Will May

M5 HW

5.1

1.) a.) Domain: {a, b, c, d, e}

b.) target: {w, x, y, z}

c.) range of f: {w, y, z}

2.)

a.) $f: X \to Y$ $f(x) = |x-2|$

X	Y
1	0
2	1
3	2
4	3
5	4

$1 \to 1$, $2 \to 0$, $3 \to 1$, $4 \to 2$, $5 \to 3$

Range: {0, 1, 2, 3}

b.) $x \in \{0,1\}^2$ $f(x) = x0$

$\{0,1\}^2$	$\{0,1\}^3$
01	000
10	001
11	011
00	111
	101
	100
	110
	010

$01 \to 010$, $10 \to 100$, $11 \to 110$, $00 \to 000$

Range: {000, 100, 110, 010}

c.) $x \{0,1\}^2$ swap two bits

$\{0,1\}^2$	$\{0,1\}^2$
01	01
10	10
11	11
00	00

$01 \to 10$, $10 \to 01$, $11 \to 11$, $00 \to 00$

Range: {01, 10, 11, 00}

4.) a.) $B = \{0,1\} \times \{0,1\}$

$= \{(0,0), (0,1), (1,0), (1,1)\}$

b.) $f(x,y) = (1-y, 1-x)$

$B \times B$	$B \times B$
(0,0)	(1,1)
(0,1)	(0,1)
(1,0)	(1,0)
(1,1)	(0,0)

$(0,0) \to (1,1)$, $(0,1) \to (1,0)$, $(1,0) \to (0,1)$, $(1,1) \to (0,0)$

c.) Range: {(1,1), (0,1), (1,0), (0,0)}

5.1 5.) a.) $f(2) = (2 \times 2) - 1 = 3$

$f(3) = (2 \times 3) - 1 = 5$

$f(4) = (2 \times 4) - 1 = 7$

$f(5) = (2 \times 5) - 1 = 9$

Range: $\{3, 5, 7, 9\}$

b.) $f(2) = 2 \times 2 = 4$

$f(3) = 3 \times 3 = 9$

$f(4) = 4 \times 4 = 16$

$f(5) = 5 \times 5 = 25$

Range: $\{4, 9, 16, 25\}$

c.) Range of $\{0,1\}^5 \to Z = \{0, 1, 2\}$

d.) Range of $\{0,1\}^5 \to Z = \{0, 1, 2, 3, 4, 5\}$

5.2 3.) a.) -4

b.) -4

c.) 5

d.) -2

4.) a.) Will be true because it will still be the same integer; not a decimal

b.) True Ex: $n = 5$ $\lfloor \frac{5}{2} \rfloor = 2$

$\frac{5-1}{2} = 2$

c.) False Ex: $\lfloor 2(1.8) \rfloor = \lfloor 3.6 \rfloor = 3$

$2\lfloor 1.8 \rfloor = 2\lfloor 1 \rfloor = 2$

d.) True Ex: $\lfloor \lceil 1.2 \rceil \rfloor = \lfloor 2 \rfloor = 2$

$\lceil 1.2 \rceil = 2$

5.3 1.) a.) $f(x,y) = z$

$\frac{z}{2} = \frac{2(x-2y)}{2}$

$\frac{z}{2}$ ∴ z must be even

∴ Since it only is limited to even #s, it is not onto.

b.) $f(x,y) = z$

$z = |x| - |y|$

$|x| = z + |y|$

∴ it is onto

c.) $f(x,y) = z$

$z = x + y + 2$

$z - 2 = x + y$

∴ Since z is only taken back two places, it is onto

d.) $f(x,y) = z$

$z = x^{|y|}$

∴ Since y might not always be a good root, it is not onto.

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5.3 2.) a.) · Not one-to-one

Ex: 5, −5 $\Rightarrow f(5) = f(-5) = 25$, but $-5 \neq 5$

· Not onto

No such element $f^{-1}(a) \in \mathbb{R}$

b.) · one-to-one

· onto

c.) · one-to one

· not onto

Ex: $h(x) = 2 \rightarrow x^3 = 2 \rightarrow x = \sqrt[3]{2} \notin \mathbb{Z}$

d.) · not one-to-one

Ex: $f(1) = \lceil \frac{1}{5} \rceil - 4 = -3$

$f(2) = \lceil \frac{2}{5} \rceil - 4 = -3$

$f(3) = \lceil \frac{3}{5} \rceil - 4 = -3$

· onto

5.4 1.) a.) Not well-defined; not one-to-one

b.) Well defined

f^{-1}:

c.) Not well-defined; not onto

2.) a.) $x + \cancel{3} = x' + \cancel{3}$

$\quad -\cancel{3} \quad -\cancel{3}$

$x = x'$ ✓ one-to-one

✓ onto

$y = x + 3$

$-3 \quad -3$

$y - 3 = x \Rightarrow f^{-1}(x) = x - 3$

$f(1) = 1 - 3 = -2$

$f(2) = 2 - 3 = -1$

$f(3) = 3 - 3 = 0$

✓ one to one

✓ onto

✓ well-def.

b.) $2x + \cancel{34} = 2x' + \cancel{34}$

$-\cancel{34} \quad -\cancel{34}$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{\cancel{2}x'}{\cancel{2}}$

$x = x'$ One-to-one ✓

Onto ✓

$y = 2x + 34$

$-34 \quad -34$

$\frac{y - 34}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$\frac{y-34}{2} = x \Rightarrow f^{-1}(x) = \frac{x-34}{2}$

$f(1) = \frac{1-34}{2} = -16.5$

$f(2) = \frac{2-34}{2} = -16$

$f(3) = \frac{3-34}{2} = -15.5$

Well-defined ✓

5.4 2.c.) $2x+\cancel{3}=2x+\cancel{3}$

$\quad -\cancel{3} \quad -\cancel{3}$

$\frac{\cancel{2}x}{\cancel{2}}=\frac{\cancel{2}x}{\cancel{2}}$

$x=x$ One-to-one ✓

Onto ✓

$y=2x+\cancel{3}$

$-3 \quad -\cancel{3}$

$\frac{y-3}{2}=\frac{\cancel{2}x}{\cancel{2}}$

$\frac{y-3}{2}=x \implies f^{-1}(x)=\frac{x-3}{2}$

$f(1)=\frac{1-3}{2}=-1$

$f(2)=\frac{2-3}{2}=-1/2$

$f(3)=\frac{3-3}{2}=0$

· one-to-one ✓

· onto ✓

d.) · Not one-to-one

Ex: $f(x)=f(y)=\underline{1}$

· Onto

∴ does not have an inverse

5.5 2.) a.) $(f \circ g)(0)=1$

b.) $(f \circ h)(52)=121$

c.) $(g \circ h \circ f)(4)=16$

d.) $\left\lceil \frac{x^2}{5} \right\rceil$

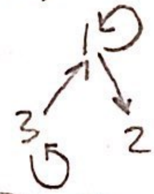
5.) a.) Range: $\{2,3\}$

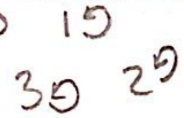
b.) domain of $h \circ g$: $\{a,b,c,\}$

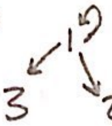
c.) $h^{-1}(4)$: 3

d.) domain of $h^{-1} \circ h$: $\{1,2,3,4\}$

5.7 1.) a.)  $\{(1,2),(2,1),(3,3)\}$

b.)  $\{(1,1),(1,2),(3,1),(3,3)\}$

c.)  $\{(1,1),(2,2),(3,3)\}$

d.)  $\{(1,1),(1,2),(1,3)\}$

2.) a.) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\{(1,1),(1,2),(1,3)\}$

b.) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\{(1,1),(1,3),(2,2),(2,3)\}$

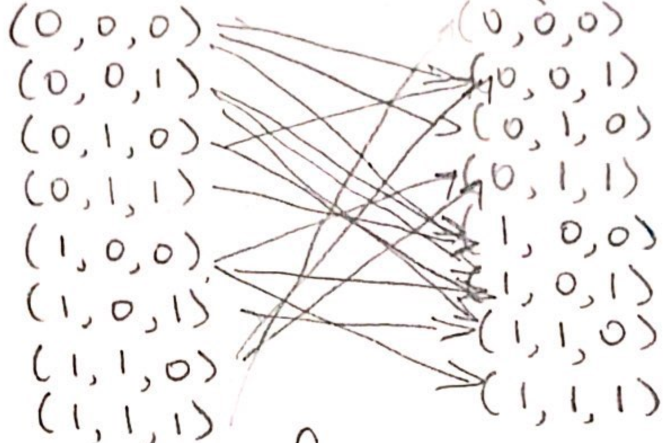
5.7 2.) c.) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$\{(1,2),(2,1),(3,1),(3,2)\}$

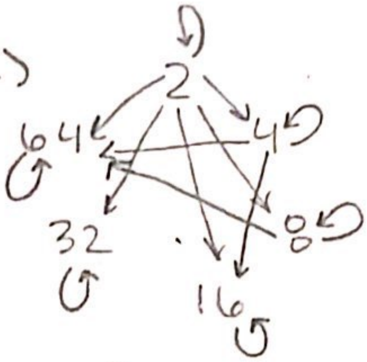
d.) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

$\{(1,2),(2,3),(3,4),(4,1)\}$

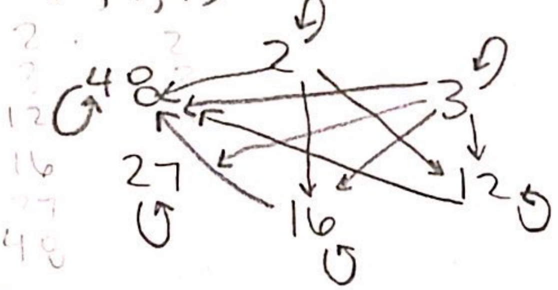
4.) a.)



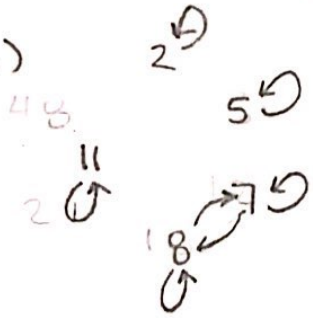
b.)



c.)



d.)



5.8 1.) a.) • for every $x \in R$, it is false that $x \geq x$
∴ L is anti-reflexive
• (y,x) does not belong to L, since it is false that $y > x$
∴ L is not symmetric
• It is true that $x \geq z$ when $x \geq y$
∴ L is transitive

b.) • For $x \in R$, it is true that $x \geq x$
∴ E is reflexive
• Since $y \geq x$ may not be true always
∴ E is symmetric
• Since it is true that $x \geq z$ when $x \geq y$ and $y \geq z$
∴ E is transitive

c.) • for every $x \in Z^+$, it is true that $x' = x$; that is (x,x) belongs to P.
∴ P is reflexive.
• (y,x) does not belong to P since $\sqrt{n}$ is not a integer
∴ P is anti-symmetric.
• $(x,z) \in Z^+$
∴ P is transitive

5.8 1.) d.) For every $x \in Z$, it is true that $x = x(1)$

$\therefore$ D is reflective

• (y, x) does not belong to D since $1/n$ is not an integer, but a real number

$\therefore$ D is not symmetric

• $z = x * (n1 \cdot n2)$, thus $(x, z) \in Z$

$\therefore$ D is transitive.

3.) a.) It is not possible to have a relation that is both reflexive & anti-reflexive.

b.) There is a possibility to be both symmetric & antisymmetric

Ex: $\{(1,1)(2,2)(3,3)\}$

c.) A relation is not symmetric nor antisymmetric if it contains an element (a,b) while it does not contain (b,a) and if it contains an element (c,d) while it also contains (d,c) with $c \neq d$

d.) Yes, it is possible

$A = \{x, y, z\}$

$R = \{(x,x), (x,y), (y,x), (y,y)\}$

5.) a.) Reflexive
Not symmetric
Transitive

b.) Not reflective
Symmetric
Not transitive

c.) Reflective
Not symmetric
Transitive

d.) Not reflective
Symmetric
Not transitive

5.9 1.) a.) 2
b.) 3
c.) c
d.) g

2.) a.) It is a circuit, not a cycle
b.) $\langle b, c, g, f, d, b \rangle$
c.) $\langle b, c, f, d, b \rangle$
d.) $\langle a, b, c, g, f, e \rangle$