Will May

M7: HW

7.1 1.) a.) The nth term is $\lceil \sqrt{n} \rceil$

$a_1 = \lceil\sqrt{1}\rceil = 1$ $a_2 = \lceil\sqrt{2}\rceil = 2$

$a_3 = \lceil\sqrt{3}\rceil = 2$ $a_4 = \lceil\sqrt{4}\rceil = 2$

$a_5 = \lceil\sqrt{5}\rceil = 3$ $a_6 = \lceil\sqrt{6}\rceil = 3$

$a_7 = \lceil\sqrt{7}\rceil = 3$ $a_8 = \lceil\sqrt{8}\rceil = 3$

$a_9 = \lceil\sqrt{9}\rceil = 3$ $a_{10} = \lceil\sqrt{10}\rceil = 4$

Sequence is non-decreasing

b.) $a_1 = 1$ $a_2 = 1$

$a_3 = 2$ $a_4 = 3$

$a_5 = 5$ $a_6 = 8$

$a_7 = 14$ $a_8 = 22$

$a_9 = 36$ $a_{10} = 58$

Sequence is increasing & non-decreasing

c.) $a_1 = 1! = 1$ $a_2 = 2! = 2$

$a_3 = 3! = 6$ $a_4 = 24$

$a_5 = 120$ $a_6 = 720$

$a_7 = 5040$ $a_8 = 40320$

$a_9 = 362880$ $a_{10} = 3628800$

Sequence is increasing & non-decreasing

d.) $a_1 = 1$ $a_2 = 1/2$

$a_3 = 1/3$ $a_4 = 1/4$

$a_5 = 1/5$ $a_6 = 1/6$

$a_7 = 1/7$ $a_8 = 1/8$

$a_9 = 1/9$ $a_{10} = 1/10$

Sequence is decreasing & non-increasing

2.) a.) $a_1 = 1^2 - 2(1) = -1$

$a_2 = 2^2 - 2(2) = 0$

$a_3 = 3^2 - 2(3) = 3$

$a_4 = 4^2 - 2(4) = 8$

Sequence is increasing & non-decreasing

b.) $a_1 = 1^2 - 3(1) = -2$

$a_2 = 2^2 - 3(2) = -2$

$a_3 = 3^2 - 3(3) = 0$

$a_4 = 4^2 - 3(4) = 4$

$a_5 = 5^2 - 3(5) = 10$

non-decreasing

c.) $a_1 = 1^2 - 4(1) = -3$

$a_2 = 2^2 - 4(2) = -4$

$a_3 = 3^2 - 4(3) = -3$

$a_4 = 4^2 - 4(4) = 0$

$a_5 = 5^2 - 4(5) = 5$

$a_6 = 6^2 - 4(6) = 12$

$a_7 = 7^2 - 4(7) = 21$

non-decreasing & non-increasing

7.1

2.) d.) $a_1 = 2^1 - 1! = 1$
$a_2 = 2^2 - 2! = 2$
$a_3 = 2^3 - 3! = 2$
$a_4 = 2^4 - 4! = -8$
$a_5 = 2^5 - 5! = -88$

non-increasing & non-decreasing

3.) a.) $a_1 = 2$ $a_2 = 6$ $a_3 = 18$
$a_4 = 54$ $a_5 = 162$ $a_6 = 486$

b.) $a_1 = 2$ $a_2 = 5$ $a_3 = 8$
$a_4 = 11$ $a_5 = 14$ $a_6 = 17$

c.) $a_1 = 27$ $a_2 = 9$ $a_3 = 3$
$a_4 = 1$ $a_5 = 1/3$ $a_6 = 1/9$

d.) $a_1 = 3$ $a_2 = 2.5$ $a_3 = 2$
$a_4 = 1.5$ $a_5 = 1$ $a_6 = 0.5$

7.2

1.) a.) $a_0 = 1$
$a_1 = 2$
$a_2 = 2 \cdot 1 = 2$
$a_3 = 2 \cdot 2 = 4$
$a_4 = 4 \cdot 2 = 8$
$a_5 = 8 \cdot 4 = 32$

b.) $a_1 = 1$
$a_2 = 5$
$a_3 = 2(5) + 3(1) = 13$
$a_4 = 2(13) + 3(5) = 41$
$a_5 = 2(41) + 3(13) = 108$
$a_6 = 2(108) + 3(41) = 339$

c.) $g_1 = 2$
$g_2 = 1$
$g_3 = 3(1) + 2 = 5$
$g_4 = 4(5) + 1 = 21$
$g_5 = 5(21) + 5 = 110$
$g_6 = 6(110) + 21 = 681$

d.) $c_1 = 4$
$c_2 = 5$
$c_3 = 5 \cdot 4 = 20$
$c_4 = 20 \cdot 5 = 100$
$c_5 = 100 \cdot 20 = 2000$
$c_6 = 2000 \cdot 100 = 200{,}000$

7.3

1.) a.) $\sum_{k=-1}^{4} k^2 \rightarrow \underset{-1}{1} + \underset{0}{0} + \underset{1}{1} + \underset{2}{4} + \underset{3}{9} + \underset{4}{16} = 31$

b.) $\sum_{k=0}^{4} 2^k \rightarrow \underset{0}{1} + \underset{1}{2} + \underset{2}{4} + \underset{3}{8} + \underset{4}{16} = 31$

c.) $\sum_{k=-3}^{2} k^3 \rightarrow \underset{-3}{-27} + \underset{-2}{-8} + \underset{-1}{-1} + \underset{0}{0} + \underset{1}{1} + \underset{2}{8} = -27$

d.) $\sum_{k=0}^{3} 3^k \rightarrow \underset{0}{1} + \underset{1}{3} + \underset{2}{9} + \underset{3}{27} = 40$

2.) a.) $\sum_{k=-2}^{7} k^5$ b.) $\sum_{k=-2}^{5} k$

c.) $\sum_{k=2}^{8} 2^k$ d.) $\sum_{0}^{17} k^3$

7.3 4.) a.) $\sum_{i=2}^{n-1} i$ b.) $\sum_{j=-1}^{n} 2^{j-1}$ c.) $\sum_{i=-1}^{n} ((i-1)^2-2(i-1)+1)$

d.) $\sum_{k=0}^{13} 2k+12$

7.4 1.) a.) $\sum_{j=1}^{3} j^2 = \underset{1}{1}+\underset{2}{4}+\underset{3}{9} = 14$ ✓

$\frac{n(n+1)(2n+1)}{6} = \frac{3(4)(7)}{6} = 14$

b.) $\sum_{j=1}^{n} j^2 = \frac{k(k+1)(2k+1)}{6}$

c.) $\sum_{j=1}^{n} j^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$

d.) Need to prove fmla is true for $n=1$

$1^2 = \frac{1(1+1)(2(1+1))}{6}$

$1 = 1$ ✓

2.) a.) $\sum_{j=1}^{n} j^3 = \left(\frac{n(n+1)}{2}\right)^2$

$\sum_{j=1}^{n} j^3 = \frac{n^2(n+1)^2}{4}$

P(1) $1^3 = \frac{1^2(1+1)^2}{4}$

✓ $1 = 1$

P(K+1) $\sum_{k=1}^{n} k^3 = \frac{(k+1)^2(k+2)^2}{4}$

$= \frac{k^2(k+1)^2}{4} + (k+1)^3$

$= (k+1)^2\left[\frac{k^2}{4} + k+1\right] = \frac{(k+1)^2(k^2+4k+4)}{4}$

$= \frac{(k+1)^2(k+2)^2}{4}$ ✓

b.) $\sum_{j=1}^{n} j\cdot 2^j = (n-1)2^{n+1}+2$

P(1) $= 1\cdot 2^1 = (1-1)2^{1+1}+2$

$2 = 2$ ✓

P(K+1) $=$

$k\cdot 2^{k+2}+2$

$= (k-1)2^{k+1}+2 + (k+1)2^{k+1}$

$= 2^{k+1}(k-1+k+1)+2$

$= 2k\cdot 2^{k+1}+2$

$= k\cdot 2^{k+2}+2$

✓

2.) c.) $\sum_{j=1}^{n} j(j-1) = \frac{n(n^2-1)}{3}$

$P(1): 1(1-1) = \frac{1(1^2-1)}{3}$

✓ $0 = 0$

$P(k+1) = \frac{(k+1)((k+1)^2-1)}{3}$

$= \frac{k(k^2-1)}{3} + (k+1)k$

$= \frac{k(k-1)(k+1)}{3} + k(k+1)$

$= \frac{(k+1)}{3}\left[k(k-1)+3k\right]$

$= \frac{(k+1)}{3}\left[k^2+2k\right] = \frac{(k+1)}{3}\left[(k+1)^2-1\right]$ ✓

d.) $\sum_{j=1}^{n} \frac{1}{j(j+1)} = 1 - \frac{1}{n+1}$

$P(1) = \frac{1}{1(1+1)} \quad 1 - \frac{1}{1+1}$

$\frac{1}{2} = \frac{1}{2}$

$P(k+1) = 1 - \frac{1}{k+2}$

$= 1 - \frac{1}{k} - \frac{1}{2}$ ✓

3.) a.) for $n=2$, $3^n = 3^2 = 9$

$2^2 + (2)^2 = 4+4 = 8$

$\therefore 3^n = 9 > 8 = 2^n + (2)^n$

$\therefore$ result is true for $n=2$

for $n=k$ let $3^k > 2^k + k^2$

For $n=k+1$, $3^{k+1} = 3^k$

$> 3(2^k + k^2)$ — By induction hypothesis

$= (2+1)(2^k + k^2)$

$= 2^{k+1} + 2^k + 2k^2 + k^2$

$> 2^{k+1} + k^2 + 2k + 1$

$\therefore 3^{k+1} > 2^{k+1} = 2^{k+1} + (k+1)^2$
$+ (k+1)2$

b.) for $n=4$

$4! = 24$

$2^4 = 16$

$n! = 24 \geq 16 = 2^n$

$n = k+1$, $n! = (k+1)!$

$= k!(k+1)$

$\geq 2^k(k+1)$

$\geq 2^k(1+1)$

$= 2^k(2)$

$= 2^{k+1}$

$= 2^n$

$\therefore (k+1)! \geq 2^{k+1}$

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7.4 3.) c.) for $n=1$, $\sum_{j=1}^{1} \frac{1}{j^2} = 1$

For $n=k+1$ $\sum_{j=1}^{k+1} \frac{1}{j^2} = \sum_{j=1}^{k} \frac{1}{j^2} + \frac{1}{(k+1)^2}$ $= 3/2$

$\leq 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$

$\leq 2 - \frac{1}{k} + \frac{1}{k+1}$

$= 2 - \left(\frac{k+1-k}{k(k+1)}\right)$

$= 2 - \frac{1}{k(k+1)}$

$\leq 2 - \frac{1}{k+1}$

$\therefore \sum_{j=1}^{k+1} \frac{1}{j^2} \leq 2 - \frac{1}{k+1}$

d.) for $n=3$, $3^3 \geq 3^3$

$27 \geq 27$

for $n=k+1$

$3^{k+1} \geq (k+1)^3$ ✓

7.5 1.) a.) Base Case:

for $n=1$

$3^{2\times1} - 1 = 9 - 1 = 8$ ← divisible by 4

Inductive Step:

$3^{2k} - 1 = 4x$

$(3^2)^k - 1 = 4x$

$9^k - 1 = 4x$

3rd Step

$3^{2(k+1)} - 1 = (9^k \cdot 9) - 1$

$= ((9^k \cdot 9) - 9) + 8$

$= 9(9^k - 1) + 8$

$= 9 \times 4x + 8$

$= 4(9x + 2)$

$= 4p$ ✓

b.) Base Case:

for $n=1$

$7^1 - 1 = 7 - 1 = 6$ ← divisible by 6

Inductive

$7^k - 1 = 6x$

3rd Step

$7^{k+1} - 1 = (7^k \cdot 7) - 1$

$= (7^k \cdot 7) - 1 - 6 + 6$

$= ((7^k \cdot 7) - 7) + 6$

$= 7(7^k - 1) - 6$

$= 7(6x) - 6$

$= 6(7x - 1)$

$= 6p$ ✓

c.) Base Case:

for $n=1$

$11^n - 7^n = 11 - 7 = 4$

$4/4$ ← divisible by 4

Inductive

$11^k - 7^k$

3rd Step

$11^{k+1} - 7^{k+1} = 42$ ✓

d.) Base case

for $n=1$

$9^1 - 2^1 = 7$ ← divisible by 7

Inductive

$9^k - 2^k$

3rd Step

$9^{k+1} - 2^{k+1} = 35$ ✓

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3.) a.) Base Case

$n=0$

$C_0=5$ and $5^{2^0}=5^1=5$

$C_0=5^{2^0}$ ✓

Induction

5^{2^k}

3rd Step

$(5^{2^k})^2$

$\therefore c_{k+1}=5^{2^k}\cdot 5^{2^k}=5^{2^k+2^k}=5^{2^k(1+1)}=5^{2\cdot 2^k}=5^{2^{k+1}}$ ✓

b.) Base Case

$n=0$

$b_0=1$ and $2^{0+1}-1=2-1=1$

$\therefore b_0=2^{0+1}-1$ ✓

Induction

$2b_k+1$

3rd Step

$=2(2^{k+1}-1)+1$

$=2\cdot 2^{k+1}-2+1$

$=2^{(k+1)+1}-1$ ✓

c.) Base case

$n=1$

$-2(1)-4+6\cdot 2^1=-2-4+12=6=a_1$ ✓

Induction

$a_k=-2k-4+6\cdot 2^k$

3rd Step

$=2[-2k-4+6\cdot 2^k]+2(k+1)\dots$

$=[-4k-8+6\cdot 2^{k+1}+2(k+1)]$

$\boxed{=-2(k+1)-4+6\cdot 2^{k+1}}$ ✓

d.) Base Case

$n=1$

$g_1=g_0+1+1$

$=g_0+2$

$=2$ ✓

Induction

$\frac{k(k+1)}{2}-1$

3rd Step

$\frac{n(n+3)}{2}$ ✓