Summer Term 2017

Homework 2 – *Nature Inspired Algorithms*

https://hpi.de/friedrich/teaching/ss17/natinsalg.html

The goal of this homework is to make sure you understand some basic tools from runtime analysis of Nature-inspired algorithms.

The homework is submitted on Moodle (`https://hpi.de/friedrich/moodle/`) by uploading a PDF file with your solutions. You are welcome to write your solutions out by hand and scan them.

Recall the MacLaurin series expansion of the exponential function that I wrote on the board:

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$$

We let ln denote the *natural logarithm* (logarithm with base e).

Exercise 2 *Show the following claim by induction.*

(a) For all $x \geq -1$ and all $k \in \mathbb{N}$, $(1+x)^k \geq 1 + kx$.

Show the following two inequalities.

(b) For all $x > -1$, $x \neq 0$: $\ln(1+x) < x$.

(c) For all $x > -1$, $x \neq 0$ and all $r > 0$: $(1+x)^r < e^{rx}$.

Exercise 3 *Consider the (1+1) EA and suppose the current solution has exactly k 0s (and, thus, $n-k$ 1s). Recall that mutation considers the current solution and flips each bit independently with probability $1/n$.*

(a) Suppose that k is a constant with respect to n. Show that the probability that the mutation creates the all-1s individual in the next step is $\Theta(n^{-k})$.

(b) Show that there is a constant c (independent of k and n) such that the probability that the mutation creates an individual with strictly less than k 0s is at least ck/n.