ORIE 5132: Pricing Analytics and Revenue Management
Assignment 2
Due on March 3rd, 11:59pm (EST)

Notes

- Each assignment must be submitted on Gradescope by the due date.
- All questions in this homework must be individual work. You are allowed to discuss the assignment with others but please mention in your write-up if you have discussed the solution with someone.
- You should also submit your programming code (Python, R, Matlab, etc) along with the answers for Problems 1,2 and 3.

Problem 1 (Network Revenue Management). An airline company offers 8 different itineraries presented in the table below. There are only four flight legs: Bos-Chi, NYC-Chi, Chi-SF, Chi-LA and each itinerary is using one or two of these flight legs. There is one aircraft per leg and all planes have 200 seats. The table presents the demand for each itinerary for two fare classes: the low fare (Q-class) and high fare (Y-class).

			Q-Class		Y-Class	
Departure	Destination	Connection	Dem	Fare	Dem	Fare
Bos	Chi	direct	25	\$200	20	\$230
Bos	SF	stop at Chi	55	\$320	40	\$420
Bos	LA	stop at Chi	65	\$400	25	\$490
NYC	Chi	direct	24	\$250	16	\$290
NYC	SF	stop at Chi	65	\$410	50	\$550
NYC	LA	stop at Chi	40	\$450	35	\$550
Chi	SF	direct	21	\$200	20	\$230
Chi	LA	direct	25	\$250	14	\$300

1. We would like to determine the optimal way to allocate the seats of each leg. Write an optimization problem that maximizes the total revenue. Solve the optimization problem using a solver (e.g. Gurobi, OR tools, etc) and report the optimal revenue.
2. If you can add one seat on one leg, which leg would it be?

Problem 2 (Pricing using a linear demand function). For this question, we use the data set in *Demand.xlsx*. Suppose the demand function is linear: $d(p) = a - bp, a, b > 0$

1. Use the data in *Demand.xlsx* to estimate a, b using linear regression.
2. Find the optimal price p^* that maximizes the expected revenue.

Problem 3 (Empirical Willingness-to-pay distribution) In the file *InSample.xlsx*, you are given data $\{(p_1, y_1), \ldots, (p_N, y_N)\}$, where p_i is the price offered for a product and y_i is the indicator function of whether the product was sold at price p_i. Suppose the willingness of paying this product is a random variable $W \sim$ cdf $F(\cdot)$, here cdf refers to the cumulative density function of W.

1. Use the data in *InSample.xlsx* to estimate the cdf $F(\cdot)$. Note F is a stair-wise function in this case. Draw a rough sketch to depict the function.

2. Find the optimal price p^* that maximizes the expected revenue. Suppose $W_1, \ldots, W_N$ are the willingness-to-pay data points in *InSample.xlsx*. Then expected revenue for any price p can be expressed as

$$\frac{1}{N} \cdot \sum_{i=1}^{N} p \cdot \mathbf{1}(W_i \geq p),$$

   where $\mathbf{1}(\cdot)$ is an indicator function that evaluates to 1 if the condition is true and 0 otherwise.

3. The file *OutOfSample.xlsx* contains willingness-to-pay data points. Each data point represents the willingness-to-pay of a certain customer. Suppose p^* is the optimal price computed in the previous question. Compute the revenue obtained by p^* if the actual demand is given by data in *OutOfSample.xlsx*.