Mathematics of Music (MUSI 175)
Yale University Department of Music
January – May 2018

Professor Richard Cohn (richard.cohn@yale.edu)
Assistant Instructor Ethan Edl (ethan.edl@yale.edu)
Class meets Monday and Wednesday, 2:30 – 3:45, in Stoeckel 106

Pre-requisite. The course is intended for students with basic arithmetic skills and the ability to read music; any other knowledge required will be provided in class. MUSI 175 has no music theory prerequisites; the course is designed to present musical concepts understandable with no prior music training and in a mathematically robust setting.

Negative pre-requisite. Students who have received credit for Music 309 are not permitted to enroll in the course.

Fit to the Curriculum. Music 175 fulfills one unit of the College-wide requirement in Quantitative Reasoning. It cannot be applied to the music major.

Activities. Each of the four units will be structured in the following way: Course Notes, in support of the lectures, will be distributed. Students will complete ungraded problem sets; you can collaborate on these. We will "workshop" solutions to these problems, and the solutions will be posted. You will complete a formal problem set at the end of each unit; you are not permitted to consult with other students. Solutions will not be posted, and discussion of these will be general. A Midterm and final examination will recast material from the earlier problem sets, and also give you an occasion to synthesize the various components of the course.

Submissions: Formal problems sets (for assessment) will be due at the beginning of class (2:30) on specified Mondays. These should primarily be submitted via hard copy, but may be scanned and emailed by 2:30 when this is not possible.

Late Assignments: You may not submit late weekly assignments, except under the following two conditions: 1) it must be submitted no later than the beginning of the immediately subsequent class meeting, and 2) you can use this option only *one time* during the semester: husband this resource well! Any other late assignments will require a Dean's Excuse.

Academic Integrity: Four graded problem sets and both exams will be completed by individuals, without input from others. Collaboration on these assignments is unacceptable. Do not show other students your work before you submit it, and do not ask to see another student's work.

Provisional Schedule of Topics

The ordering of topic is fixed but the day-by-day schedule, and the due dates for problem sets, are subject to change.

UNIT 1. Set-Theoretic Models of Cyclic Rhythms and Musical Scales

January 17 - 19. A deep analogy between musical pitch and musical time. Arithmetic models of pitch and time, in linear spaces. Scale and Interval.

January 22 -24 . Cyclical universes. Set theory and its musical applications. Cyclical models of duple and triple meter.

January 29 - 31. Models of cyclic rhythms, up to six beats per cycle. Modular arithmetic. Unit and prime generators.

Feburary 5 - 7. Models of cyclic rhythms, seven and eight beats per cycle. A Cyclic model of the diatonic scale.

First Problem Set Due February 12

February 12 - 14. Combinations and Pascal's Triangle. A cyclic model of Afro-Caribbean rhythm. Prime generated, maximally even, and hyperdiatonic rhythms.

UNIT 2. Algebraic Models of Tuning and Intonation

February 19 - 21. Introduction to number theory and its musical applications . Why are there 7 notes in a scale, and 12 notes to an octave? Pythagorean models of tuning

Second Problem Set Due February 26

February 26 - 28. Just intonation and the Eulerian *Tonnetz*. Equal temperament.

Midterm Exam, March 5

UNIT 3. Graph-Theoretic Models of Chords

March 7. Introduction to graph theory and its musical applications.

Two-week break

March 26 - 28. The circle of fifths, the toroidal *Tonnetz*, and other graphs of chordal space.

April 2. Graph Dualism

UNIT 4. Geometric and Topological Models of Chordal Spaces

Third Problem Set Due April 9

April 4 - 9. A Möbius-strip model of 2-note chord space.

April 11 - 16. A topologically unusual model of 3-note chord space.

Fourth Problem Set Due April 23

April 18 -23. Generalization to higher-dimensional chordal spaces.

April 25. Temporal Reserve Fund.

Final Exam May 5

ASSESSMENT TASK	***WEIGHTING***
Four Graded Problem	60% (15% each set)
Midterm Examination	15%
Final Examination	25%