STA 602 Lab 5

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Exercise 1

- Exp(1) cannot be used as a candidate distribution to draw samples from N(0,1) on the interval (-3, 3), because it has no support on the negative plane.
- Between the interval of (-2, 2), N(0, 1) can be used to draw standard Cauchy samples because we have a finite ceiling constant. The standard Cauchy has the highest density, $\frac{1}{\pi}$ at the point of x=0; same for the standard normal with a density of $\frac{1}{\sqrt{2\pi}}$ at the same location. Hence, we can always find a scaling constant, M, for the rejection sampling.
- Yes, knowing the kernel is enough for rejection sampling. We can draw the samples and then normalize it using the finite samples. It is a Yes for a 100-variate distribution. However, the efficiency is not ideal.

Exercise 2

Given just the kernel is the possible M' are anything that is ≥ 1 for a uniform candidate distribution. The range is certainly different if apply to the true density function.

Exercise 3

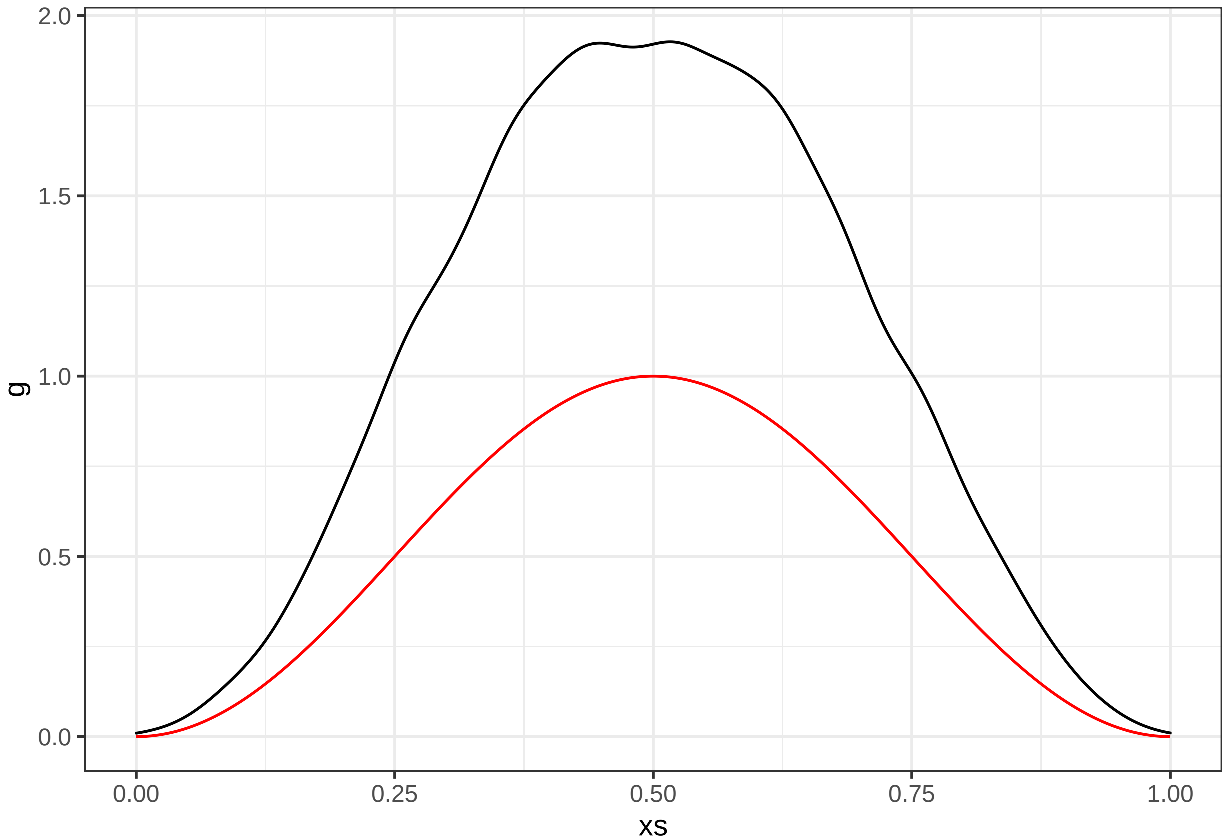
```r
# target density function
f <- function(x) {
  sin(pi * x)^2
}

set.seed(23523)
nsim <- 10000
M <- 1
accepted_count <- 0
total_count <- 0
res <- numeric()

while (accepted_count < nsim){
  x <- runif(1)
  r <- f(x) / (M * dunif(x))
  if (runif(1) <= r) {
    res <- append(res, x)
    accepted_count <- accepted_count + 1
  }

  total_count <- total_count + 1
}
```

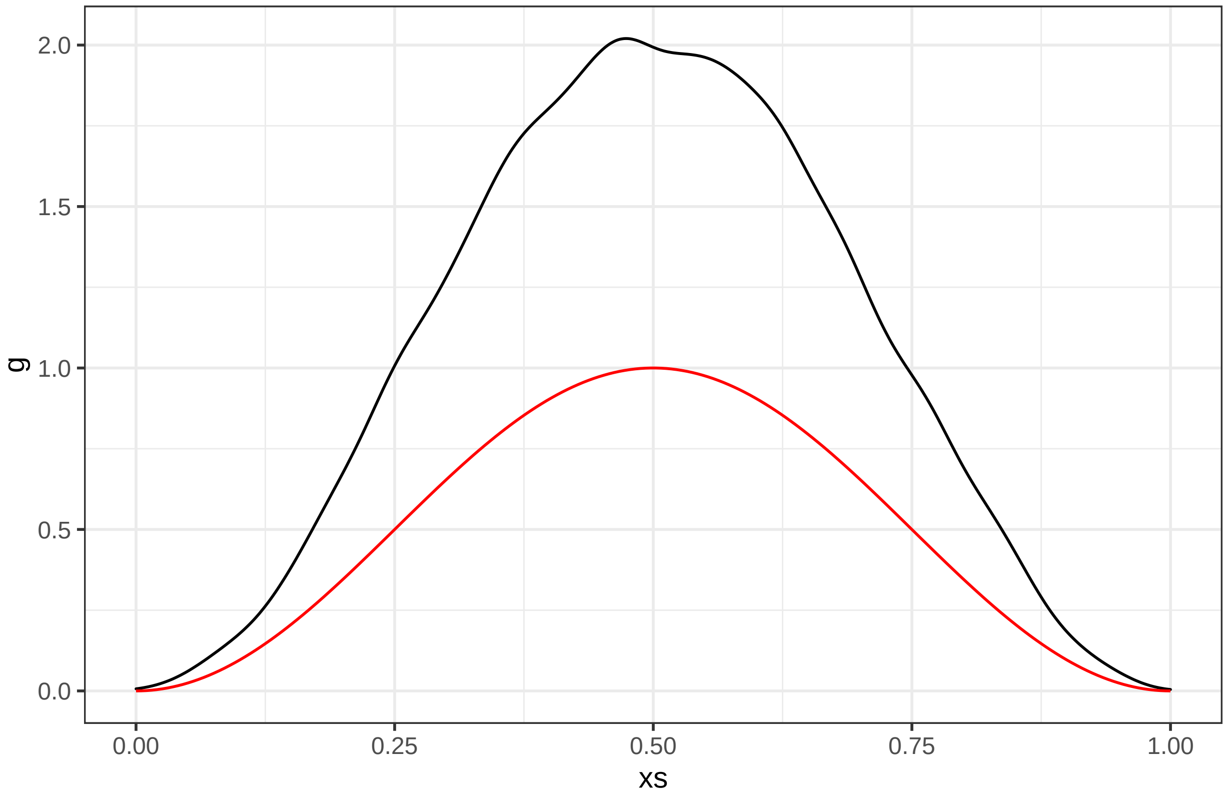
```r
theta <- seq(0, 1, length.out = 200)
g_theta <- sin(pi * theta)^2
ggplot() +
  geom_density(data=data.frame(xs = res), aes(xs)) +
  geom_line(data=data.frame(theta=theta, g=g_theta), aes(theta, g), color="red")
```



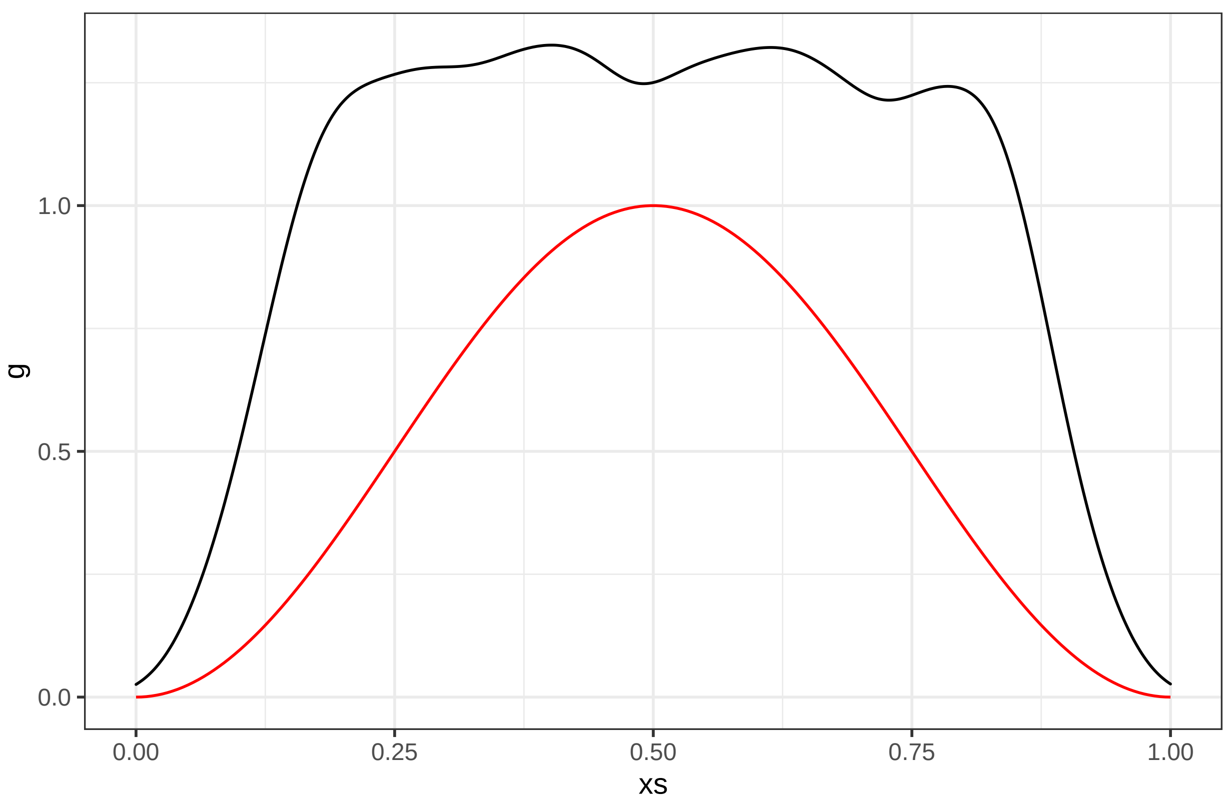
Exercise 4

- Because $sin^2(\pi x)$ is no the real density, just a kernel, of course the samples we obtained from the rejection sampler will differ. But the overall shape is simliar.
- At $M = 1$, I had about 50% acceptance rate. A total counts of 20,026 for 10,000 accepted samples.
- The higher the M' is, the lower the acceptance rate. On the contrary, if M' is too small, the candidate q will not cover the entire support of the target distribution, we will not obtain good samples.

M = 3


2.0
1.5
1.0
0.5
0.0
g
0.00
0.25
0.50
0.75
1.00
xs

M = 0.25


1.0
0.5
0.0
g
0.00
0.25
0.50
0.75
1.00
xs

Exercise 5

Beta(2, 2) is a nice choice compare to the uniform candidate. Using this, the acceptance rate increased from 50% to around 70%.