Research Review

Game Tree Searching by Min / Max Approximation by Ron L. Rivest, Laboratory for Computer Science, MIT, Cambridge, MA 02139, U.S.A.; August, 1986

Summary of the paper's goals or techniques introduced.

The goals of this paper

In this paper, the author, Mr. Rivest introduce a technique for searching in game trees which is based on a generalized mathematical mean-value theorem to approximating of the min and max operators.

Mr. Rivest's wishes to create a new game tree searching idea is not alone, there are other researches also believes that alpha-beta pruning have been essential in reducing the computational burden of exploring a game tree; however other new technique is still needed to enhance the process. His method to generate a numeric value as a reliable reference, so that the computer will know what the next leaf node to be opened most effectively is really novel like.

The introduced techniques

The new introduced techniques is called "min/max approximation." It focuses the computer's attention on the important lines of play, and allows computer to define the "expandable tip upon whose value the backed-up value at the root most heavily depends" in a nontrivial manner.

How's the method created and how it implemented?

The method is based on generalized means-value theorem (Lagrange).

The author noted "Min and Max" functions value runs from 0 to –inf. and 1 to inf. respectively, so it is a continuous functions, and it is differentiable ; therefore generalized means value is suitable for a "sensitivity analysis" in estimated the node value rather than just the min or max functions.

Summary of the paper's results

The author used Connect Four, a well-known common game to demonstrate the introduced techniques. The following denotes the details of this experiment, MM represent the Min/Max approximation method, AB represents the Alpha Beta Pruning method.

- A static evaluator was used as the move and un-move operators to implement the static evaluation function. This static evaluation function was used by all game-playing strategies, so that differences in playing ability would not be due to differences in the static evaluators.
- In the playoffs, each strategy was allocated a fixed amount of resources to use in computing its move. This was a fixed bound per turn; a strategy could not save on the computation for one turn and use it later. Two different resource bounds were used: elapsed CPU time (measured in seconds), and calls to the basic "move" subroutine (measured in thousands of calls).
- The implementation of the min/max heuristic worked as follows: (1) The 'reverse approximation" was used; the value computed for a node was its true backed-up rain/max value, based on the tree computed so far. (2) The penalty on an edge was computed to be 0.05 plus the absolute value of the difference between the natural logarithm of the value of the node

and the natural logarithm of the value of his "best" The table below shown the game result of comparing “Min Max search by approximation” with the “Minimax search with alpha-beta pruning”.

Resource bound per turn	MM wins	AB wins	Ties
1 second	41	46	11
2 second	40	42	16
3 seconds	36	44	18
4 seconds	39	52	7
5 seconds	30	55	13
Total	186	239	65
1000 moves	47	35	16
2000 moves	50	35	13
3000 moves	42	47	9
4000 moves	49	42	7
5000 moves	61	31	6
Total	249	190	51

Clearly, Min/Max approximation definitely doing better than AB pruning in the “move bound” than the CPU time resource bound.