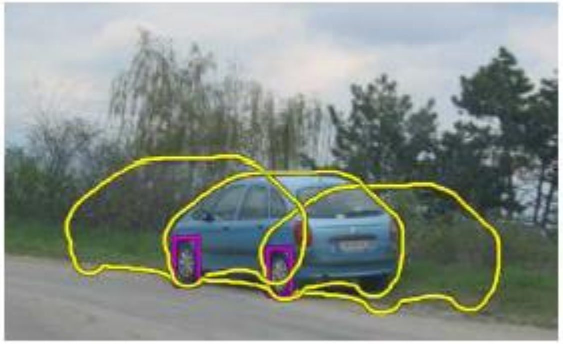
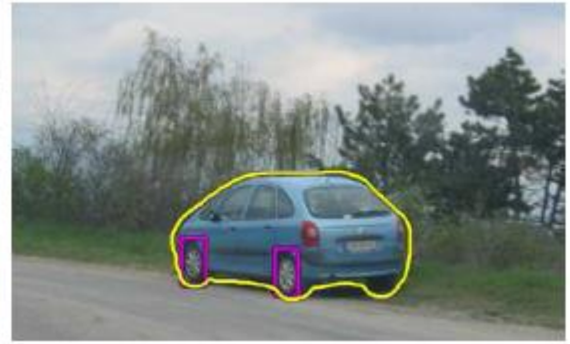
 

Fitting:
Voting and the Hough Transform

Tuesday, Sept 22, 2020

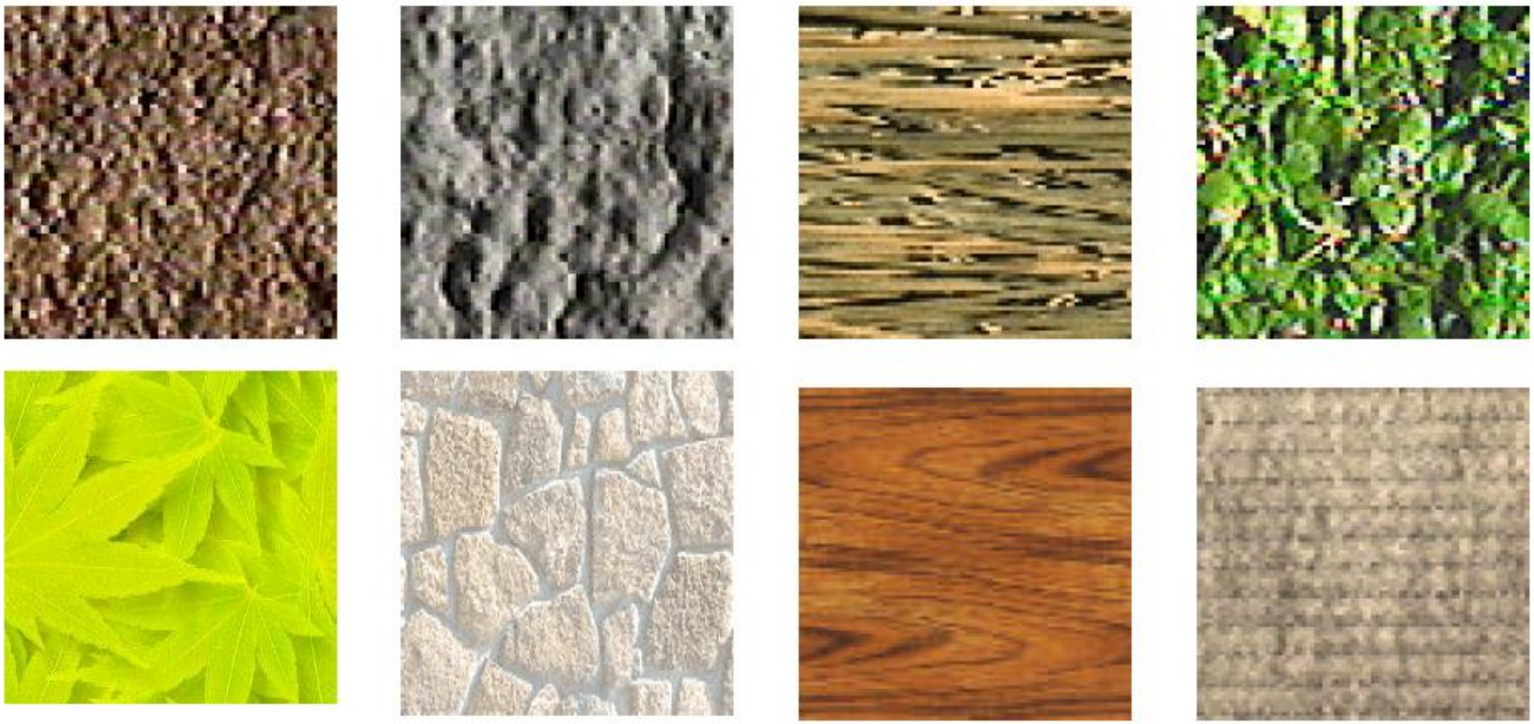
Richang Hong, Hefei University of Technology

Binary images

- Pros
  - Can be fast to compute, easy to store
  - Simple processing techniques available
  - Lead to some useful compact shape descriptors

- Cons
  - Hard to get “clean” silhouettes
  - Noise common in realistic scenarios
  - Can be too coarse of a representation
  - Not 3d

Last Time: Texture



What defines a texture?

Why analyze texture?

Importance to perception:

- Often indicative of a material's properties
- Can be important appearance cue, especially if shape is similar across objects
- Aim to distinguish between shape, boundaries, and texture

Technically:

- Representation-wise, we want a feature one step above "building blocks" of filters, edges.

Texture-related tasks

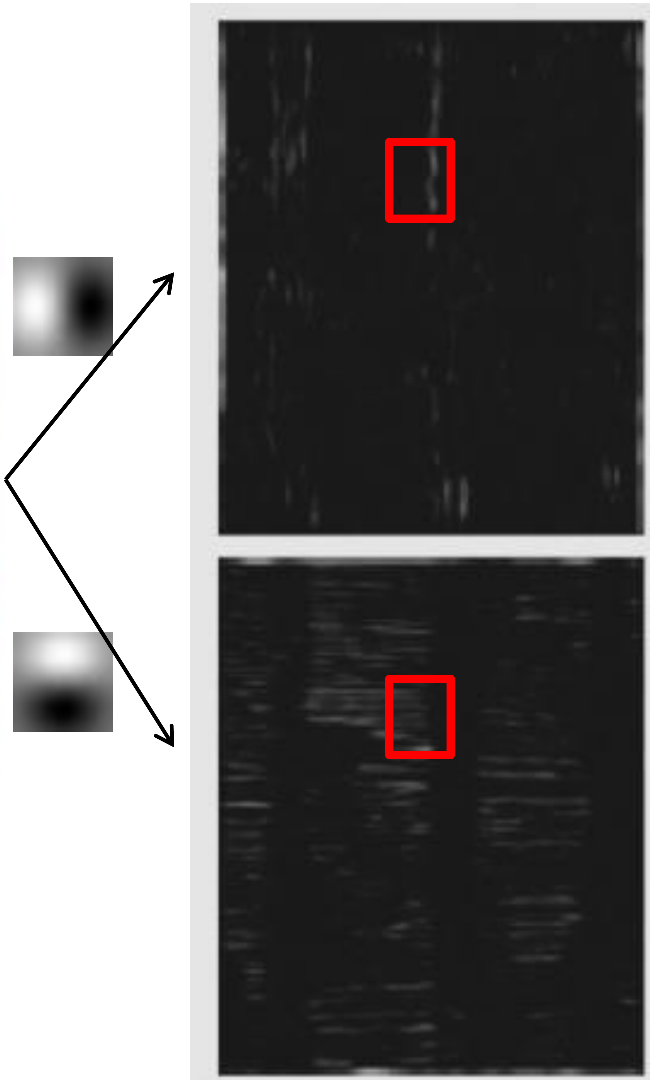
- **Shape from texture**
  - Estimate surface orientation or shape from image texture
- **Segmentation/classification** from texture cues
  - Analyze, represent texture
  - Group image regions with consistent texture
- **Synthesis**
  - Generate new texture patches/images given some examples

Texture representation

- Textures are made up of repeated local patterns, so:
  - Find the patterns
    - Use filters that look like patterns (spots, bars, raw patches…)
    - Consider magnitude of response
  - Describe their statistics within each local window, e.g.,
    - Mean, standard deviation
    - Histogram
    - Histogram of “prototypical” feature occurrences

Texture representation: example

original image



derivative filter responses, squared

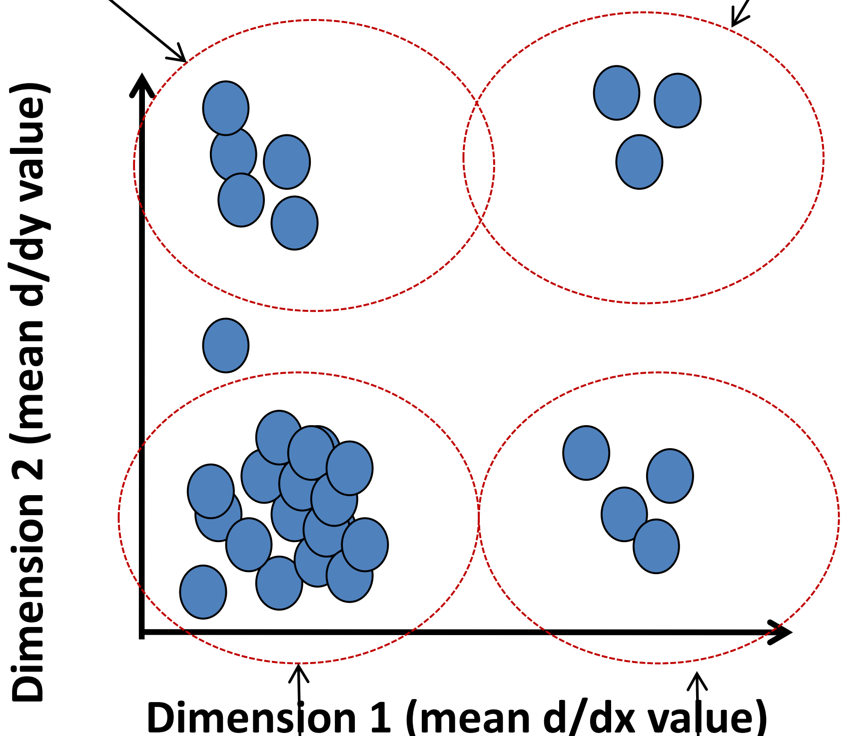
	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2	18	7
⋮		
Win.#9	20	20

⋮

statistics to summarize patterns in small windows

Slide credit: Kristen Grauman

Texture representation: example



	mean d/dx value	mean d/dy value
Win. #1	4	10
Win.#2 ⋮	18	7
Win.#9	20	20

⋮

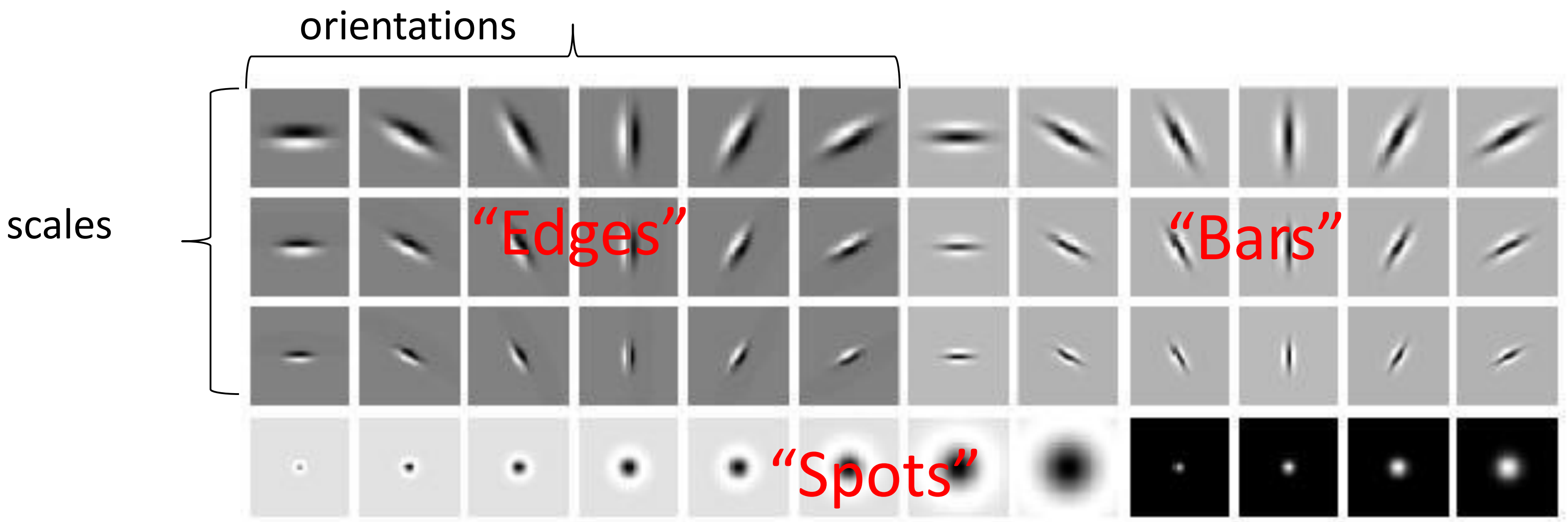
statistics to summarize patterns in small windows

Slide credit: Kristen Grauman

Filter banks

- Our previous example used two filters, and resulted in a 2-dimensional feature vector to describe texture in a window.
  - x and y derivatives revealed something about local structure.
- We can generalize to apply a collection of multiple (*d*) filters: a "filter bank"
- Then our feature vectors will be *d*-dimensional.
  - still can think of nearness, farness in feature space

Filter banks

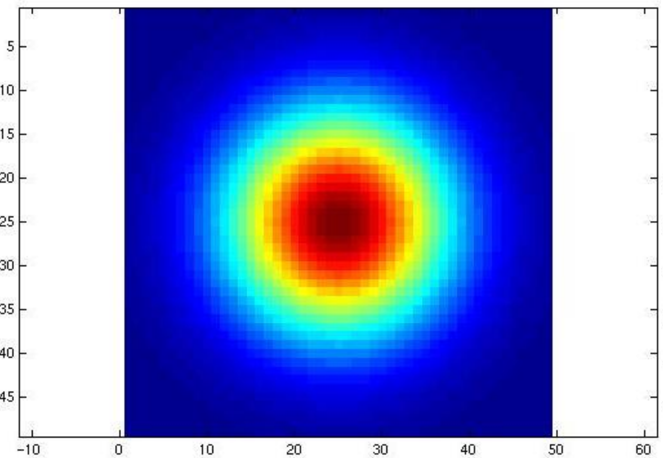


- What filters to put in the bank?
  - Typically we want a combination of scales and orientations, different types of patterns.

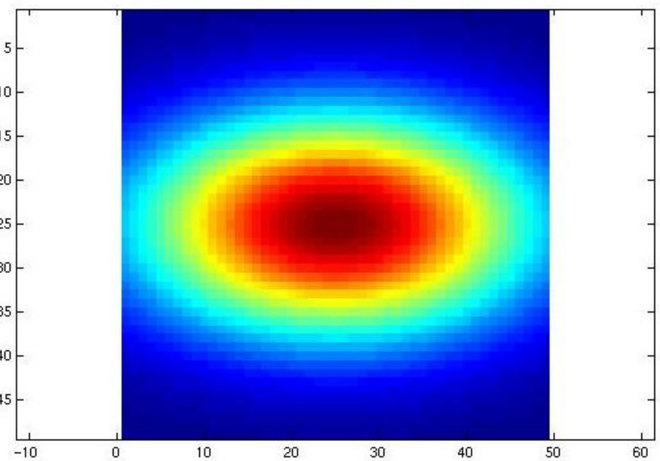
Matlab code available for these examples:
http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html

Multivariate Gaussian

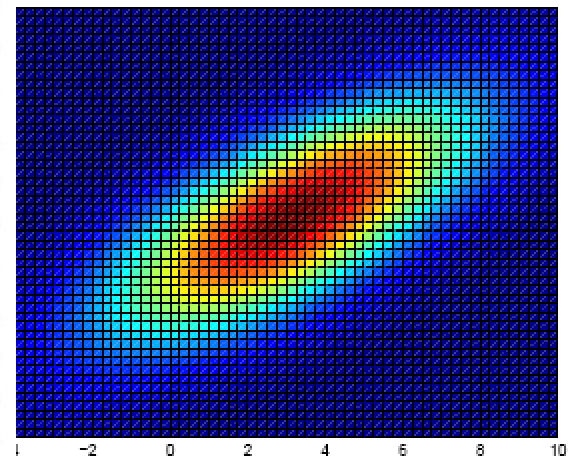
$$p(x;\mu,\Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right).$$



$$\Sigma = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$



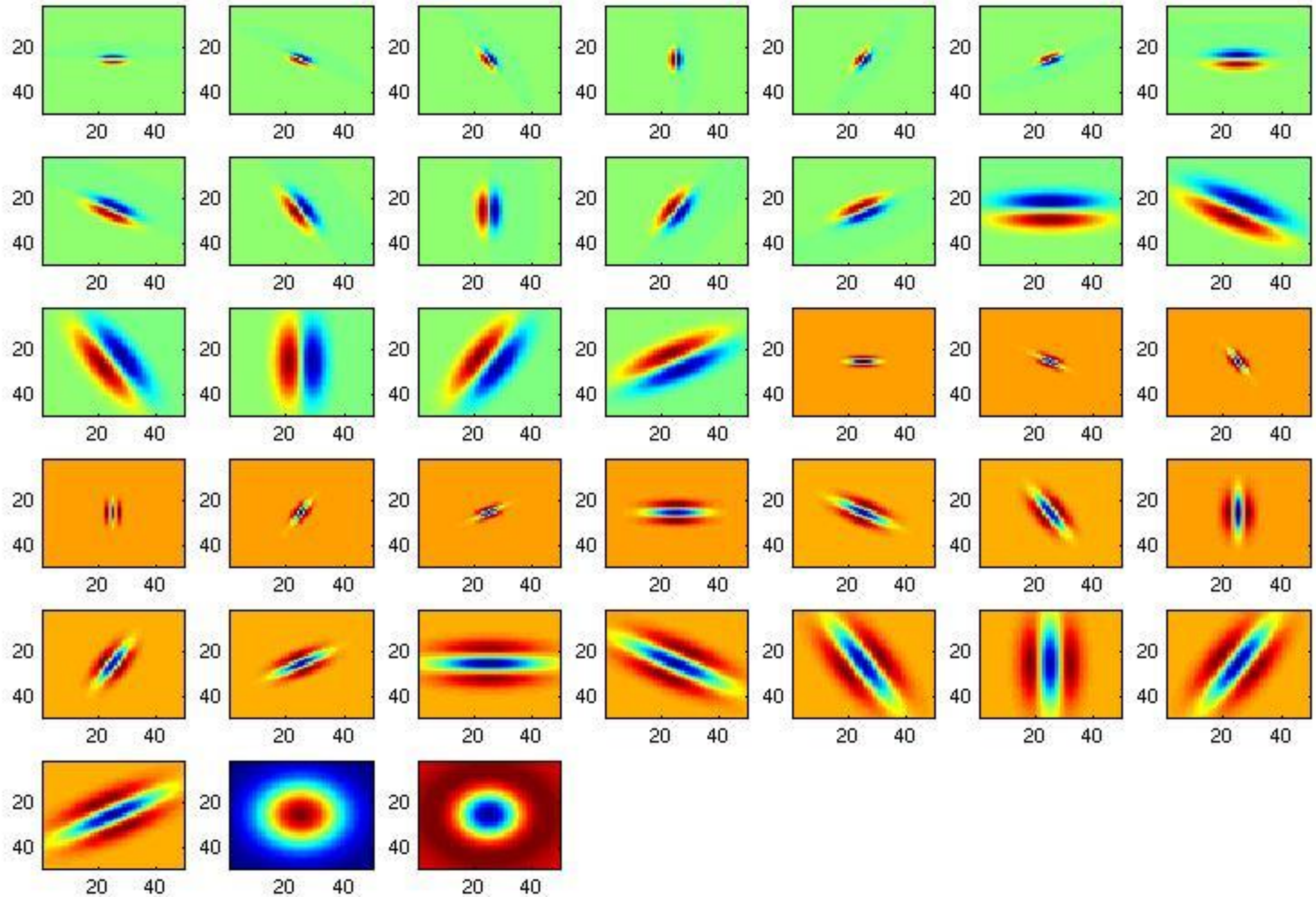
$$\Sigma = \begin{bmatrix} 16 & 0 \\ 0 & 9 \end{bmatrix}$$



$$\Sigma = \begin{bmatrix} 10 & 5 \\ 5 & 5 \end{bmatrix}$$

Slide credit: Kristen Grauman

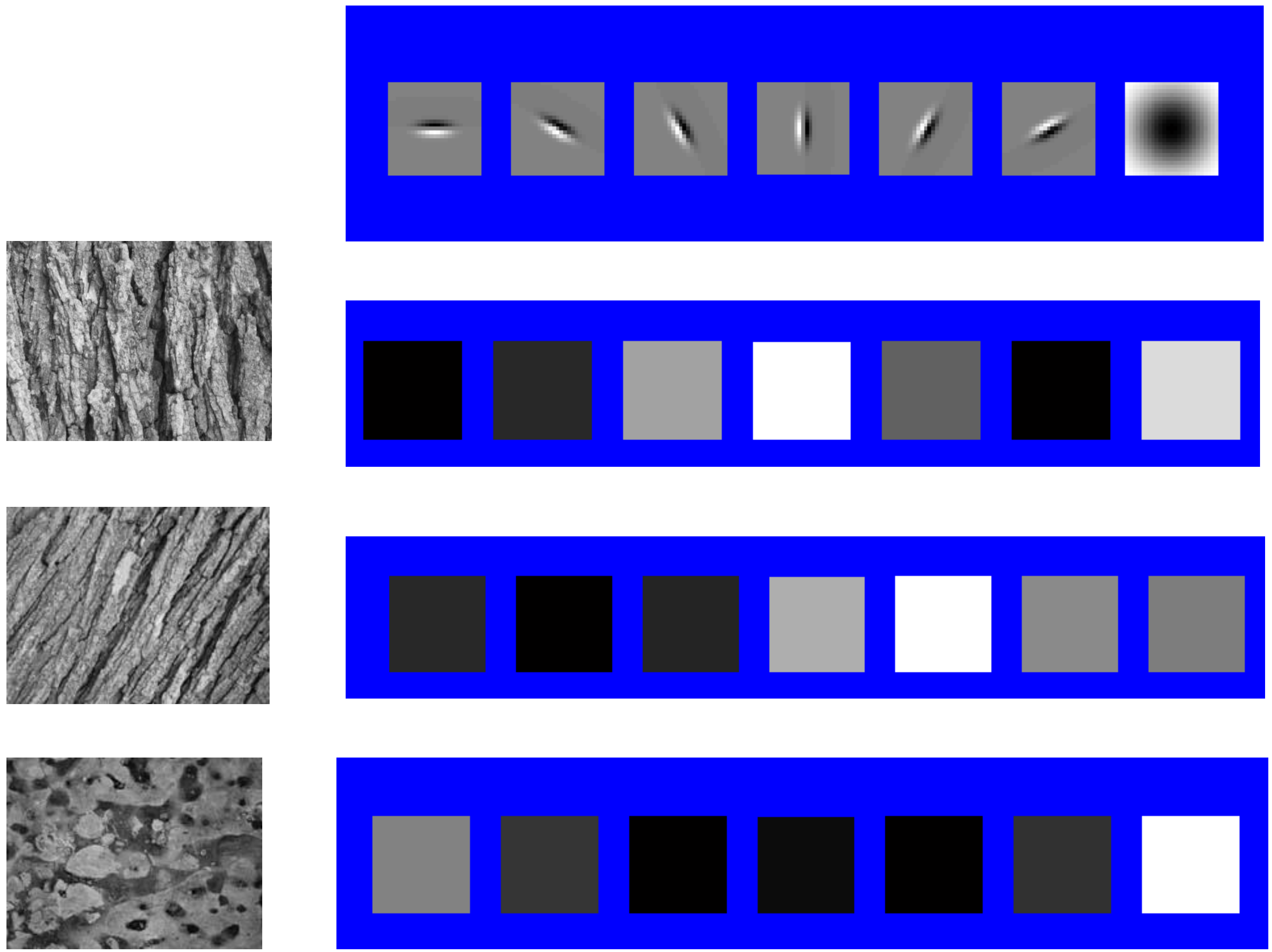
Filter bank



Slide credit: Kristen Grauman

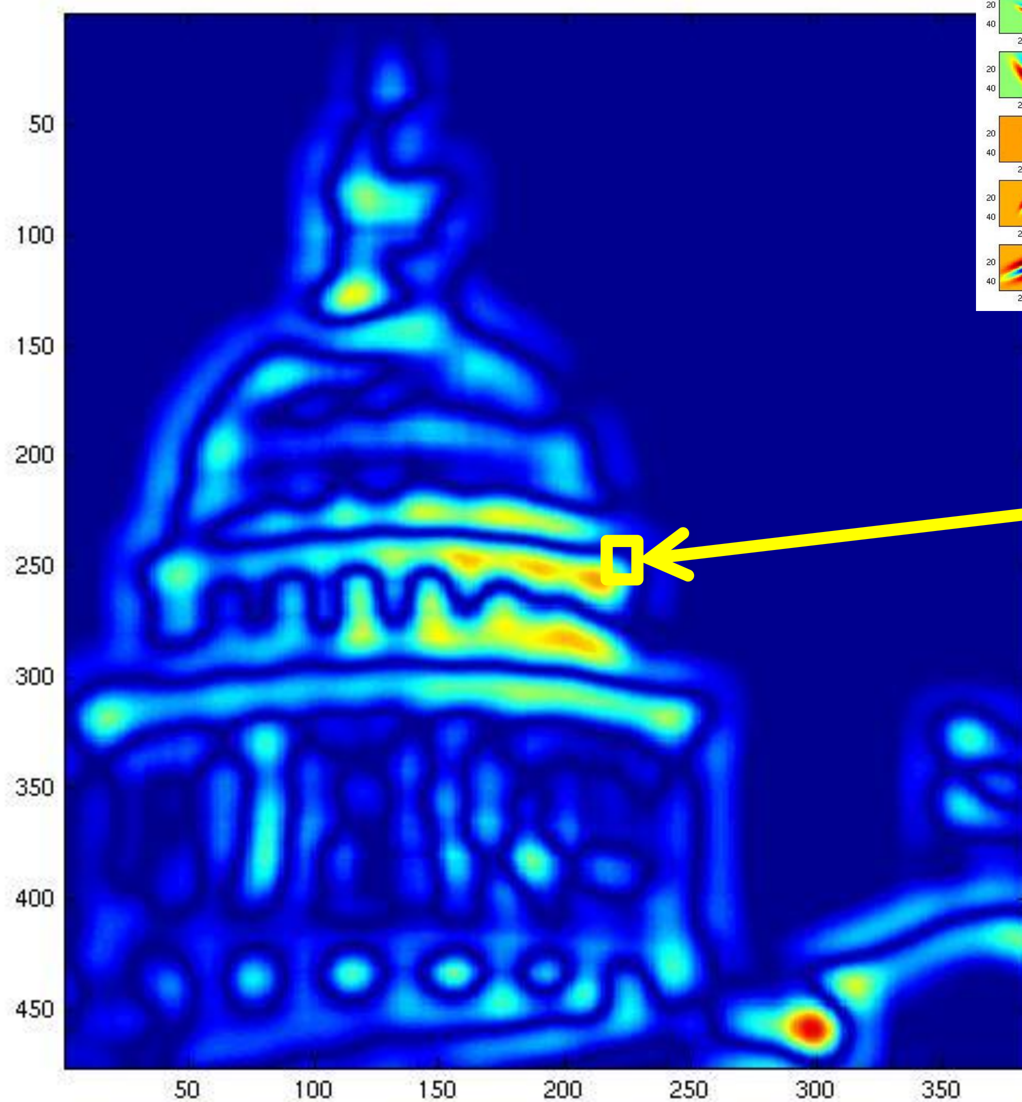
Representing texture by mean abs response

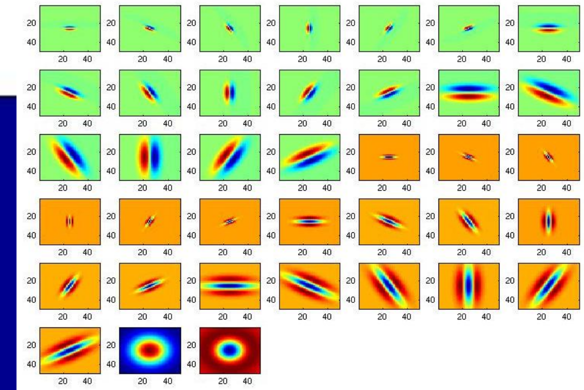
Filters



Mean abs responses

Derek Hoiem





[r1, r2, …, r38]

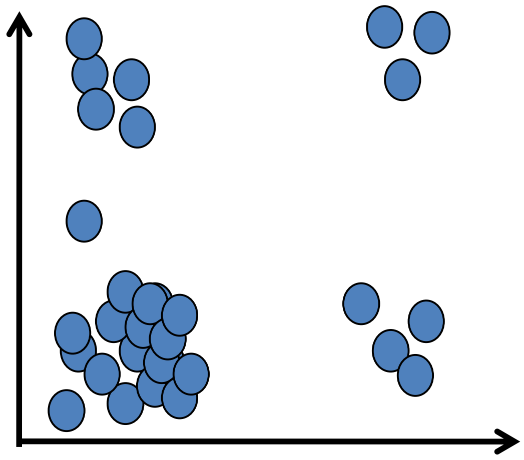
We can form a feature vector from the list of responses at each pixel.

Slide credit: Kristen Grauman

d-dimensional features

$$D(a,b) = \sqrt{\sum_{i=1}^{d} (a_i - b_i)^2}$$

Euclidean distance (L_2)



2d

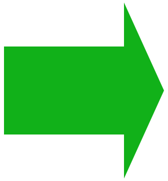
Slide credit: Kristen Grauman

Texture-related tasks

- **Shape from texture**
  - Estimate surface orientation or shape from image texture
- **Segmentation/classification** from texture cues
  - Analyze, represent texture
  - Group image regions with consistent texture
- **Synthesis**
  - Generate new texture patches/images given some examples

Texture synthesis

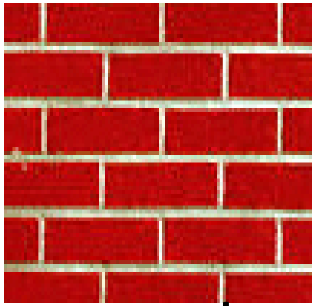
- Goal: create new samples of a given texture
- Many applications: virtual environments, hole-filling, texturing surfaces



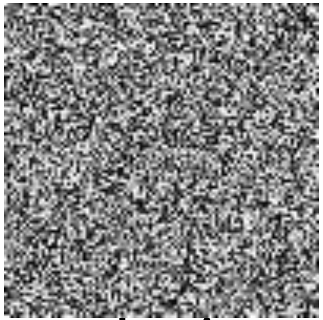
The Challenge

- Need to model the whole spectrum: from repeated to stochastic texture

Alexei A. Efros and Thomas K. Leung, “Texture Synthesis by Non-parametric Sampling,” Proc. International Conference on Computer Vision (ICCV), 1999.



repeated



stochastic

Both?

Markov Random Field

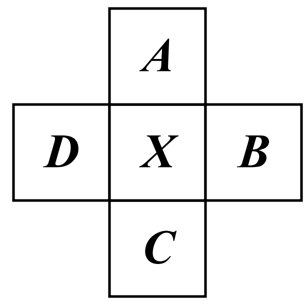
A Markov random field (MRF)

- generalization of Markov chains to two or more dimensions.

First-order MRF:

- probability that pixel *X* takes a certain value given the values of neighbors *A*, *B*, *C*, and *D*:

$$P(\mathbf{X}|\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D})$$

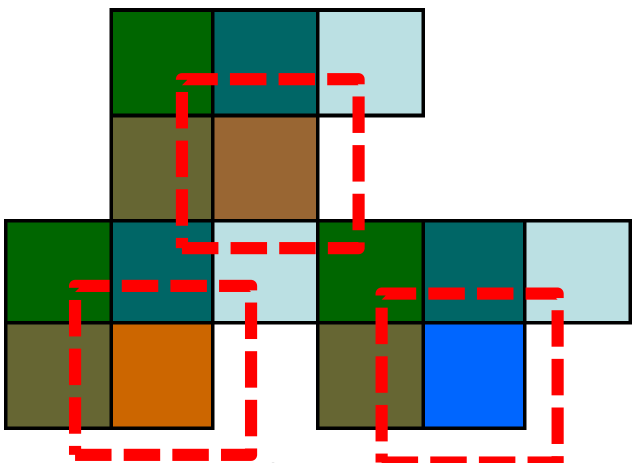


Source: S. Seitz

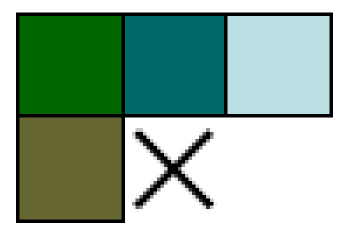
Texture synthesis: intuition

Before, we inserted the next word based on existing nearby words…

Now we want to insert **pixel intensities** based on existing nearby pixel values.



Sample of the texture ("corpus")

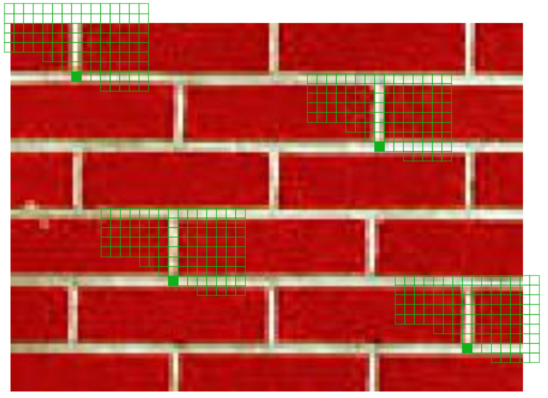


Place we want to insert next

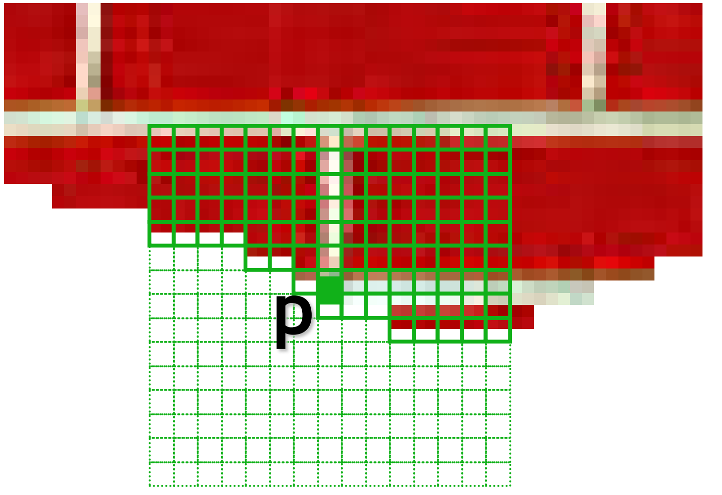
Distribution of a value of a pixel is conditioned on its neighbors alone.

Slide credit: Kristen Grauman

Synthesizing One Pixel



input image

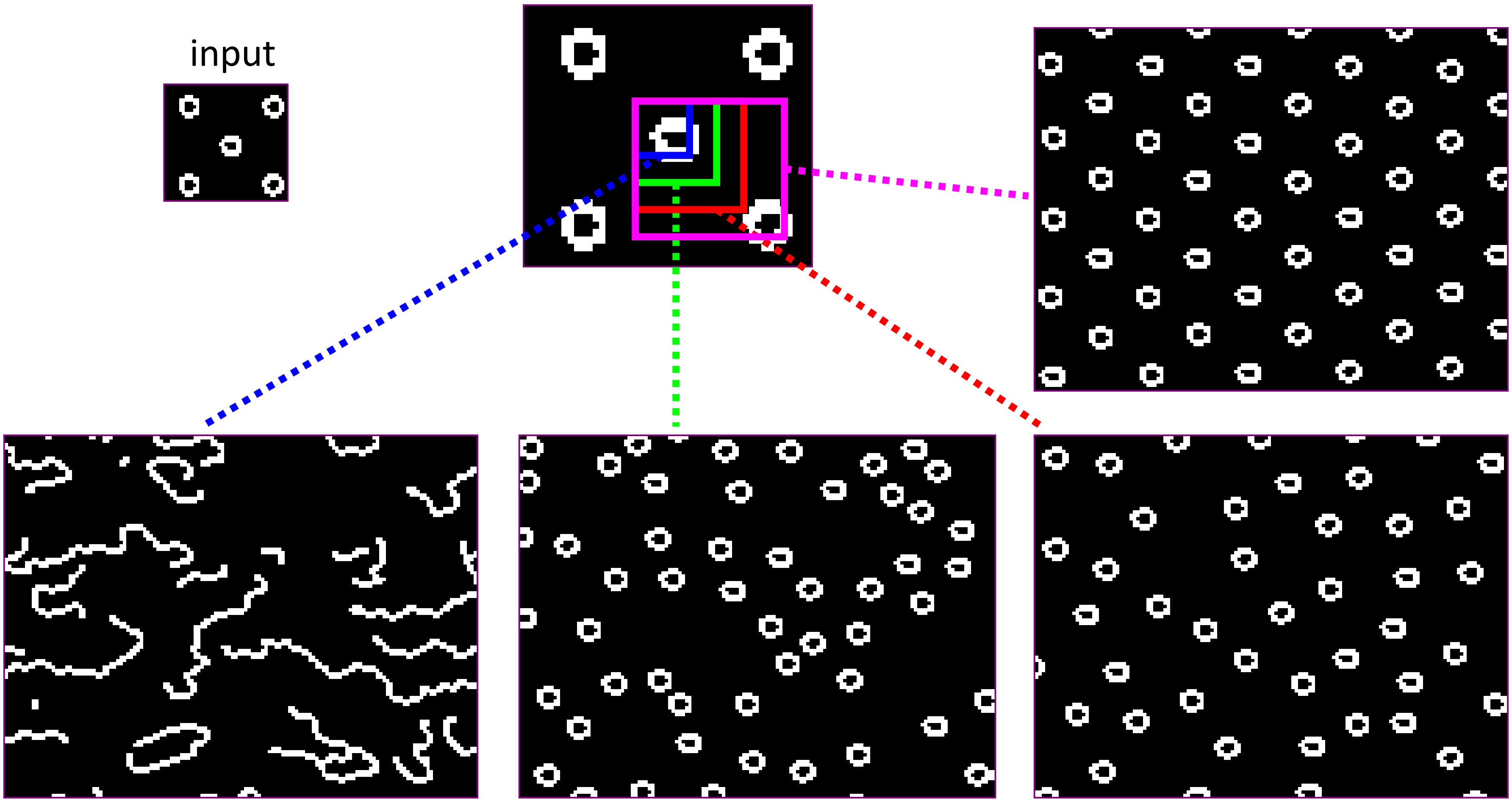


synthesized image

- What is $P(\mathbf{x}|\text{neighborhood of pixels around x})$
- Find all the windows in the image that match the neighborhood
- To synthesize **x**
  - pick one matching window at random
  - assign **x** to be the center pixel of that window
- An **exact** neighbourhood match might not be present, so find the **best** matches using **SSD error** and randomly choose between them, preferring better matches with higher probability

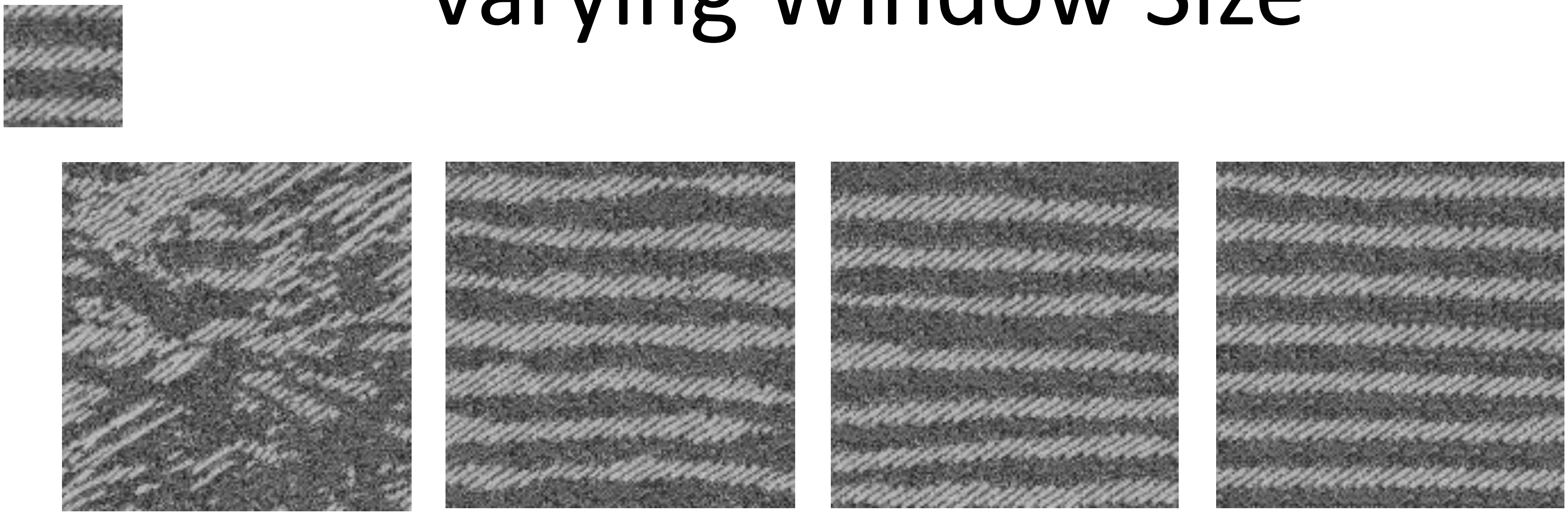
Slide from Alyosha Efros, ICCV 1999

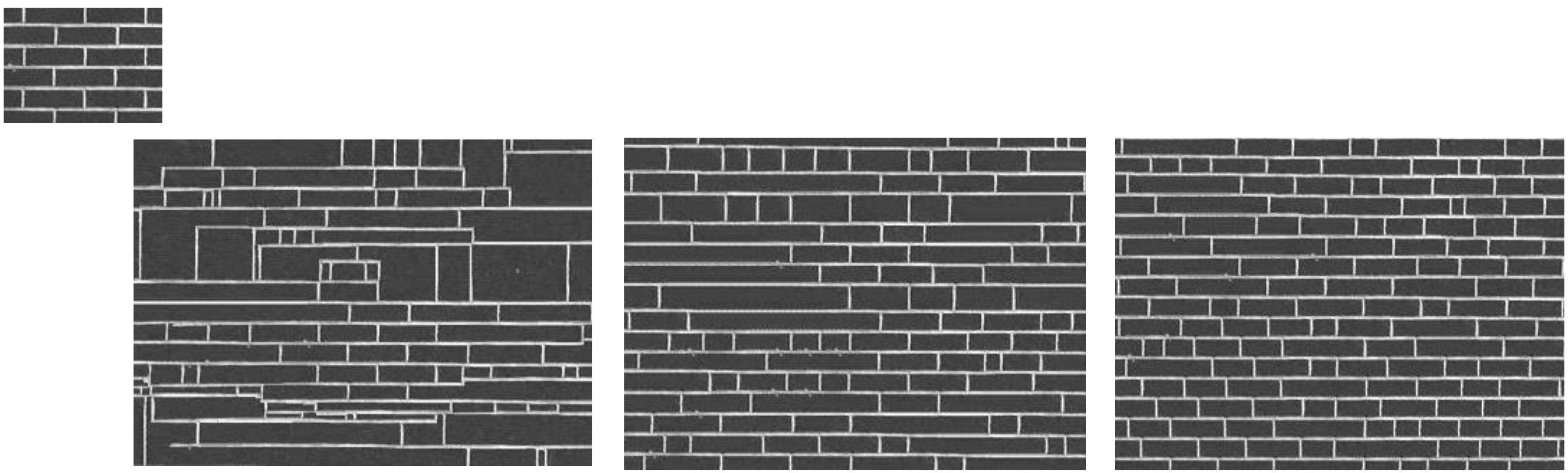
Neighborhood Window



Slide from Alyosha Efros, ICCV 1999

Varying Window Size



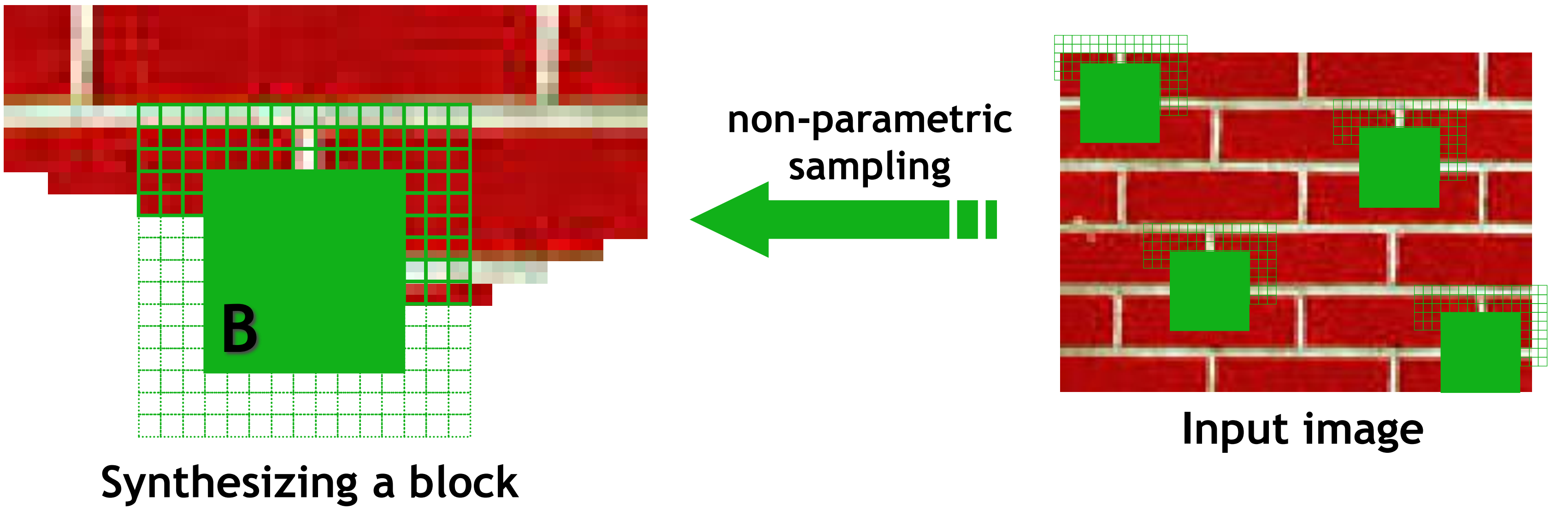


Increasing window size

Slide from Alyosha Efros, ICCV 1999

- The Efros & Leung algorithm
  - Simple
  - Surprisingly good results
  - Synthesis is easier than analysis!
  - …but very slow

Image Quilting [Efros & Freeman 2001]



- <u>Observation:</u> neighbor pixels are highly correlated

<u>Idea:</u> unit of synthesis = block

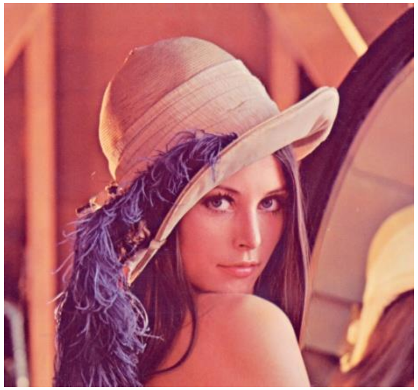
- Exactly the same but now we want P(**B**|N(**B**))
- Much faster: synthesize all pixels in a block at once

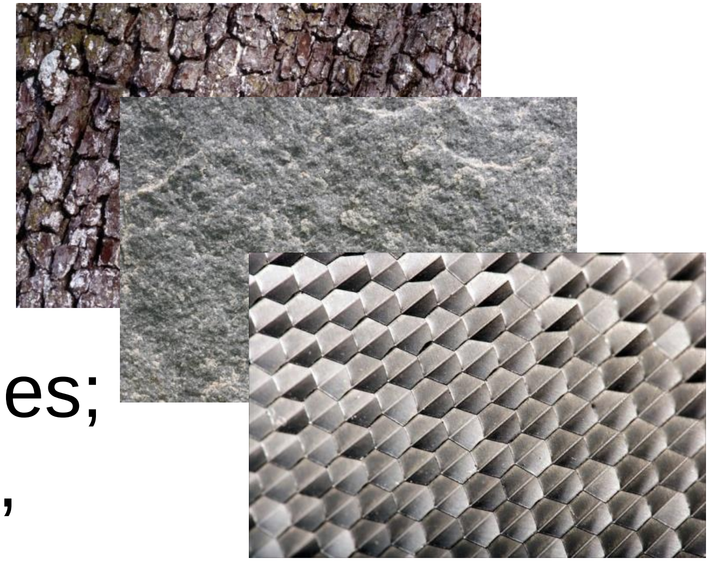
Slide from Alyosha Efros, ICCV 1999

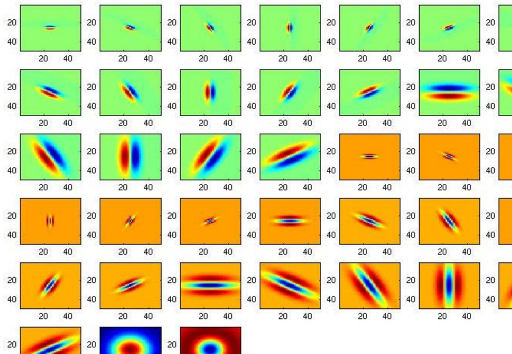
Summary

- Texture is a useful property that is often indicative of materials, appearance cues
- **Texture representations** attempt to summarize repeating patterns of local structure
- **Filter banks** useful to measure redundant variety of structures in local neighborhood
  - Feature spaces can be multi-dimensional
- Neighborhood statistics can be exploited to “sample” or **synthesize** new texture regions
  - Example-based technique

So far: features and filters



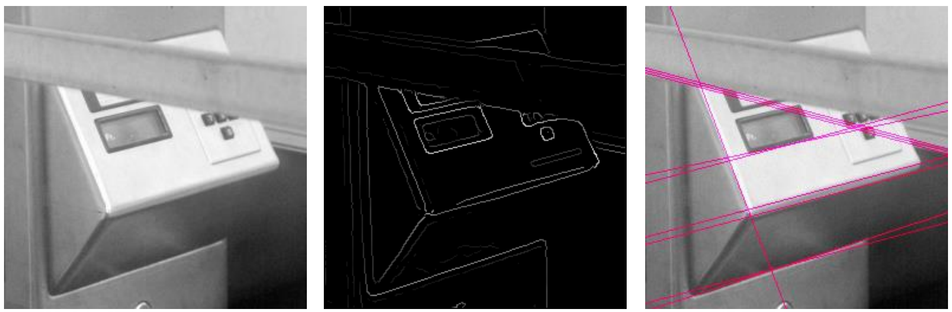




Transforming images; gradients, textures, edges, flow

Now: Fitting

- Want to associate a model with observed features



[Fig from Marszalek & Schmid, 2007]

For example, the model could be a line, a circle, or an arbitrary shape.

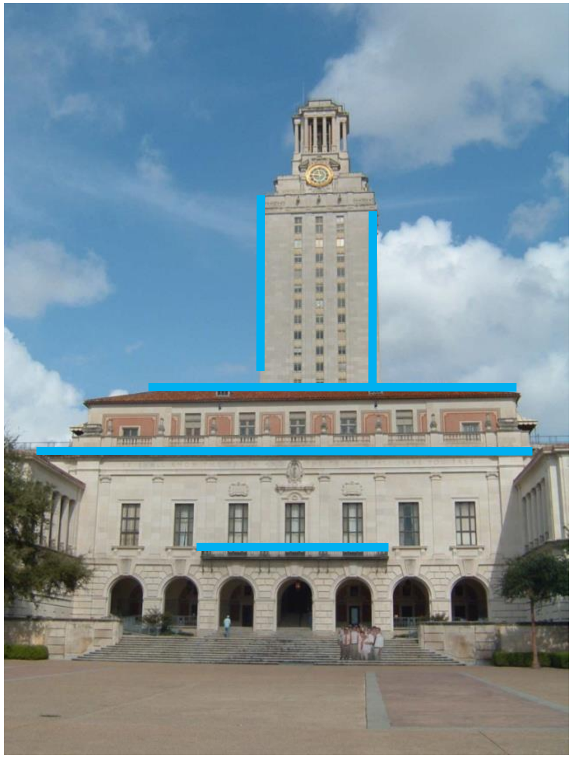
Fitting: Main idea

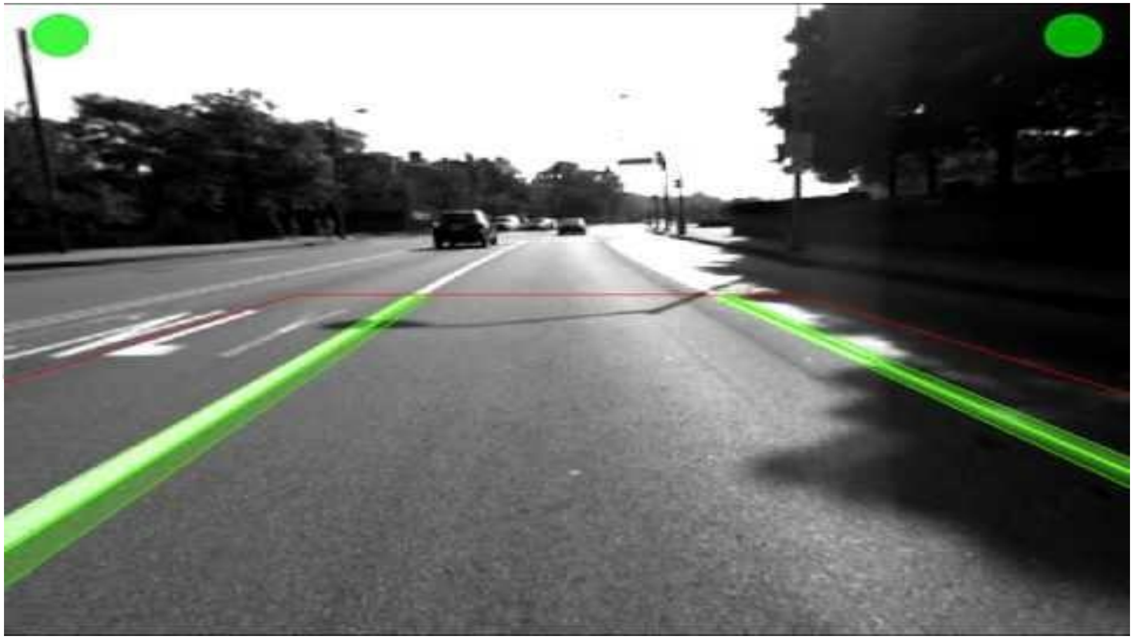
- Choose a parametric model to represent a set of features
- Membership criterion is not local
  - Can’t tell whether a point belongs to a given model just by looking at that point
- Three main questions:
  - What model represents this set of features best?
  - Which of several model instances gets which feature?
  - How many model instances are there?
- Computational complexity is important
  - It is infeasible to examine every possible set of parameters and every possible combination of features

Slide credit: L. Lazebnik

Case study: Line fitting

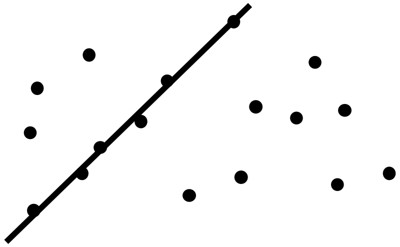
- Why fit lines?
  Many objects characterized by presence of straight lines

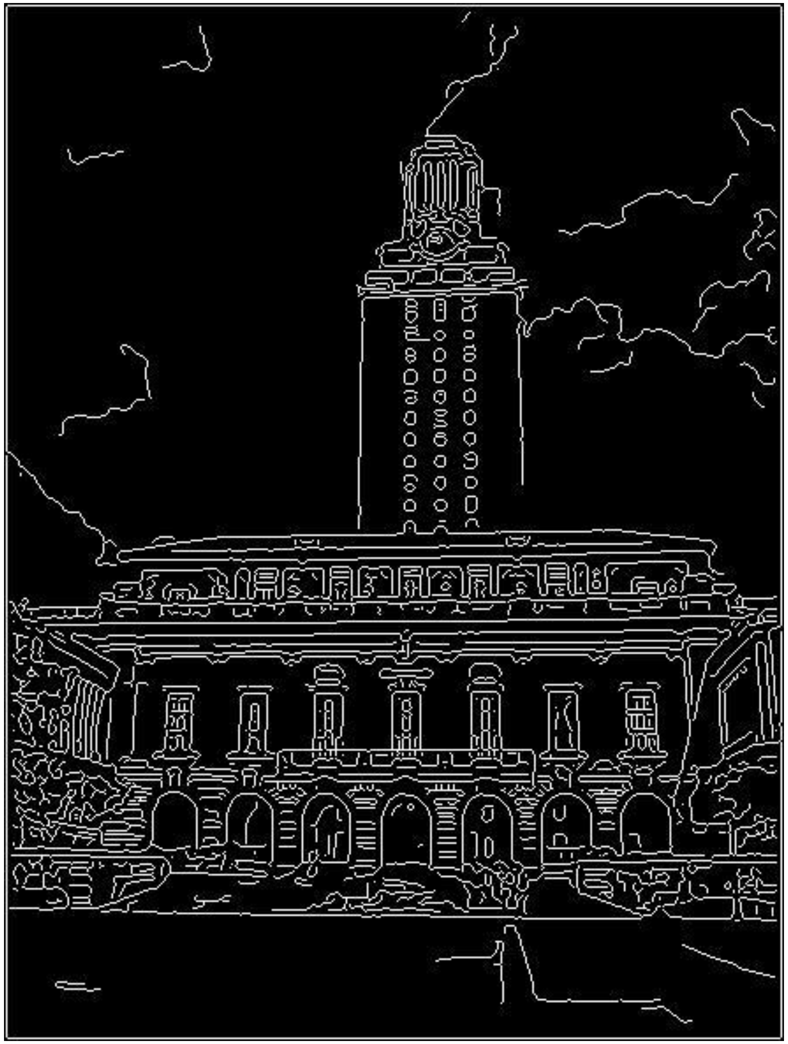




- Wait, why aren't we done just by running edge detection?

Difficulty of line fitting





- **Extra** edge points (clutter), multiple models:
  - which points go with which line, if any?
- Only some parts of each line detected, and some parts are **missing:**
  - how to find a line that bridges missing evidence?
- **Noise** in measured edge points, orientations:
  - how to detect true underlying parameters?

Voting

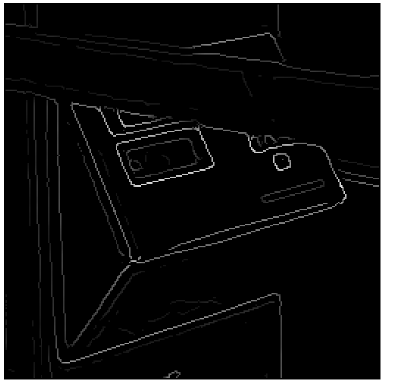
- It's not feasible to check all combinations of features by fitting a model to each possible subset.

- **Voting** is a general technique where we let the features *vote for all models that are compatible with it*.
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive a lot of votes.

- Noise & clutter features will cast votes too, *but* typically their votes should be inconsistent with the majority of "good" features.

Fitting lines: Hough transform

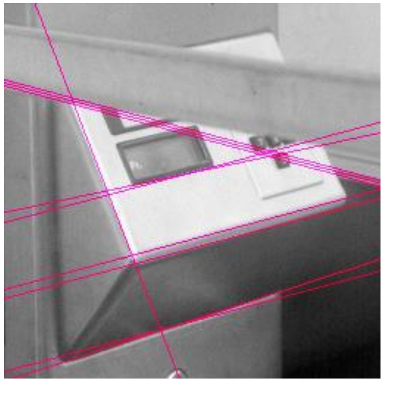
- Given points that belong to a line, what is the line?
- How many lines are there?
- Which points belong to which lines?



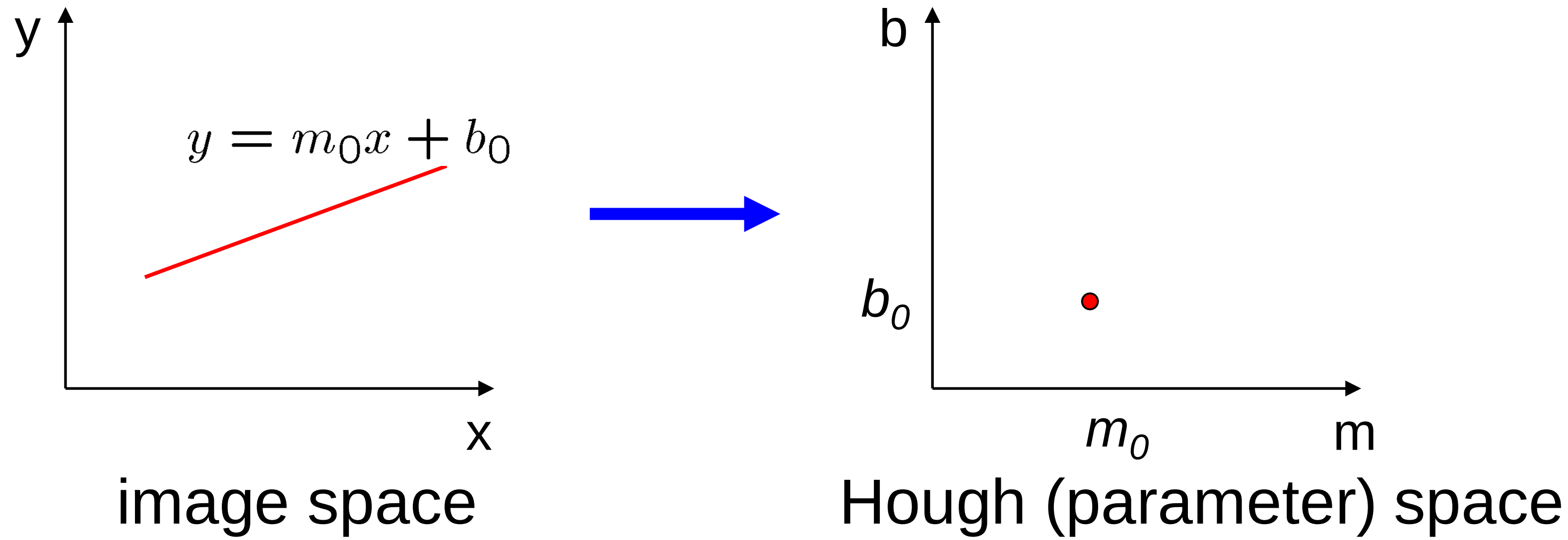
- **Hough Transform** is a voting technique that can be used to answer all of these questions.

<u>Main idea</u>:

1. Record vote for each possible line on which each edge point lies.
2. Look for lines that get many votes.



Finding lines in an image: Hough space

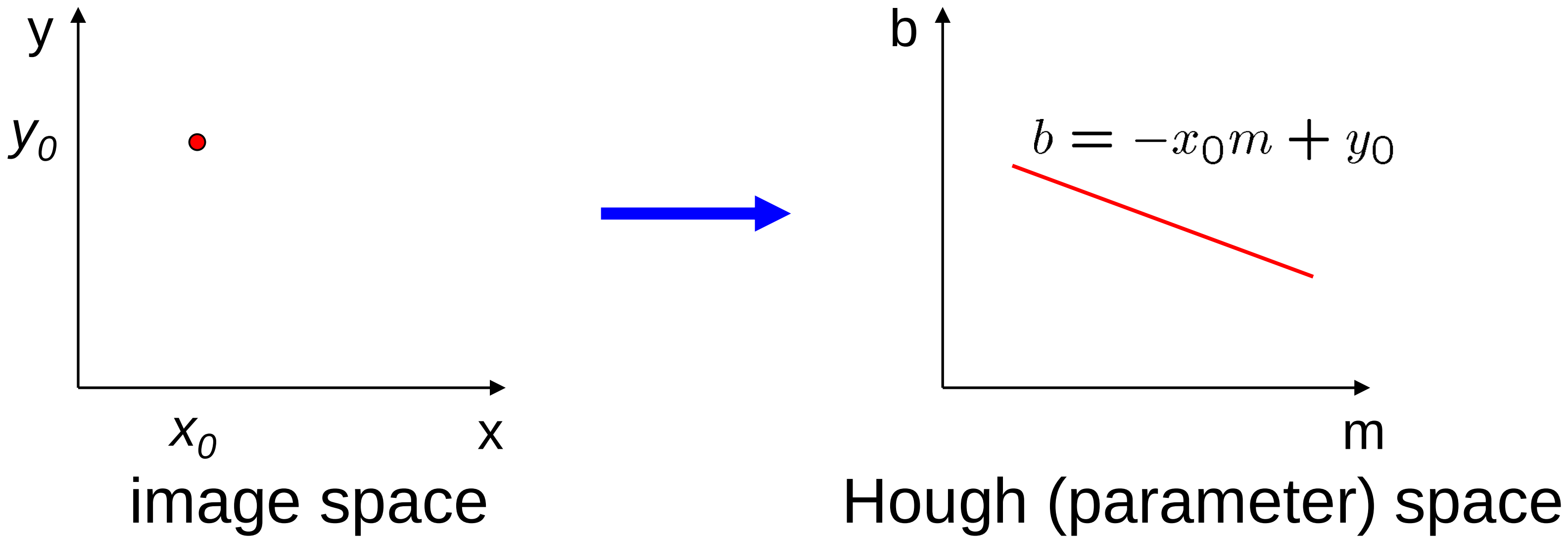


Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points (x,y), find all (m,b) such that y = mx + b

Slide credit: Steve Seitz

Finding lines in an image: Hough space



Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points (x,y), find all (m,b) such that y = mx + b
- What does a point (x_0, y_0) in the image space map to?
  - Answer: the solutions of $b = -x_0m + y_0$
  - this is a line in Hough space

Slide credit: Steve Seitz

Finding lines in an image: Hough space

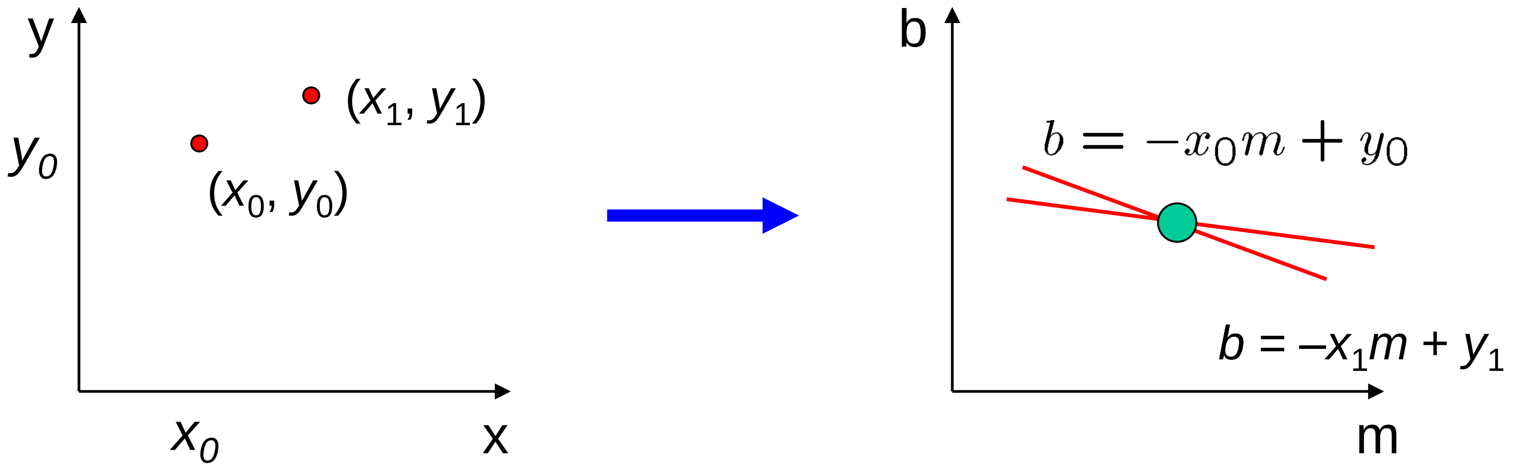


image space

Hough (parameter) space

What are the line parameters for the line that contains both (x_0, y_0) and (x_1, y_1)?

- It is the intersection of the lines $b = -x_0 m + y_0$ and $b = -x_1 m + y_1$

Finding lines in an image: Hough algorithm

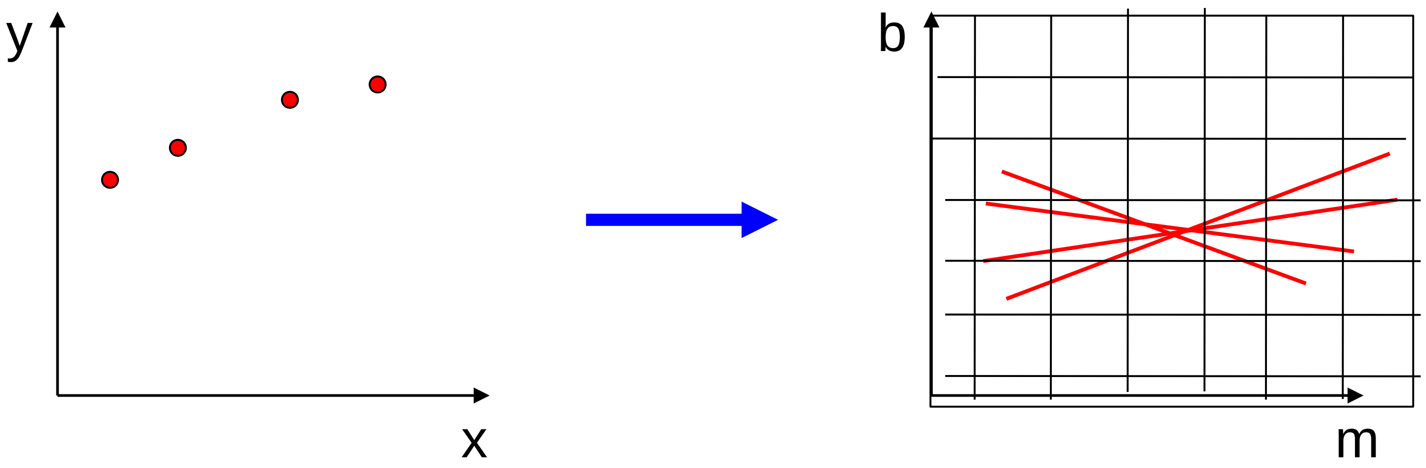


image space

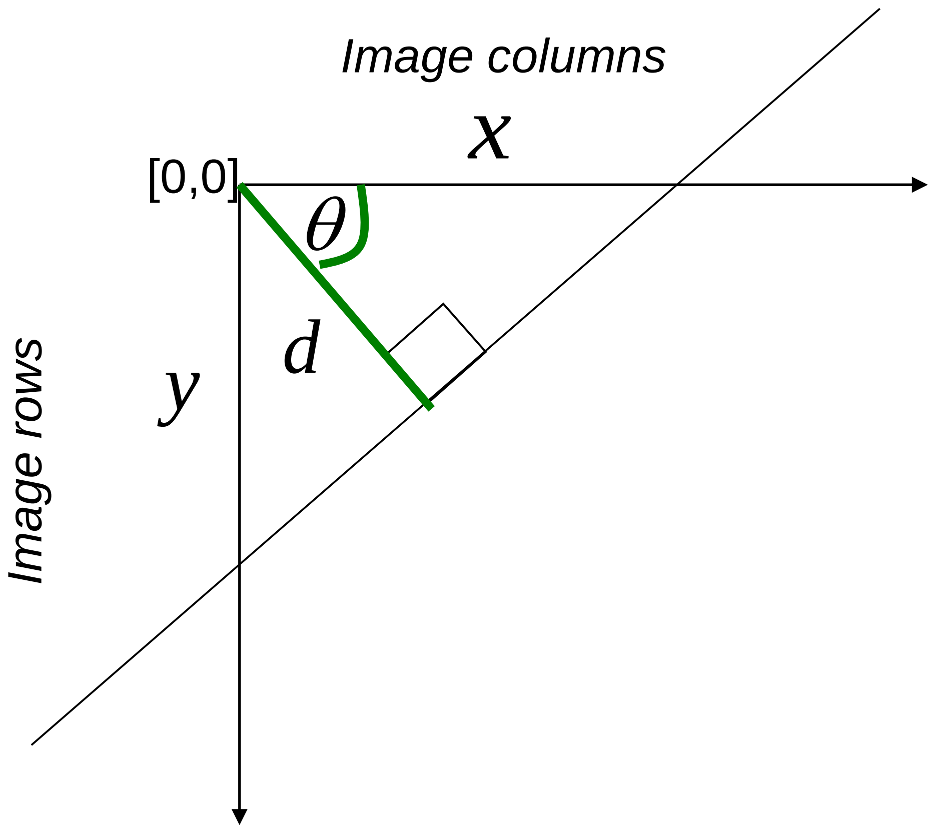
Hough (parameter) space

How can we use this to find the most likely parameters (m,b) for the most prominent line in the image space?

- Let each edge point in image space *vote* for a set of possible parameters in Hough space
- Accumulate votes in discrete set of bins*; parameters with the most votes indicate line in image space.

Polar representation for lines

Issues with usual (*m,b*) parameter space: can take on infinite values, undefined for vertical lines.



d : perpendicular distance from line to origin

θ : angle the perpendicular makes with the x-axis

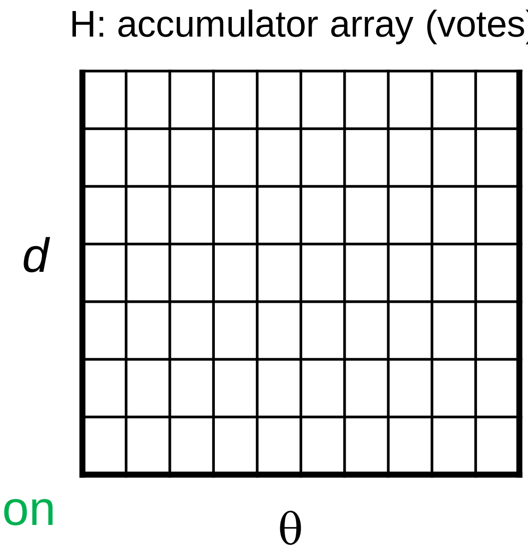
$$x\cos\theta - y\sin\theta = d$$

Point in image space → sinusoid segment in Hough space

Hough transform algorithm

Using the polar parameterization:

$$x\cos\theta - y\sin\theta = d$$



Basic Hough transform algorithm

1. Initialize H[d, θ]=0
2. for each edge point I[x,y] in the image

   for θ = [θ_{min} to θ_{max}] // some quantization

   $$d = x\cos\theta - y\sin\theta$$

   H[d, θ] += 1
3. Find the value(s) of (d, θ) where H[d, θ] is maximum
4. The detected line in the image is given by $d = x\cos\theta - y\sin\theta$

Time complexity (in terms of number of votes per pt)?

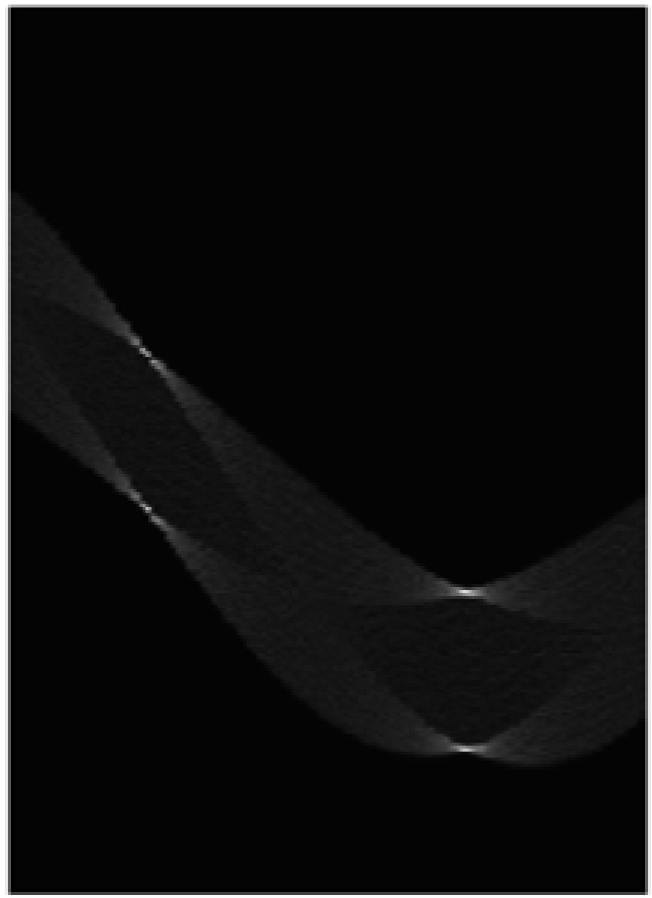
Source: Steve Seitz


How Hough
Transform
works

Example: What was the shape?

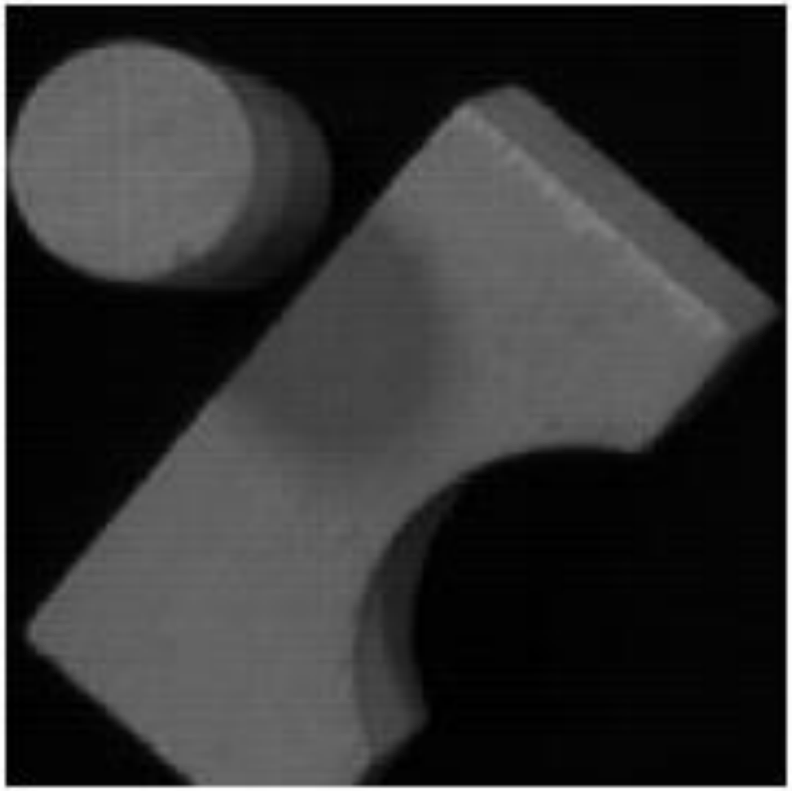
Square :

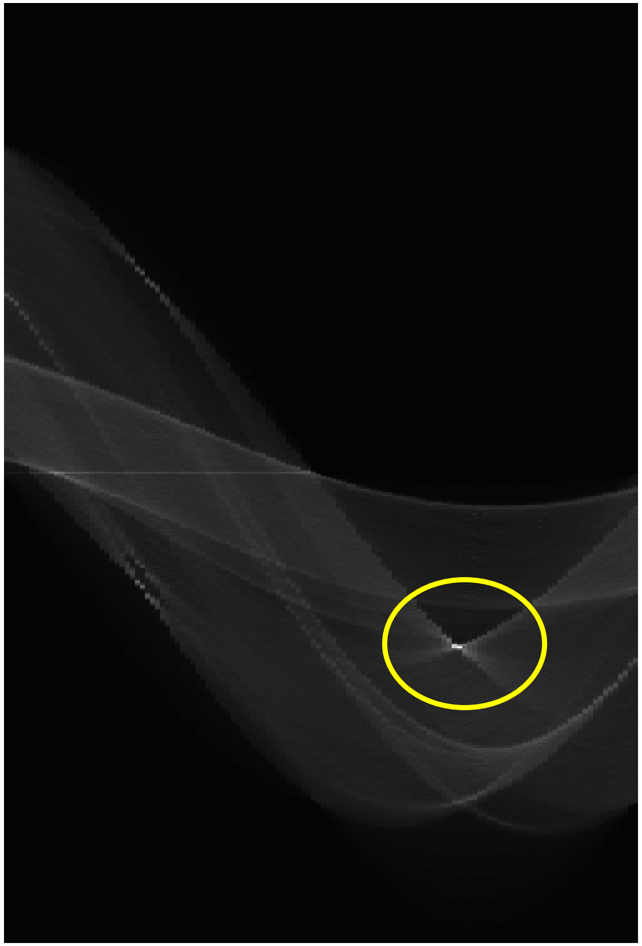
d



θ

Example: Hough transform for straight lines

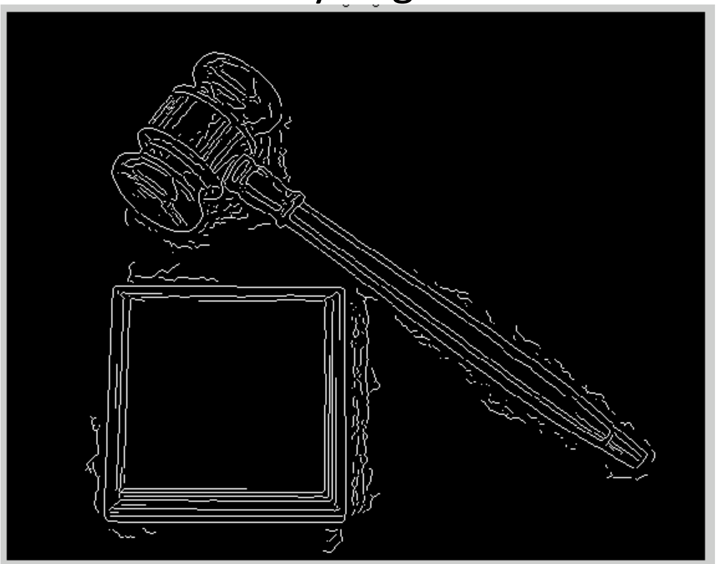




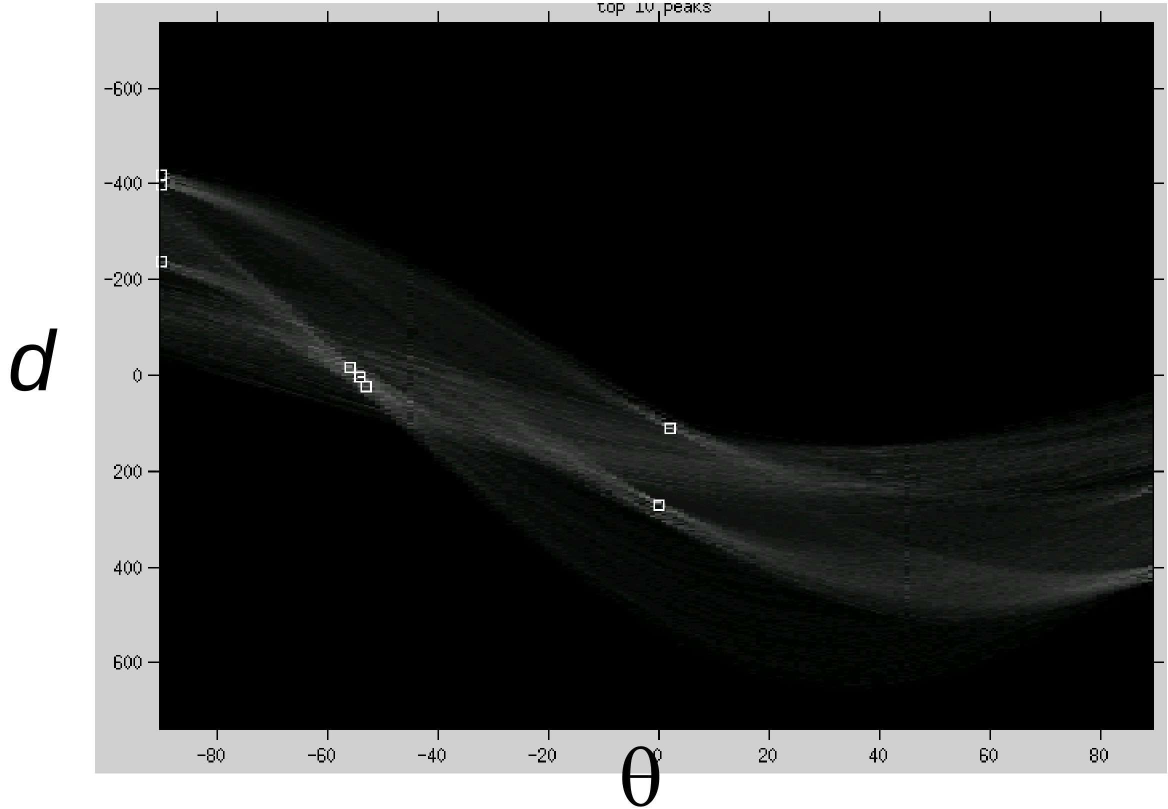
Which line generated this peak?

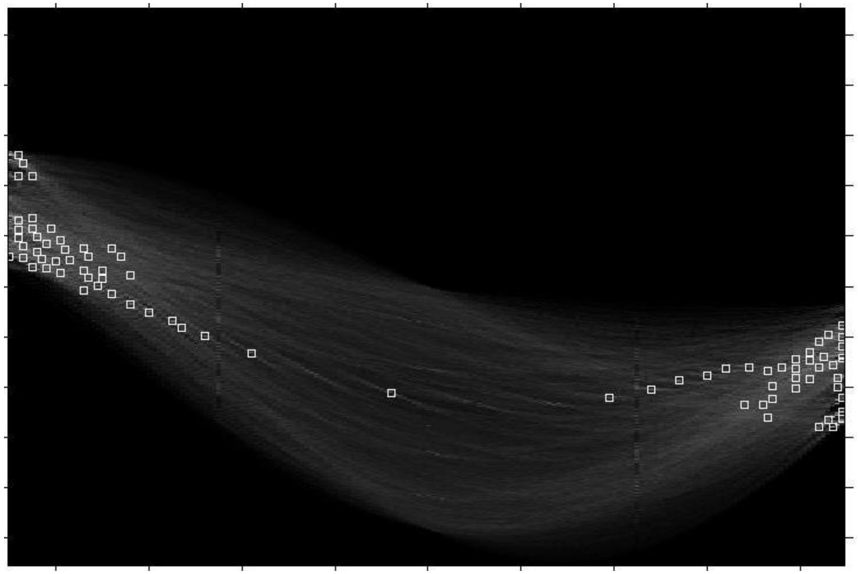
Original image

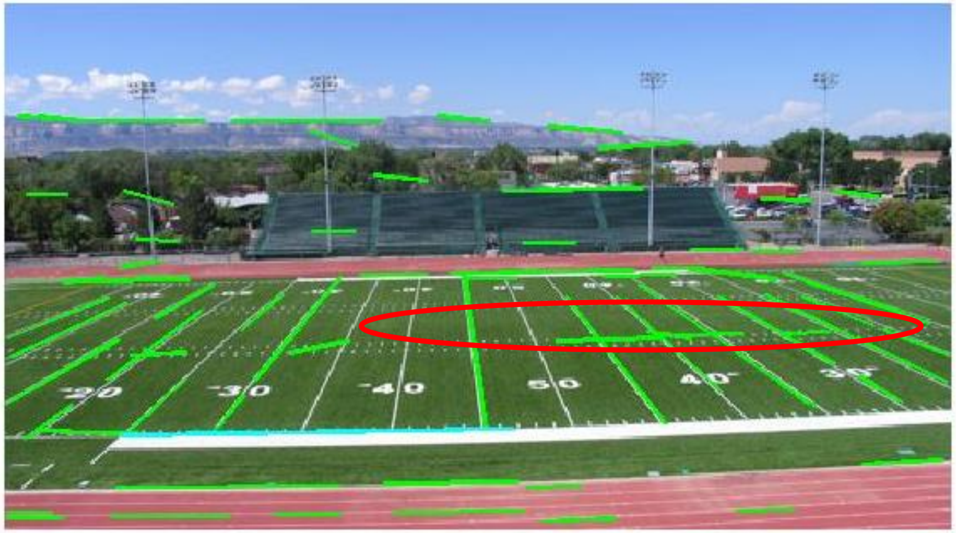
Canny edges



Decode the vote space.

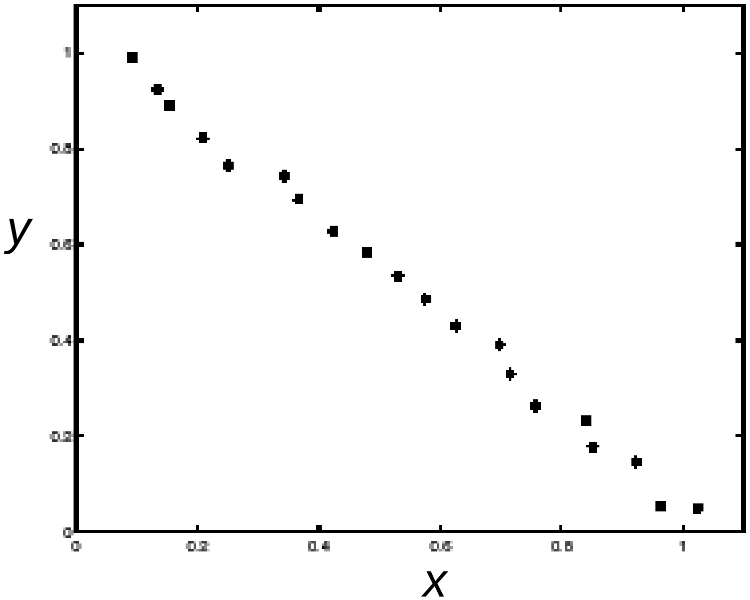






Showing longest segments found

Impact of noise on Hough



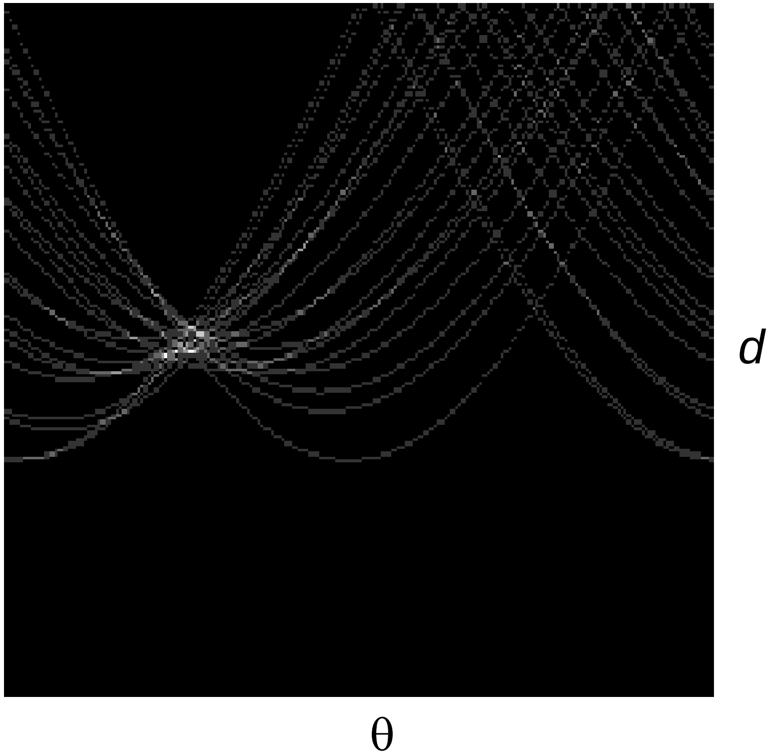
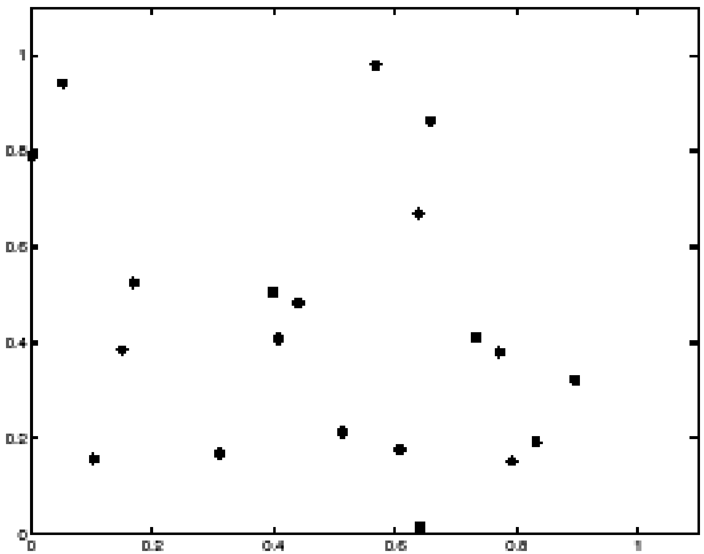


Image space edge coordinates

Votes

What difficulty does this present for an implementation?

Impact of noise on Hough



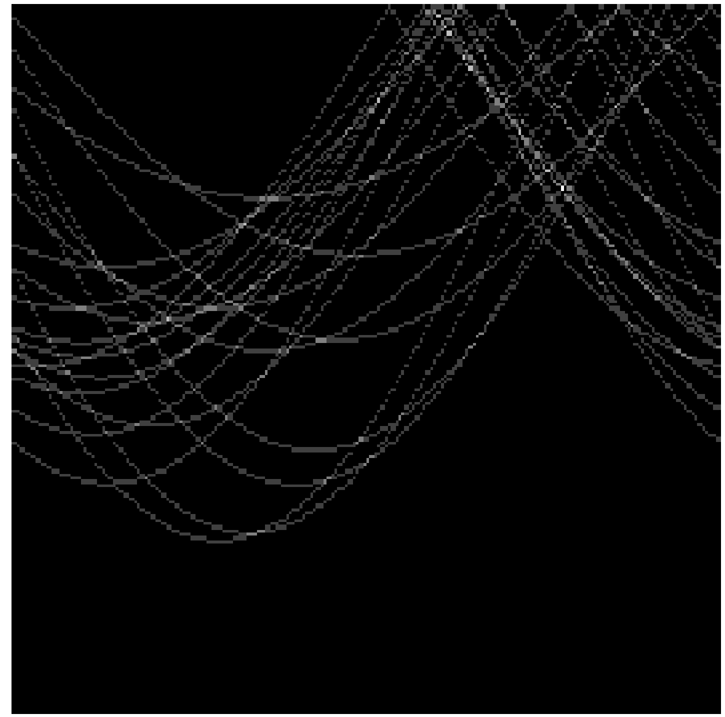


Image space edge coordinates

Votes

Here, everything appears to be “noise”, or random edge points, but we still see peaks in the vote space.

Extensions

Extension 1: Use the image gradient

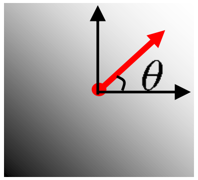
1. same
2. for each edge point I[x,y] in the image

   θ = gradient at (x,y)

   $$d = x \cos\theta - y \sin\theta$$

   H[d, θ] += 1
3. same
4. same

(Reduces degrees of freedom)



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

Extensions

Extension 1: Use the image gradient

1. same
2. for each edge point I[x,y] in the image

   compute unique (d, θ) based on image gradient at (x,y)

   $H[d, \theta]$ += 1
3. same
4. same

(Reduces degrees of freedom)

Extension 2

- give more votes for stronger edges (use magnitude of gradient)

Extension 3

- change the sampling of (d, θ) to give more/less resolution

Extension 4

- The same procedure can be used with circles, squares, or any other shape…

Source: Steve Seitz

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r, unknown gradient direction

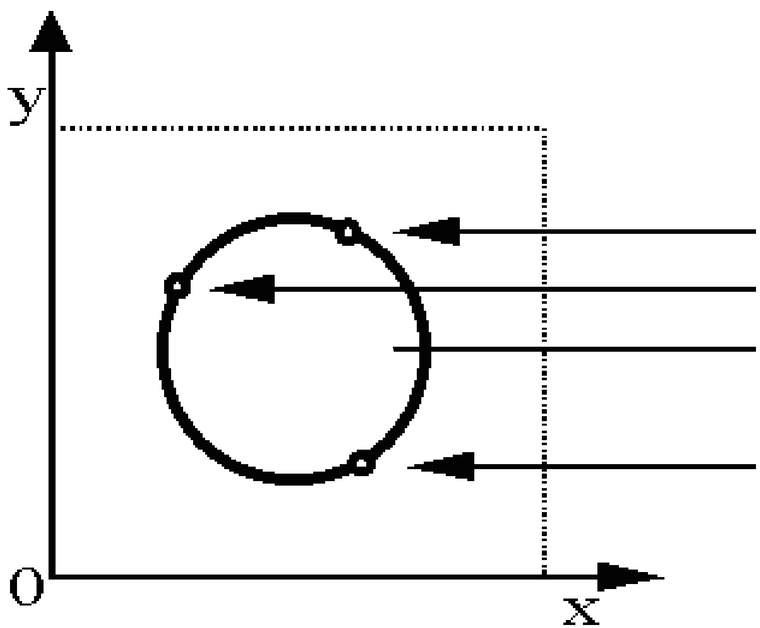
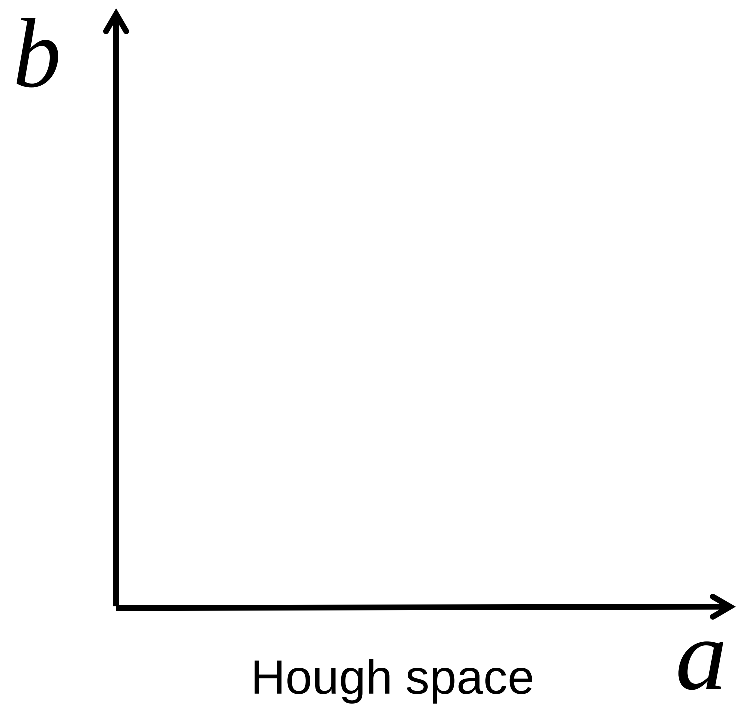


Image space



Hough space

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r, unknown gradient direction

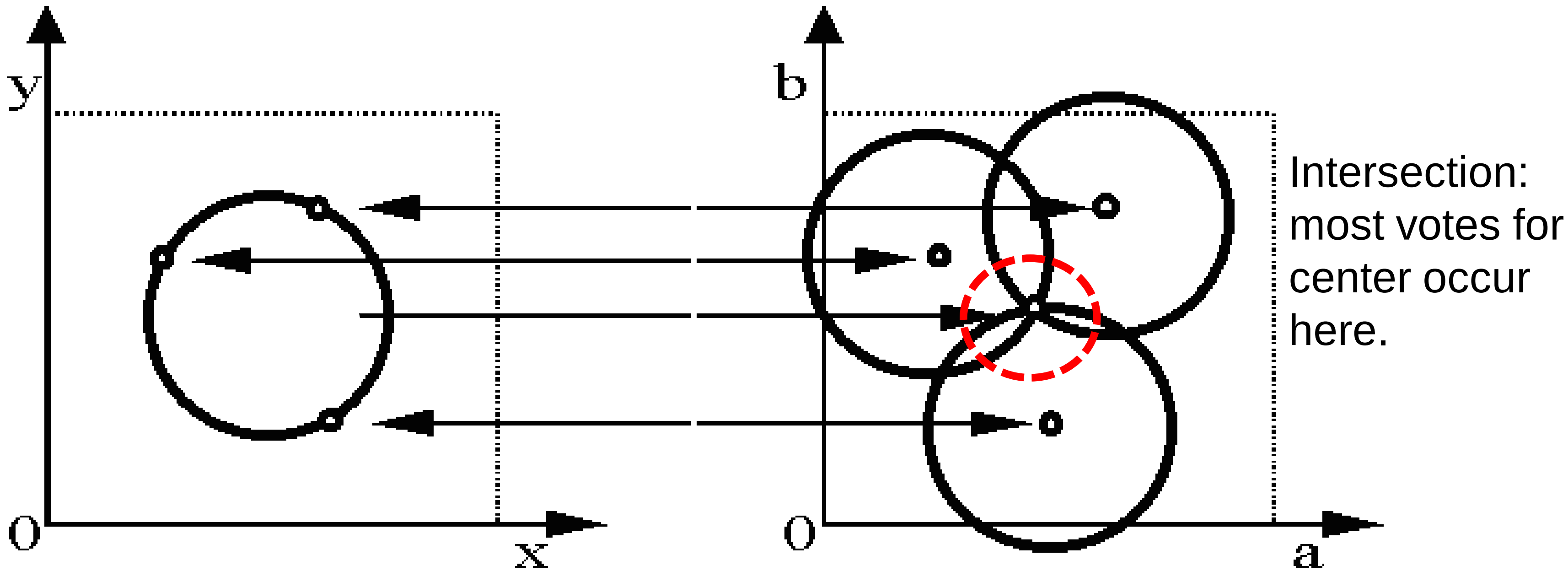


Image space

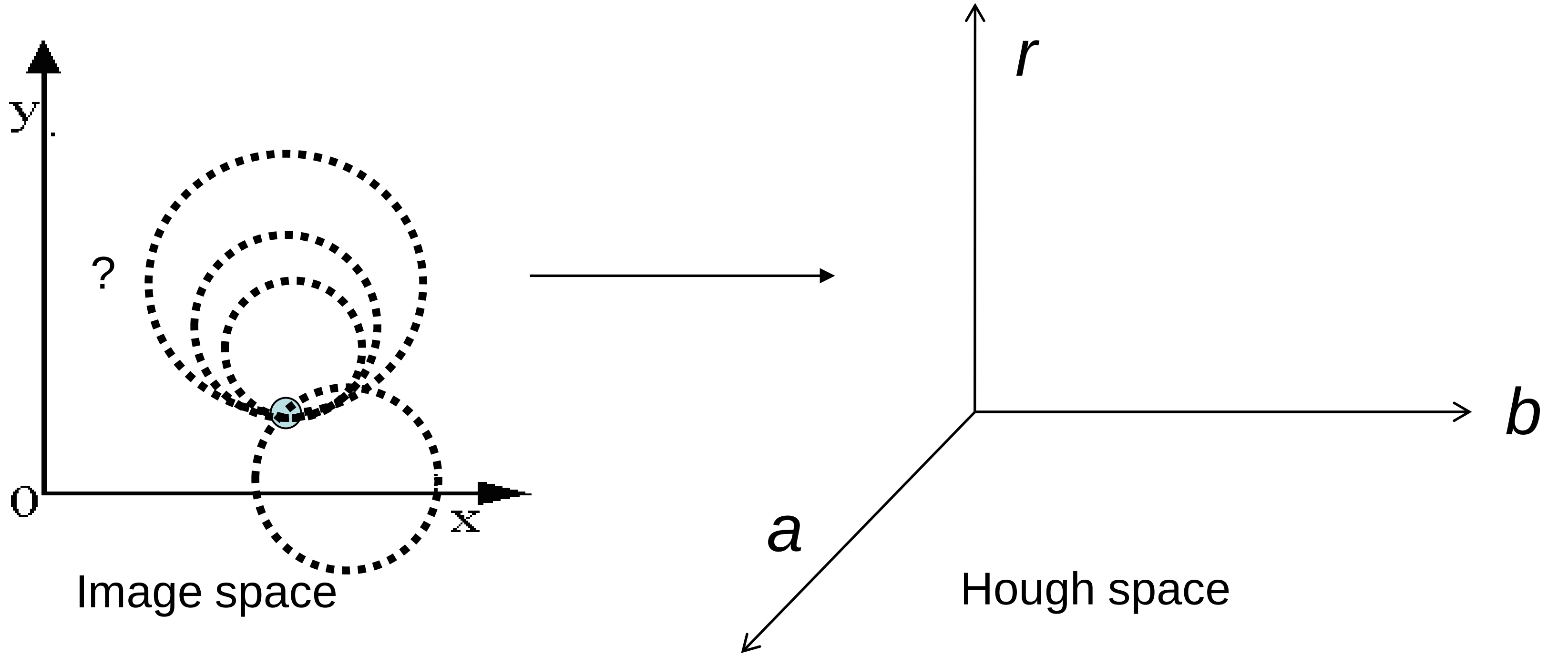
Hough space

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r, unknown gradient direction



Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r, unknown gradient direction

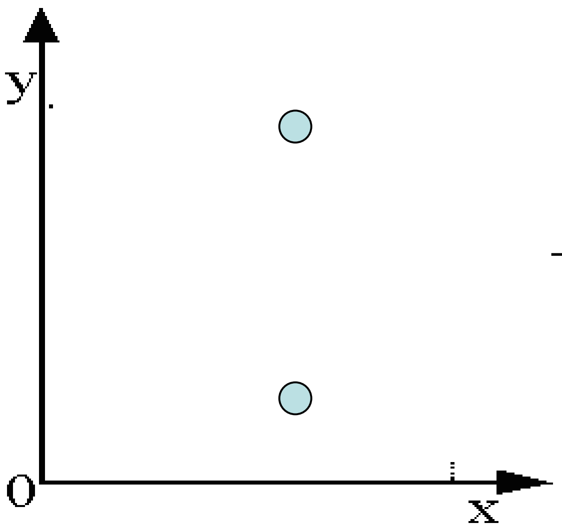
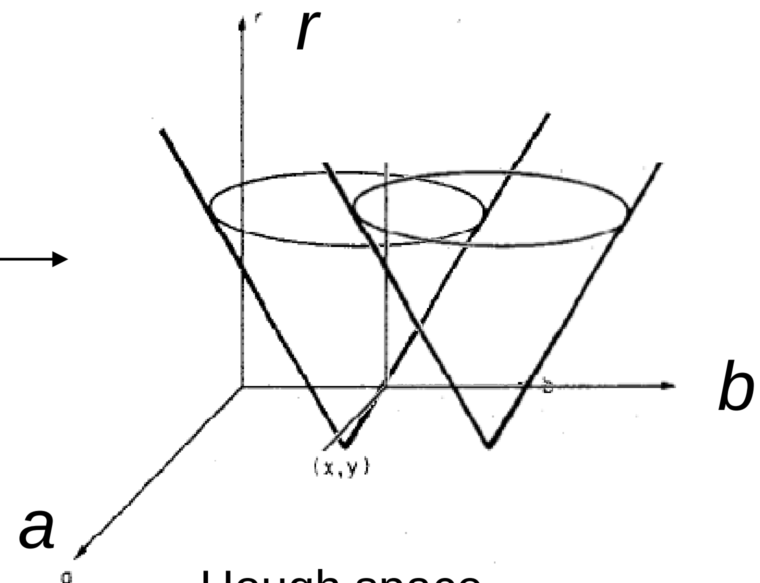


Image space



Hough space

Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For an unknown radius r, **known** gradient direction

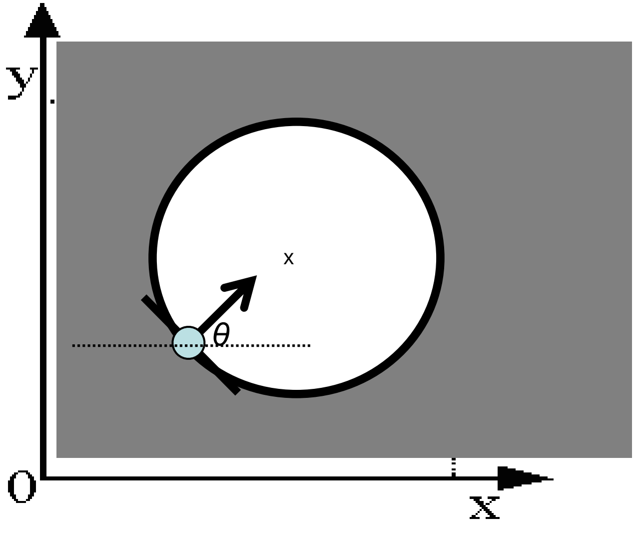
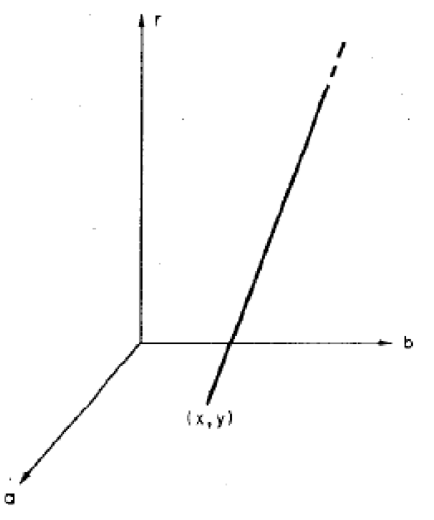


Image space



Hough space

Hough transform for circles

For every edge pixel (x,y) :

For each possible radius value r:

For each possible gradient direction θ:

// or use estimated gradient at (x,y)

$a = x + r\cos(\theta)$ // column

$b = y - r\sin(\theta)$ // row

H[a,b,r] += 1

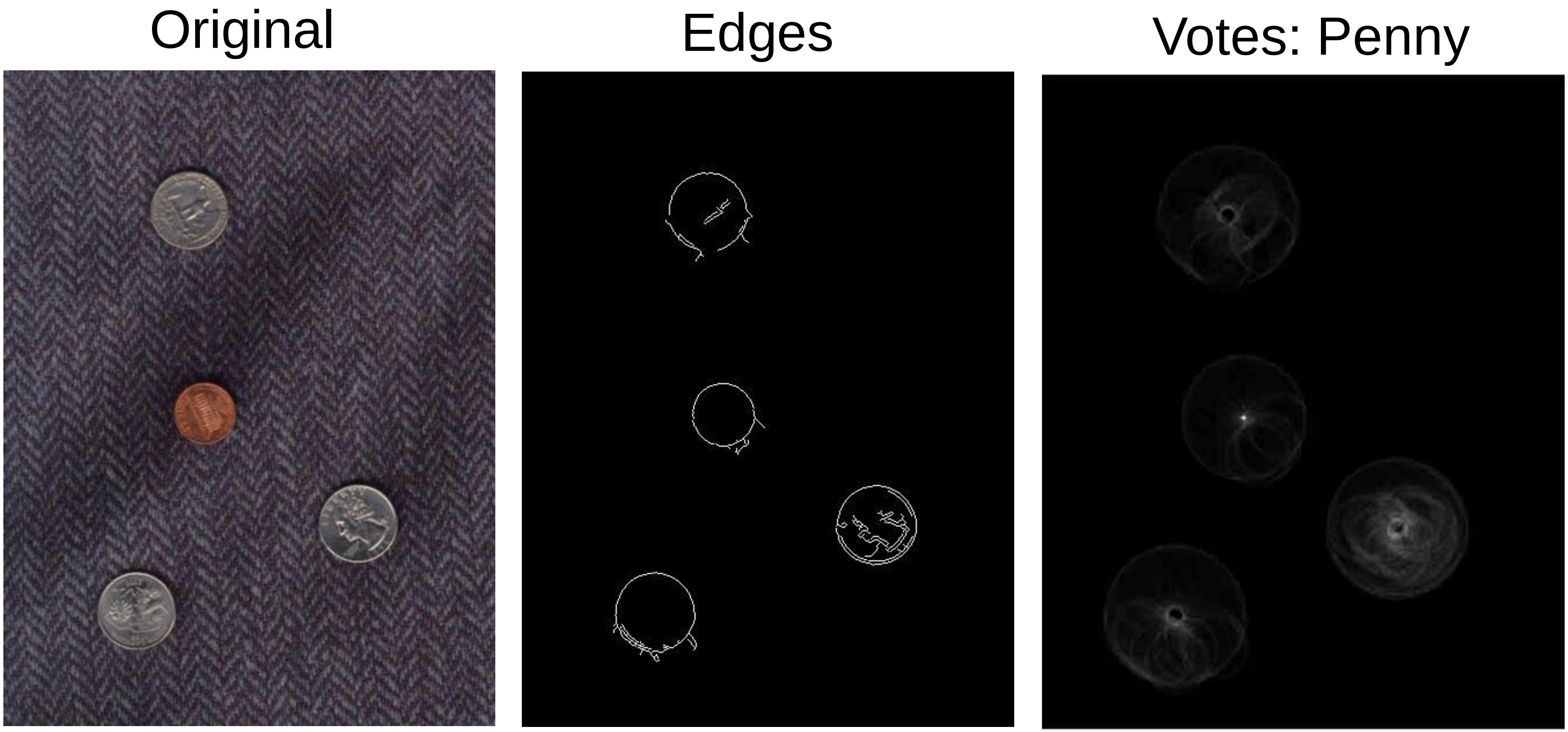
end

end

Time complexity per edge pixel?

- Check out online demo : http://www.markschulze.net/java/hough/

Example: detecting circles with Hough

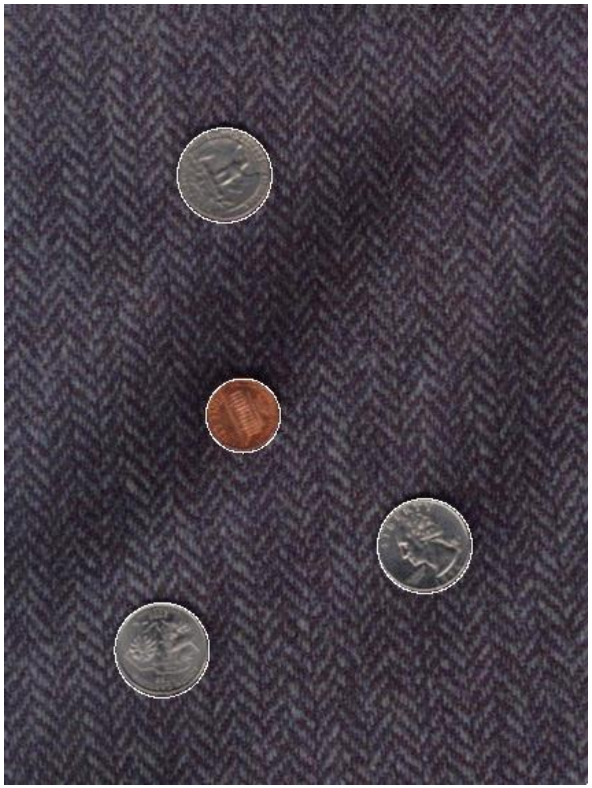


Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).

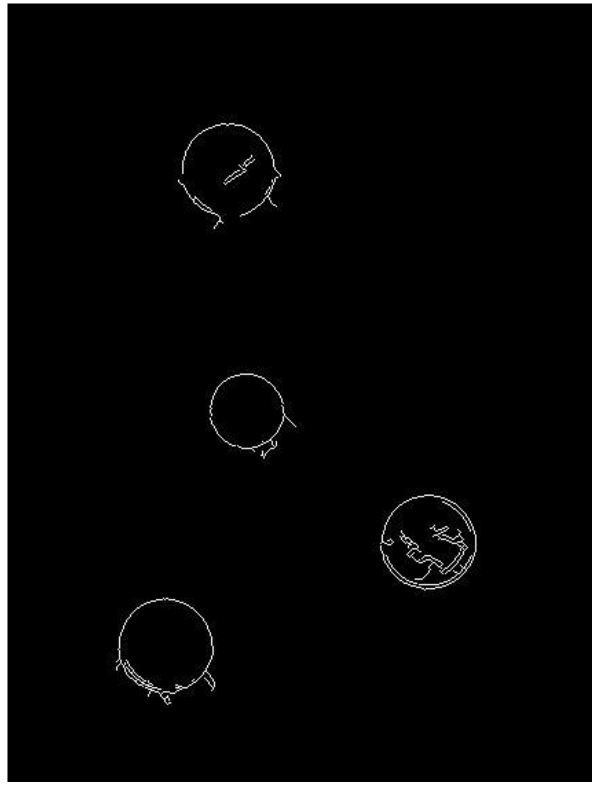
Example: detecting circles with Hough

Original

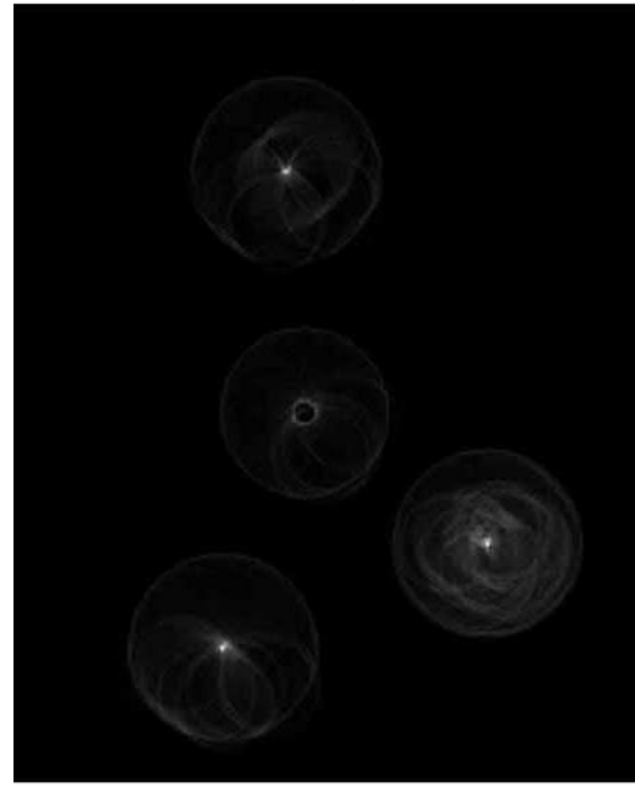
Combined detections



Edges



Votes: Quarter



Coin finding sample images from: Vivek Kwatra

Example: iris detection

Gradient+threshold

Hough space
(fixed radius)

Max detections

- Hemerson Pistori and Eduardo Rocha Costa
  http://rsbweb.nih.gov/ij/plugins/hough-circles.html

Example: iris detection

Figure 2. Original image

Figure 3. Distance image

Figure 4. Detected face region

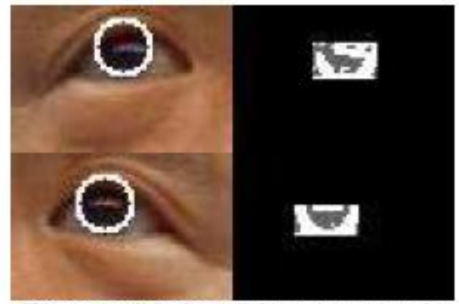


Figure 14. Looking upward

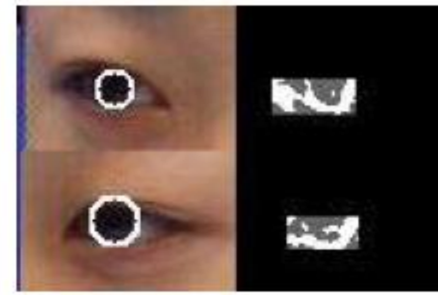


Figure 15. Looking sideways

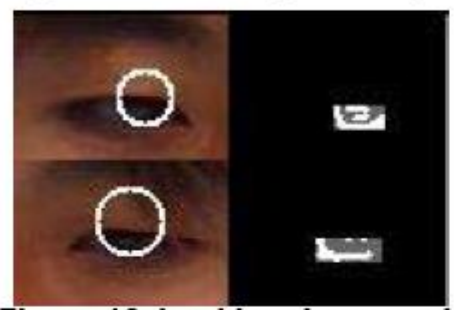


Figure 16. Looking downward

- An Iris Detection Method Using the Hough Transform and Its Evaluation for Facial and Eye Movement, by Hideki Kashima, Hitoshi Hongo, Kunihito Kato, Kazuhiko Yamamoto, ACCV 2002.

Voting: practical tips

- Minimize irrelevant tokens first
- Choose a good grid / discretization

Too fine ? Too coarse

- Vote for neighbors, also (smoothing in accumulator array)
- Use direction of edge to reduce parameters by 1
- To read back which points voted for “winning” peaks, keep tags on the votes.

Hough transform: pros and cons

Pros

- All points are processed independently, so can cope with occlusion, gaps
- Some robustness to noise: noise points unlikely to contribute *consistently* to any single bin
- Can detect multiple instances of a model in a single pass

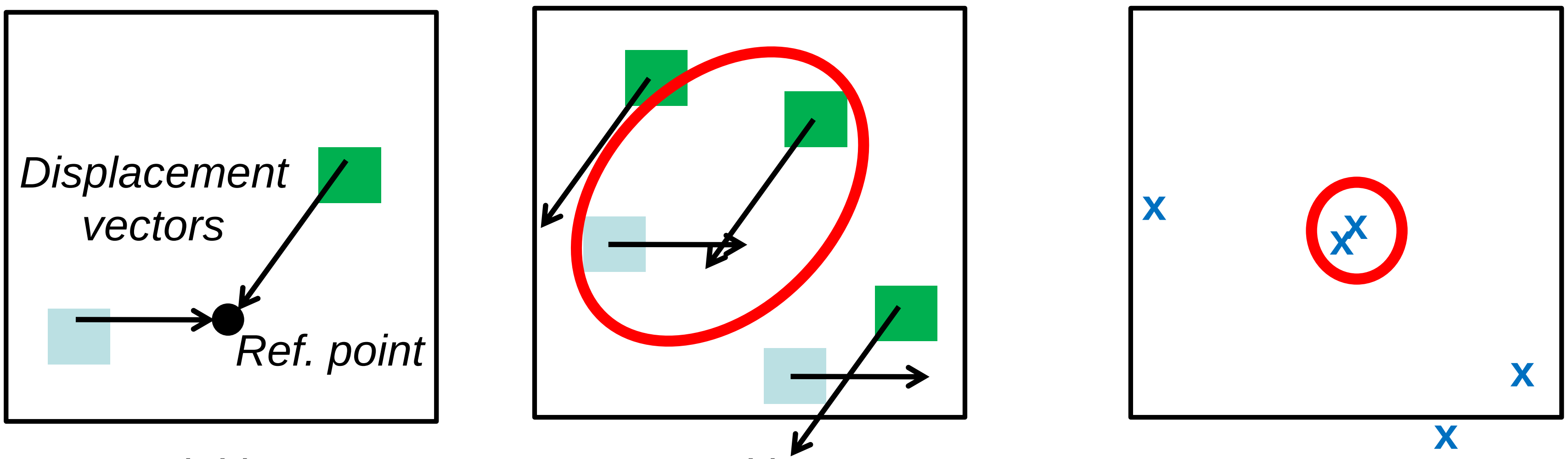
Cons

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- Quantization: can be tricky to pick a good grid size

Generalized Hough Transform

- What if we want to detect arbitrary shapes?

Intuition:



Model image

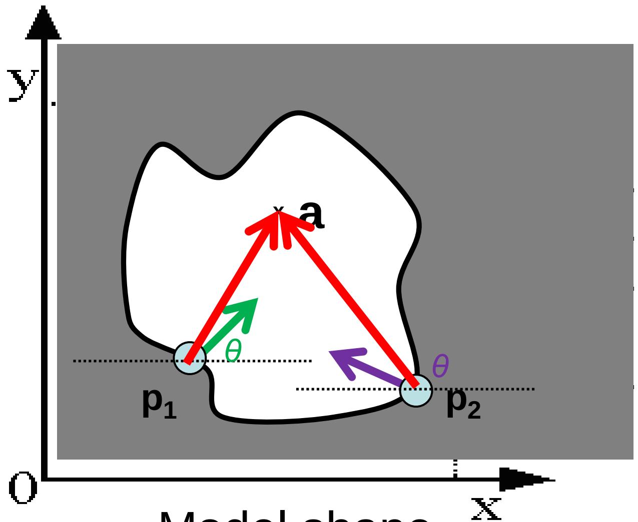
Novel image

Vote space

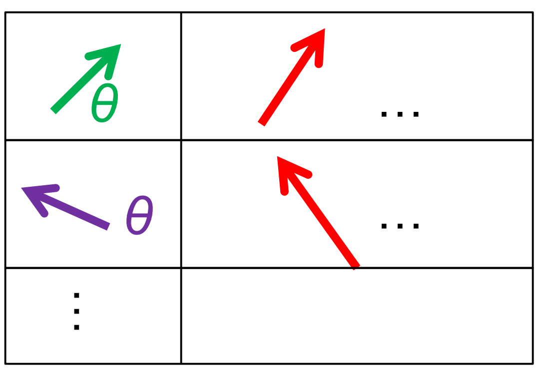
Now suppose those colors encode gradient directions…

Generalized Hough Transform

- Define a model shape by its boundary points and a reference point.



Model shape



<u>Offline procedure:</u>

At each boundary point, compute displacement vector: $\mathbf{r} = \mathbf{a} - \mathbf{p}_i$.

Store these vectors in a table indexed by gradient orientation θ.

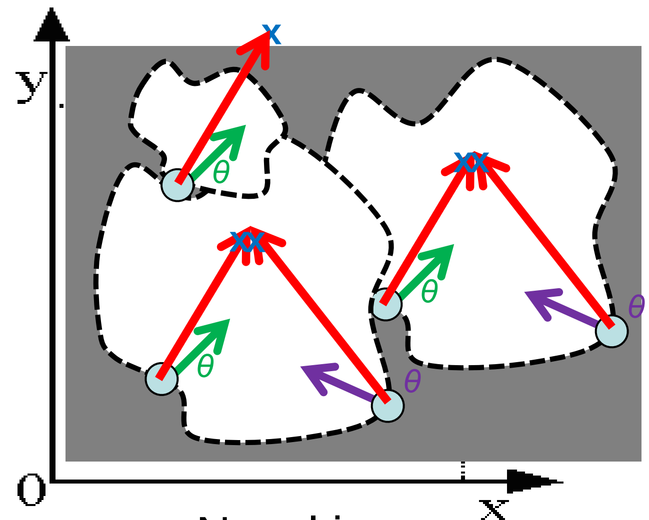
[Dana H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980]

Generalized Hough Transform

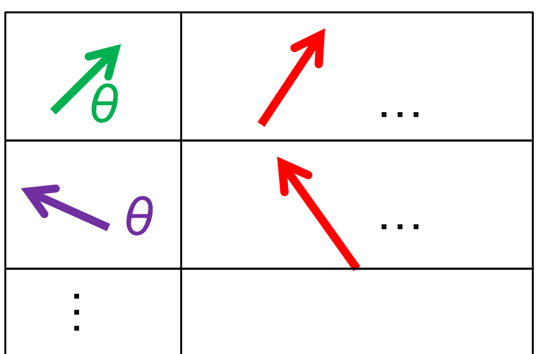
Detection procedure:

For each edge point:

- Use its gradient orientation θ to index into stored table
- Use retrieved **r** vectors to vote for reference point



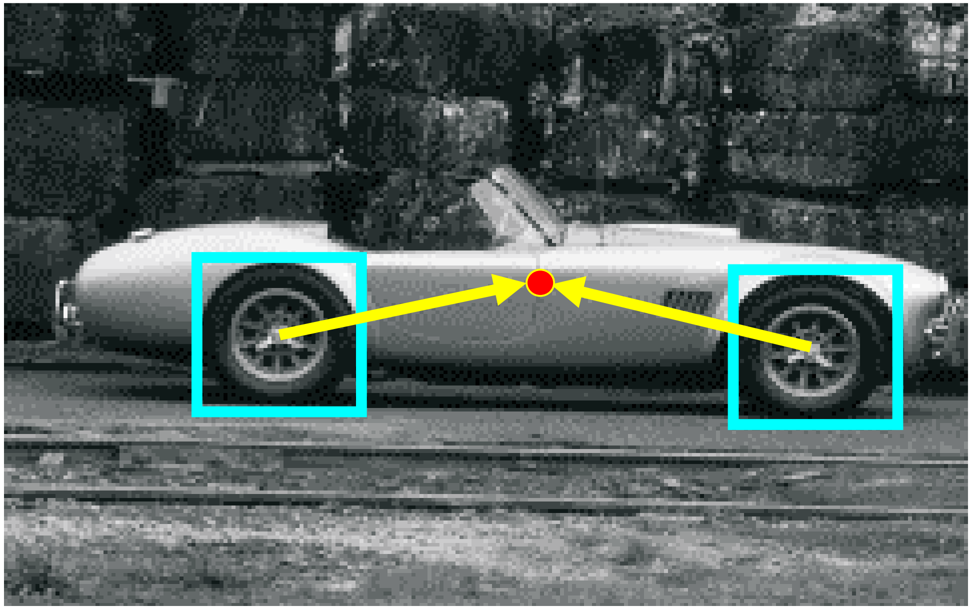
Novel image



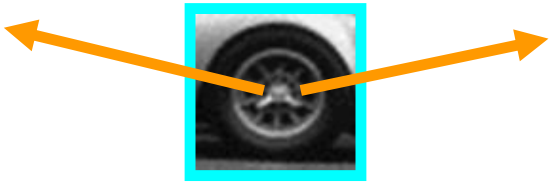
Assuming translation is the only transformation here, i.e., orientation and scale are fixed.

Generalized Hough for object detection

- Instead of indexing displacements by gradient orientation, index by matched local patterns.



training image



“visual codeword” with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Source: L. Lazebnik

Generalized Hough for object detection

- Instead of indexing displacements by gradient orientation, index by "visual codeword"

test image

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Source: L. Lazebnik

Summary

- **Fitting** problems require finding any supporting evidence for a model, even within clutter and missing features.
  - associate features with an explicit model

- **Voting** approaches, such as the Hough transform, find likely model parameters without searching all combinations of features.
  - Hough transform approach for lines, circles, …, arbitrary shapes defined by a set of boundary points, recognition from patches.

Assignments

- **编写哈夫变换实现图像中直线检测的伪码（可参考书中习题4.7）。重要步骤需要加注。**
- 阅读论文：An Iris Detection Method Using the Hough Transform and Its Evaluation for Facial and Eye Movement, by Hideki Kashima, Hitoshi Hongo, Kunihito Kato, Kazuhiko Yamamoto, ACCV 2002.