Analytic of VPT in iron-based superconductor

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From the analysis， we want to understand change the sign of the A_z will cause the topological phase transition from two to one.For simpilify we choose vortex along the z direction.

In the basics $|p_z,\uparrow\rangle, |p_z,\downarrow\rangle, |d_{xz+iyz},\downarrow\rangle, |d_{xz-iyz},\uparrow\rangle, |d_{xz+iyz},\uparrow\rangle, |d_{xz-iyz,\downarrow}\rangle$， we consider the$k_z = 0$

$$H = \begin{pmatrix} M(k) & 0 & 0 & -iAk_- & -iAk_+ & 0 \\ 0 & M(k) & -iAk_+ & 0 & 0 & -iAk_- \\ 0 & iAk_- & -M(k) & 0 & 0 & 0 \\ iAk_+ & 0 & 0 & -M(k) & 0 & 0 \\ iAk_- & 0 & 0 & 0 & -M(k)+\delta & 0 \\ 0 & iAk_+ & 0 & 0 & 0 & -M(k)+\delta \end{pmatrix} \tag{1}$$

$M(k) = M_0 + M_1(k_x^2 + k_y^2), k_+ = k_x + ik_y, k_- = k_x - ik_y$ we can do a transformation to diagonalization the Hamiliton

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Finally we can get the block from is

$$H_k = \begin{pmatrix} M(k) & -iAk_- & -iAk_+ & & & \\ iAk_+ & -M(k) & 0 & & & \\ iAk_- & 0 & -M(k)+\delta & & & \\ & & & M(k) & -iAk_+ & -iAk_- \\ & & & iAk_- & -M(k) & 0 \\ & & & iAk_+ & 0 & -M(k) \end{pmatrix} \tag{3}$$

we can get the electron in the basics $\{|p_z,\uparrow\rangle_e, |d_{xz-iyz},\uparrow\rangle_e, |d_{xz+iyz},\uparrow\rangle_e\}$,the Hamiliton is

$$H_{e\uparrow}(k)=\begin{pmatrix} M(k) & -iAk_- & -iAk_+ \\ iAk_+ & -M(k) & 0 \\ iAk_- & 0 & -M(k) \end{pmatrix} \tag{4}$$

In the $\{|p_z,\downarrow\rangle_e, |d_{xz+iyz},\downarrow\rangle_e, |d_{xz-iyz},\downarrow\rangle_e\}$ the Hamiliton is

$$H_{e\downarrow}(k)=\begin{pmatrix} M(k) & -iAk_+ & -iAk_- \\ iAk_- & -M(k) & 0 \\ iAk_+ & 0 & -M(k) \end{pmatrix} \tag{5}$$

The we can do the particle-hole transformation to get the Hamiliton in the hole space $\{|p_z,\downarrow\rangle_h, |d_{xz+iyz},\downarrow\rangle_h, |d_{xz-i}$

$$H_{h\downarrow}(k)=-H^*_{e\downarrow}(-k)=-\begin{pmatrix} M(k) & -iAk_+ & -iAk_- \\ iAk_- & -M(k) & 0 \\ iAk_+ & 0 & -M(k) \end{pmatrix}=-H_{e\uparrow}(k) \tag{6}$$

Then we can solve the eigenvalue and eigenwave function for $H_{e\uparrow}(k)$,for simpilify,we can write it in the polar coordinate,we can get the $H_{e\uparrow}(k)$ is

$$H_{e\uparrow}(k)=\begin{pmatrix} M(k) & -iAke^{-i\theta} & -iAke^{i\theta} \\ iAke^{i\theta} & -M(k) & 0 \\ iAke^{-i\theta} & 0 & -M(k) \end{pmatrix} \tag{7}$$

we can get the three eigenvalue

$$E_{\pm}=\pm\sqrt{M(k)^2+2A^2k^2} \qquad E_0=-M(k) \tag{8}$$

We can see that the eigenvalue is only relate to the length of k,independent of theta,so we can definit a angular momentum

$$J_z=-i\partial_\theta+J_{basis}=-i\partial_\theta+\begin{pmatrix} \frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & \frac{3}{2} \end{pmatrix} \tag{9}$$

For the orbit angular momentum,we can write it in p appearance

$$L_z=(\vec{r}\times\vec{p})_z=xp_y-yp_x \tag{10}$$

$$=ip_y\partial_{p_x}-ip_x\partial_{p_y} \tag{11}$$

In the coordinate of the p

$$\partial_{p_x} = cos(\theta)\partial_p - \frac{1}{p}sin(\theta)\partial_\theta \tag{12}$$

$$\partial_{p_y} = sin(\theta)\partial_p + \frac{1}{p}cos(\theta)\partial_\theta \tag{13}$$

Then we can get

$$L_z = ip_y\partial_{p_x} - ip_x\partial_{p_y} \tag{14}$$

$$= ipsin(\theta)[cos(\theta)\partial_p - \frac{1}{p}sin(\theta)\partial_\theta] - ipcos(\theta)[sin(\theta)\partial_p + \frac{1}{p}cos(\theta)\partial_\theta] \tag{15}$$

$$= -i\partial_\theta \tag{16}$$

Due to $[H_{e\uparrow}(k), J_z] = 0$,we can get the common eigen states of the $H_{e\uparrow}(k)$ and J_z

$$\psi(k,\theta)^j = \begin{pmatrix} e^{i(j-\frac{1}{2})\theta}u_1(k) \\ e^{i(j+\frac{1}{2})\theta}u_2(k) \\ e^{i(j-\frac{3}{1})\theta}u_3(k) \end{pmatrix} \tag{17}$$

Then we can definite a unitary transformation

$$U = \begin{pmatrix} e^{i(j-\frac{1}{2})\theta} & & \\ & e^{i(j+\frac{1}{2})\theta} & \\ & & e^{i(j-\frac{3}{2})\theta} \end{pmatrix} \tag{18}$$

The we can get

$$U^\dagger H_{e\uparrow}(k)U = \begin{pmatrix} M(k) & -iAk & -iAk \\ iAk & -M(k) & 0 \\ iAk & 0 & -M(k) \end{pmatrix} \tag{19}$$

we can solve the eigen states

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \qquad |\psi_\pm\rangle = \frac{1}{a_\pm}\begin{pmatrix} -i(M(k)\pm E) \\ Ak \\ Ak \end{pmatrix} \tag{20}$$

The$E = \sqrt{M(k)^2 + 2A^2k^2}$, $a_\pm$is normalization constant，then we can get the eigen states of angular momentum j is

$$|\psi_0^j\rangle_e = \frac{1}{2\sqrt{\pi}}\begin{pmatrix} 0 \\ -e^{i(j+\frac{1}{2})\theta} \\ e^{i(j-\frac{3}{2}\theta)} \end{pmatrix} \qquad |\psi_\pm^j\rangle_e = \frac{1}{a_\pm\sqrt{2\pi}}\begin{pmatrix} -i(M(k)\pm E)e^{i(j-\frac{1}{2})\theta} \\ Ake^{i(j+\frac{1}{2})\theta} \\ Ake^{i(j-\frac{3}{2})\theta} \end{pmatrix} \tag{21}$$

For the hole part，because$H_{h\downarrow}(k) = -H_{e\uparrow}(k)$,use the same way to calculate，we can find it's eigen value is negative to the electronic part, it's eigen states will correspondenced to electronic part.

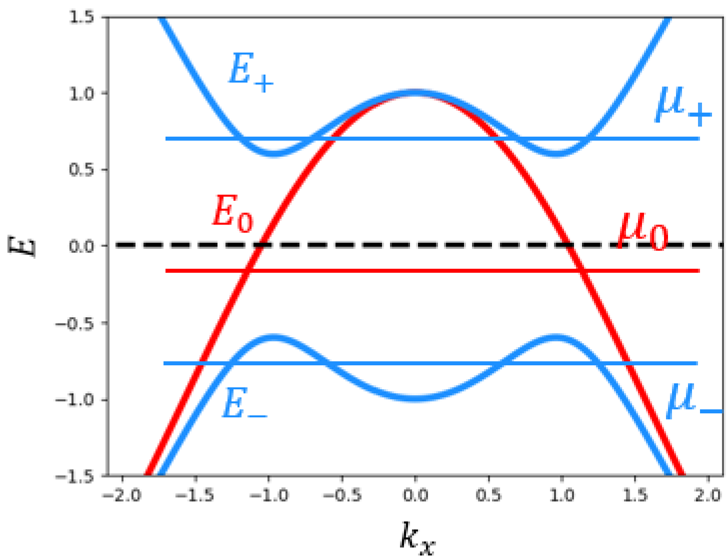
$$|\psi_0^j\rangle_h = \frac{1}{2\sqrt{\pi}}\begin{pmatrix} 0 \\ -e^{i(j+\frac{1}{2})\theta} \\ e^{i(j-\frac{3}{2}\theta)} \end{pmatrix} \qquad |\psi_\pm^j\rangle_h = \frac{1}{a_\pm\sqrt{2\pi}}\begin{pmatrix} -i(M(k)\pm E)e^{i(j-\frac{1}{2})\theta} \\ Ake^{i(j+\frac{1}{2})\theta} \\ Ake^{i(j-\frac{3}{2})\theta} \end{pmatrix} \tag{22}$$

For simpilify,we can choose$j = \frac{1}{2}$，we can get

$$|\psi_0\rangle_e = \frac{1}{2\sqrt{\pi}}\begin{pmatrix} 0 \\ -e^{i\theta} \\ e^{-i\theta} \end{pmatrix} \qquad |\psi_\pm\rangle_e = \frac{1}{a_\pm\sqrt{2\pi}}\begin{pmatrix} -i(M(k)\pm E) \\ Ake^{i\theta} \\ Ake^{-i\theta} \end{pmatrix} \tag{23}$$

$$|\psi_0\rangle_h = \frac{1}{2\sqrt{\pi}}\begin{pmatrix} 0 \\ -e^{i\theta} \\ e^{-i\theta)} \end{pmatrix} \qquad |\psi_\pm\rangle_h = \frac{1}{a_\pm\sqrt{2\pi}}\begin{pmatrix} -i(M(k)\pm E) \\ Ake^{i\theta} \\ Ake^{-i\theta} \end{pmatrix} \tag{24}$$

We can choose the parameter $M_0 = -1, M_1 = 1, A = 0.5$,we can plot the band structure along the k_x direction(because we consider the system has circle rotation symmetry,so along the any direction of the $k_x - k_y$plane,we will get the same band structure.) When the Fermi surface near the μ_0,Only one



band near Fermi surace, so we can project H_{BdG} to this band,it means that we can write H_{BdG} on the $\{|\psi_0\rangle_e, |\psi_0\rangle_h\}$ basics.

$$H_{BdG}^{00} = \begin{pmatrix} E-\mu & i\Delta_e e^{-i\theta}(\partial_k - \frac{i\partial_\theta + A_\theta^{00}}{k}) \\ i\Delta_e e^{i\theta}(\partial_k + \frac{i\partial_\theta + A_\theta^{00}}{k}) & \mu - E \end{pmatrix} \tag{25}$$

The vortex will not break the rotation symmetry on the $x - y$ plane,so we can define the angular momentum

$$J_z = L_z + J_{basis} + J_{vortex} = -i\partial_\theta + \frac{1}{2} + \frac{1}{2}\tau_z \tag{26}$$

The eigne wave function is

$$\psi(k,\theta) = \frac{1}{\sqrt{k}} \begin{pmatrix} e^{i(j-1)\theta} u_1(k) \\ -ie^{ij\theta} u_2(k) \end{pmatrix} \tag{27}$$

so we can do the unitary transformation

$$U = \begin{pmatrix} e^{i(j-1)\theta} & \\ & -ie^{ij\theta} \end{pmatrix} \tag{28}$$

Then we can get

$$(H_{BdG}^{00})^j = U^\dagger H_{BdG}^{00} U = \begin{pmatrix} E-\mu & \Delta_e(\frac{j-\frac{1}{2}-A_\theta^{00}}{k} + \partial_k) \\ \Delta_e(\frac{j-\frac{1}{2}-A_\theta^{00}}{k} - \partial_k) & \mu - E \end{pmatrix} \tag{29}$$

$$= (E-\mu)\sigma_z + i\Delta_e\partial_k + \frac{\Delta_e}{k}(j - \frac{1}{2} - A_\theta^{00}) \tag{30}$$

The resulting Hamiltonian is equivalent to the Jackiw-Rebbi model,so we can get

$$E_j = \frac{\Delta_e}{k}(j - \frac{1}{2} - A_\theta^{00}) \tag{31}$$

we can calculate $A_\theta^{00} = i_e \langle\psi_0|\, \partial_\theta \,|\psi_0\rangle_h = -1$,so E_j is impossible to be zero,so we can conclude that it's will not have vortex transition point near the μ_0

When the Fermi surface near the μ_-,there will have two band E_0, E_- near the Fermi surface ,we can project H_{BdG} to this two band. For simipilify calculte,we can do a transformation from $(k,\theta)to(E,\theta)$,Then we can write H_{BdG} in the basics $\{|\psi_0(E,\theta)\rangle_e, |\psi_-(E,\theta)\rangle_e, |\psi_0(E,\theta)\rangle_h, |\psi_-\rangle(E,\theta)_h\}$

$$\begin{pmatrix} E-\mu & & \Delta_{11} & \Delta_{12} \\ & E-\mu & \Delta_{21} & \Delta_{22} \\ \Delta_{11}^\dagger & \Delta_{12}^\dagger & \mu - E & \\ \Delta_{21}^\dagger & \Delta_{22}^\dagger & & \mu - E \end{pmatrix} \tag{32}$$

The elements is

$$\Delta_{11} = i\Delta_e e^{-i\theta}(\frac{dE}{dk_-}\partial_E - \frac{i\partial_\theta + A_\theta^{00}}{k_-(E)}) \tag{33}$$

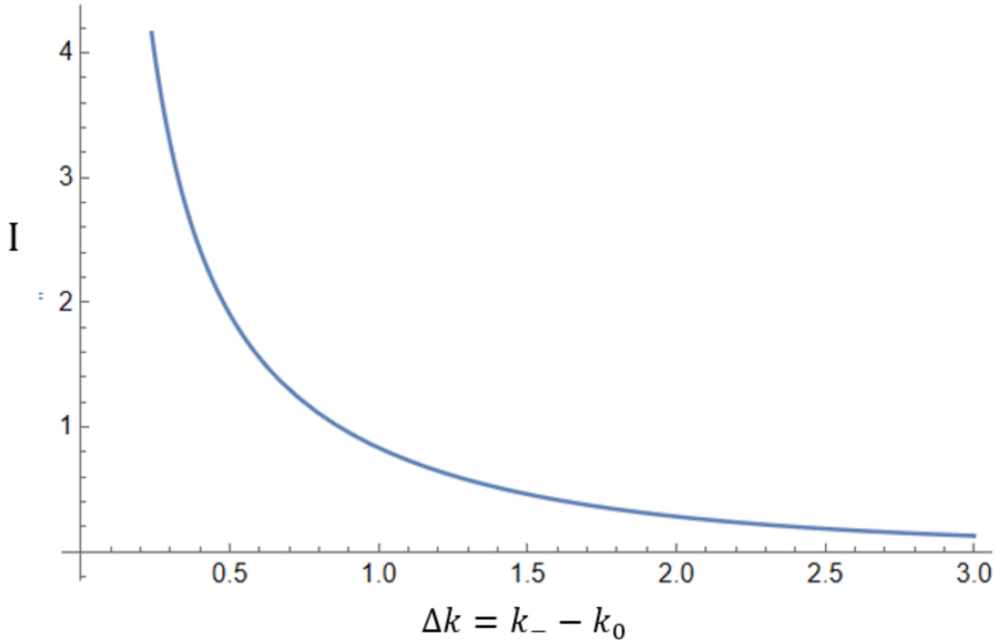
$$= i\Delta_e e^{-i\theta}(\hbar v_{k_-}\partial_E - \frac{i\partial_\theta + A_\theta^{00}}{k_-(E)}) \tag{34}$$

$$\Delta_{12} = {}_e\langle\psi_0(k_0,\theta)|\,\Delta_0 tanh(\frac{r}{\xi})e^{-i\theta}\,|\psi_-(k_-,\theta)\rangle_h \tag{35}$$

$$= {}_e\langle\psi_0(k_0,\theta)|\,\Delta_e r e^{-i\theta}\frac{tanh(\frac{r}{\xi})}{\frac{r}{\xi}}\,|\psi_-(k_-,\theta)\rangle_h \tag{36}$$

$$= -\frac{\Delta_e}{k_-}A^{12}_\theta\,{}_e\langle\psi_0(k_0,\theta)|\,\frac{tanh(\frac{r}{\xi})}{\frac{r}{\xi}}\,|\psi_-(k_-,\theta)\rangle_h \tag{37}$$

The right integral can be calculate by numeral calculations,we can get



The horizontal axis representation the difference of the k_0 and k_-,we can see that the more difference,the little integral.So we can only consider the interaction of the nearst k.For simpilify we can treat this integral as a correction on k_-,we can change k_- to $k_m = \frac{k_0+k_-}{2}$,Then we can get

$$\Delta_{12} = -\frac{\Delta_e}{k_m}A^{12}_\theta \tag{38}$$

At the same time

$$\Delta_{22} = i\Delta_e e^{-i\theta}(\hbar v_{k_-}\partial_E - iA^{22}_E - \frac{i\partial_{\theta+A^{22}_\theta}}{k}) \tag{39}$$

we can do the gauge transfomation $|\psi_-(E,\theta)\rangle \rightarrow e^{i\int^E A^{22}_{E'}}dE'$ to eliminate A^{22}_E,The we can get the form of the project H_{BdG},For easy to calculate,we can write the H_{BdG} in the basics $\{|\psi_0(E,\theta)\rangle_e, |\psi_0(E,\theta)\rangle_h, |\psi_-(E,\theta)\rangle_e,$

$$\begin{pmatrix} E-\mu & i\Delta_e e^{-i\theta}(\hbar v_{k_-}\partial_E - \frac{i\partial_\theta + A_\theta^{00}}{k_-(E)}) & 0 & -\frac{\Delta_e}{k_m}e^{-i\theta}A_\theta^{0-} \\ i\Delta_e e^{i\theta}(\hbar v_{k_-}\partial_E + \frac{i\partial_\theta + A_\theta^{00}}{k}) & \mu - E & -\frac{\Delta_e}{k_m}e^{i\theta}A_\theta^{-0} & 0 \\ 0 & -\frac{\Delta_e}{k_m}e^{-i\theta}A_\theta^{0-} & E-\mu & i\Delta_e e^{-i\theta}(\hbar v_{k_-}\partial_E - \frac{i\partial_\theta + A_\theta^{--}}{k_-(E)}) \\ -\frac{\Delta_e}{k_m}e^{i\theta}A_\theta^{-0} & 0 & i\Delta_e e^{i\theta}(\hbar v_{k_-}\partial_E + \frac{i\partial_\theta + A_\theta^{--}}{k}) & E-\mu \end{pmatrix} \tag{40}$$

The H_{BdG} has the rotation symmetry,so we can definite the angular momentum

$$J_z = -i\partial_\theta + \frac{1}{2} + \frac{1}{2}\tau_z \tag{41}$$

The egie wave function is

$$\psi(E,\theta) = \frac{1}{\sqrt{k_-(E)}}\begin{pmatrix} e^{i(j-1)\theta}u_1(k) \\ -ie^{ij\theta}u_2(k) \\ e^{i(j-1)\theta}u_3(k) \\ -ie^{ij\theta}u_4(k) \end{pmatrix} \tag{42}$$

So we can do a unitary transformation

$$U = \frac{1}{\sqrt{k_-(E)}}\begin{pmatrix} e^{i(j-1)\theta} & & & \\ & -ie^{ij\theta} & & \\ & & e^{i(j-1)\theta} & \\ & & & -ie^{ij\theta} \end{pmatrix} \tag{43}$$

Then we can get

$$\begin{pmatrix} E-\mu & \Delta_e(\hbar v_{k_-}\partial_E + \frac{j-\frac{1}{2}-A_\theta^{00}}{k}) & 0 & -\Delta_e\frac{A_\theta^{0-}}{k} \\ -\Delta_e(\frac{j-\frac{1}{2}-A_\theta^{00}}{k} - \hbar v_{k_{\_\partial_E}}) & \mu-E & -\Delta_e\frac{A_\theta^{0-}}{k} & 0 \\ 0 & -\Delta_e\frac{A_\theta^{-0}}{k} & E-\mu & \Delta_e(\hbar v_{k_-}\partial_E + \frac{j-\frac{1}{2}-A_\theta^{--}}{k}) \\ -\Delta_e\frac{A_\theta^{-0}}{k} & 0 & \Delta_e(\frac{j-\frac{1}{2}-A_\theta^{--}}{k} - \hbar v_{k_-}\partial_E) & \mu-E \end{pmatrix} \tag{44}$$

$$= (E-\mu)s_0\tau_z + i\Delta_e\hbar v_{k_-}\partial_E s_0\tau_y + \frac{\Delta_e}{k_-(E)}(j-\frac{1}{2})s_0\tau_x - \begin{pmatrix} 0 & \frac{\Delta_e}{k_-(E)}A_\theta^{00} & 0 & \frac{\Delta_e}{k_m}A_\theta^{0-} \\ \frac{\Delta_e}{k_-(E)}A_\theta^{00} & 0 & \frac{\Delta_e}{k_m}A_\theta^{0-} & 0 \\ 0 & \frac{\Delta_e}{k_m}A_\theta^{-0} & 0 & \frac{\Delta_e}{k_-(E)}A_\theta^{--} \\ \frac{\Delta_e}{k_m}A_\theta^{-0} & 0 & \frac{\Delta_e}{k_-(E)}A_\theta^{--} & 0 \end{pmatrix} \tag{45}$$

$$= (E-\mu)s_0\tau_z + i\Delta_e\hbar v_{k_-}\partial_E s_0\tau_y + \frac{\Delta_e}{k_-(E)}[(j-\frac{1}{2})s_0 - \begin{pmatrix} A_\theta^{00} & \frac{k_-(E)}{k_m}A_\theta^{0-} \\ \frac{k_-(E)}{k_m}A_\theta^{-0} & A_\theta^{--} \end{pmatrix}]\tau_x \tag{46}$$

The resulting Hamiltonian is equivalent to the $4*4$Jackiw-Rebbi model,we can solve the zero solution of the first two items

$$\psi_0^1(E)=\frac{1}{2}e^{\int^E\frac{\mu-E'}{\Delta_e\hbar v_{k\_}}dE'}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\tag{47}$$

$$\psi_0^2(E)=\frac{1}{2}e^{\int^E\frac{\mu-E'}{\Delta_e\hbar v_{k\_}}dE'}\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}\tag{48}$$

we can calculate that

$$A_\theta^{00}=0\qquad A_\theta^{--}=0\qquad A_\theta^{0-}=\frac{\sqrt{2}Ak\_(E)}{a\_}\tag{49}$$

The energy can be determined by

$$H_j=\frac{\Delta_e}{k\_(E)}(j-\frac{1}{2})s_0\tau_x-\frac{\Delta_e}{k_m}A_\theta^{0-}s_x\tau_x\tag{50}$$

Then we can get the eigen value

$$E_j=\frac{\Delta_e}{k\_(E)}(j-\frac{1}{2}\pm\frac{k\_(E)}{k_m}A_\theta^{0-})\tag{51}$$

It's eigen value is $(1,1,1,1)^T$ and $(1,1,-1,-1)^T$ corresponed to the zero solution of the Jackiw-Rebbi model.If we want to get the zero solution,we need to require $\frac{k\_(E)}{k_m}A_\theta^{0-}$ be a half integer,we can easily to analysis that $\frac{k\_(E)}{k_m}A_\theta^{0-}$ more than zero and less than 1,so the only possible is to be $\frac{1}{2}$,Then we can get the zero solution's angular must be 0 or 1,Then we can get the zero solution

$$\psi_0^{j=0}(E,\theta)=\frac{1}{2\sqrt{k\_(E)}}e^{\int^E\frac{\mu-E'}{\Delta_e\hbar v_{k\_}}dE'}\begin{pmatrix}e^{-i\theta}\\-i\\e^{-i\theta}\\-i\end{pmatrix}\tag{52}$$

$$\psi_0^{j=1}(E,\theta)=\frac{1}{2\sqrt{k\_(E)}}e^{\int^E\frac{\mu-E'}{\Delta_e\hbar v_{k\_}}dE'}\begin{pmatrix}1\\-ie^{i\theta}\\-1\\ie^{i\theta}\end{pmatrix}\tag{53}$$

At the same time,we can calculate the vortex phase transition energy,we can get

$$\frac{k\_(E)}{k_m}A_{\theta}^{0-}=\frac{k\_(E)}{\frac{k_0(E)+k\_(E)}{2}}\frac{\sqrt{2}Ak\_(E)}{a\_}=\frac{1}{2} \tag{54}$$

we can bring the parameter into equation,it's the $M0=-1,M1=1,A=0.5$,for simpilify,we can estimate $\frac{k\_(E)}{\frac{k_0(E)+k\_(E)}{2}}$ to be a constant 0.81,Finally we can get it's vortex phase transition at E=-0.79,At the same time,we can get When the Fermi surface near the μ_+,it's vortex phase transition point at E=0.66.

So from the analysis,we can get there will have four vortex phase transition point,two of them have the 0 angular momentum,other have +1 angular momentum, and the vortex phase transition energy at $E_+=0.66$,$E\_=-0.79$,there are all correspondence to numerical calculation.

```
Solve[0.81 * Sqrt[2] * 0.5 * x / Sqrt[((-1 + x^2 + Sqrt[(-1 + x^2)^2 + 0.5 * x^2])^2 + 0.5 * x^2)] == 0.5, x]
解方程          平方根

{{x → 1.09035}}

x2 = 1.090347871187568`

1.09035

-Sqrt[(-1 + x2^2)^2 + 0.5 * x2^2]
 平方根

-0.793786
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Solve[0.81 * Sqrt[2] * 0.5 * x / Sqrt[((1 - x^2 + Sqrt[(-1 + x^2)^2 + 0.5 * x^2])^2 + 0.5 * x^2)] == 0.5, x]
解方程          平方根

{{x → 0.917138}}

x1 = 0.9171384898572194`
Sqrt[(-1 + x1^2)^2 + 0.5 * x1^2]
平方根

0.917138

0.667688
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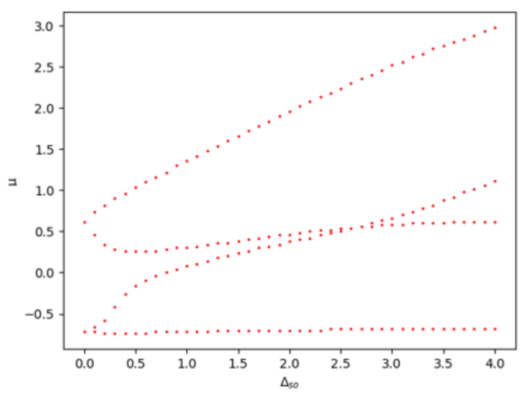


When change the sign of parameter of the k_y,the Hamiliton change to
When we change the sign of ky's parameter in Dirac semimetal,the electron Hamiliton become

$$H_{e\uparrow}(k) = \begin{pmatrix} M(k) & -iAke^{-i\theta} & -iAke^{-i\theta} \\ iAke^{i\theta} & -M(k) & 0 \\ iAke^{i\theta} & 0 & -M(k) \end{pmatrix} \tag{55}$$

We can definite the angular momentum
Now the angular momentum become

$$J_z = -i\partial_\theta + \begin{pmatrix} \frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & -\frac{1}{2} \end{pmatrix} \tag{56}$$

We can use the same way to get the eige states of the $H_{e\uparrow}(k)$
Use the same way,we can solve the eigen states of the $H_{e\uparrow}(k)$

$$|\psi_0^j\rangle_e = \frac{1}{2\sqrt{\pi}} \begin{pmatrix} 0 \\ -e^{i(j+\frac{1}{2})\theta} \\ e^{i(j+\frac{1}{2}\theta)} \end{pmatrix} \qquad |\psi_\pm^j\rangle_e = \frac{1}{a_\pm\sqrt{2\pi}} \begin{pmatrix} -i(M(k)\pm E)e^{i(j-\frac{1}{2})\theta} \\ Ake^{i(j+\frac{1}{2})\theta} \\ Ake^{i(j+\frac{3}{2})\theta} \end{pmatrix} \tag{57}$$

For the hole part

$$|\psi_0^j\rangle_h = \frac{1}{2\sqrt{\pi}} \begin{pmatrix} 0 \\ -e^{i(j+\frac{1}{2})\theta} \\ e^{i(j+\frac{1}{2}\theta)} \end{pmatrix} \qquad |\psi_\pm^j\rangle_h = \frac{1}{a_\pm\sqrt{2\pi}} \begin{pmatrix} -i(M(k)\pm E)e^{i(j-\frac{1}{2})\theta} \\ Ake^{i(j+\frac{1}{2})\theta} \\ Ake^{i(j+\frac{1}{2})\theta} \end{pmatrix} \tag{58}$$

For simpilify to calculate,we can choose $j=-\frac{1}{2}$

$$|\psi_0\rangle_e = \frac{1}{2\sqrt{\pi}} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \qquad |\psi_\pm\rangle_e = \frac{1}{a_\pm\sqrt{2\pi}} \begin{pmatrix} -i(M(k)\pm E)e^{-i\theta} \\ Ak \\ Ak \end{pmatrix} \tag{59}$$

$$|\psi_0\rangle_h = \frac{1}{2\sqrt{\pi}} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \qquad |\psi_\pm\rangle_h = \frac{1}{a_\pm\sqrt{2\pi}} \begin{pmatrix} -i(M(k)\pm E)e^{-i\theta} \\ Ak \\ Ak \end{pmatrix} \tag{60}$$

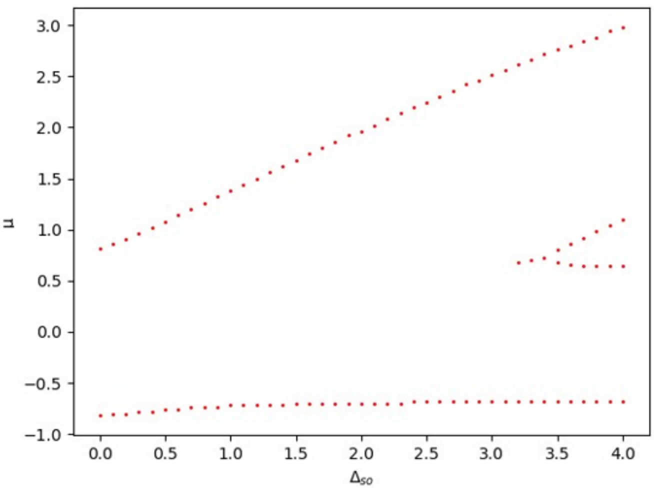
We can notice that $|\psi_0\rangle_e$ is independent of the k and θ,so the it's connection of the other band must zero,so we just to consider one band model.For example, When Fermi energy near the μ_-,the eigne energy is

$$E_j = \frac{\Delta_e}{k}(j - \frac{1}{2} - A_\theta^{--}) \tag{61}$$

If we want to get the zero solution,we need the A_θ^{--} is the half of integer,we can get

$$A_\theta^{--} = \frac{(-M(k)+E)^2}{(-M(k)+E)^2 + 2A^2k^2} \tag{62}$$

From easily analysis,if the A_θ^{--} is the half of integer,it must be $\frac{1}{2}$.we can get when $M(k) = 0$,the $A_\theta^{--} = \frac{1}{2}$.We can take the parameter into this equation,we can get the vortex phase traansition energy is ±0.707,and the angular momentum is $j = 1$,so the C_{2z} is -1.This result is corresponding the numerical result too.



When change the parameter of k_y to zero,it says that the Hamiliton change to

$$H_{e\uparrow}(k) = \begin{pmatrix} M(k) & -iAk_- & -iAk_x \\ iAk_+ & -M(k) & 0 \\ iAk_x & 0 & -M(k) \end{pmatrix} \tag{63}$$

we can solve it's eigenvalue and eigenstates

$$E_0 = -M(k) \qquad E_\pm = \pm\sqrt{M(k)^2 + 2A^2k_x^2 + A^2k_y^2} \tag{64}$$

$$\psi_0 = \frac{1}{\sqrt{3\pi}}\begin{pmatrix} 0 \\ -cos\theta \\ e^{-i\theta} \end{pmatrix} \qquad \psi_\pm = \frac{1}{a_\pm}\begin{pmatrix} -i(M(k)\pm E) \\ Ake^{i\theta} \\ Akcos\theta \end{pmatrix} \tag{65}$$

The normalization coefficient is

$$a_{\pm} = \int_0^{2\pi} A^2k^2cos\theta^2 + A^2k^2 + (M(k) \pm E)^2 d\theta \tag{66}$$

$$= 3\pi A^2k^2 + 2\pi M(k) + \int_0^{2\pi} \sqrt{M(k)^2 + A^2k^2 + A^2k^2cos\theta^2} \tag{67}$$

At this time,we can find the E_0 is isotropic,but the $E_{\pm}$ is anisotropy.For simpilify to estimate,we can just think $E_{\pm}$ is isotropic.The we can use the same way of the calculation, we assume that the fermi surface is near the μ_0 we can get the

$$A_{\theta}^{00} = i \langle\psi_0| \partial_\theta |\psi_0\rangle = \frac{2}{3} \tag{68}$$

so it can't be a half of integer , so the middle band can't have a vortex phase transition point.

we next assume that the fermi surface is near the μ_-,we can get if the vortex phase transition happened,we need $\frac{k_-(E)}{k_m}A_{\theta}^{0-}$ to be $\frac{1}{2}$