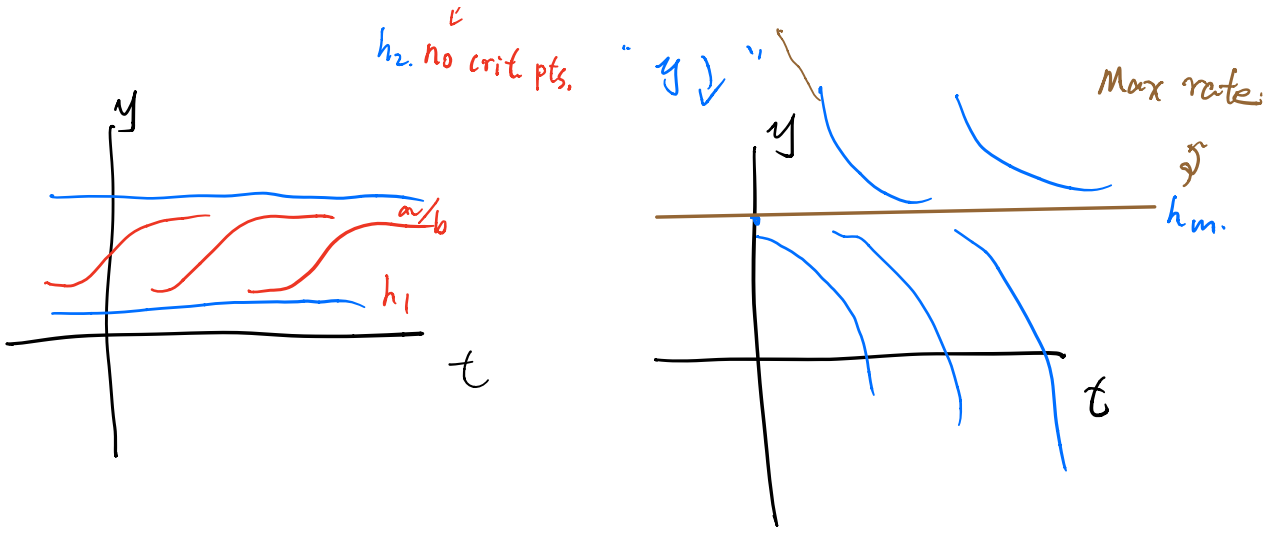
with harvesting → 收割

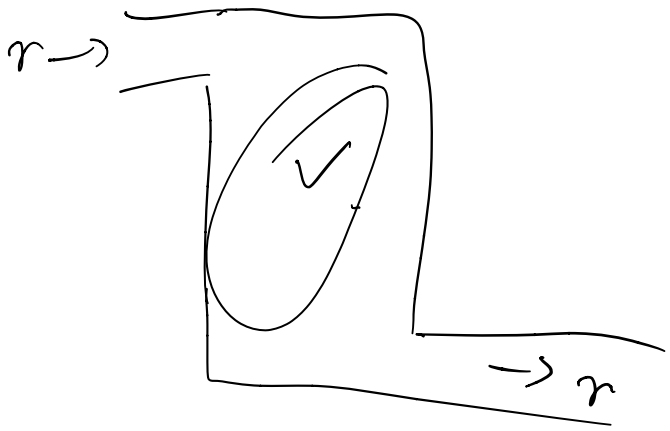
harvest: at constant time rate.

$$\frac{dy}{dt} = ay - by^2 - h.$$

解: ①. Crit. pts:




h_2. no crit. pts.
y
a/b
h_1
t
y ↓
Max rate:
y
h_m.
t



$C_{in}(t)$

$\chi(t)$

$\chi = x(t)$

9/e

$$\frac{dx}{dt} = r \cdot C - r \cdot C_{out}$$

$$= r \cdot b - r \cdot \frac{x}{V}$$

$$C(t) = b\left(1 - e^{-b^{-4}t}\right)$$

$$5 = b\left(1 - be^{-\frac{t}{t_{min}}}\right)$$

$$\frac{1}{2} \quad be\, e^{-\frac{t}{t_{min}}} = \frac{1}{2}$$

$$\rightarrow \frac{t}{100} = \ln \frac{1}{2}$$

$I = 0.05 \Rightarrow 5\%$ / year.

q.

a: "income of fund" $\dfrac{dy}{dt} = Iy - q$.

b:

$$\frac{dy}{Iy - q} = dt.$$

$$\frac{1}{I} \cdot \ln(Iy - q) = t + c$$

$$Iy - q = e^{It} \cdot e^{c} \Rightarrow Iy = \frac{q}{I} + Ce^{It}$$

$|Z$ trust $= \dot{y} = 0. \Rightarrow Iy = Q. \Rightarrow y = \frac{a}{I}.$

$\Rightarrow y = \$24000$

$$y = \frac{\frac{500000}{1mm}}{0.95}$$

$$\left(\frac{dy}{dt}\right) =$$

$$y(T) = 0$$

$T = 20$ in

$$x(T) = \frac{a}{I} + Ce^{IT}.$$

$$\Rightarrow L = -\left(\frac{a}{I}\right) e^{-IT}$$

a). $St \in$ half-life t_s.

Mi $\in$ Endium. $t_s \pm t_m$.

$t_m \mid \mu$

$\in n$

$St \rightarrow x(t)$ $\quad x(0) = 1$

$Mi \rightarrow y(t)$ $\quad y(0) = 0$

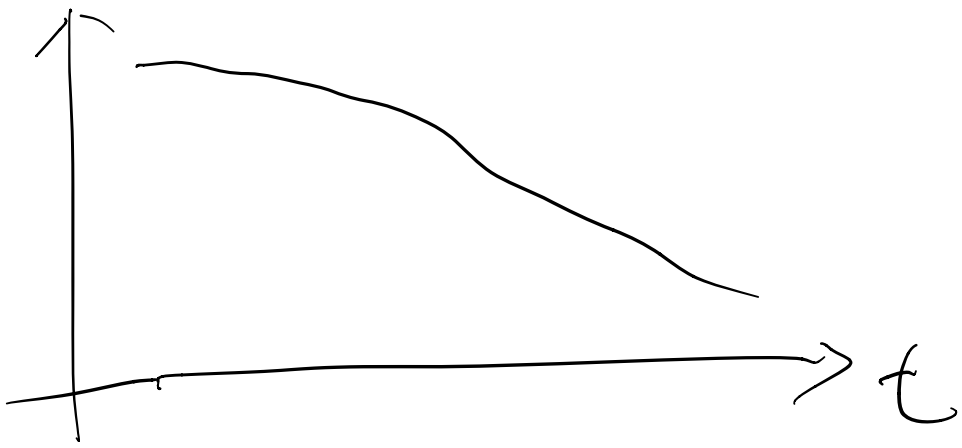
$\in n \rightarrow z(t)$ $\quad z(0) = 0$

$$\frac{dx}{dt} = \frac{x}{2}$$

$x(t) + s(t_s) = \frac{x(t)}{2}$

as $t \rightarrow \infty$.

$x = \left(\frac{x}{2}\right)^{\infty} t.$



$$x + y + z = n.$$

$$-\frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = 0$$

$$\begin{cases} \dfrac{dx}{dt} = -\sigma x \\ \dfrac{dy}{dt} = \dfrac{\sigma x}{2} - \mu y \end{cases}$$

$$\frac{dv}{dt} = \mu g + \frac{\sigma x}{2}$$

$$\frac{dx}{x} = -\sigma dt$$

$$\ln x = -\sigma t \Rightarrow x = e^{-\sigma t}$$

$$\sigma = \frac{\ln 2}{t_s}$$

$$\frac{x(0)}{2} = x(0) \cdot e^{-\sigma t_s} \Rightarrow \sigma = \ln 2 / t_s$$

$$\mu = \ln 2 / t_m$$

$$x = e^{-\ln 2 / t_s \cdot t}$$

$x = e^{-\sigma t}$.

$$\frac{dy}{dt} + \mu y = \frac{\sigma x}{2} = \frac{\sigma}{2} \cdot e^{-\sigma t}$$

$\mu = \sqrt{\frac{\ln 2}{t_m}}$

$$e^{\int \mu dt} = e^{\mu t}$$

$$(e^{\mu t} y)' = \frac{\sigma}{2} \cdot e^{-\sigma t} \cdot e^{\mu t}$$

$$= \frac{\sigma}{2} e^{t \cdot (\mu - \sigma)}$$

$$e^{\mu t} y = \frac{\sigma}{2(\mu - \sigma)} \cdot e^{t(\mu - \sigma)}$$

$$y = \frac{\sigma}{2\mu - \sigma} e^{-\sigma t} + C e^{-\mu t}$$

$z = 1 - x - y =$ ——— .

$\frac{dy}{dt} = 0 \Rightarrow$.

$x(t) = e^t$.

$t \cdot \frac{dx}{dt} + 2x = q(t)$

$\frac{dx}{dt} + \frac{2}{t} x = \frac{q(t)}{t}$

$e^{\int \frac{2}{t} dt} = e^{2 \cdot \ln t} = t e^2$

$(te^2 \cdot x)' = te^2 \cdot \frac{q(t)}{t}$

$\int e^2 q(t) \, dt$

$te^2 \cdot x(t) = \frac{1}{2} e^2 \int q(t) \, dt + C$

$x(t) = \frac{1}{2t} \int q(t) \, dt = e^t$

$\int q(t) \, dt = 2te^t$

$$q(t) = 2$$

$$\frac{1}{2t}\int q(t)\,dt + \frac{c_1}{2t} = e^t$$

$$\int q(t)\,dt = 2te^t + c_1$$

$$q(t) = \frac{te^t}{2te^t} + c_3$$