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Will May M3 HW

3.1

1.) a.)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

← invalid

b.)

p	q	$p \leftrightarrow q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	T	F

← valid

c.)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

← valid

d.)

p	q	$p \vee q$	$\neg q$	$p \leftrightarrow q$
T	T	T	F	T
T	F	T	T	F
F	T	T	F	F
F	F	F	T	T

← invalid

2.) a.)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

← invalid

b.)

p	q	$p \rightarrow q$	$\neg p$	$\neg q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

← invalid

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3.2 1.) a.) Invalid Sally had side effects = T
Sally took the medication = T

b.) Invalid Sally had side effects = T
Sally took the medication = F

c.) Valid Addition

d.) Invalid Sally had side effects = F
Sally took the medication = T

2.) a.) ① $\sim r$ Hypothesis
② $q \to r$ Hypothesis
③ $\sim q$ Modulus tollens 1, 2
④ $p \to q$ Hypothesis
⑤ $\sim p$ Modulus tollens 3, 4

b.) ① $q \cap r$ Hypothesis
② $p \to q$ Simplification 1
③ $\sim q$ Hypothesis
④ $\sim p$ Modulus tollens 2, 3

c.) ① $(p \wedge q) \to r$ Hypothesis
② $\sim r$ Hypothesis
③ $\sim (p \wedge q)$ Modulus tollens 1, 2
④ $\sim p \vee \sim q$ De Morgan 3
⑤ q Hypothesis
⑥ $\sim p$ Disjunctive Syllogism 4, 5

d.) ① p Hypothesis
② $p \vee q$ Addition 1
③ $(p \vee q) \to r$ Hypothesis
④ r Simplification 2, 3

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Buy a new car = q

3.2 5.) a.) Get a job = p Buy a new house = r

$p \to (q \wedge r)$
$\neg r$
$\therefore \neg p$

1. $p \to (q \wedge r)$ Hypothesis
2. $p \to (r \wedge q)$ Communitive 1
3. $p \to r$ Simplification 2
4. $\neg r$ Hypothesis
5. $\neg p$ Modulus tollens 3, 4

b.) $p \to (q \vee r)$
$\neg r$
$\neg p$

Invalid

c.) $(q \wedge r) \to p$
$\neg p$
$\neg q$

1. $(q \wedge r) \to p$ Hypothesis
2. $q \to p$ Simplification 1
3. $\neg p$ Hypothesis
4. $\neg q$ Modulus tollens 2, 3

d.) $(q \wedge r) \to p$
$\neg p$
r
$\neg q$

Invalid

3.3 1.) a.)

1. $\exists x (P(x) \wedge Q(x))$ Hypothesis
2. $P(y) \wedge Q(y)$ Existential inst
3. $P(y)$ Simplification
4. $Q(y)$ Simplification
5. $\exists x\, Q(x)$ Combination
6. $\exists x\, P(x)$ Simplific.
7. $\exists x\, Q(x) \wedge \exists x\, P(x)$ Comb.

b.)

1. $\exists x\, Q(x) \wedge \exists x\, P(x)$ Hypothesis
2. $\exists x\, Q(x)$ Simplification
3. $Q(y)$ Existential instantiation
4. $\exists x\, P(x)$ Simplification
5. $P(y)$ Existential instantiation
6. $P(y) \wedge Q(y)$ Combination
7. $\exists x [P(x) \wedge Q(x)]$ existential generali.

c.)

1. $(\forall x)(P(x) \wedge Q(x))$ Hypothesis
2. $P(y) \wedge Q(y)$ Universal instantiation
3. $P(y)$ Simplification
4. $Q(y)$ Simplification
5. $\forall x\, Q(x)$ Universal generalization
6. $\forall x\, P(x)$ universal gener.
7. $\forall x\, Q(x) \wedge \forall x\, P(x)$ conclusion

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d.) ① $\forall x (P(x) \vee Q(x))$ Hypothesis
② $P(y) \vee Q(y)$ Universal inst.

2.) a.) Invalid – because $\neg A(j)$ could be true
b.) Invalid – false → true is also true

4.) a.) valid
① $(\exists x)[P(x) \wedge Q(x)]$ Hypothesis
② $P(y) \wedge Q(y)$ Existential Instantiation
③ $P(y)$ Simplification
④ $Q(y)$ Simplification
⑤ $(\exists x)P(x)$ Existential Generalization
⑥ $(\exists x)Q(x)$ Existential Generalization
⑦ $((\exists x)Q(x)) \wedge ((\exists x)P(x))$ conclusion

b.) not valid
① $((\exists x)Q(x)) \wedge ((\exists x)(P(x))$ Hypothesis
② $(\exists x)Q(x)$ Simplification
③ $(\exists x)P(x)$ Simplification
④ $Q(y)$ Existential Instantiation
⑤ $P(z)$ Existential Instantiation
⑥ $P(z) \wedge Q(y)$ conclusion

c.) valid
① $\forall x(P(x) \wedge Q(x))$ Hypothesis
② $P(y) \wedge Q(y)$ Universal instantiation
③ $P(y)$ Simplification
④ $Q(y)$ Simplification
⑤ $\forall x P(x)$ Universal Generalization
⑥ $\forall x Q(x)$ Universal generalization
⑦ $\forall x Q(x) \wedge \forall x P(x)$ Conclusion

d.) not valid
① $\forall x(P(x) \vee Q(x))$ Hypothesis
② $P(c) \vee Q(c)$ Universal Instantiation

3.4 4.) a.) 1.
b.) 1, 7
c.) 1, 2, 4, 8
d.) 1, 3, 5, 15, 25, 75

5.) a.) Prime
b.) Prime
c.) Prime
d.) Prime

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3.5 2.) a.) Case 1: $n=0$ $(n+1)^2=1$ $>$ 0
Case 2: $n=1$ $(n+1)^2=4$ $>$ 1
Case 3: $n=2$ $(n+1)^2=9$ $>$ 9

b.) Case 1: $n=0$ $2^{(n+2)}=4$ $>$ 1
Case 2: $n=1$ $2^{(n+2)}=8$ $>$ 3
Case 3: $n=2$ $2^{(n+2)}=16$ $>$ 9
Case 4: $n=3$ $2^{(n+2)}=32$ $>$ 27

c.) Case 1: $n=0$ $(n+1)^3$ $\geq$ 1
Case 2: $n=1$ $(n+1)^3$ $\geq$ 3
Case 3: $n=2$ $(n+1)^3$ $\geq$ 9
Case 4: $n=3$ $(n+1)^3$ $\geq$ 27
Case 5: $n=4$ $(n+1)^3$ $\geq$ 32

3.) a.) February has either 28 or 29 days
b.) 2 is an integer and 2^2 is divisible by 4 but 2 is not divisible by 4.
c.) $n=2$ then $2^3=8$ is not less than $2^2=4$.
d.) $n=3$; no integers satisfy.

5.) a.) $x=1$, $y=1$
$\frac{1}{1}+\frac{1}{1}=2$ = True

b.) $x=3$
Positive integers less than 3 = $1+2$

c.) $m=0$ $n=1$
$\sqrt{m+n}=\sqrt{0+1}=1$
$\sqrt{m}+\sqrt{n}=\sqrt{0}+\sqrt{1}=1$
$\therefore \sqrt{m+n}=\sqrt{m}+\sqrt{n}$

d.) $c=-2$ $d=3$
$7c+5d=7(-2)+5(3)$
$=-14+15$
$=1$

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3.6 1.) a.) Theorem If a, b, and c are integers such that $a^3 | b$ and $b^2 | c$ and $a^6 | c$

Proof Here a, b, and c are integers.

If $a^3 | b$ i.e. b is divisible by $a^3 \Rightarrow a^3$ divides b

So, $b = Ka^3$ for some integer K.

Again if $b^2 | c$ i.e. c is divisible by $b^2 \Rightarrow b^2$ divides c

So, $c = jb^2$ for some integer J.

Now $c = jb^2$

$c = j(Ka^3)^2 = JK^2a^6$ for some integer JK^2

So this implies c is divisible by a^6 i.e. a^6 divides c.

So $a^6 | c$

Therefor for integer a, b, c if $a^3 | b$ & $b^2 | c$ then $a^6 | c$.

Hence, proved ■

b.) Theorem If m and n are integers such that $m | n$, then $m | (5n^3 - 2n^2 + 3n)$

Proof $m | n$

$\Rightarrow \exists$ integers K ∴ $n = mK$

So, $5n^3 - 2n^2 + 3n$

$= 5(mK)^3 - 2(mK)^2 + 3(mK)$

$= 5m^3K^3 - 2m^2K^2 + 3mK$

$= m[5m^2K^3 - 2mK^2 + 3K]$

$= mp$

Hence, $p = 5m^2K^3 - 2mK^2 + 3K$ is an integer ■

c.) Theorem If n is an odd integer, then 4 divides $n^2 - 1$

Proof $n^2 - 1 = (2K+1)^2 - 1$

$= 4K^2 + 4K + 1 - 1$

$= 4(K^2 + K)$

Since, $n^2 - 1 = 4(K^2 + K)$ and $(K^2 + K)$ is an integer

So, $4 | (n^2 - 1)$

Hence, if it is an odd integer then $4 | (n^2 - 1)$ ■

d.) Theorem the sum of the squares of any two consecutive integers is odd.

Proof Now, $x^2 + (x+1)^2$

$= x^2 + x^2 + 2x + 1$

$= 2(x^2 + x) + 1$

$= 2n + 1$

Since, $x^2 + (x+1)^2$ is represented of $(2n+1)$

So, the sum of the squares of any two consecutive intervals is odd.

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3.6 2.) a.) In this proof some integer is taken ... K
($x = kw$ and $z = ky$)
Two different integer should be taken.
b.) Taking m as an integer such as $xz = m.wy$ is invalid
x and z should be multiplied first.
c.) $xz = (kw)(jy)$
$xz = (kj)(wy)$
The second step is forgotten
d.) This is the correct proof.

3.7 1.) a.) Theorem The sum of an odd and even integer is odd.
Proof odd numbers $= 2k+1$
Even numbers $= 2k$
$2k + 2k + 1 = 4k+1$
$4k+1$ = remainder 1
results in odd number ∎

b.) Theorem The sum of two odd integers is an even integer
Proof odd numbers $= 2k+1$
odd numbers $= 2m+1$
$2k+1+2m+1$
$= 2k+2m+2$
$= 2(k+m+1) \quad / 2 \quad = k+m+1$
hence, sum of two odds is even. ∎

c.) Theorem The square of an odd integer is an odd integer
Proof odd numbers $= 2k+1$
square $= (2k+1)(2k+1)$
$= 4k^2+4k+1 \quad / 2 \quad = 2k^2+2k$ remainder 1
hence, the square of an odd is a odd ∎

d.) Theorem The product of two odd integers is an odd integer.
Proof odd numbers $= 2k+1$
odd numbers $= 2m+1$
product $= (2k+1)(2m+1)$
$= 4km + 2k + 2m + 1$
$= 2(2km + k + m) + 1 \quad / 2 \quad = 2km+k+m$ r: 1
Hence, the product of two odds is an odd ∎

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3.7 2.) a.) <u>Theorem</u> the product of two rational numbers is a rational number

<u>Proof</u> Definition of rational numbers

$x = \frac{a}{b}$, for some integer a, b, with $b \neq 0$

$s = \frac{c}{d}$, for some integer c, d with $d \neq 0$

$x \cdot s = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

Since ac and bc are integers and $bd \neq 0$ integers is also a integer.

Hence, the product of two rational numbers is a rational number. ∎

b.) <u>Theorem</u> The quotient of a rational number and a non-zero rational number is a rational number

<u>Proof</u> Definition of rational numbers

$x = \frac{a}{b}$, for some integer a, b with $b \neq 0$

$s = \frac{c}{d}$, for some integer c, d with $d \neq 0$

$x \div s = \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$

Since ad and bc are integers and $bd \neq 0$

Hence the quotient of a rational number and a non-zero is a rational number

c.) <u>theorem</u> If x, y are rational numbers then $3x + 2y$ is also a rational number

<u>Proof</u> Since x, y are rational numbers,

$\exists P_1, P_2, V_1, V_2 \quad V_1 \neq 0 \quad V_2 \neq 0$

$x = \frac{P_1}{V_1} \qquad y = \frac{P_2}{V_2}$

$3x + 2y = 3\left(\frac{P_1}{V_1}\right) + 2\left(\frac{P_2}{V_2}\right) = \frac{3P_1V_2 + 2P_2V_1}{V_1V_2}$

So $\frac{a}{b}$ is rational, $3x + 2y$ is also rational ∎

d.) <u>Theorem</u> If x and y are rational numbers then $3x^2 + 2y$ is also a rational number

<u>Proof</u> Definition of rational number

$x = \frac{1}{2}, \quad y = \frac{1}{3}$

then $3x\left(\frac{1}{2}\right)^2 + 2x\left(\frac{1}{3}\right)$

$= \frac{3}{4} + \frac{2}{3} = \frac{9+8}{12} = \frac{17}{12}$

Hence plugging in any rational numbers equates to also a rational number ∎

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3.7 4.) a.) Theorem If x and y are even integers, then $x+y$ is an even integer

Proof Even number means it is divisible by 2

$x = 2a$ for an integer a

$y = 2b$ for an integer b

So, $x+y = 2a+2b$

$= 2(a+b) \quad /2 = a+b$ ✓: 0

Hence, even integer ∎

b.) False Counter example.

8 is an even integer

$8 = 5+3 = x+y$

$x=5$, $y=3$

both are odd integers

c.) False counter example:

$x=-7$

$y=7$

$x^2 = y^2 = 49$

but $x \neq y$

d.) True

3.8 1.) a.) Theorem For every integer n, if n^2 is odd, then n is odd.

Proof Assume that n is not odd

This implies n is even

$n = 2K$ for some $K \in \mathbb{Z}$

$n^2 = (2K)^2$

$= 4K^2$ which is even

If n is even, then n^2 is even

Hence, for every integer n, if n^2 is an odd, then n is odd ∎

b.) Theorem For every integer n, if n^3 is even, then n is even

Proof n is odd

$n = 2K+1$

$n^3 = (2K+1)^3$

$= 8K^3 + 1 + 12K^2 + 6K$

$= 2(4K^3 + 6K^2 + 3K) + 1$

If n is odd, then n^3 is odd

Hence, for every integer n, if n^3 is even, then n is even.

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5.8 1.) c.) <u>Theorem</u> for every integer n, if $5n+3$ is even, then n is odd

<u>Proof</u> n is not odd $\therefore$ n is even

$n = 2K$ for some integer

$$5n+3 = 5(2K)+3$$
$$= 10K+3$$
$$= 2(5K+1)+1 = \text{odd}$$

If n is odd, then $5K+3$ is even

Hence, for every integer n, if $5n+3$ is even, then n is odd. ■

d.) <u>Theorem</u> for every integer n, if n^2-2n+7 is even, then n is odd

<u>Proof</u> n is not odd $\therefore$ n is even

$n = 2K$ for some integer

$$n^2-2n+7 = (2K)^2-2(2K)+7$$
$$= 4K^2-4K+7$$
$$= 2(2K^2-2K+3)+1$$

If n is even, then n^2-2n+7 is even

Hence, for every integer n, if n^2-2n+7 is even, then n is odd. ■

2.) a.) <u>Theorem</u> If x and y are integers such that $3 \mid xy$ then $3 \mid x$

<u>Proof</u> Let us assume $3 \mid x$ for some integer

$x = 3K$ for some integer

$xy = 3Ky$

Let $Ky = g$ for some integer

$xy = 3g$

$3 \mid x$ then $3 \mid xy$

Hence, by contraposition we have proved that if x & y are integers such that $3 \mid xy$ then $3 \mid x$. ■

b.) <u>Theorem</u> For any integers x, y, and z if $x \mid y$ and $x \mid z$, then $x \mid (y+z)$

<u>Proof</u> Suppose $x \mid y+z$

$x \mid y \Rightarrow y = m_1 x$, for some integer m_1

$x \mid (y+z) \Rightarrow y+z = m_2 x$, for some integer m_2.

$\therefore y = m_1 x$ and $y+z = m_2 x$

$\therefore m_1 x + z = m_2 x$

$\therefore z = m_2 x - m_1 x = (m_2 - m_1)x$

$m_2 - m_1$ is integer as m_1 & m_2

$\therefore x \mid z$

Hence, for any integers x, y, and z if $x \mid y$ and $x \mid z$, then $x \mid y+z$ ■

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3.9 2.) c.) Theorem If x and y are integers such that $x > 1$, and x divides y, then $x \mid (y+1)$

Proof $x \mid y+1 \Rightarrow x \nmid y$

So, if $x \mid y+1 \Rightarrow y+1 = kx$

and $y = kx - 1$

$1 = kx - y$

Hence if x divides y, then $x \mid (y+1)$ ∎

5.) a.) Theorem the product of any integer and an even integer is even

Proof Assume x is even ; y is an integer

$xy = 2ky$

Since k and y are both integers ky is also an integer. Since xy can be expressed as 2 times an integer, xy is an even integer ∎

b.) Theorem If $p > 2$ and p is a prime number, then p is odd

Proof p is even

$p = 2k$ for some integer k

$2 \mid p$ = contradiction

Since only divisor of p is p and 1

Hence, p is odd ∎

c.) Theorem For every non-zero real number x, if x is irrational, then $1/x$ is also irrational

Proof Suppose $\frac{1}{x}$ is rational

$\frac{1}{x} = \frac{a}{b}$ for some integer a, b

$x = \frac{b}{a}$ for some integer a, b

x is a rational number

Thus if x is irrational then $1/x$ is irrational is proved ∎

d.) Theorem If x is a real number such that $x^3 + 2x < 0$ then $x < 0$

Proof x is a real number

$x^3 + 2x < 0$

If possible let $x \geq 0$

Case 1.) $x = 0$

$x^3 + 2x = 0^3 + 2 \cdot 0$

$= 0 < 0$ = contrad.

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d.) Case 2.) $x > 0$

$x^2 > 0$ and $x^3 > 0$

$x + x > 0$

$2x > 0$

From C1 / C2 .) $x^3 + 2x > 0$

$x + y$ is even hence multiple of 2 ∎

3.9 1.) a.) Theorem $\sqrt{2}/2$ is irrational

Proof $\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$

Reciprocal of an irrational number is also irrational

$\frac{\sqrt{2}}{2}$ is irrational ∎

b.) Theorem $2 - \sqrt{2}$ is irrational

Proof Assuming $2 - \sqrt{2}$ is rational

$2 - \sqrt{2} = \frac{x}{y}$

$-\sqrt{2} = \frac{x}{y} - 2$

$-\sqrt{2} = \frac{x - 2y}{y}$

this contradicts the fact that $-\sqrt{2}$ is irrational

Hence, $2 - \sqrt{2}$ is irrational ∎

c.) Theorem Is it true that the sum of two positive irrational #s is also irrational?

Proof π is irrational

$\pi + \pi = 2\pi$ which is irrational

Hence it is true ∎

d.) Theorem Is it true that the product of two irrational numbers is also irrational?

Proof π is irrational

$\pi \cdot \pi = \pi^2$ which is irrational

Hence, it is true ∎

5.) a.) Theorem If x & y are a pair of consecutive integers, then x & y have opposite parity

Proof by Contrapos. If x & y does not have opposite parity then $x + y$ are not pair of consecutive integer ∎

Proof by Contradic Assume that x & y are pair of consecutive integers and x & y does not have opposite parity ∎

contradiction

b.) Theorem For all integers n, if n^2 is odd, then n is also odd.

Proof n^2 is of the form $2j + 1$

$n^2 = (2j+1)^2$

$n = 4K^2 + 4K + 1$

$n = 2K + 1$

n is also odd ∎

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3.9 6.) a.) Theorem a group of 9 kids won total of 100 trophies

Proof $9 * 11 = 99$

If 9 won equally w/ 11 trophies each, then 1 must have won by a kid.

So, 12 trophies are won by at least one kid ■

b.) Theorem 400 cups of coffee in one year

Proof 366 days in most years

If we evenly distribute each coffee for a day, we are left w/ 34 coffee left. So, one day the person has bought at least 2 cups ■

c.) Theorem Three real number x, y, z

Proof Average $= \frac{x+y+z}{3}$

If x is least amongst all, the average will be greater than x.

If all have the same value, then average is equal to that value only. ■

d.) Theorem There is no smallest integer

Proof If x is integer then $x-1$ is also integer.

$x-1 < x$ ← contradicts

The assumption is false ■

3.10 1.) a.) Theorem For every real number x, $x^2 \geq 0$

Proof Case 1: $x < 0$

$x^2 > 0$ implies that $x^2 \geq 0$

Case 2: $x = 0$

$x^2 = 0$ implies that $x^2 \geq 0$

Case 3: $x > 0$

$x^2 > 0$ implies that $x^2 \geq 0$ ■

b.) Theorem For every integer n, $n^2 \geq n$

Proof Case 1: $n^2 > n$ which satisfies $n^2 \geq n$

Case 2: $n = 0$

$0^2 = 0$

$n^2 = n$ which satisfies $n^2 \geq n$

Case 3: $n \geq 1$

$n^2 \geq n$ which satisfies $n^2 \geq n$ ■

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1.) c.) Theorem If x is a real number such that $x^2+2x-3<0$ then $-3<x<1$

Proof $x(x+3)-(x+3)<0$

$(x-1)(x+3)<0$

then either $x-1<0$ or $x+3>0$

$x<1$ or $x>-3$

$-3<x<1$ ∎

d.) Theorem If x is a real number such that $x^2-3x-10<0$ then $-2<x<5$

Proof $x(x-5)+2(x-5)<0$

$(x-5)(x+2)<0$

either $x-5<0$ or $x+2>0$

$x-5<0$ $x>-2$

$-2<x<5$ ∎

2.) a.) Theorem If x is an integer, then x^2+5x-1 is odd

Proof: Case 1: $x=2k$

$x^2+5x-1=(2k)2+5(2k)-1$

$=4(2k)+10k-1$

$=2(4k+5k)-1$ ← odd

Case 2: $x=2k+1$

$x^2+5x-1=(2k+1)+5(2k+1)-1$

$=4(2k)+4k+1+10k+5-1$

$=4(2k)+14k+4+1$

$=2(2k2+7k+2)+1$ ← odd ∎

b.) Theorem same parity, even

Proof: Case 1: $x=2m$ $y=2n$

$x+y=2m+2n$

$=2(m+n)$ ← even ∎

c.) theorem opposite parity

Proof: x is even

$x=2k$

$y=2k+1$ ← odd ∎

d.) theorem x & y are odd ...

Proof: $x=2n$

$xy=2ny=2(ny)$ ← contradiction ∎

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3.10 3.) a.) Theorem x, $|x| \geq 0$

Proof every real # $x \geq 0$
true ∎

b.) Theorem any real #

Proof if $x > 0$
then $|x| = x > -x$
if $x < 0$
then $|x| = -x > x$
If $x = 0$
then $|x| = x = -x$.

Hence, for any real number x
$|x| \geq x$ and $|x| \geq -x$ ∎

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