Study Of Fractional Order Systems
BTP2014

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Motivation

We are familiar with derivatives of integer order.

$$\frac{d}{dx} f(x) \quad \text{or} \quad D^1 f(x) \qquad \frac{d^2}{dx^2} f(x) \quad \text{or} \quad D^2 f(x)$$

But what would be the meaning of notations like fractional integer differentiation like

$$\frac{d^{1/2}}{dx^{1/2}} f(x) \quad or \quad D^{1/2} f(x)$$

Today a vast literature exists on fractional calculus but this idea was only in mathematics until eighteenth century when people like leibniz and L'Hopital used it. Idea is to study systems which is concerned with such equations and properties of such systems.

Introduction

In field of dynamical systems and control theory, a fractional order system is a dynamical system that can be modeled by a differential equation containing derivatives of non-integer order.

Fractional - order systems are useful in studying

- Dynamical behaviour in Electrochemistry.
- Radioactivity.
- Chaotic systems.
- Biology.

Work Schedule

Weekly meetings and implementation of algorithms in Matlab programming language as prescribed by Sir.

Expected Deliverables

Report on project and implementation of algorithms in Matlab described by Sir.

Mathematical Formulae

Let $f(x) = x^m$

then

$$\frac{d}{dx}f(x) = m\times(x^{m-1}), \quad \frac{d^2}{dx^2}f(x) = m\times(m-1)\times(x^{m-2}), \ \ldots$$

So,

$$\frac{d^n}{dx^n}f(x) = m\times(m-1)\ldots(m-n+1)x^{m-n}$$

Now, By the Grünwald–Letnikov definition we can write the derivative of non-integer order as

$$ {}_a\mathscr{D}_t^\alpha f(t) = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t-jh) $$

On simplifying by putting $w_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}$, where omega alpha is a recursive relation.

$$ w_0^{(\alpha)} = 1, \quad w_j^{(\alpha)} = \left(1 - \frac{\alpha+1}{j}\right) w_{j-1}^{(\alpha)}, \quad j = 1, 2, \ldots . $$

So the final formula for the recursive relation becomes for calculation which, we've written program

$$ {}_a\mathscr{D}_t^\alpha f(t) = \lim_{h\to 0} \frac{1}{h^\alpha} \sum_{j=0}^{[(t-a)/h]} (-1)^j \binom{\alpha}{j} f(t-jh) \approx \frac{1}{h^\alpha} \sum_{j=0}^{[(t-a)/h]} w_j^{(\alpha)} f(t-jh) $$

Proceed to Code

Thank you.