

Summer Term 2017

## Homework 1 – *Nature Inspired Algorithms*

<https://hpi.de/friedrich/teaching/ss17/natinsalg.html>

The goal of this homework is to make sure you follow the basic probability theory from Lecture 3.

The homework is submitted on Moodle (<https://hpi.de/friedrich/moodle/>) by uploading a PDF file with your solutions. You are welcome to write your solutions out by hand and scan them.

Let  $\Omega$  be a countable set of elementary events and  $P$  be a probability measure on  $\Omega$ . Recall that a *random variable* is a mapping  $X: \Omega \rightarrow \mathbb{R}$ . The *expected value* of a random variable  $X$  is

$$E(X) = \sum_{\omega \in \Omega} P(\omega)X(\omega),$$

and its variance is

$$\text{Var}(X) = E((X - E(X))^2).$$

**Exercise 1** *Prove the following.*

- (a) *For any two arbitrary random variables  $X$  and  $Y$ , we have  $E(X+Y) = E(X) + E(Y)$ . **Hint: it helps to write**  $P(X = r) = \sum_{\omega: X(\omega)=r} P(\omega)$ .*
- (b) *For any two independent random variables  $X$  and  $Y$ , we have  $E(XY) = E(X)E(Y)$ . Is it necessary to assume independence? Why or why not?*
- (c) *For any two independent random variables  $X$  and  $Y$ , we have  $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ . Is it necessary to assume independence? Why or why not?*