# PLANAR BIAXIAL PILOT TESTS III

# MSCM V1 Implementation in LS-DYNA

User-defined recruitment stretch bounds

**Key Features:**

- Double integration over:
  - Fiber angles $\theta \in [-90, 90]$
  - Recruitment stretch $\lambda_s \in [\lambda_{lb}, \lambda_{ub}]$
- Gaussian distributions for:
  - Angular dispersion $\Gamma_n(\theta; \sigma_n)$
  - Recruitment $\Gamma_s(\lambda_s; \mu_s, \sigma_s)$
- Neo-Hookean matrix
- Tension-only fibers

**Material Parameters:**

- $\bar{c}_m = \mu_m/2$ matrix modulus (kPa)
- $\bar{\gamma}_f$ = fiber modulus (kPa)
- $\sigma_\theta$ = angular dispersion
- $\mu_s$ = mean recruitment stretch
- $\sigma_s$ = recruitment standard deviation
- $\lambda_{lb}, \lambda_{ub}$ = bounds
- $K = d_{\text{pen}}/2$ bulk modulus (kPa)

# Computational Implementation

2nd Piola-Kirchhoff Stress Computation

$$\mathbf{S}(\mathbf{C}) = \bar{\gamma}_f \int_{-\pi/2}^{\pi/2} \int_{\lambda_{lb}}^{\lambda_f} \frac{\Gamma_n(\theta)}{\lambda_f} \cdot \Gamma_s(\lambda_s) \cdot \frac{P_f}{\lambda_s} \, d\lambda_s \, (\mathbf{n} \otimes \mathbf{n}) \, d\theta + \bar{c}_m(\mathbf{I} - C(3,3)\mathbf{C}^{-1}) + K \ln(J)\mathbf{C}^{-1} \tag{2}$$

**Key Implementation Details:**

- Tension-only: $\lambda_f > 1.0$
- Recruitment: $\lambda_f > \lambda_{lb}$
- Gauss quadrature over fiber orientation $(\theta)$: 21-po
- Gauss quadrature over slack distribution $(\lambda_s)$: 5-p
- History variables:
  - F tensor
  - Stress components (fiber and matrix)
  - Total Fiber Recruitment in percentile scale

# Additional Formulation and Implementation Details

## Test Configuration:

- Biaxial tension test
- $3.5 \times 3.5$ mm specimen
- 4 suture points per side
- Material: Bovine pericardium
- Parameters from Fan & Sacks (2014)

## Material Parameters Used:

| | |
|---|---|
| $\bar{c}_m =$ | 50 kPa |
| $d_{\text{pen}} =$ | 50000 kPa |
| $\bar{\gamma}_f =$ | 22966.7 kPa |
| $\sigma_\theta =$ | $10^\circ$, $33^\circ$ w.r.t the $X_1$ direction |
| $\mu_s =$ | 1.15 |
| $\lambda_{lb} =$ | 1.01 |
| $\lambda_{ub} =$ | 1.24 |

## Expected Behavior:

- **Pre-recruitment** $(\lambda < 1.01)$:
  - Isotropic response
  - $S_{11}/S_{22}$ 1
- **Active recruitment** $(1.01 < \lambda < 1.15)$:
  - Increasing anisotropy
  - $S_{11}/S_{22}$ ratio 3-4
- **Full recruitment** $(\lambda > 1.15)$:

## Performance:

- speedup with Intel MKL
- Stable explicit integration

# Fiber Orientation Distribution Function (ODF): Simplified

Standard Gaussian Distribution

$$\Gamma(\theta) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left(-\frac{\theta^2}{2\sigma_n^2}\right) \tag{3}$$

Normalization for Physical Domain $[-\pi/2, +\pi/2]$

1. Compute normalization factor:

$$z_{\text{norm}} = \text{erf}\left(\frac{\pi}{2\sqrt{2}\sigma_n}\right) - \text{erf}\left(-\frac{\pi}{2\sqrt{2}\sigma_n}\right) \tag{4}$$

2. Apply normalization:

$$\Gamma_n = \frac{\Gamma}{z_{\text{norm}}} \tag{5}$$

# ODF Formulation

Integrated Normalization

$$\Gamma_n(\theta) = \frac{\exp\left(-\frac{\theta^2}{2\sigma_n^2}\right)}{\text{erf}\left(\frac{\pi}{2\sqrt{2}\sigma_n}\right) \cdot \sqrt{2\pi}\sigma_n} \tag{6}$$

- Uses symmetry property: $\text{erf}(x) - \text{erf}(-x) = 2\text{erf}(x)$
- Single-step normalization

Optional Mixture Model

For partially aligned fibers:

$$\Gamma(\theta) = d \cdot \Gamma_n(\theta) + (1-d) \cdot \frac{1}{\pi} \tag{7}$$

where:

- $d = 1$: Purely Gaussian (aligned fibers), $d = 0$: Uniform distribution (random fibers), $0 < d < 1$: Partial alignment

# Post-Processing: Fiber Reorientation

Reference Configuration

For each fiber direction $\theta \in [-\pi/2, \pi/2]$:

- Reference unit vector: $\mathbf{n}_{\text{ref}} = (\cos\theta, \sin\theta)^T$
- Discretization: $\theta = -\frac{\pi}{2} + \pi\frac{i-0.5}{n_\theta}$ for $i = 1, ..., n_\theta$

Deformed Configuration

Apply deformation gradient $\mathbf{F}$:

$$\mathbf{n}_{\text{def}} = \mathbf{F} \cdot \mathbf{n}_{\text{ref}} \tag{8}$$

Current fiber angle:

$$\beta = \arctan\left(\frac{n_{\text{def},2}}{n_{\text{def},1}}\right) \tag{9}$$

Output

Store $\beta$ (in degrees) in history variables

# Post-Processing: ODF Transformation

**Evolution of Fiber Distribution During Deformation:**

Reference ODF (Fan & Sacks Normalization)

$$\Gamma_0(\theta) = \frac{\exp\left(-\frac{\theta^2}{2\sigma_n^2}\right)}{\text{erf}\left(\frac{\pi}{2\sqrt{2}\sigma_n}\right) \cdot \sqrt{2\pi}\sigma_n} \tag{10}$$

Transformed ODF in Deformed Configuration

Using kinematic constraint:

$$\Gamma_t(\beta) = \Gamma_0(\theta) \cdot \frac{\sqrt{\mathbf{n}^T\mathbf{C}\mathbf{n}}}{J_{2D}} \tag{11}$$

where:

- $J_{2D} = \det(\mathbf{F})$: 2D Jacobian
- $\mathbf{n}^T\mathbf{C}\mathbf{n}$ : Fiber stretch squared $(\lambda^2)$

# Post-Processing: Fiber Ensemble Fiber Recruitment (FEFR)

## Recruitment Distribution

Gaussian distribution for recruitment stretch $\lambda_s$:

$$\Gamma_s(\lambda_s) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(\lambda_s - \mu_s)^2}{2\sigma_s^2}\right) \tag{12}$$

Normalized over recruitment range $[\lambda_{lb}, \lambda_{ub}]$

## FEFR Calculation:

For fiber stretch $\lambda_f > \lambda_{lb}$:

$$\text{FEFR} = \int_{\lambda_{lb}}^{\min(\lambda_f, \lambda_{ub})} \Gamma_s(\lambda_s) d\lambda_s \tag{13}$$

- Computed using 5-point Gauss quadrature
- FEFR $= 0$ if $\lambda_f \leq \lambda_{lb}$ (fiber not recruited)
- FEFR $= 1$ if $\lambda_f \geq \lambda_{ub}$ (fully recruited)

# Recruitment Distribution: UMAT Implementation

Gaussian Distribution Formulation

$$\Gamma_s(\lambda_s) = \frac{1}{\sigma_s\sqrt{2\pi}} \exp\left(-\frac{(\lambda_s - \mu_s)^2}{2\sigma_s^2}\right) \tag{14}$$

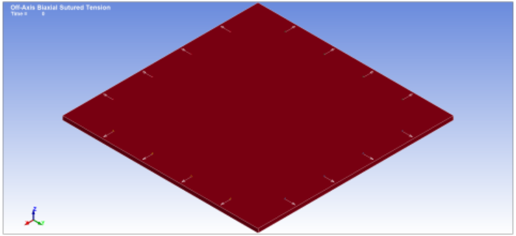
Normalized over $[\lambda_{lb}, \lambda_{ub}]$

**Key Features:**

- Stretch-based: $\lambda_s$ is recruitment stretch
- Parameters:
  - $\mu_s$: Mean recruitment stretch
  - $\sigma_s$: Standard deviation (spread)
  - $[\lambda_{lb}, \lambda_{ub}]$: Recruitment bounds

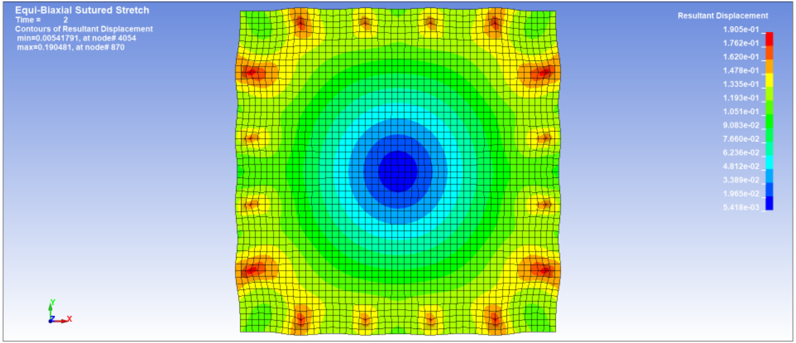
# Geometry and Boundary Conditions

- Specimen: Four pins on the each side of the specimen representing sutured points; load applied along the global $X_1$ and $X_2$.
- Dimensions are $3.5 \times 3.5 \times 0.070$ mm.
- Stretch applied through the sutured points equi-biaxlially up to 0.165 unit (velocity-based stretch for better stability).



# Displacement Sum Contour

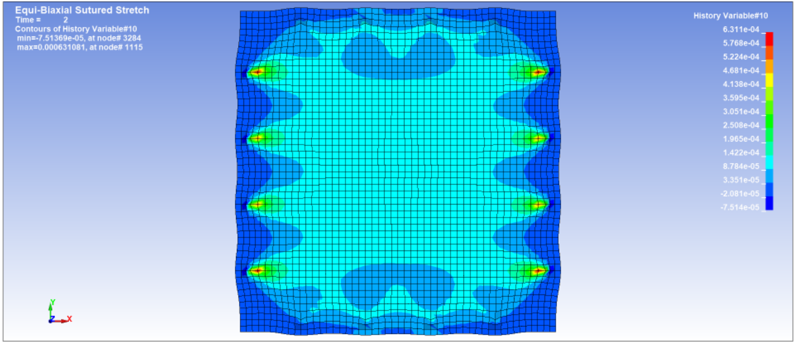
$\sigma_\theta = 33^\circ$




Displacement sum that evolves over simulation from isotropic to anisotropic pattern with activating / recruiting fibers

# $S_{11}$ Contour Along $X_1$ Direction

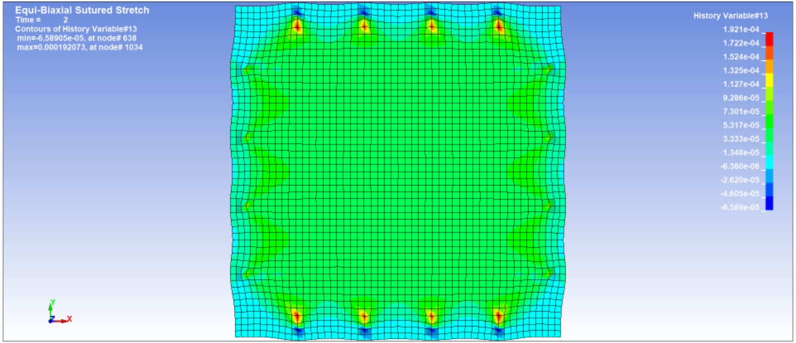
$\sigma_\theta = 33^\circ$




Total stress aligned the preferred fiber direction.

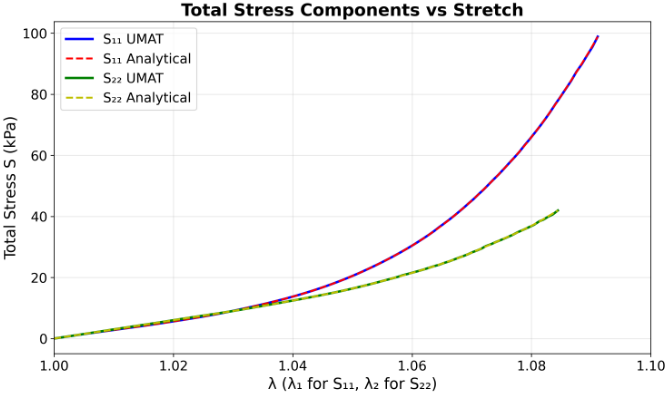
# $S_{22}$ Contour Along $X_2$ Direction

$\sigma_\theta = 33^\circ$




Total stress crossing the preferred fiber direction.

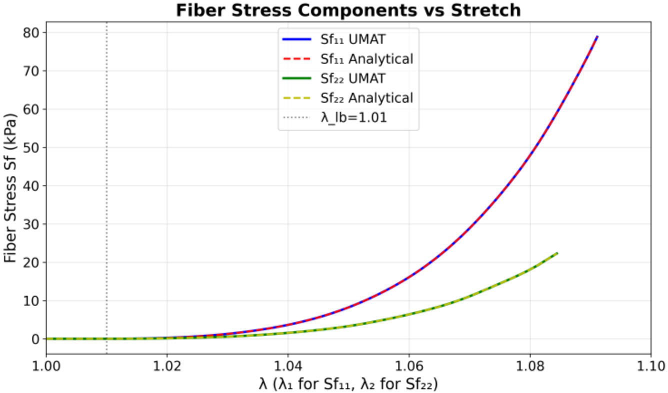
# Total Stress Components vs Stretch




Total 2nd PK stresses $S_{11}$ and $S_{22}$ showing excellent agreement between UMAT and analytical solutions

- Anisotropic response: $S_{11} > S_{22}$ due to fiber orientation

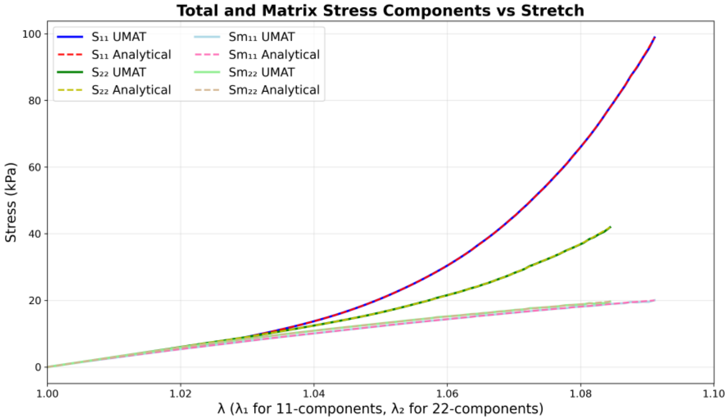
# Fiber Stress Components vs Stretch




Fiber stresses $Sf_{11}$ and $Sf_{22}$ with recruitment threshold at $\lambda_{lb}$

- Exponential stiffening after fiber recruitment ($\lambda > 1.01$)
- Direction-dependent fiber contribution: $Sf_{11} \approx 4 \times Sf_{22}$

# Stress Decomposition: Total and Matrix Components

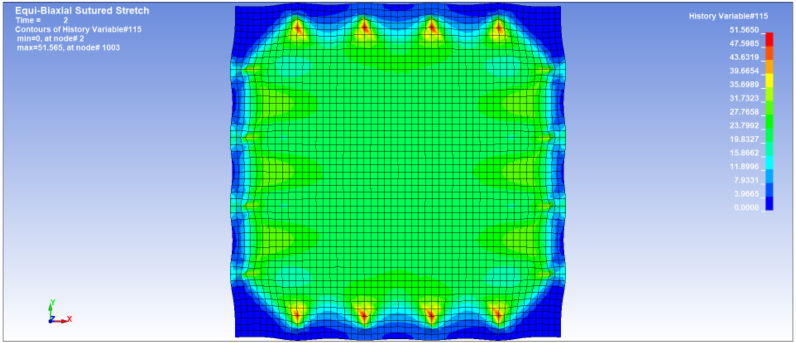



Comparison of total stresses ($S_{11}$, $S_{22}$) and matrix contributions ($Sm_{11}$, $Sm_{22}$)

- Matrix contribution: Nearly linear Neo-Hookean response
- Fiber dominance: At $\lambda = 1.09$, fibers contribute $\sim$75% of total stress

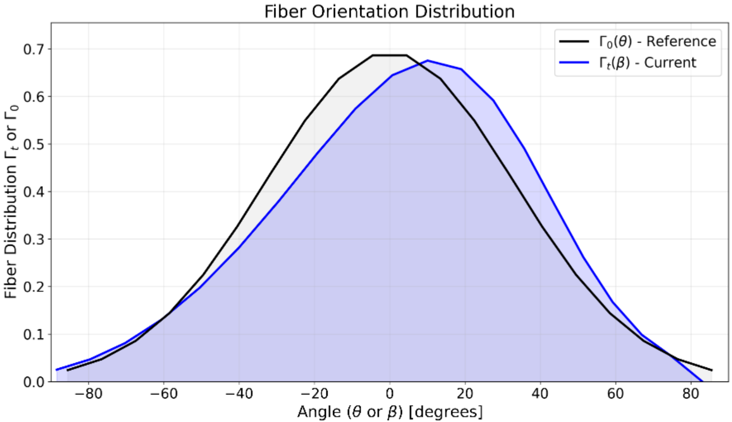
# Total Fiber Recruitment (TFR) Distribution in Percentile Scale

$\sigma_\theta = 33^\circ$




Total fiber recruitment under off-axis biaxial sutured tension, showing stress-induced recruitment at suture boundaries (max: 51.565%).

# Deformed Fiber Orientation Distribution




Evolution of fiber distribution from reference $\Gamma_0(\theta)$ to deformed $\Gamma_t(\beta)$ configuration. The overlapping curves indicate a slight fiber rotation under equi-biaxial loading for element 1063 located on top-left corner region (away from center).