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Terrain Traversability and Optimal Path Planning in 3D Uneven Environment for an Autonomous Mobile Robot

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Abstract A key issue in mobile robot applications involves building a map of the environment that may be used by the robot for path planning. In this paper, a new approach for uneven environment reconstruction based on the information supplied by a millimetre wave (MMW) radar has been proposed. The returned raw data from the MMW are stored in a database and then transferred to the CAD model, and represented as objects using an algorithm. The non-uniform rational B-splines have been used to extract the polygonal mesh decomposition equivalent to the 3D uneven environment and obstacle surfaces. The polygonal mesh decomposition helps to locate the different obstacles and to build the optimal path to reach the target starting from any initial position, by taking into consideration the terrain traversability. The optimal path is obtained by applying distance calculation method and speed limits. Indeed, the returned path is smooth which is a crucial performance consideration, especially for practical service robots.

Keywords Environment modelling · 3D uneven environment · Polygonal mesh · Optimal path · NURBS · Speed limits

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الخلاصة

تشمل إحدى القضايا الرئيسية في تطبيقات الروبوت المتحرك تكوين خريطة للبيئة التي يمكن أن يستخدمها الروبوت لتخطيط مساره. وفي هذه المقالة ، نقترح مقاربة جديدة لإعادة بناء البيئة الغير مستوية استنادا على المعلومات التي قدمها الرادار ألميليمتري يتم تخزين البيانات الخام المقدمة من طرف الرادار ألميليمتري في قاعدة البيانات ، ونقلها إلى نموذج باستخدام النظام التصميم بمساعدة الحاسوب ، ومن ثم تمثيلها بوصفها كائنات باستخدام خوارزمية. كذلك استخدمت الأسطح المنحنية غير المنتظمة لاستخراج الشبكة المضلعة المعادلة للبيئة غير المستوية الثلاثية الأبعاد وسطح العقبات. إن التحليل إلى الشبكة المضلعة يساعد على تحديد موقع العقبات المختلفة وبناء المسار الأمثل للوصول إلى الهدف بدءا بأي موضع ابتدائي ، مع الآخذ بعين الاعتبار معبرة التضاريس. ويتم الحصول على المسار الأمثل من خلال تطبيق طريقة حساب المسافة والحدود القصوى للسرعة. وفي واقع الأمر فإن المسار الذي تم الحصول عليه أملس ، وهذا يعبر عن كفاءة الطريقة، وبخاصة بالنسبة لروبوتات الخدمة.

1 Introduction

To accurately predict the robot response on the uneven terrain, the environment representation is needed in a manner that shows well the geometry and characteristics of the robot surroundings.

In the literature, there are several research papers in the field of environment model estimation; their aims are to improve the accuracy and reliability of the predicted environment geometry using available sensor data. Digital elevation maps (DEMs) have been used to create a discrete geometric representation of the environment surfaces; however, some works have been investigated to create a more complete model using DEMs, i.e., to estimate elevation in regions where little or no data [1–3] are available. Nevertheless, these techniques have not yet been applied to build accurate land maps in uneven terrain. In regions of little or no data, interpolation techniques can be used to estimate elevation. In [4], a method for estimating missing information in incomplete data sets, by using Gaussian process (GP) regression, has been proposed. By representing the land as an elevation map, the amount of stored data can be scaled using the grid size, which is favourable in applications where memory and computational resources are limited. In that study, the task of interpretation of the sensing data consists in analysing the land traversability. This terrain traversability analysis allows the robot to provide a map of difficult, unstructured terrain, where a part of terrain with highest difficulty will be considered as an obstacle, which is necessary to the path planner. GP is unsuitable for uneven environment geometry estimation even it is currently in the practice, and has some drawbacks. The first drawback is that GP is implicitly continuous and assumes single output value, which may lead to misrepresentations of boundaries' terrain features. The second drawback is the problem of spatial correlation, which is a common limitation among interpolation methods. This has the effect of smoothing out terrain features and thus reduces the accuracy of the terrain geometry estimation. Moreover, the radar observations (like high-resolution LIDAR) will provide large data at each observation over time, where GP cannot be implemented without making-large approximations.

Consequently, a theoretical model describing the geometric and intensity properties of the ground echo in radar images has been developed [5]. It provides prediction of the range spread of the ground return along with the expected power spectrum. The described model serves as a basis for the development of a novel method for radar ground segmentation, which allows classification of observed ground returns in three broad categories, namely, ground, non-ground, and unknown. Hence, fast and reliable algorithms capable of extracting features from a large set of noisy data are critical for detection and segmentation of the outdoor environment ground in a sensor generated image. The research work of [6] deals with the problem of segmentation of sparse 3D data using ground models of non-constant resolution either providing a continuous probabilistic surface or a terrain mesh built from the structure of a range image, both representations providing close to real-time performance. The algorithm can process any 3D point cloud sparse or dense potentially formed of data accumulated from several sensors, and a mesh-based technique was optimized for the processing of range images. Recently, a method that combines radar and monocular vision for ground and non-ground modelling and scene segmentation by a mobile robot operating in outdoor environments has been developed [7]. This work aims to build a model of the ground online.

In this paper, a method for 3D uneven environment modelling that is based on data acquired from millimetre-wave radar (MMW radar) has been proposed. The environment is rebuilt by using non-uniform rational B-splines (NURBS) curves to obtain the polygon mesh representing the surface of the uneven environment.

NURBS curves are used for trajectory reconstruction since they have been proven to be the best parametric curves for path planning both for 2D mobile robots and 3D curve approximation [21]. NURBS curves provide a flexible way to represent both standard analytic and free-form curves, their evaluation is fast, and they can be manipulated either by directly modifying the position of the points lying on the curve or by indirectly changing the configuration of the control points. Moreover, NURBS curves can exploit powerful geometric tool kits such as knot insertion and removal. Furthermore, the use of NURBS ensures that the generated trajectory connecting the starting point and the target is continuous and derivative of second degree without dealing with quantisation.

After modelling the uneven environment, this work deals with path planning problem. The objective is to find a collision free path from a given start position to a predefined target point by taking into consideration the terrain traversability. A complete path planning algorithm may guarantee that the robot can reach the target if possible, or indicate that the target cannot be reached. Several researches have addressed the problem of 3D path planning such as paper [8] which gives a path planning algorithm based on energy function of neural network architecture for mobile robot. But this method is mainly for the regular obstacles not for the irregular ones. And [9] proposed a path planning algorithm to planning both global and local optimum path by estimating running costs in known 3D environment. However, this algorithm was simulated in known 3D environment not in unknown 3D uneven environment, and the problem of obstacles avoidance was not addressed.

A very crucial performance consideration may be taken, i.e., the robot should move smoothly. This smoothness purpose is of key importance especially for practical service robots, by considering the physical limits of robot actuators, safety issues and that some service robots are required to carry sensitive loads. Many researchers have studied the smoothness taking in hand path planning problem for a wheeled robot [10,11]. A trajectory that approximates a given ordered sequence of points on the plane and satisfies certain smoothness requirements and curvature constraints is constructed. Moreover, the path must be optimized, i.e., the free path linking the start point to the target must be as short as possible. In the literature, there exist several optimization methods such as simulated annealing (SA) and genetic algorithm (GA), which have been used for solving optimization problems. However, the computational processing time is crucial for the real-time applications such as mobile robots. It is proved that SA method [12] can offer better performance on both path length and processing time than the GA method [13]. A multi-operator based SA (MSA) approach incorporating with additional four mathematical operators that can find the optimal path for robots in dynamic environments has been proposed in [14]. Compared to the normal SA and GA, the MSA approaches are effective for getting path solution and computationally efficient in deriving the path solution. However, this approach gives a near-optimal path solution, and the obtained path is not smooth. Hence, these methods are not used in case of 3D surfaces, as the 3D path planning issue is quite difficult, since the robots navigating in uneven environment have fast and complex dynamics, and the paths have to be generated and modified in real-time. Furthermore, the path generated using SA, GA or any other standard form of dynamic programming existing in the literature, does not ensure a smoothness criterion.

Moreover, to ensure the robot mobility, two basic requirements are necessary: the appropriate forces and the effective contact. However, the mobile robot has to be stable during its motion. So, avoiding tip-over and slippage will be the crucial issues to keep the robot stable. In our work, for the optimization purpose of the obtained path a method which is based on distance calculation and speed limits has been used. It is very efficient for solving the local minima problem, as well as the developed technique provides the results rapidly.

2 Environment Sensing

Among active imaging sensors, MMW radars have been employed in autonomous vehicle systems, since they provide relatively accurate measurements of obstacles in low visibility conditions, including dust, fog, and rain. Radar also provides a rich source of information allowing for multiple objects detection within a single beam, whereas other range sensors are generally limited to one target return per emission. MMW radar with its narrow beam pattern and wide available bandwidth provides consistent range measurements for the environmental imaging needed to perform autonomous operations in poorly lit environments. Thus, the MMW radar has recently appeared as a LIDAR alternative. Some MMW radar characteristics compared to that of the LIDAR are listed in Table 1. The frequency given in Table 1 refers to that of the MMW radar built at the Australian Centre for Field Robotics, and it reports the amplitude of echoes at ranges between 1 to 120 m [5].

MMW radar also produces a large beam, so radar returns may be interpreted using the interaction of the beam with a finite but relatively large region of the environment. Returned images represent the convolution of the environment with the emission, so it is more complicated to infer the structure of the environment from the return [15]. Figure 1 shows the used configuration, where the radar is focussed at the front of the vehicle with a grazing angle φ, illustrated in the explanatory scheme. The proximal and distal borders of the footprint area illuminated by the divergence beam are denoted with b_{p} and b_{d}, respectively. D represents the distance between the origin of the beam (at the centre of the antenna) and the ground. The slant range of the radar bore sight is d_0, and the scan angle is θ.

In this work, the algorithm described in [15] to extract the ground raw position data is used. Three main features to define the land model are employed such as the intensity associated with the slant range, C, the goodness of fit, f, and the shape factor, S. This set of features has been used in [15] for self-learning classification framework. Whereas in [16], similar features have been used in a form of logical "expert system", with manually tuned thresholds to classify ground returns. It can be noticed that it is a good descriptor of the ground appearance and to extract the raw data. The first feature defining the land appearance is constructed upon a set of intensity and shape features that are obtained by fitting the ground model, expressed by the power return of the ground echo [$P_{\mathrm{r}}(d)$] to radar data as function of the range d, is given by,

$$P_{\mathrm{r}}(d) = C\frac{A(d, d_0)^2}{\cos\varphi} \tag{1}$$

where C is a constant quantity, d_0 is the slant range, A is the antenna gain and φ is the grazing angle. The second feature is goodness of fit, and it is defined by [17],

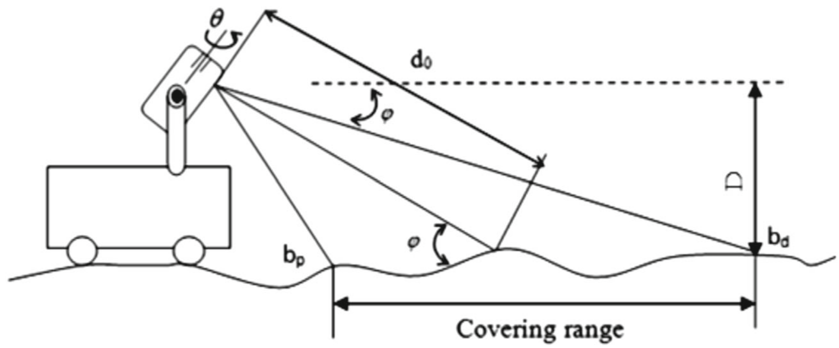
$$f = 1 - \frac{\sum(x-y)^2}{\sum(x-\bar{x})^2} \tag{2}$$

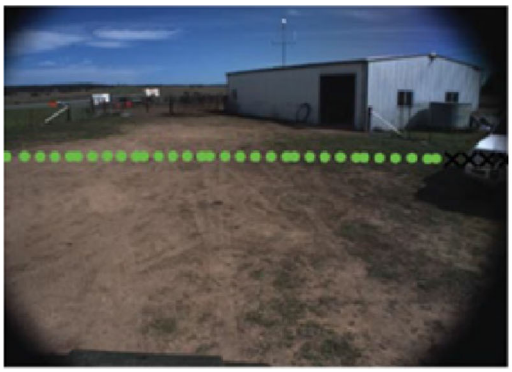
where x and $\bar{x}$ represent the position point and the mean of the data, and y the output from the regression model. The best possible value is when $f \in]-\infty, 1]$.

Table 1 Characteristics of MMW radar compared to that of LIDAR

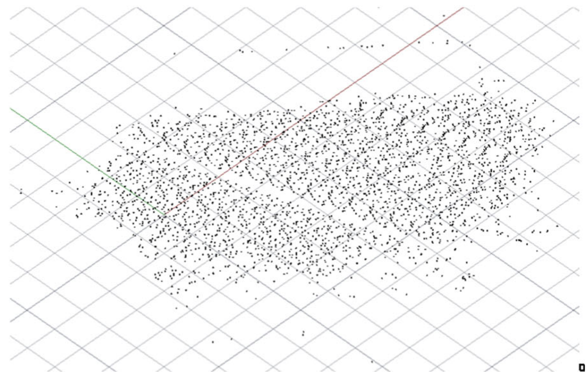
Item	Value for MMW radar [5]	Value for LIDAR [18]
Scanning angle rate	≈3 rps	10 scans/s
Range accuracy	0.051 m	0.02 m
Horizontal field of view	360°	360°
Maximum detection range	120 m	40 m
Beam width	2°–3°	0.1°
Wavelength	3 mm	905 nm
Frequency-modulated continuous wave	About 95 GHz	–

Fig. 1 Millimetre-wave radar parameters used to control the uneven environment





(a)



(b)

Fig. 2 **a** Image of the real uneven environment introduced in [15]. **b** 3D geo-referenced positions for the labeled points $P_i(x_i, y_i, z_i)$ provided from the MMW radar extracted information for the uneven environment

We consider the initial intensity (I_0) and final intensity (I_f) value of the ground echo as the third feature. A normal ground echo should have similar initial and final intensities due to the physical interaction between the radar emission and the ground. So a shape factor S can be defined by,

$$S = \left| \frac{I_0 - I_f}{I_0} \right|. \tag{3}$$

When a single radar observation i is successfully labelled as ground, an estimate of its range distance $d_{0,i}$ is also returned by the fitting process. When combined with the localization estimation of the vehicle, this provides a 3D geo-referenced position for the labelled point $P_i(x_i, y_i, z_i)$, and the set of point positions P_i is shown in Fig. 2, which can be used to build the environment model in our approach.

3 Uneven Environment Surface Reconstruction

In this paper, the approach of reconstructing 3D models from real uneven environment generated generally from any 3D point cloud (e.g., from a laser scanner, stereo-camera or radar etc.) is proposed. However, in this work the approach is based on combing data acquired by MMW radar as explained previously. Indeed, the points P_i are transferred to the CAD model, and represented as objects using standard CAD tools. Thus, the algorithm is coded by using Visual Basic macro.

Data extraction for environment modelling desired for robot navigation has specific requirements such that: all geometry model needs to be effectively represented, and sufficient graphical detail must be acquired for surface rendering at the required resolution. So, performance of the algorithm depends on the resolution of the model. Thus, small details may be important in certain area, and in obstacles representation.

The radar system provides a set of ground raw data. Knowing the geometry of the radar system, we can compute the 3D position corresponding to each radar data. This 3D data constitutes a 3D representation of the uneven terrain, but higher level representation is required to interpret it. It is desirable that the reconstructed surface describes the environment geometry as simple as possible (using simple geometric form models), while preserving its precision.

While our first objective is the reconstruction of a model for an uneven environment, an algorithm which is based on the use of the 3D position data as control points is proposed. Then, these control points are linked by NURBS in x and y directions in such a way that a polygonal mesh representing the surface of the environment is constructed. This method is more suitable for graphical and visualisation application. The algorithm is described as follows:

1. Use a set of features to define the ground model and to separate the raw data position points corresponding to the ground from the non-ground data,
2. Create a selection set which covers the ground data position points P_i, where each point is represented as an object for getting the boundary extent of the ground,
3. Set y coordinates to zero,
4. Search the points P_i existing in the xz plane and link them by a NURBS, (as P_i are used to build NURBS they are called control points).
5. Increment y coordinates by one step. Since the ground region is often assumed to be larger than that of obstacles, large y steps for ground reconstruction can be considered, which facilitates the estimation of the ground homograph in short time.
6. Repeat the steps 4 and 5 until the y coordinates attain the boundary of the ground region.
7. Set x coordinates to zero;
8. Search the control points (P_i) existing in the yz plane and link them by a NURBS,
9. Increment x coordinates by one step, the same step as the one used to increment the y coordinates is used.
10. Repeat the steps 8 and 9 until the x coordinate attend the boundary of the ground region.

After running the steps mentioned above, the polygonal mesh representing the form of the ground surface of the uneven environment is obtained, as illustrated in Fig. 3. Moreover, this approach was applied to an extended data base extracted from the radar system used in [15], and we get the map represented in Fig. 4.

In order to reconstruct the form of the obstacles existing on the ground, the same algorithm is used, but another selection set which groups the non-ground data position points is created. Moreover, as the obstacles dimensions are very small as compared to the ground surface, and maybe little details are important for obstacles modelling; so at this stage small steps are suitable to increase x and y coordinates. Figure 5 illustrates obstacles from reconstruction.

4 Optimal Path Planning

Path planning over uneven environment requires the search for a feasible path, and the computation of some velocity profile along that path. The path is generated first to ensure that the mobile robot is statically stable using a kinematic planner, and taking into account distance optimisation criteria. Then, the velocity profile along that path is computed to ensure that the vehicle is also dynamically stable. The robot is considered to be statically stable when no velocity profile along the path is computed. However, dynamic stability reflects the vehicle's ability to traverse uneven terrain at high speeds. It is determined from the set of admissible speeds and tangential accelerations of the center of mass along the path, subject to the ground force constraints and the geometric path constraints [19].

4.1 Feasible Path Construction

The technique of path planning in 2D and 3D environment using parametric curves is explained in details in our published work [20,21]; where, path planning for a robot navigating in planar environment using NURBS was proposed. Here, the technique of path planning in a 3D uneven environment is described. First, the starting point S and the target point T are selected; so that, the starting point is the current position of the robot and the target is defined by the user. Then, the following steps will be executed to extract the free path:

– link S to the nearest control point q_{Si} toward T, and T is linked to the nearest control point q_{Ti} toward S. The nearest control points are determined by calculating the Euclidian distance between S (or T) and the vicinity control points (see the flowchart in Fig. 6);
– Connect these points together by the NURBS passing by the control points constituting the surface of the environment. The NURBS linking S to T is given by,

$$Q(s) = \frac{\sum_{i=0}^{n-1} B_{i,k}(s) \cdot w_i \cdot q_i}{\sum_{i=0}^{n-1} B_{i,k}(s) \cdot w_i} \tag{4}$$

where n is the number of control points between S and T, q_i are the control points and k represents the NURBS degree and each control point q_i is associated with a weight w_i. $B_{i,k}(t)$ are the B-spline blending functions.

$Q(s)$ is a parametric function which depends on the parameter s. Setting rational basis function *R(s) as* follows,

$$R(s) = \frac{B_{i,k}(s) \cdot w_i}{\sum_{i=0}^{n-1} B_{i,k}(s) \cdot w_i}. \tag{5}$$

This allows writing:

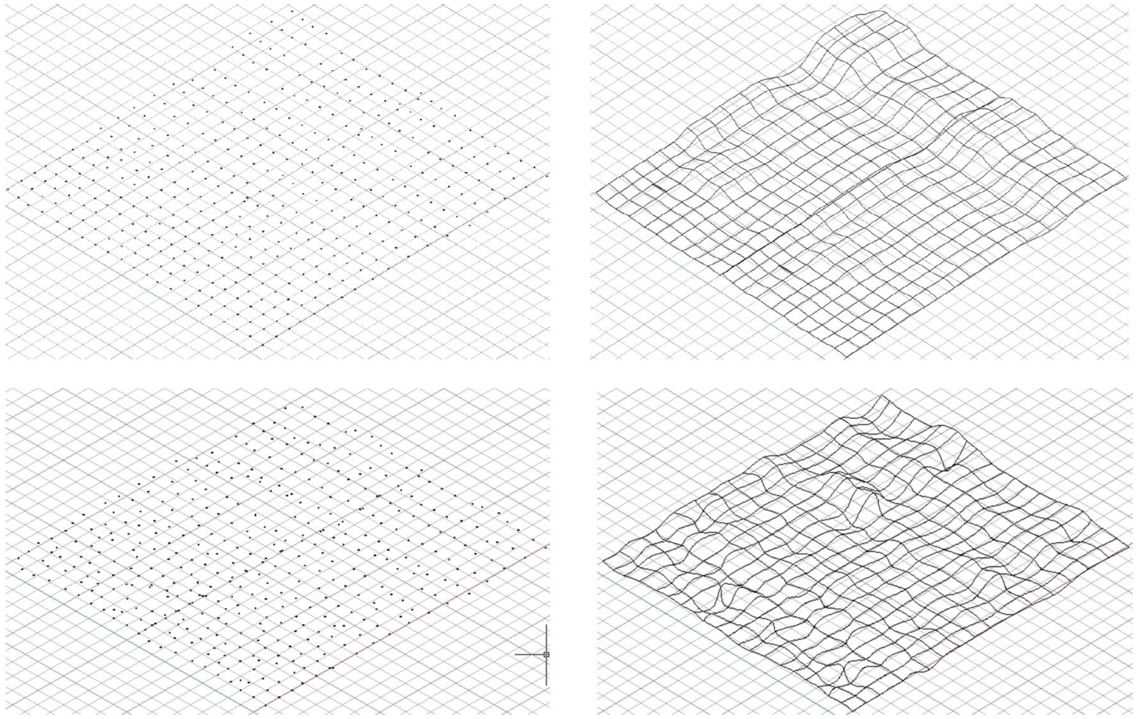
$$Q(s) = \sum_{i=0}^{n-1} R(s) q_i. \tag{6}$$

The obstacle contours are also modelled by NURBS function, so, an obstacle contour belonging to the ground surface is defined by the parametric function $O(t)$ which is given by,

$$O(t) = \sum_{j=0}^{m-1} R(t) p_j \tag{7}$$

where t is the obstacle NURBS parameter.

To determine if $Q(s)$ collides an object, an interference checking of the NURBS $Q(s)$ with the obstacle contours



(a) **(b)**

Fig. 3 The application of this approach for ground model reconstruction. **a** The ground data position points P_i, **b** the polygonal mesh representing the form of the ground surface of the uneven environment obtained by using NURBS curves

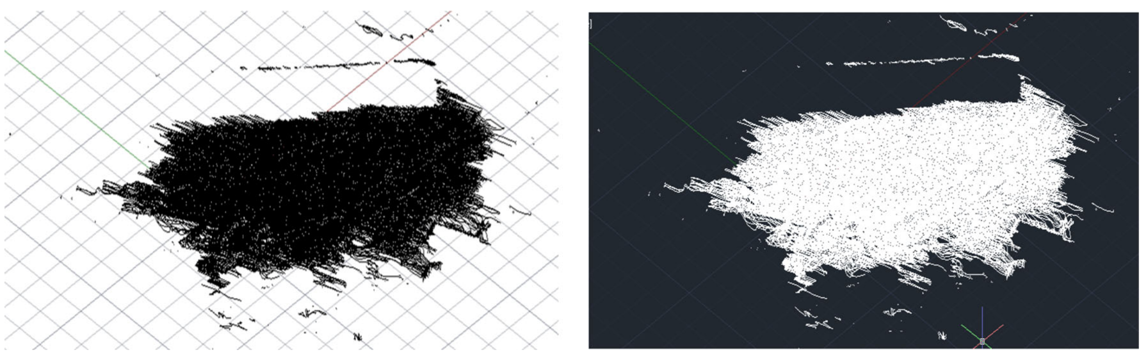


Fig. 4 The ground map building resulted from the application of the approach on the radar raw data introduced in [15] (the map is built from a database containing 66,396 position points)

$O(t)$ is carried out. $Q(s)$ interfere with $O(t)$ means that there exists one or more control point q_i which coincide with p_j. Thus, in this case, it can be said that $Q(s)$ is not a free path, and the coordinates of the collision points can be calculate from the following equality equation,

$$Q(s) = O(t) \Rightarrow \sum_{i=0}^{n-1} R(s) q_i = \sum_{j=0}^{m-1} R(t) p_j. \quad (8)$$

Let's consider that the collision is at the lth control point thus:

$$\sum_{i=0}^{n-1} R(s) q_i = \sum_{\substack{j=0 \\ j \neq l}}^{m-1} R(t) p_j + R(t) q_l. \quad (9)$$

By resolving Eq. 9, the following equation can be obtained,

$$q_l = \frac{R(t) \sum_{\substack{j=0 \\ j \neq l}}^{m-1} p_j - R(s) \sum_{\substack{i=0 \\ i \neq l}}^{n-1} q_i}{R(s) - R(t)}. \quad (10)$$

So, the coordinates of q_l have been replaced by other free neighbouring control points from the ground surface which

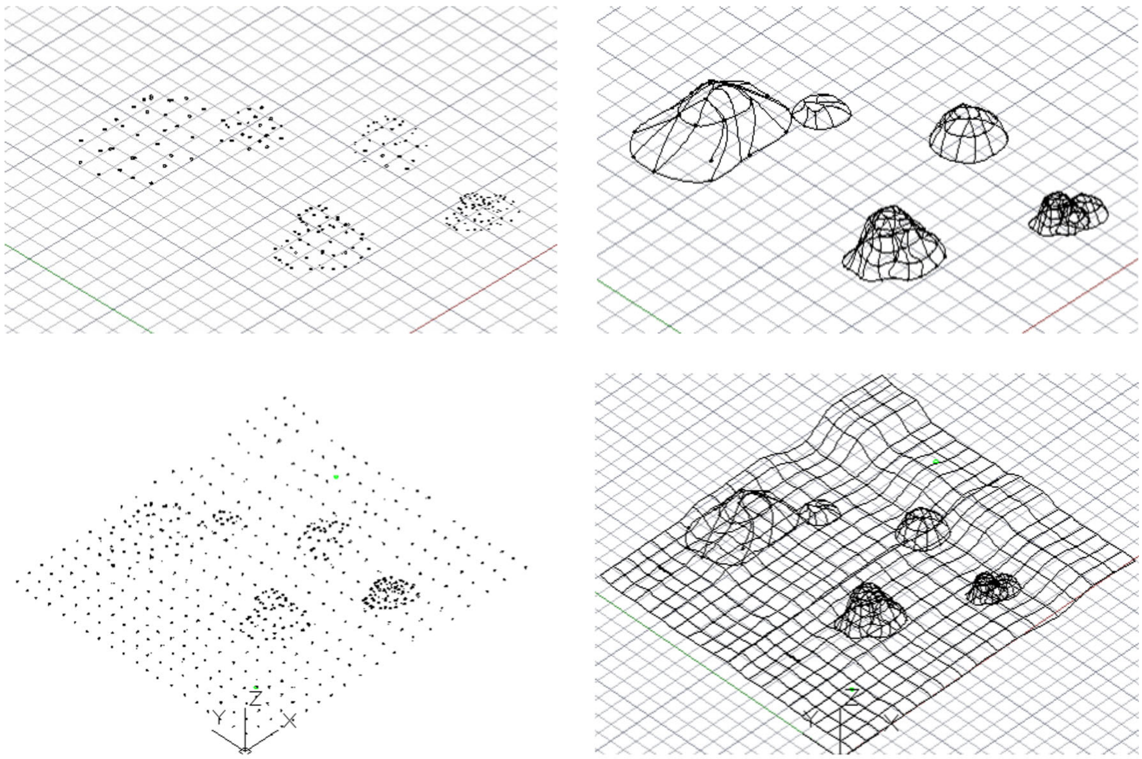


Fig. 5 The application of the approach for non-ground and obstacles model reconstruction, **a** an example of obstacle data position points P_i, **b** the polygonal mesh representing the form of the ground and the obstacle surfaces obtained by using NURBS curves

will be inserted as new control points to $Q(s)$. So, the control points existing in the right and the left of the obstacles are used to build all the feasible free paths [$Q_r(s)$ and $Q_l(s)$ respectively]. The flowchart in Fig. 6 explains the approach of feasible path construction. Figure 7 illustrates the approach of free feasible path construction, where the blue NURBS in Fig. 7a illustrate that the robot cannot follow a straight way from S to T (problem of collision), where the NURBS in Fig. 7b show all the feasible free paths linking any starting point S to any target position T.

4.2 Speed Constraints

To ensure the robot mobility, two basic requirements are necessary: the appropriate forces and the effective contact. However, the mobile robot has to be stable during its motion. So, avoiding tip-over and slippage will be the crucial issues to keep the robot stable.

Since we seek the time optimal trajectory, the global search selects paths along which the robot can sustain high speeds without violating dynamic constraints such as rollover, excessive side slip, and maintaining ground contact. Velocity limits (above which some of the dynamic constraints may be violated) are computed by mapping the dynamic constraints to constraints on the vehicle's speed and tangential acceleration. As we are looking for long range planning, so the robot is modelled as a suspended point mass.

Under the assumption that the contacts between the wheels and the floor are point contacts, the contact forces will have the following form:

$$\vec{F} = f_u\vec{u} + f_v\vec{v} + f_r\vec{r} \tag{11}$$

where: f_u is the friction force component tangent to the path; f_v is the friction component normal to the path and the ground surface and f_r is the contact forces component normal to the ground surface. Whereas, u, v and r denote projections along the path coordinate frame.

By expressing these three external forces in terms of the robot's speed and tangential acceleration we get [22]:

$$f_u\vec{u} + f_v\vec{v} + f_r\vec{r} - mg.\vec{g} = m\rho\dot{q}^2.\vec{\kappa} + m\ddot{q}.\vec{u}$$
$$\Rightarrow \begin{cases} f_u = mg.\vec{g}_u + m\ddot{q}.\vec{u} \\ f_v = mg.\vec{g}_v + m\rho\dot{q}^2.\vec{\kappa}_v \\ f_r = mg.\vec{g}_r + m\rho\dot{q}^2.\vec{\kappa}_r \end{cases} . \tag{12}$$

Such that, ρ is the path curvature, g is a unit vector pointing opposite of the gravity force, κ is a unit vector pointing in the direction of the path centre of curvature.

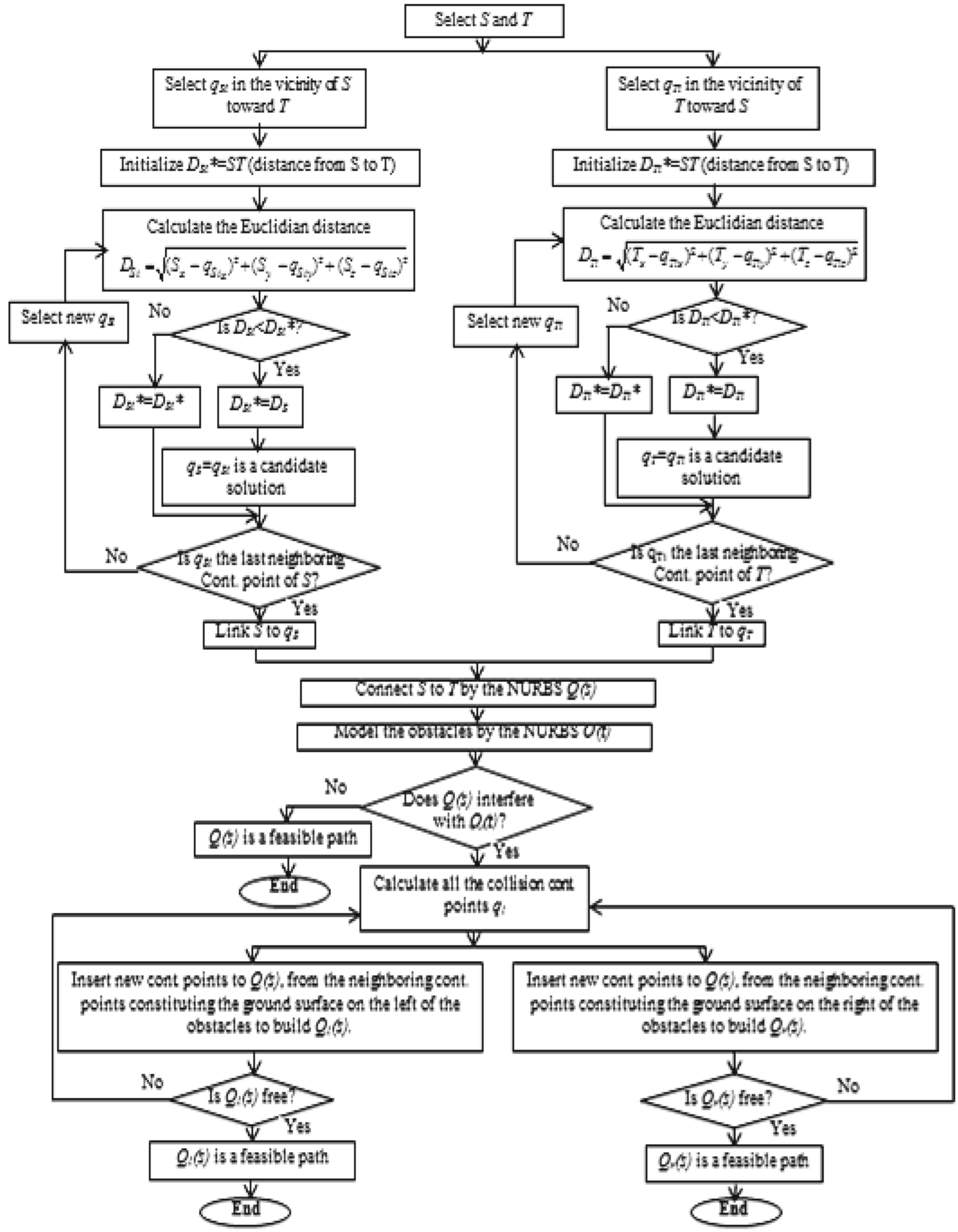


Fig. 6 Flowchart summarizing the feasible path construction approach

According to Coulomb's law and considering the sliding and contact constraints, inequality constraints have the following form:

$$\begin{cases} f_u^2 + f_v^2 \leq \mu^2 f_r^2 \\ f_r \geq 0 \end{cases}. \tag{13}$$

The tip-over constraint becomes:

$$f_v^2 \leq \left(f_r \frac{b}{h} \right)^2 \tag{14}$$

where: μ is the coefficient of friction, h is the height of the centre of mass and b is the lateral distance between the

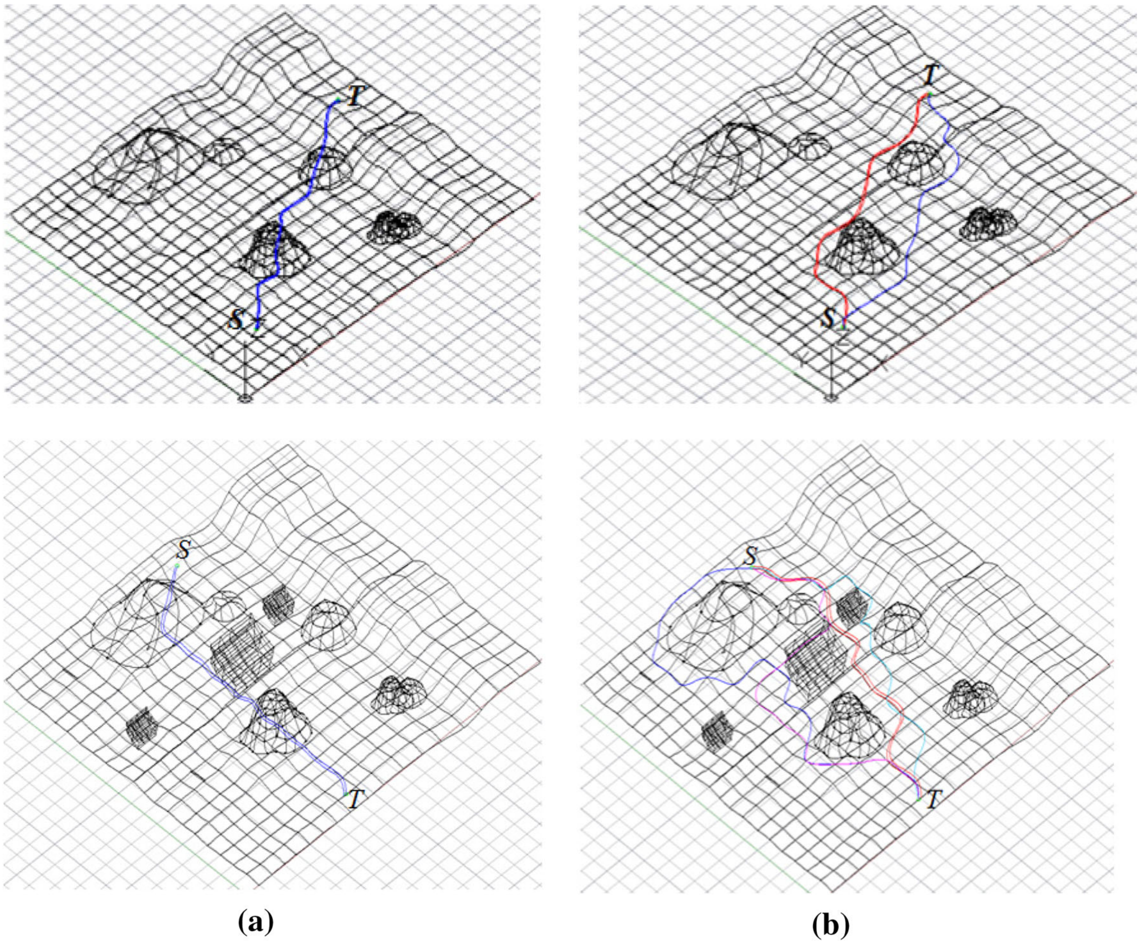


Fig. 7 Path planning in uneven environment, **a** the NURBS linking S to T (problem of collision), **b** the feasible free paths, and the *red one* is the optimal path basing on the shortest distance and the maximum safe speed

wheels. Substituting (12) into the dynamic constraints (13) yields three constraints (sliding, contact and tip-over constraint) on vehicle speed [22]. Thus, the sliding constraint yields the following velocity constraint:

$$\begin{aligned}&\rho^2\left(\kappa_v^2-\mu^2\kappa_r^2\right)\dot{q}^4+2g\rho\left(g_v\kappa_v-\mu^2g_r\kappa_r\right)\dot{q}^2\\&\quad+\ddot{q}^2+2gg_u\ddot{q}+g^2\left(g_u^2+g_v^2-\mu^2g_r^2\right)\geq 0\\&\qquad\Rightarrow\alpha.\dot{q}^4+2\beta.\dot{q}^2+\delta\geq 0.\end{aligned}\tag{15}$$

$$\text{With}\begin{cases}\alpha=\rho^2\left(\kappa_v^2-\mu^2\kappa_r^2\right)\\ \beta=g\rho\left(g_v\kappa_v-\mu^2g_r\kappa_r\right).\\ \delta=\ddot{q}^2+2gg_u\ddot{q}+g^2\left(g_u^2+g_v^2-\mu^2g_r^2\right)\end{cases}$$

Such that, the coefficients α, β and δ are determined from the terrain geometry, path direction and curvature. keeping the speed below its limits obtained from the three dynamic constraints ensures that the robot does not roll, skid, or lose contact with the terrain (for more details refer to [22]).

4.3 Optimal Path Extraction

After obtaining the feasible paths, and computing the velocity limits, the robot may follow the shortest one to reach its target without exceeding the maximum allowed speed to ensure the robot stability. Thus, a problem of optimization is faced. A technique based on distance calculation is proposed in this work.

The maximum velocity $\dot{q}_{\max i}$, which cannot be exceeded by the robot, is computed for each path segment and considered as a candidate for measuring traversability since it takes into account the effects of robot dynamics, terrain topography, and surface friction. When, $\dot{q}_{\max i}$ is zero this means that the robot is statically unstable, so the given path segment is hence not traversable. While, a nonzero value of $\dot{q}_{\max i}$ implies that the given path portion is traversable at such speed. Therefore, the maximum velocity can indicate if the path segment separating two control points is traversable or not.

So, the cost function is defined by dividing the path segment separating two control points by the maximum velocity. This cost function has units of time and can be computed for each path segment of each feasible path.

The distance between two control points in 3D (the distance along a path segment) can be calculated by:

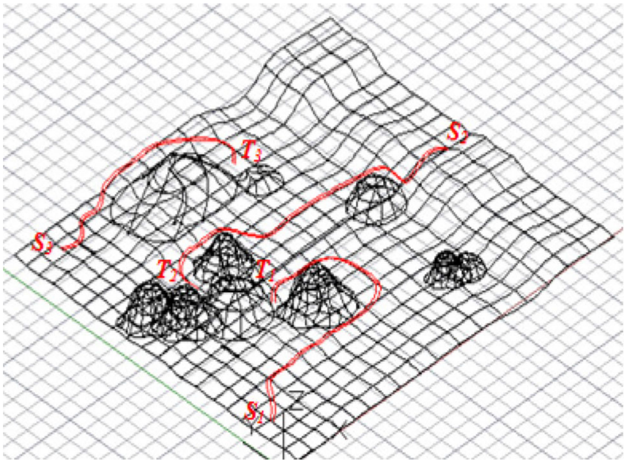


Fig. 8 The *red* NURBS illustrate examples of optimal path generation without being trapped in local minima problem; with the selection of deferent starting (S) and target (T) positions

$$d_i = \sqrt{(q_{ix} - q_{(i-1)_x})^2 + (q_{iy} - q_{(i-1)y})^2 + (q_{iz} - q_{(i-1)z})^2} \quad (16)$$

Distances d_i' s separating two control points are computed starting from S towards T. Thus, the total distance between S and T is the sum of distances separating all the control points can be obtained from Eq. 17,

$$D_{\text{feasible}} = \sum_{i=1}^{n-1} d_i. \quad (17)$$

Thus, the cost function is defined for each feasible path is defined by H_i such that:

$$H_i = \sum_{i=1}^{n-1} \frac{d_i}{\dot{q}_{\max i}}. \quad (18)$$

Therefore, as at least two feasible paths can be obtained, and by assuming that the robot travels at its maximum safe speed and following the shortest distance, so the optimal path can be calculated by:

$$H_{\text{optimal}} = \min\{H_i\}. \quad (19)$$

The efficiency of this approach is proved by the ability to select a traversable path at high speeds and following the shortest path without the need for an expensive search in the state-space. Illustration of optimal path reconstruction is shown in Fig. 7b.

It can be noticed that the local minimums does not cause problems in path generation (see Fig. 8), as the coordinates of the points P_i adjacent to each obstacle are known, thus the selection of the NURBS control points does not cause any problem.

In all the simulations, the paths have been successfully generated and the obtained curves satisfy all the necessary smoothness conditions as the paths are generated directly by using NURBS.

5 Conclusion

In this work, the problem of autonomous robot navigation in uneven environment is studied. The new approach of polygonal mesh decomposition to reconstruct the uneven environment in front of the robot has been proposed. The radar sensor returns the data related to the environment and the obstacles, and then the polygonal mesh is used for modelling the environment and localizing the obstacles. This allows the robot to navigate in any environment (simple or complex one) with obstacles avoidance. The raw data returned by radar sensor are identified and classified to ground and non-ground by the previous published algorithm [15] by assuming that the obstacles must be stationary objects. The advantage of this method is its good performance in terms of data size reduction and accurate geometric information, which are needed by the next stages such as localisation, path planning and obstacles avoidance etc.

Besides, the algorithm for optimal path planning in 3D uneven environment has been proposed. Simulated results show that the algorithm can produce a short path; and as NURBS have been used, the produced path is smooth, a fact that improves security. Furthermore, the insertion of new control points to the NURBS to avoid obstacles does not alter the whole path. The approach avoids the problem of local minimum, it is simple and robust. Hence, the efficiency of this approach is proved by the ability to select a traversable path at high speeds and following the shortest path without the need for an expensive search in the state-space. As we are looking for long range planning, the robot is modelled as a suspended point mass. The approach can be improved by taking into account other optimization factors and by considering sensitive load dynamics and dynamic obstacles avoidance.

References

1. Lang, T.; Plagemann, C.; Burgard, W.: Adaptive non-stationary kernel regression for terrain modeling. In: Robotics: Science and Systems (RSS), Seattle (2009)
2. Vasudevan, S.; Ramos, F.; Nettleton, E.; Durrant-Whyte, H.: Large-scale terrain modeling from multiple sensors with dependent Gaussian processes. In: Proceedings of the International Conference on Intelligent Robots and Systems (IROS), Taipei (2010)
3. Hadsell, R.; Bagnell, J.A.; Huber, D.; Hebert, M.: Accurate rough terrain estimation with space-carving kernels. In: Robotics: Science and Systems (RSS), Seattle (2009)
4. Ho, K.; Peynot, T.; Sukkarieh, S.: Analysis of terrain geometry representations for traversability of a Mars Rover. In: Proceedings of the 11th NCSS/NSSA Australian Space Science Conference, Canberra (2011)

5. Reina, G.; Underwood, J.; Brooker, G.; Durrant-Whyte, H.: Radar-based perception for autonomous outdoor vehicles. J. Field Robot. **28**(6), 894–913 (2011)
6. Douillard, B.; Underwood, J.; Kuntz, N.; Vlaskine, V.; Quadros, A.; Morton, P.; Frenkel, A.: On the segmentation of 3D LIDAR point clouds. In: Proceedings of the IEEE International Conference on Robotics and Automation Shanghai International Conference Center, Shanghai (2011)
7. Milella, A.; Reina, G.; Underwood, J.; Douillard, B.: Visual ground segmentation by radar supervision. Robot. Auton. Syst. (2012). doi:10.1016/j.robot.2012.10.001
8. Yu, J.; Cheng, S.; Sun, Z.; et al.: An optimal algorithm of 3D path planning for mobile robots[J]. J. Cent. South Univ. (Sci. Technol.) **40**(2), 471–476 (2009)
9. Liang, J.: A path planning algorithm of mobile robot in known 3D environment. Adv. Control Eng. Inf. Sci. Procedia Eng. **15**, 157–162, 1877–7058 © 2011 Published by Elsevier Ltd. (2011)
10. Gilimyanov, R.F.; Pesterev, A.V.; Rapoport, L.B.: Smoothing curvature of trajectories constructed by noisy measurements in path planning problems for wheeled robots. J. Comput. Syst. Sci. Int. **45**(5), 812–819 (2008)
11. Sezer, V.; Gokasan, M.: A novel obstacle avoidance algorithm: follow the gap method. Robot. Auton. Syst. **60**, 1123–1134 (2012)
12. Miao, H.; Tian, Y.C.: Robot path planning in dynamic environments using a simulated annealing based approach. In: Proceedings of the 10th International Conference on Control, Automation, Robotics and Vision (ICARCV'08), Hanoi, pp. 1253–1258 (2008)
13. Wang ,Y.; Sillitoe, P.W.; Mulvaney, J.: Mobile robot path planning in dynamic environments. In: Proceedings of the International Conference on Robotics and Automation, vol. 1, pp. 71–76. IEEE, Roma (2007)
14. Miao, H.: A multi-operator based simulated annealing approach for robot navigation in uncertain environments. Int. J. Comput. Sci. Secur. **4**(1), 50–61 (2010)
15. Reina, G.: Self-learning classification of radar features for scene understanding. Robot. Auton. Syst. **60**, 1377–1388 (2012)
16. Reina, G.; Underwood, J.; Brooker, G.: Short-range radar perception in outdoor environments. In: Towards Autonomous Robotics Systems (TAROS), pp. 265–276 (2011)
17. Nash, J.; Sutcliffe, J.: River flow forecasting through conceptual models part 1: a discussion of principles. Int. J. Robot. Res. **28**(2), 172–190 (2009)
18. Kidono, K.; Miyasaka, T.; Watanabe, A.; Naito, T.; Miura, J.: Pedestrian recognition using high-definition LIDAR. In: Proceedings of the IEEE Intelligent Vehicles Symposium (IV), Germany (2011)
19. Mann, M.P.; Shiller, Z.: Dynamic stability of off-road vehicles: a geometric approach. In: Robotics and Automation, ICRA, pp. 3705–3710, 1050–4729 (2006)
20. Belaidi, H.; Bentarzi, H.; Belaidi, A.: Application of splines to three-dimensional map building for mobile robot. In: Proceedings of the International Conference on Applied and Computational Mathematics, Ankara (2012)
21. Belaidi, H.; Bentarzi, H.; Belaidi, A.: 2D/3D optimal path planning for an autonomous mobile robot using NURBS and distance calculation. In: Proceedings of the XIII Workshop en Agentes Fisicos (WAF'12), Santiago de Compostela (2012)
22. Shiller, Z.; Gwo, Y.R.: Dynamic motion planning of autonomous vehicles. IEEE Trans. Robot. Autom. **7**(2), 241–249 (1991)