1. | Framing | cabinet | profit | production |
   |---|---|---|---|
   | 1 | 3 | 10 | 500 |
   | 2 | 2 | 8 | 300 |
   | 2 | 1 | 5 | 200 |

   500 300 200 cabinet decreases 600 framing increases 100

   Originally type1 uses 500 framings, 1500 cabinets, type2 uses 600 framings, 600 cabinets, type3 uses 400 framings, and 200 cabinets. Usual month profit is 5000 + 2400 + 1000 = 8400. There are 1500 framings and 2300 cabinets.
   This month has 1600 framings and 1700 cabinets

   LP: assume type1 x, type2 y, type3 z.
   max(10x+8y+5z)
   Subject to
   x+2y+2z<=1600
   3x+2y+z<=1700
   x>=0, y>=0, z>=0

   The maximum profit for this month is 6700, one possible way is to produce 200 type1, 400 type2, and 300 type3. The maximum profit can be achieved as long as 2x=z+100, and y = 900-5x/2, and x,y,z >=0

2. min(y1+3y3)
   Subject to
   -y1+y2 >= 2
   -y1-y2+2y3>=-5
   Y1,y2,y3 >= 0

3. min(sum of W_k for k from 1 to m) W_k = 1 if F_k is used, otherwise 0
   Subject to
   Sum of X_ak = 1 a from 1 to n j from 1 to m (Every sation must be assigned a frequency)
   X_ik + X_jk <= 1 for every i, j in [n] k from 1 to m (for adjacent sations, they cannot share the same frequency)
   Wk , X _ik, X_jk are either 1 or 0

4. 1. False. A can reduce to B, and B is NP-hard. It means A is at most as hard as B. But A can also be NP. so we are not sure A is in NP-hard.
   2. True. A can reduce to B, and B is NP. every point in A has to map to B, A can be NP or P, but they both belong to NP, so A has to be in NP.
   3. True. if 3 SAT can be reduced to 2 SAT in polynomial time, then P = NP. assume P != NP, then there is a polynomial reduction from 3SAT to 2SAT, then 2SAT has to be NPC, but we know 2SAT is solvable in polynomial time, so P has to equal NP.

4. True. any NP problem can be verified in polynomial time poly(n), and for every possible solution there are 2 states (2 decisions), YES or NO, so every NP problem can be solved in O(2^poly(n)) time

5.False, A can be mapped to B in polynomial time. But it does not mean every point in B can be mapped to A in polynomial time.

5. Set a boolean formula clause1 ^ clause2 ^ clause3^...^clause4...^clausem, with variables x1, x2, x3...xn.

   If the boolean formula clause1 ^ clause2 ^ clause3^...^clause4...^clausem is false, then there is no such assignment. If it is true, then there is a satisfying assignment.
   Then we look at each variable, if x1=true makes the boolean formula true, then x1 is true, else not x1 is true, after setting the boolean value for x1, we use the same way to set the boolean value for x2, x3,...,xn. After giving all the variables a boolean value, we have a satisfying assignment for the 3SAT problem.

   Proof of its correctness: if it can find a satisfying assignment, the 3SAT problem must be true. So we consider one varibale at a time, if the boolean value of the variable makes 3SAT true, then it is the correct boolean value. If every variable can make the 3SAT true, then the algorithm must find a satisfying assignment.

6. Hamiltonian path is to find a path that only visits every vertex once. It means that hamiltonian path is a path that only visites every vertex once so that the weight of unvisited vertices is at most 0, we can reduce it to a decision whether we can find a path that only visits every vertex once so that the weight of unvisited vertices is at most K.

   Prove that multi-lane highway is NP problem. This problem can not be solved in polynomial time because there are 2^n possible ways to construct the highway. However, if we are given a solution, we can verify it in polynomial time.
   Run BFS on all the roads in the country, and run BFS on the highway. The difference is the remoteness of the highway. Since BFS can be done in polynomial time. The problem is a NP problem.

   Prove multi-lane highway is NP Hard. since the hamiltonian path is NP-hard and it can reduce to the multi-lane highway problem, so the multi-lane highway problem is at least as hard as hamiltonian path problem.

   Conclusion: since multi-lane high way is NP hard and it belongs to NP, so the multi-lane problem is NP-complete.