2.5 $a^T x = b_1 \qquad a^T x = b_2$

let x_1 be a point on $a^T x = b_1$

$L = x_1 + at$ will be perpendicular to $a^T x = b_1$ and $a^T x = b_2$

and intersect $a^T x = b_2$ at x_2

$$a^T(x_1 + at) = b_2$$

$$a^T x_1 + a^T a t = b_2$$

$$t = \frac{b_2 - a^T x_1}{a^T a} \qquad x_2 = x_1 + at.$$

distance between two hyperplane $d = \| x_2 - x_1 \|$

$$d = \left\| x_1 + a \cdot \frac{b_2 - a^T x_1}{a^T a} - x_1 \right\|$$

$$= \left\| \frac{b_2 - a^T x_1}{a^T a} \cdot a \right\| = \frac{|b_2 - b_1|}{\|a\|}$$

2.7

$$x : \{ x \mid \|x-a\|_2 \leq \|x-b\|_2 \}$$

$$\sqrt{\sum_{i=1}^{n} (x_i - a_i)^2} \leq \sqrt{\sum_{i=1}^{n} (x_i - b_i)^2}$$

$$\sum_{i=1}^{n} (x_i - a_i)^2 \leq \sum_{i=1}^{n} (x_i - b_i)^2$$

$$\sum_{i=1}^{n} x_i^2 - 2a_i x_i + a_i^2 \leq \sum_{i=1}^{n} x_i^2 - 2b_i x_i + b_i^2$$

$$\sum_{i=1}^{n} 2(b_i - a_i) x_i \leq \sum_{i=1}^{n} b_i^2 - a_i^2$$

$$2(b-a)^T x \leq \|b-a\|$$

$$(b-a)^T x \leq \frac{\|b\| - \|a\|}{2}$$

2.12

(a) $\{x \in R^n \mid \alpha \leq a^T x \leq \beta\}$

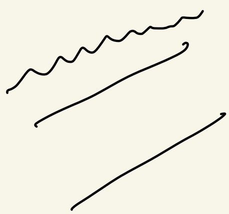
Joint of half planes is convex.

(b) each dimension are convex. The joint of every dimension is convex.

(c) intersection of half planes is convex

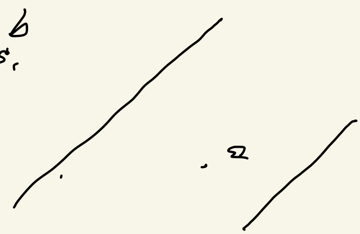
(d) convex. intersection of half space

(e) not convex. The set x can take any shape.



(f) convex.

(g) convex. Intersection of two halfplanes.



2.28 $n=1$

$X_1 \succeq 0$ 则 $x_1 \geq 0$

$n=2$

$$\begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \end{bmatrix} \succeq 0$$

$\lambda_1 + \lambda_2 \geq 0$

$\lambda_1 \lambda_2 \geq 0$

$x_1 + x_3 \geq 0$

$x_1 x_3 - x_2^2 \geq 0$

$n=3$

$$\begin{array}{c} + \\ - \\ + \end{array} \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & x_4 & x_5 \\ x_3 & x_5 & x_6 \end{bmatrix} \succeq 0$$

$\lambda_1 + \lambda_2 + \lambda_3 \geq 0$

$\lambda_1 \lambda_2 \lambda_3 \geq 0$

$x_1 \geq 0.$

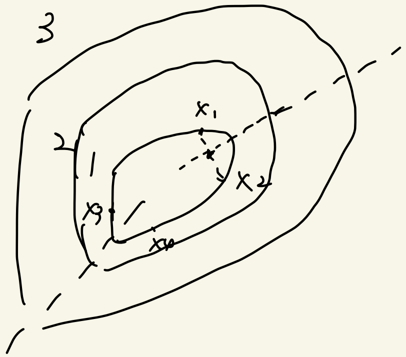
$x_1 x_4 - x_2^2 \geq 0.$

$x_1 x_4 x_6 - x_1 x_5^2 - x_2^2 x_6 + x_2 x_3 x_5 + x_3 x_2 x_5 - x_3^2 x_4 \geq 0$

$x_1 + x_4 + x_6 \geq 0.$

3.2

f is quasiconvex

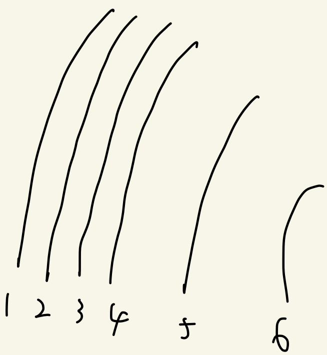


when x_1, x_2 on the $f(x)=1$,

$$f(\theta x_1 + (1-\theta) x_2) \geq \theta f(x_1) + (1-\theta) f(x_2)$$

when x_3, x_4 on the $f(x)=1$

$$f(\theta x_3 + (1-\theta) x_4) \leq \theta f(x_3) + (1-\theta) f(x_4)$$



f is concave. since for any x, y on f

$$f(\theta x + (1-\theta) y) \geq \theta f(x) + (1-\theta) f(y)$$

3.3

f is convex dom g is also convex
$= f(x)$

$g(\theta f(x_1) + (1-\theta) f(x_2))$

$$\theta g(f(x_1)) + (1-\theta) g f(x_2) = \theta x_1 + (1-\theta) x_2$$

$$f(\theta x_1 + (1-\theta) x_2) \leq \theta f(x_1) + (1-\theta) f(x_2)$$

$$g(f(\theta x_1 + (1-\theta) x_2)) \leq g(\theta f(x_1) + (1-\theta) f(x_2))$$

$$\Downarrow$$

$$\theta x_1 + (1-\theta) x_2 \leq g(\theta f(x_1) + (1-\theta) f(x_2))$$

g is concave

Add. Ex

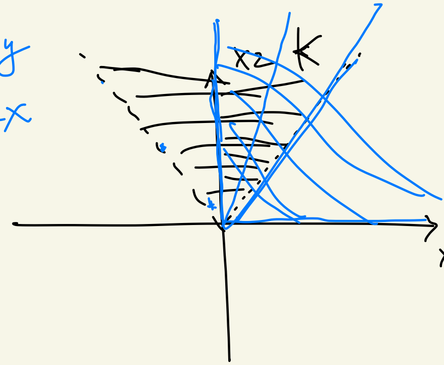
1. $K = \{0\}$

   Dual cone is R^2

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $(-1, 2)$

$2x = y$

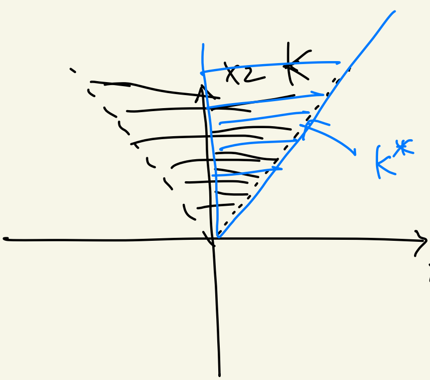
$y = 2x$



2. $K = R^2$

   Dual cone is $\emptyset$

3. $K = \{(x_1, x_2) \mid |x_1| \leq x_2\}$

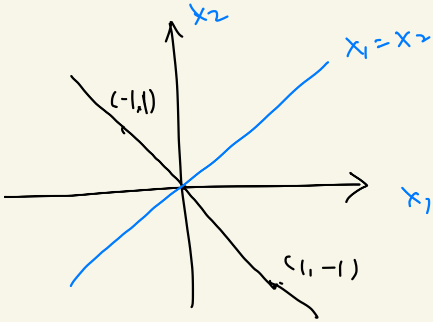


$y^T x \geq 0$ for all $x \in K$

$K^* = \{(x_1, x_2) \mid x_2 \geq x_1 \geq 0\}$

4.

$K = \{(x_1, x_2) \mid x_1 + x_2 = 0\}$



$(1, -1)$

$K^* = \{(x_1, x_2) \mid x_1 = x_2\}$