Computational Heat Transfer: Final Project

Ran Li

January 3, 2017

1 Problem Statement

Problem 8.8:

Consider a heated cylindrical rod emerging at time t = 0, from a die at temperature T_0, as shown in Fig. 8.5b. Assuming axisymmetry, develop a numerical scheme to solve for the time-dependent temperature distribution in the material, if the rod loses energy by convection to ambient fluid at a constant value of heat transfer coefficient.

2 Mathematical Analysis

This is a modified 1-D heat conduction problem with convection terms at each point on the entity. Setup of the problem is shown down below. The basic idea is to take the changing length of the rod as constant over certain time interval Δt so that by solving for transient behavior of the beam we can approximate the real phenomena.

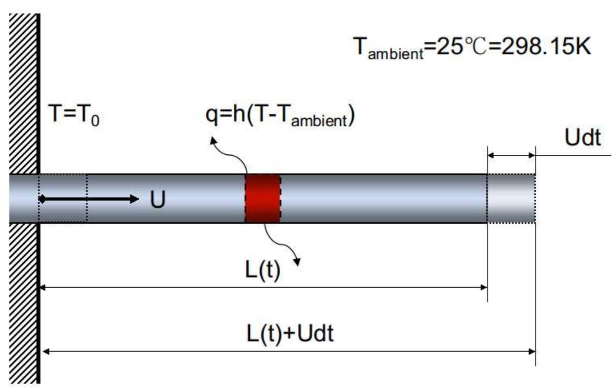

Figure 1: Problem Setup

Mathematically, governing equation for such problem is:

$$\rho C\left(\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}\right)=k\frac{\partial^2 T}{\partial x^2}-\frac{hP}{A}\left(T-T_a\right) \tag{1}$$

Only convection term is taken into consideration as mentioned in the problem description. Boundary condition could be specified as:

$$x=0:T\left(x\right)=T_0 \tag{2}$$

$$x=L\left(t\right):-k\frac{\partial T}{\partial x}=h\left(T-T_{ambient}\right) \tag{3}$$

Since $L\left(t\right)$ is constantly changing with time:

$$L\left(t\right)=Ut \tag{4}$$

Thus we take $L\left(t\right)$ as constant over certain time interval Δt and by taking heat conduction within each interval as common transient problem of 1-D beam the problem could be solved. The iteration scheme is shown in figure down below.

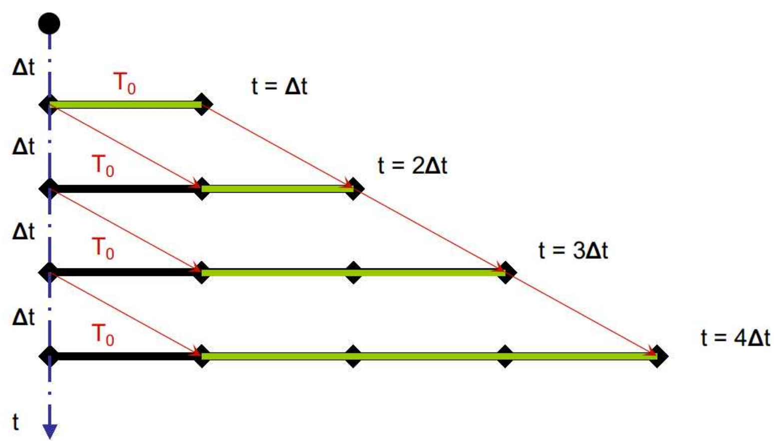

Figure 2: Iteration Scheme of the Problem

3 Method Applied and Grid Generation

For this problem I selected FTCS finite difference method. The reason of my choice is that this problem could be devided into superposition of multiple simple heat conduction problems which could be easily solved using explicit method. Also code for explicit method is much more easier to compose and modify.

This problem is a 1-D problem and thus it's discretization is done by dividing the longtitude region of the beam into multiple discrete nodes. Since it's quite simple and straight forward I incoupled the mesh generation within my solver.

MATLAB code composed for this problem is as shown:

Listing 1: MATLAB function

```
%% Extrusion Heat Transfer Problem
%% Iteration parameters
dt = 0.001;% Time interval
U = 2;
dx=U*dt;

duration=10; % Total time length 10s

%% Physical parameters of the rod
D = 0.001; % Diameter of the rod

a = 0.001;  %    Diffusion coeff
k = 25;  %    Conductivity coeff
rhoC = k/a;

P = pi*D;
A = 0.25*pi*D^2;

Fo = a*dt/(dx^2);

%% Enviromental condition
h = 2500; %    Convection film coeff

T0 = 783.15; % initial temeprature : 500 degree Celsius assumed
Ta = 298.15; % ambient temeprature : 25 degree Celsius assumed

%% Indicators as shown in prof. Jaluria's case
C1=(h*D)/k;
C2=(U*D)/a;

%% Initialization
T=zeros(duration/dt,duration/dt);
T(:,1) = T0;% T(3,1)=T0;T(3,2)=T0;T(3,3)=T0;

[m,n]=size(T);

%% ITERATION START

for i=2:m
    for j=2:i
        if i==2 % first time interval
            T(i,j) = T(i,j-1) - (T(i-1,j-1)-Ta)*(h*dx/k);
        elseif j==i % last node of the rode
            T(i,j) = T(i,j-1) - (T(i-1,j-1)-Ta)*(h*dx/k);
```

```
        else % general case
            T(i,j)= (Fo )*T(i-1,j)+(1-2*Fo - h*P*dt/(rhoC*A))*T(i-1,j-1)+(Fo )*T
            %- U*dt/(2*dx)
            %+ U*dt/(2*dx)
        end
    end
end
```

4 Numerical Scheme Setup

Discretizing the physical domain using finite difference approach:(within certain Δt)

$$\rho C\left[\frac{T_i^{(n+1)}-T_i^{(n)}}{\Delta t}+U\frac{T_{i+1}^{(n)}-T_{i-1}^{(n)}}{2\Delta x}\right]=k\frac{T_{i+1}^{(n)}-2T_i^{(n)}+T_{i-1}^{(n)}}{\Delta x^2}-\frac{hP}{A}\left(T_i^{(n)}-T_a\right) \tag{5}$$

Rearranging terms and thus the forward iteration could be derived:

$$T_i^{(n+1)}=\left(F_o-\frac{U\Delta t}{2\Delta x}\right)T_{i+1}^{(n)}+\left(1-2F_o-\frac{hP\Delta t}{\rho CA}\right)T_i^{(n)}+\left(F_o+\frac{U\Delta t}{2\Delta x}\right)T_{i-1}^{(n)}+\frac{hP\Delta t}{\rho CA}T_a \tag{6}$$

Yet in our computation, at each time step we are assuming that current length of the rod is a constant. Thus terms including variable U will be all cancelled, which result in our final governing equation for computation:

$$T_i^{(n+1)}=(F_o)\,T_{i+1}^{(n)}+\left(1-2F_o-\frac{hP\Delta t}{\rho CA}\right)T_i^{(n)}+(F_o)\,T_{i-1}^{(n)}+\frac{hP\Delta t}{\rho CA}T_a \tag{7}$$

This is our formular to solve for tempreture distribution within certain Δt. For each time step, extrusion points (first node of the rod) will be initialized as constant tempreture of T_0.

5 Results and Compare

In my computation, physical parameters could be found in section 3 where I inserted MATLAB code, while my inputs for function are chosen as shown in the table below:

Input Name	Meaning	Value
dt	Time Step for Iteration	$1 \times 10^{-3} s$
dx	Grid Size	Udt
U	Extrusion Speed	$2m/s$
D	Rod Diameter	$1 \times 10^{-3} m$
duration	Total Time	$1s$

Table 1: Input Parameters

The result from my code could be hardly called satisfactory, yet I believe it does explained about behavior of rod when L is big enough. This figure shows tempreture distribution when time is quite short.

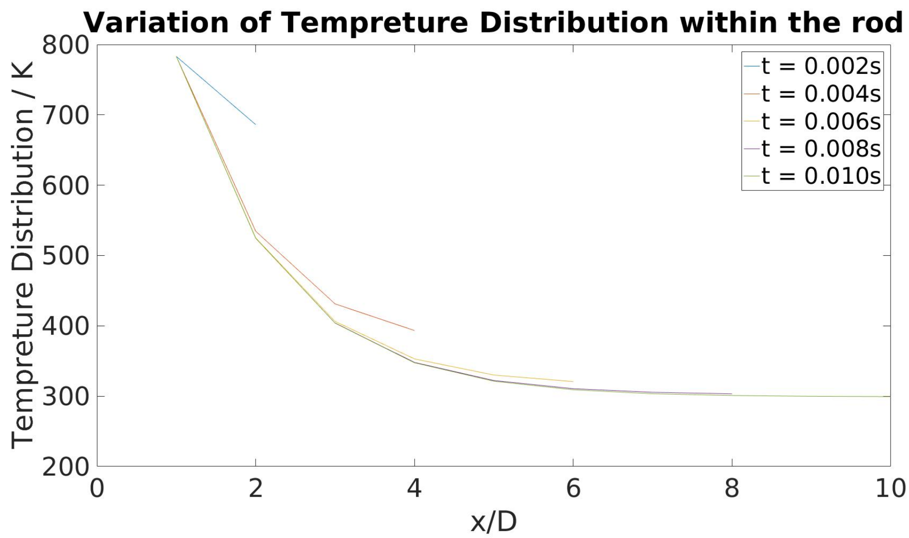

Figure 3: Tempreture Distribution w.r.t. Time

In comparison with what I found in the textbook(down below), it is clear that my calculation is about the same with standard result.

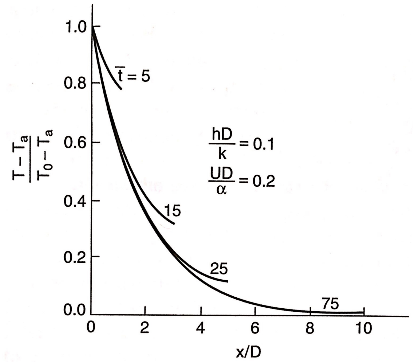

Figure 8.6 Temperature distributions in a continuously moving cylindrical rod of diameter D, at various time intervals after the start of the process, assuming one-dimensional conduction [From Jaluria and Singh (1983). Reprinted with permission of Elsevier/North-Holland.]

Figure 4: Standard Solution from Textbook

When it comes to the long term behavior, it seems that the solution is quite alike the standard result. According to analytical analysis, when $L(t)$ is big enough, the governing equation will collapse into:

$$\rho C \left(U \frac{\partial T}{\partial x} \right) = k \frac{\partial^2 T}{\partial x^2} - \frac{hP}{A} (T - T_a) \tag{8}$$

This solution will be quite alike to standard 1-D beam heat conduction problem. What I got from my code also shows this property. The result I got for long time scale is shown down below:

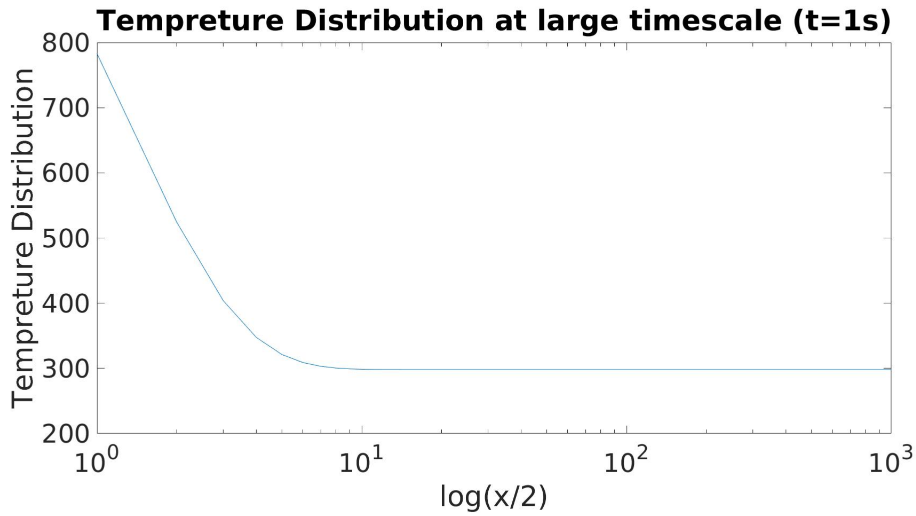

Figure 5: Result for long timescale

6 Conclusion

In this program I practiced setting up speciallized FDM scheme from governing equation for certain problem. It took some time even after the semester for me to get it right but I did learnt a lot from this project. This document is a make up for my previous mistake in final project.