



# TECHNICAL NOTES

Health Equity Assessment Toolkit (HEAT and HEAT Plus)

VERSION 5.0



World Health  
Organization



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# Contents

1	Introduction	4
2	Disaggregated data	5
2.1	Indicators.....	6
2.2	Dimensions of inequality .....	6
3	Summary measures	8
3.1	Absolute concentration index (ACI) .....	11
3.2	Between-group standard deviation (BGSD).....	12
3.3	Between-group variance (BGV).....	14
3.4	Coefficient of variation (COV) .....	15
3.5	Difference (D) .....	16
3.6	Index of disparity (unweighted) (IDISU).....	20
3.7	Index of disparity (weighted) (IDISW) .....	21
3.8	Mean difference from best performing subgroup (unweighted) (MDBU) .....	23
3.9	Mean difference from best performing subgroup (weighted) (MDBW) .....	24
3.10	Mean difference from mean (unweighted) (MDMU).....	25
3.11	Mean difference from mean (weighted) (MDMW).....	27
3.12	Mean log deviation (MLD) .....	28
3.13	Population attributable fraction (PAF).....	29
3.14	Population attributable risk (PAR) .....	31
3.15	Ratio (R) .....	33
3.16	Relative concentration index (RCI).....	38
3.17	Relative index of inequality (RII).....	39
3.18	Slope index of inequality (SII) .....	41
3.19	Theil index (TI).....	43
	Annex	45
	Annex 1 Summary measures: overview .....	45

# 1 Introduction

Equity is at the heart of the United Nations 2030 Agenda for Sustainable Development, which aims to “leave no one behind”. This commitment is reflected throughout the 17 Sustainable Development Goals (SDGs) that Member States have pledged to achieve by 2030.

Monitoring inequalities (observable differences across population subgroups) is essential for tackling inequities (differences that are deemed unfair, avoidable or remediable): it allows identifying vulnerable population subgroups that are left behind and helps inform equity-oriented policies, programmes and practices that can close existing gaps.

The World Health Organization (WHO) is committed to achieving equity in health and has developed a number of tools and resources for monitoring health inequalities, including the Health Equity Assessment Toolkit.

The **Health Equity Assessment Toolkit** is a free and open-source software application that facilitates the exploration, analysis and reporting of health inequalities. Through innovative and interactive data visualizations, the software makes it easy to assess and communicate data about health inequalities. Disaggregated data and summary measures are visualized in a variety of graphs and tables that can be customized according to users’ needs. Results can be exported to communicate findings to different audiences and inform evidence-based decision making.

The software is available in **two editions**:



**HEAT** (built-in database edition), which contains datasets of disaggregated data from the WHO Health Inequality Data Repository,



**HEAT Plus** (upload database edition), which allows users to upload their own datasets of disaggregated data.

Together, HEAT and HEAT Plus are powerful tools that help make data about inequalities accessible and bring key messages to decision-makers to tackle inequities and achieve the SDGs.

These **technical notes** provide information about the data presented in HEAT and HEAT Plus, including disaggregated data (Section 2) and summary measures of inequality (Section 3). Following a general introduction to disaggregated data, Section 2 provides details about the types and characteristics of indicators and inequality dimensions (Sections 2.1 and 2.2). Section 3 first gives a general overview of summary measures and then lists detailed information about the 19 summary measures calculated in HEAT and HEAT Plus (Sections 3.1–3.19). For each summary measure, information about the definition, calculation, and interpretation are provided; examples illustrate the use and interpretation of each summary measure. A summary table of all summary measures is available in Annex 1.

Throughout the technical notes, **blue boxes** highlight links to further resources and summarize the most salient points of each section. **Orange boxes** highlight further information on HEAT. **Green boxes** provide useful tips for using HEAT Plus.

You may want to read these technical notes sequentially and in its entirety, or consult different sections as required. You are also encouraged to consult the other documents that accompany the software, including the user manual, which provide detailed information about its features and functionalities. Moreover, you may want to supplement these resources with materials that provide further information on the theoretical and/or practical steps of inequality monitoring, such as the WHO’s *Handbook on health inequality monitoring* and *National health inequality monitoring: a step-by-step manual*. Many resources are publicly available through the WHO Health Inequality Monitor, and although with a focus on health, the approaches may be applied to any topic.



## LINKS

- [WHO Health Inequality Monitor](#)
- [WHO Health Inequality Data Repository](#)
- [Health Equity Assessment Toolkit \(HEAT and HEAT Plus\)](#)

## 2 Disaggregated data

Assessing within-country inequalities requires the use of data that are disaggregated according to relevant dimensions of inequality. Disaggregated data break down overall averages, revealing differences between different population subgroups. They are useful to identify patterns of inequality in a population and vulnerable subgroups that are being left behind.

Two types of data are required for calculating disaggregated data: data about “indicators” that describe an individual’s experience and data about “dimensions of inequality” that allow populations to be organized into subgroups according to their demographic, socioeconomic and/or geographic characteristics.

The following two sections provide more information about indicators (Section 2.1) and inequality dimensions (Section 2.2).



## DISAGGREGATED DATA

- ✓ **Disaggregated data** are data on indicators disaggregated by relevant dimensions of inequality (demographic, socioeconomic or geographic factors)



## HEAT

- ✓ HEAT contains datasets of disaggregated data from the **WHO Health Inequality Data Repository**. Please refer to the Indicator Metadata in the About menu for detailed information about the data available in HEAT.



## HEAT PLUS

HEAT Plus allows you to upload your own datasets of disaggregated data. Datasets have to be in a specific format and stored as comma separated values (csv) or Microsoft Excel (xls orxlsx) files in order to be uploaded to HEAT Plus. The **HEAT Plus Template** illustrates the required structure. The **HEAT Plus Validation Tool** helps you prepare your data according to the template. Please refer to the user manual for further information.

## 2.1 Indicators

There are different types of indicators, which may be reported at different scales. Differentiating between the **different indicator types and scales** is important as these characteristics have implications for the calculation of summary measures (see Section 3).

Indicators can be divided into favourable and adverse indicators. **Favourable indicators** measure desirable events that are promoted through public action. For example, health intervention indicators, such as antenatal care coverage, and desirable health outcome indicators, such as life expectancy, are favourable indicators. For these indicators, the ultimate goal is to achieve a maximum level, either in health intervention coverage or health outcome (for example, complete coverage of antenatal care or the highest possible life expectancy). **Adverse indicators**, on the other hand, measure undesirable events, that are to be reduced or eliminated through public action. Undesirable health outcome indicators, such as stunting prevalence in children aged less than five years or under-five mortality rate, are examples of adverse indicators. Here, the ultimate goal is to achieve a minimum level in health outcome (for example, a stunting prevalence or mortality rate of zero).

Furthermore, indicators can be reported at different **indicator scales**. For example, while total fertility rate is usually reported as the number of births *per woman* (indicator scale = 1), coverage of skilled birth attendance is reported as a *percentage* (indicator scale = 100) and neonatal mortality rate is reported as the number of deaths *per 1000 live births* (indicator scale = 1000).



### INDICATORS

- ✓ **Indicators** describe an individual's experience
- ✓ Different indicators have different **characteristics**
  - **Favourable indicators** measure desirable events, while **adverse indicators** measure undesirable events
  - Indicators are reported at different **indicator scales**



### HEAT PLUS

In the HEAT Plus Template you must provide information about the indicator type (favourable vs. adverse) and the indicator scale for each indicator by filling in the variables 'favourable\_indicator' and 'indicator\_scale'. Please refer to the FAQs in the user manual or the template legend for instructions on how to correctly fill in these variables.

## 2.2 Dimensions of inequality

There are **different types of inequality dimensions with different characteristics**. It is important to take these characteristics into account as they have implications for the calculation of summary measures, too (see Section 3).

At the most basic level, dimensions of inequality can be divided into **binary dimensions**, i.e. dimensions that compare the situation in two population subgroups (e.g. females and males), versus

**dimensions that look at the situation in more than two population subgroups** (e.g. economic status quintiles).

In the case of dimensions with more than two population subgroups it is possible to differentiate between dimensions with ordered subgroups and non-ordered subgroups. **Ordered dimensions** have subgroups with an inherent positioning and can be ranked. For example, education has an inherent ordering of subgroups in the sense that those with less education unequivocally have *less* of something compared to those with more education. **Non-ordered dimensions**, by contrast, have subgroups that are not based on criteria that can be logically ranked. Subnational regions are an example of non-ordered groupings.

For ordered dimensions, subgroups can be ranked from the most-disadvantaged to the most-advantaged subgroup. The **subgroup order** defines the rank of each subgroup. For example, if education is categorized in three subgroups (no education, primary school, and secondary school or higher), then subgroups may be ranked from no education (most-disadvantaged subgroup) to secondary school or higher (most-advantaged subgroup).

For binary and non-ordered dimensions, while it is not possible to rank subgroups, it is possible to identify a **reference subgroup**, that serves as a benchmark. For example, for subnational regions, the region with the capital city may be selected as the reference subgroup in order to compare the situation in all other regions with the situation in the capital city.



## DIMENSIONS OF INEQUALITY

- ✓ **Dimensions of inequality** allow populations to be organized into subgroups according to their demographic, socioeconomic, and/or geographic characteristics
- ✓ Different inequality dimensions have different **characteristics**
  - Dimensions may have 2 subgroups (binary dimensions) or >2 subgroups
  - Dimensions with >2 subgroups may be ordered or non-ordered: **ordered dimensions** have subgroups with an inherent positioning, while subgroups of **non-ordered dimensions** cannot be ranked
  - Subgroups of ordered dimensions have a specific **subgroup order**
  - For non-ordered dimensions, one subgroup may be identified as a **reference subgroup**



## HEAT PLUS

In the HEAT Plus template you must provide information about the dimension type (ordered vs. non-ordered), subgroup order and reference subgroup by filling in the variables 'order\_dimension', 'subgroup\_order' and 'reference\_subgroup'. Please refer to the FAQs in the user manual or the template legend for instructions on how to correctly fill in these variables.

### 3 Summary measures

Summary measures build on disaggregated data and present the level of inequality across multiple population subgroups in a single numerical figure. They are useful to compare the situation between different indicators and inequality dimensions and assess changes in inequality over time.

Many different summary measures exist, each with different strengths and weaknesses. Knowing the characteristics of the different summary measures is important so that you can decide which summary measure is suitable for the analysis and interpret results correctly.

Summary measures of inequality can be divided into absolute measures and relative measures. For a given indicator, **absolute inequality measures** indicate the magnitude of difference between subgroups. They retain the same unit as the indicator.<sup>1</sup> **Relative inequality measures**, on the other hand, show proportional differences among subgroups and have no unit.

Furthermore, summary measures may be weighted or unweighted. **Weighted measures** take into account the population size of each subgroup, while **unweighted measures** treat each subgroup as equally sized. Importantly, simple measures are always unweighted and complex measures may be weighted or unweighted.

**Simple measures** make pairwise comparisons between two subgroups, such as the most and least wealthy. They can be calculated for all indicators and dimensions of inequality. The characteristics of the indicator and dimension determine which two subgroups are compared to assess inequality. Contrary to simple measures, **complex measures** make use of data from all subgroups to assess inequality. They can be calculated for all indicators, but they can only be calculated for dimensions with more than two subgroups.<sup>2</sup>

Complex measures can further be divided into ordered complex measures and non-ordered complex measures of inequality. **Ordered measures** can only be calculated for dimensions with more than two subgroups that have a natural ordering. Here, the calculation is also influenced by the type of indicator (favourable vs. adverse). **Non-ordered measures** are only calculated for dimensions with more than two subgroups that have no natural ordering.<sup>3</sup>

The software enables the assessment of inequalities using 19 different summary measures of inequality, which are calculated based on the disaggregated data. The following sections give detailed information about the definition, calculation and interpretation of each summary measure. Examples are provided to illustrate how each summary measure can be used and interpreted.

Figure 1 provides an overview of the 19 summary measures and figure 2 presents a decision tree for the selection of appropriate summary measure(s) for the analysis. Annex 1 contains a summary table of all summary measures along with their basic characteristics, formulas and interpretation.

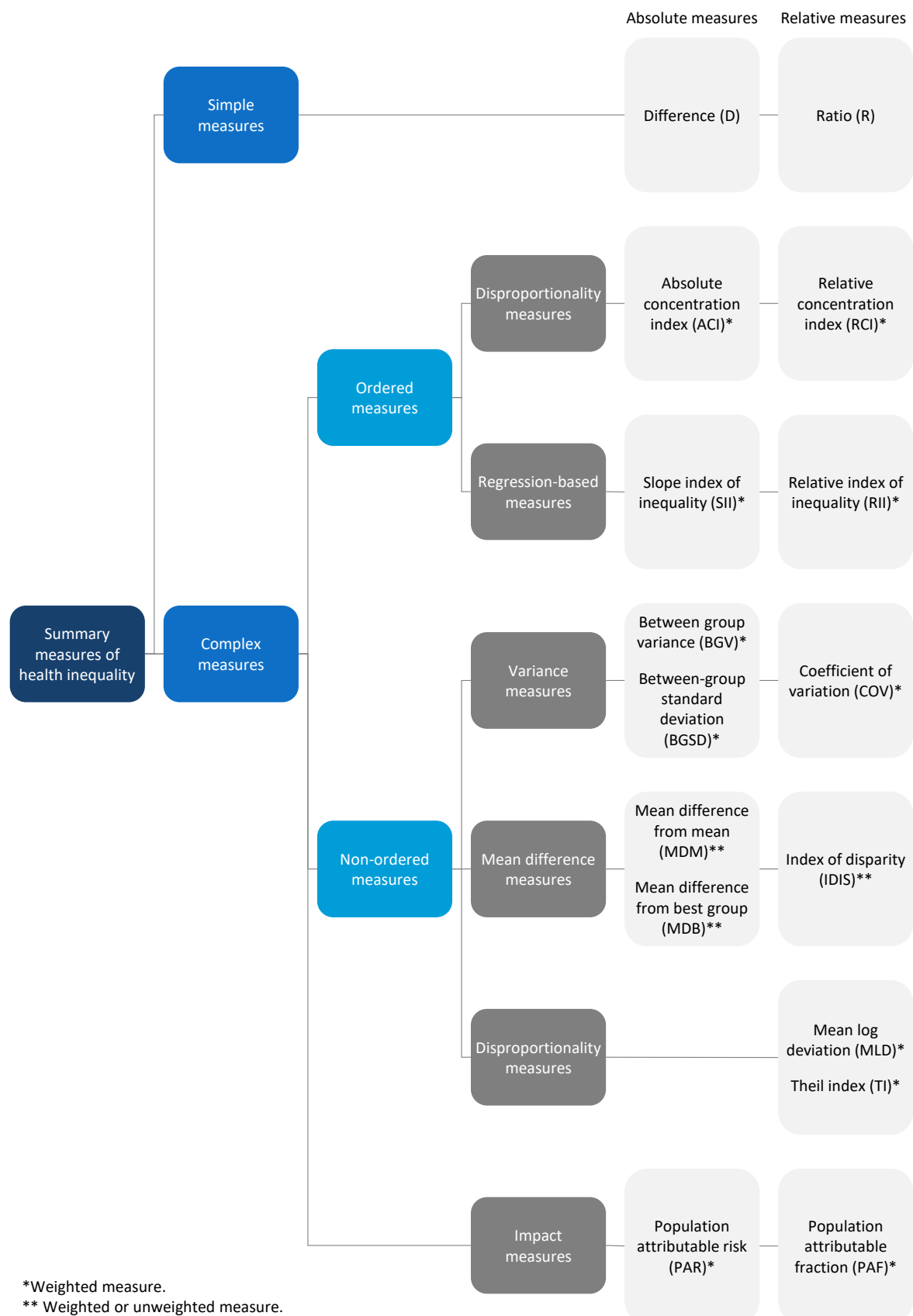
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1 One exception to this is the between-group variance (BGV), which takes the squared unit of the indicator.

2 Exceptions to this are the population attributable risk (PAR) and the population attributable fraction (PAF), which can be calculated for all dimensions of inequality.

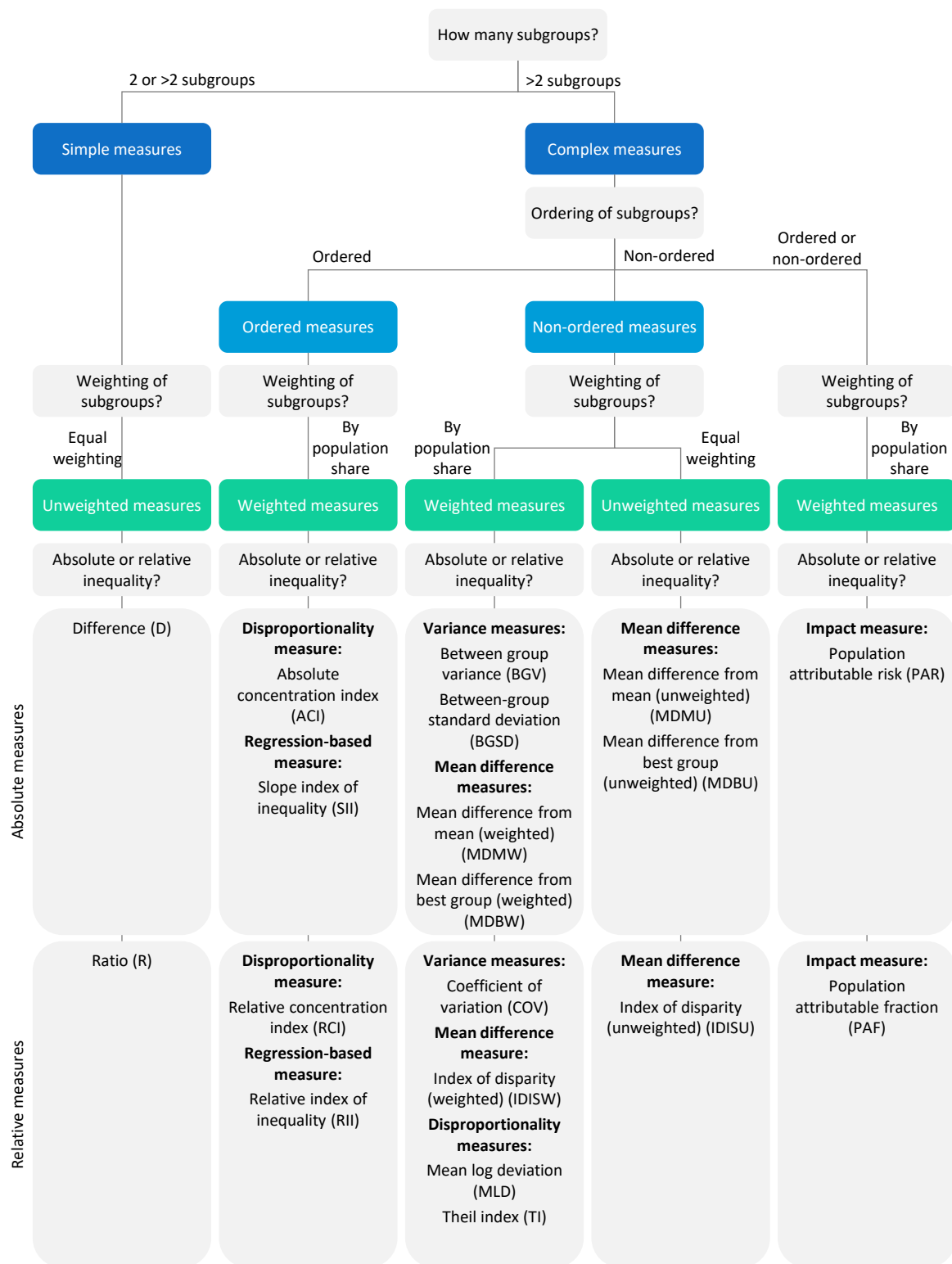
3 Non-ordered complex measures could also be calculated for ordered dimensions, however, in practice, they are not used for such dimensions and are therefore only reported for non-ordered dimensions.



**Figure 1** Overview of summary measures

Source: Schlottheuber A, Hosseini AR. Summary Measures of Health Inequality: A Review of Existing Measures and Their Application. International Journal of Environmental Research and Public Health. 2022; 19(6):3697. <https://doi.org/10.3390/ijerph19063697>

**Figure 2** Decision tree for selecting appropriate summary measures



Source: Schlotheuber A, Hosseinpoor AR. Summary Measures of Health Inequality: A Review of Existing Measures and Their Application. International Journal of Environmental Research and Public Health. 2022; 19(6):3697. <https://doi.org/10.3390/ijerph19063697>



## SUMMARY MEASURES

- ✓ **Summary measures** build on disaggregated data and present the level of inequality across multiple population subgroups in a single numerical figure
- ✓ Different summary measures have different **characteristics**
  - **Absolute measures** assess absolute differences; **Relative measures** capture proportional differences between subgroups
  - **Weighted measures** take into account the population size of each subgroup; **Unweighted measures** treat each subgroup as equally sized
  - **Simple measures** compare the situation between two subgroups; **Complex measures** consider all subgroups
  - **Ordered measures** are calculated for ordered inequality dimensions with >2 subgroups; **Non-ordered measures** are calculated for non-ordered inequality dimensions with >2 subgroups

### 3.1 Absolute concentration index (ACI)

#### Definition

ACI shows the gradient across population subgroups, on an absolute scale. It indicates the extent to which an indicator is concentrated among disadvantaged or advantaged subgroups.

ACI is an absolute measure of inequality that takes into account all population subgroups. It is calculated for ordered dimensions with more than two subgroups, such as economic status. Subgroups are weighted according to their population share. ACI is missing if at least one subgroup estimate or subgroup population share is missing.

#### Calculation

The calculation of ACI is based on a ranking of the whole population from the most-disadvantaged subgroup (at rank 0) to the most-advantaged subgroup (at rank 1), which is inferred from the ranking and size of the subgroups. The relative rank of each subgroup is calculated as:  $X_j = \sum_j p_j - 0.5p_j$ . Based on this ranking, ACI can be calculated as:

$$ACI = \sum_j p_j (2X_j - 1)y_j$$

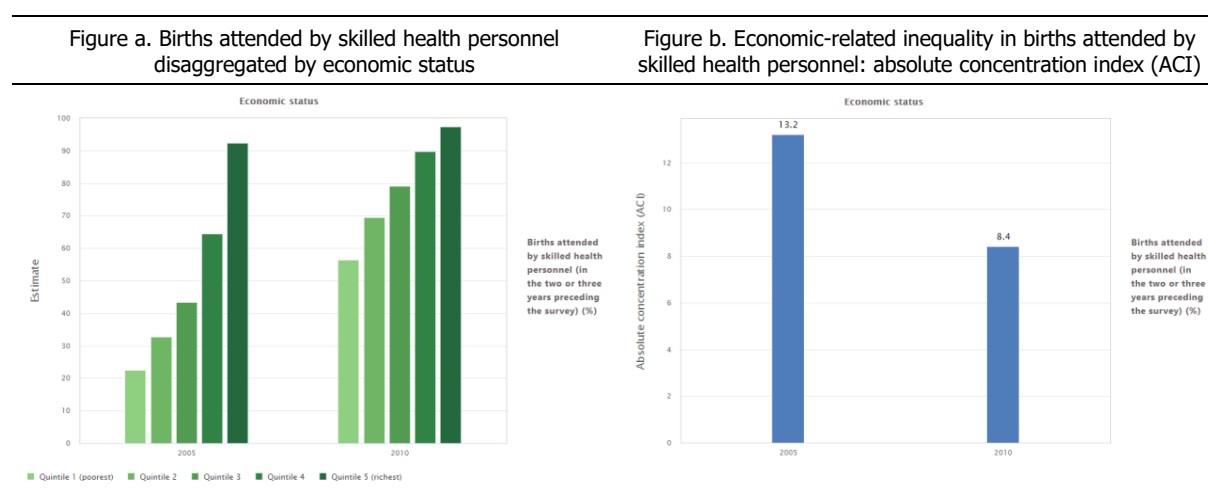
where  $y_j$  indicates the estimate for subgroup  $j$ ,  $p_j$  the population share of subgroup  $j$  and  $X_j$  the relative rank of subgroup  $j$ .

#### Interpretation

If there is no inequality, ACI takes the value zero. Positive values indicate a concentration of the indicator among the advantaged, while negative values indicate a concentration of the indicator among the disadvantaged. The larger the absolute value of ACI, the higher the level of inequality.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. ACI quantifies the level of inequality in each year. Figure b shows that absolute economic-related inequality, as measured by the ACI, reduced from 13.2 percentage points in 2005 to 8.4 percentage points in 2010.



### ABSOLUTE CONCENTRATION INDEX (ACI)

Measures the extent to which an indicator is concentrated among disadvantaged or advantaged population subgroups.

Takes the value zero if there is no inequality. Positive values indicate a concentration among advantaged, negative values among disadvantaged subgroups. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **ordered inequality dimensions**, such as economic status (ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.2 Between-group standard deviation (BGSD)

### Definition

BGSD is an absolute measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. BGSD is missing if at least one subgroup estimate or subgroup population share is missing.

## Calculation

BGSD is calculated as the square root of the weighted average of squared differences between the subgroup estimates  $y_j$  and the setting average  $\mu$ . Squared differences are weighted by each subgroup's population share  $p_j$ :

$$BGSD = \sqrt{\sum_j p_j (y_j - \mu)^2}$$

## Interpretation

BGSD takes only positive values, with larger values indicating higher levels of inequality. BGSD is zero if there is no inequality. BGSD is more sensitive to outlier estimates as it gives more weight to the estimates that are further from the setting average.

## Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. BGSD quantifies the level of inequality in each year. Figure b shows that absolute subnational regional inequality, as measured by the BGSD, reduced from 20.5 percentage points in 2005 to 14.7 percentage points in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

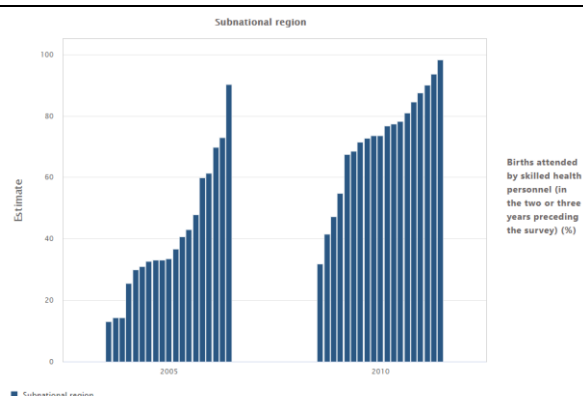
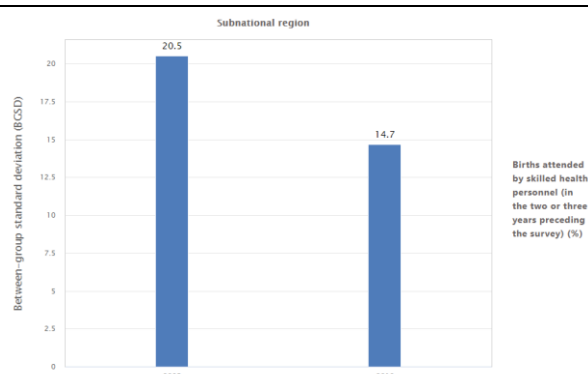


Figure b. Subnational regional inequality in births attended by skilled health personnel: between-group standard deviation (BGSD)





## BETWEEN-GROUP STANDARD DEVIATION (BGSD)

Measures the square root of the weighted average of squared differences between each population subgroup and the setting average.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

### 3.3 Between-group variance (BGV)

#### Definition

BGV is an absolute measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. BGV is missing if at least one subgroup estimate or subgroup population share is missing.

#### Calculation

BGV is calculated as the weighted average of squared differences between the subgroup estimates  $y_j$  and the setting average  $\mu$ . Squared differences are weighted by each subgroup's population share  $p_j$ :

$$BGV = \sum_j p_j (y_j - \mu)^2$$

#### Interpretation

BGV takes only positive values with larger values indicating higher levels of inequality. BGV is zero if there is no inequality. BGV is more sensitive to outlier estimates as it gives more weight to the estimates that are further from the setting average.

#### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. BGV quantifies the level of inequality in each year. Figure b shows that absolute subnational regional inequality, as measured by the BGV, reduced from 421.7 squared percentage points in 2005 to 214.8 squared percentage points in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

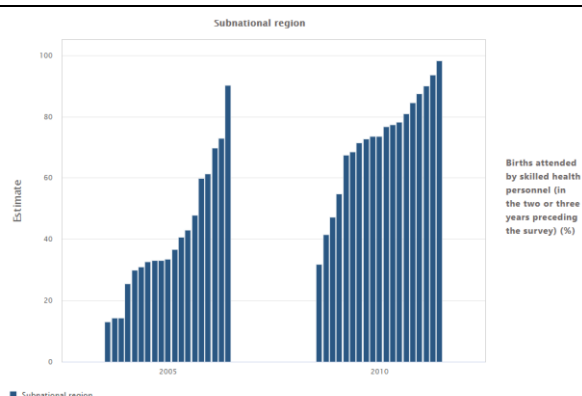
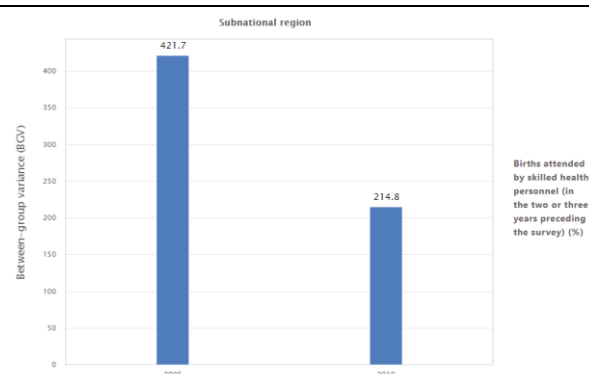


Figure b. Subnational regional inequality in births attended by skilled health personnel: between-group variance (BGV)



### BETWEEN-GROUP VARIANCE (BGV)

Measures the weighted average of squared differences between each population subgroup and the setting average.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.4 Coefficient of variation (COV)

### Definition

COV is a relative measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. COV is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

COV is calculated by dividing the between-group standard deviation (BGSD) by the setting average  $\mu$  and multiplying the fraction by 100:

$$COV = \frac{BGSD}{\mu} * 100$$

### Interpretation

COV takes only positive values, with larger values indicating higher levels of inequality. COV is zero if there is no inequality.

## Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. COV quantifies the level of inequality in each year. Figure b shows that relative subnational regional inequality, as measured by the COV, reduced from 38.7% in 2005 to 13.3% in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

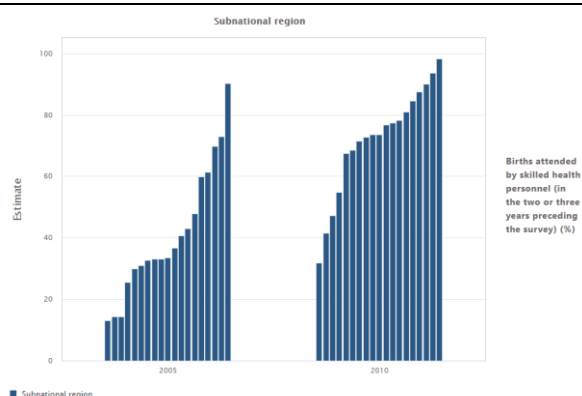
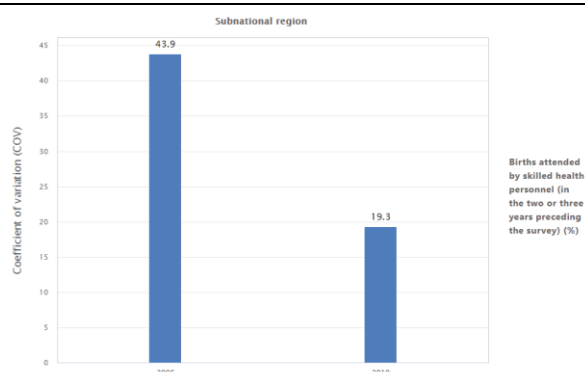


Figure b. Subnational inequality in births attended by skilled health personnel: coefficient of variation (COV)



## COEFFICIENT OF VARIATION (COV)

Measures the square root of the weighted average of squared differences between each population subgroup and the setting average (the between-group standard deviation) as a fraction of the setting average.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.5 Difference (D)

### Definition

D is an absolute measure of inequality that shows the difference between two population subgroups. It is calculated for all inequality dimensions, provided that subgroup estimates are available for the two subgroups used in the calculation of D.

### Calculation

D is calculated as the difference between two population subgroups:

$$D = y_{high} - y_{low}$$



Note that the selection of  $y_{high}$  and  $y_{low}$  depends on the characteristics of the inequality dimension and the type of indicator, for which D is calculated.

Table 1 provides an overview of the calculation of D.

**Table 1** Calculation of the Difference (D)

Dimension type	Reference subgroup selected?	Indicator type	
		Favourable indicator	Adverse indicator
Binary dimension	Yes	Reference group – Other group	Other group – Reference group
	No	Highest – Lowest	Highest – Lowest
Ordered dimension	N/A	Most-advantaged – Most-disadvantaged	Most-disadvantaged – Most-advantaged
Non-ordered dimension	Yes	Reference group – Other group (that maximizes the difference)	Other group (that maximizes the difference) – Reference group
	No	Highest – Lowest	Highest – Lowest

## Interpretation

If there is no inequality, D takes the value zero. Greater absolute values indicate higher levels of inequality.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. The difference quantifies the level of inequality in each year. Figure b shows that the difference between quintile 5 and quintile 1 reduced from 70.0 percentage points in 2005 to 41.0 percentage points in 2010.

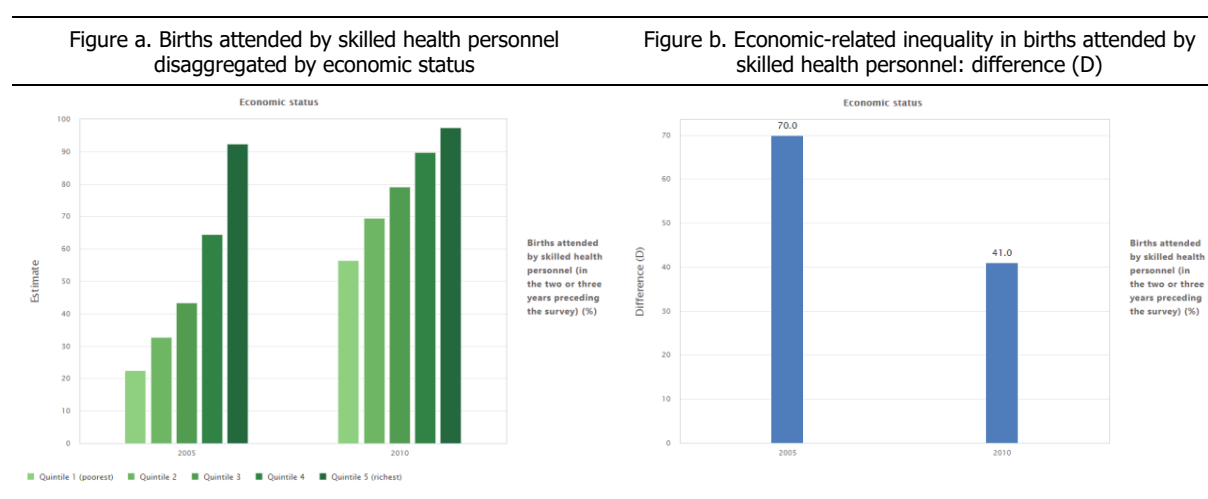


Figure c shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. The difference quantifies the level of inequality in each year. Figure d shows that the difference between the best and the worst performing region reduced from 77.1 percentage points in 2005 to 66.5 percentage points in 2010.

Figure c. Births attended by skilled health personnel disaggregated by subnational region

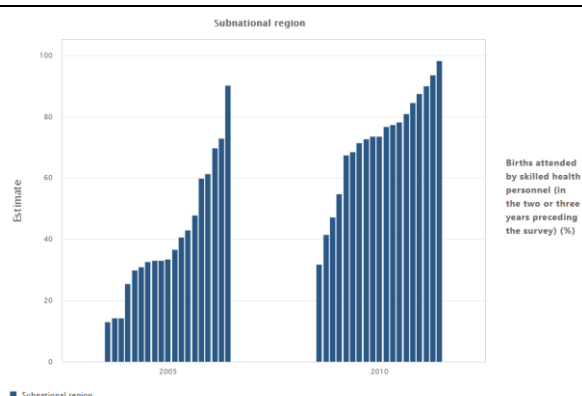
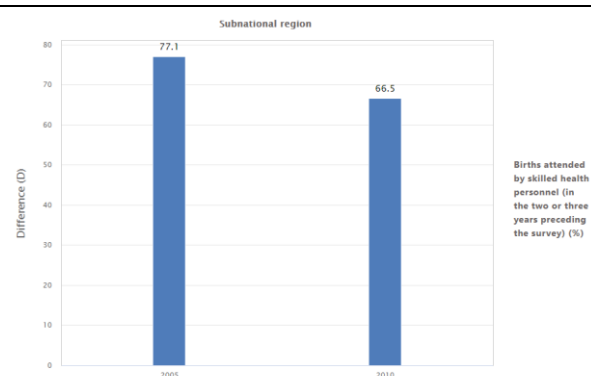


Figure d. Subnational regional inequality in births attended by skilled health personnel: difference (D)



## Other difference measures

In addition to the difference measure described above, variations of the difference are calculated for non-ordered inequality dimensions with many subgroups, such as subnational region. The following difference measures are calculated for

- Dimensions with more than 30 subgroups:
  - **Difference between percentile 80 and percentile 20.** The difference between percentile 80 and percentile 20 is calculated by identifying the subgroups that correspond to percentiles 20 and 80 and subtracting the estimate for percentile 20 from the estimate for percentile 80:  $D_{p80p20} = y_{p80} - y_{p20}$
  - **Difference between mean estimates in quintile 5 and quintile 1.** The difference between mean estimates in quintile 5 and quintile 1 is calculated by dividing subgroups into quintiles, determining the mean estimate for each quintile and subtracting the mean estimate in quintile 1 from the mean estimate in quintile 5:  $D_{q5q1} = y_{q5} - y_{q1}$
- Dimensions with more than 60 subgroups:
  - **Difference between percentile 90 and percentile 10.** The difference between percentile 90 and percentile 10 is calculated by identifying the subgroups that correspond to percentiles 10 and 90 and subtracting the estimate for percentile 10 from the estimate for percentile 90:  $D_{p90p10} = y_{p90} - y_{p10}$
  - **Difference between mean estimates in decile 10 and decile 1.** The difference between mean estimates in decile 10 and decile 1 is calculated by dividing subgroups into deciles, determining the mean estimate for each decile and subtracting the mean estimate in decile 1 from the mean estimate in decile 10:  $D_{d10d1} = y_{d10} - y_{d1}$
- Dimensions with more than 100 subgroups:
  - **Difference between percentile 95 and percentile 5.** The difference between percentile 95 and percentile 5 is calculated by identifying the subgroups that correspond to percentiles 5 and 95 and subtracting the estimate for percentile 5 from the estimate for percentile 95:  $D_{p95p5} = y_{p95} - y_{p5}$
  - **Difference between mean estimates in the top 5% and the bottom 5%.** The difference between mean estimates in the top 5% and the bottom 5% is calculated by dividing subgroups into vigintiles, determining the mean estimate for each vigintile and

subtracting the mean estimate in the bottom 5% from the mean estimate in the top 5%:

$$D_{v20v1} = y_{v20} - y_{v1}$$

For dimensions with many subgroups, these measures may be a more accurate reflection of the level of inequality than measuring the range between the maximum and minimum values using the (range) difference, as they avoid using possible outlier values. They are displayed in the 'Summary measures' tab of the selection menu for horizontal bar graphs showing disaggregated data under the 'Explore inequality' component of the tool.



## Difference (D)

Measures the difference between two population subgroups.

Takes the value zero if there is no inequality. The larger the absolute value, the higher the level of inequality.

Other difference measures are calculated for non-ordered inequality dimensions with many subgroups. These measures avoid using possible outlier values.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **all inequality dimensions**
- ✓ Takes into account **two population subgroups** (simple measure)
- ✓ Does not take into account the **population size** of subgroups (unweighted measure)

## 3.6 Index of disparity (unweighted) (IDISU)

### Definition

IDISU shows the unweighted average difference between each population subgroup and the setting average, in relative terms.

IDISU is a relative measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. IDISU is missing if at least one subgroup estimate or subgroup population share is missing.<sup>4</sup>

### Calculation

IDISU is calculated as the average of absolute differences between the subgroup estimates  $y_j$  and the setting average  $\mu$ , divided by the number of subgroups  $n$  and the setting average  $\mu$ , and multiplied by 100:

$$IDISU = \frac{\frac{1}{n} * \sum_j |y_j - \mu|}{\mu} * 100$$

Note that the 95% confidence intervals calculated for IDISU are simulation-based estimates.

### Interpretation

IDISU takes only positive values, with larger values indicating higher levels of inequality. IDISU is zero if there is no inequality.

### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. IDISU quantifies the level of inequality in each year. Figure b shows that relative subnational regional inequality, as measured by the IDISU, reduced from 39.2 in 2005 to 16.7 in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

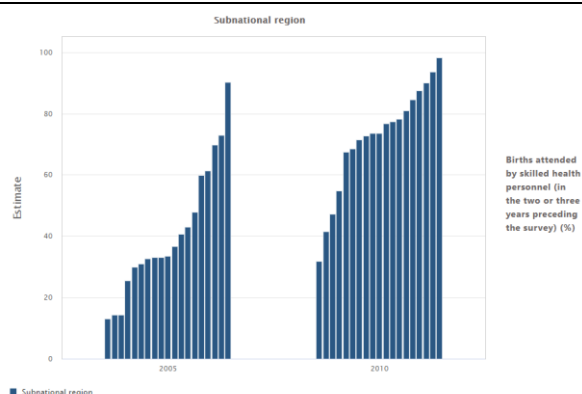
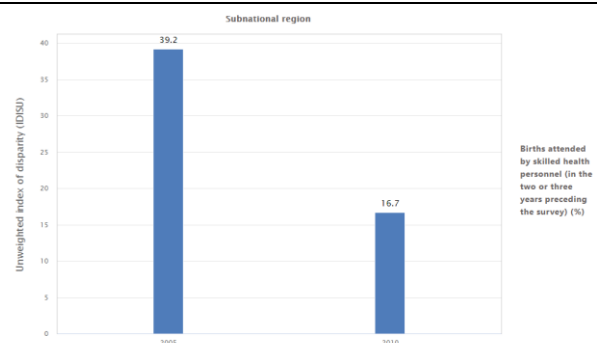


Figure b. Subnational inequality in births attended by skilled health personnel: index of disparity (unweighted) (IDISU)



<sup>4</sup> While IDISU is an unweighted measure, the setting average is calculated as the weighted average of subgroup estimates. Subgroups are weighted by their population share. Therefore, if any subgroup population share is missing, the setting average, and hence IDISU, cannot be calculated.



## INDEX OF DISPARITY (UNWEIGHTED) (IDISU)

Shows the unweighted average difference between each population subgroup and the setting average, in relative terms.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Does not take into account the **population size** of subgroups (unweighted measure)

## 3.7 Index of disparity (weighted) (IDISW)

### Definition

IDISW shows the weighted average difference between each population subgroup and the setting average, in relative terms.

IDISW is a relative measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. IDISW is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

IDISW is calculated as the weighted average of absolute differences between the subgroup estimates  $y_j$  and the setting average  $\mu$ , divided by the setting average  $\mu$ , and multiplied by 100. Absolute differences are weighted by each subgroup's population share  $p_j$ :

$$IDISW = \frac{\sum_j p_j |y_j - \mu|}{\mu} * 100$$

Note that the 95% confidence intervals calculated for IDISW are simulation-based estimates.

### Interpretation

IDISW takes only positive values, with larger values indicating higher levels of inequality. IDISW is zero if there is no inequality.

### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. IDISW quantifies the level of inequality in each year. Figure b shows that relative subnational regional inequality, as measured by the IDISW, reduced from 36.5 in 2005 to 13.9 in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

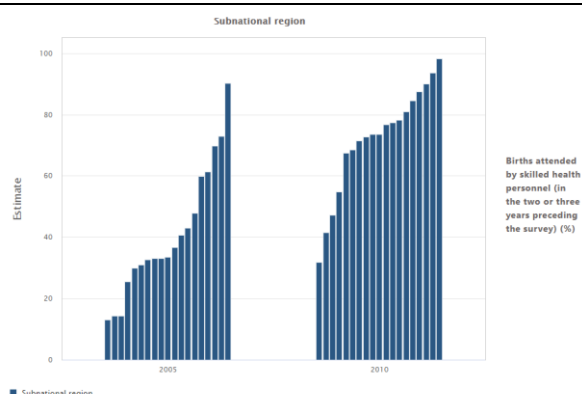
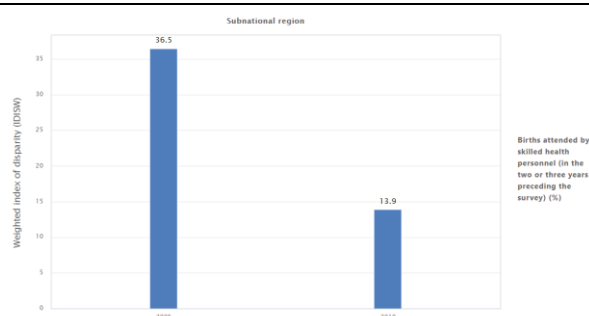


Figure b. Subnational inequality in births attended by skilled health personnel: index of disparity (weighted) (IDISW)



### Index of disparity (weighted) (IDISW)

Shows the weighted average of difference between each population subgroup and the setting average, in relative terms.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.8 Mean difference from best performing subgroup (unweighted) (MDBU)

### Definition

MDBU shows the unweighted mean difference between each population subgroup and a reference subgroup.

MDBU is an absolute measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. MDBU is missing if at least one subgroup estimate is missing.

### Calculation

MDBU is calculated as the average of absolute differences between the subgroup estimates  $y_j$  and the estimate for the reference subgroup  $y_{ref}$ , divided by the number of subgroups  $n$ :

$$MDBU = \frac{1}{n} * \sum_j |y_j - y_{ref}|$$

$y_{ref}$  refers to the subgroup with the highest estimate in the case of favourable indicators and to the subgroup with the lowest estimate in the case of adverse indicators.

Note that the 95% confidence intervals calculated for MDBU are simulation-based estimates.

## Interpretation

MDBU takes only positive values, with larger values indicating higher levels of inequality. MDBU is zero if there is no inequality.

## Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. MDBU quantifies the level of inequality in each year. Figure b shows that absolute subnational regional inequality, as measured by the MDBU, reduced from 49.0 percentage points in 2005 to 26.2 percentage points in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

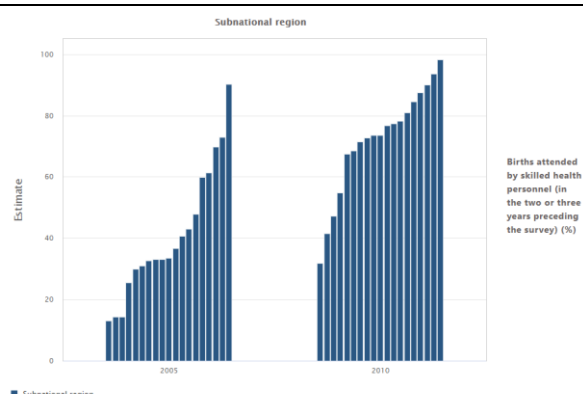
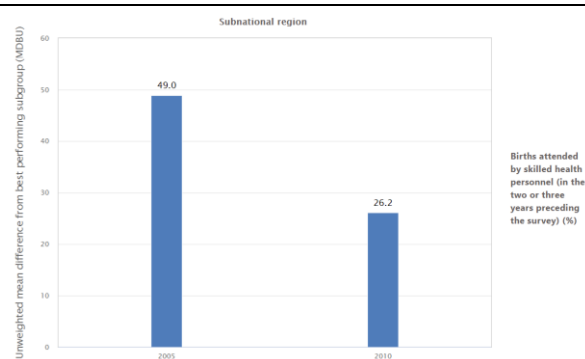


Figure b. Subnational regional inequality in births attended by skilled health personnel: mean difference from best performing subgroup (unweighted) (MDBU)



## MEAN DIFFERENCE FROM BEST PERFORMING SUBGROUP (UNWEIGHTED) (MDBU)

Shows the unweighted mean difference between each population subgroup and a reference subgroup.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Does not take into account the **population size** of subgroups (unweighted measure)

### 3.9 Mean difference from best performing subgroup (weighted) (MDBW)

#### Definition

MDBW shows the weighted mean difference between each population subgroup and a reference subgroup.

MDBW is an absolute measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. MDBW is missing if at least one subgroup estimate or subgroup population share is missing.

#### Calculation

MDBW is calculated as the weighted average of absolute differences between the subgroup estimates  $y_j$  and the estimate for the reference subgroup  $y_{ref}$ . Absolute differences are weighted by each subgroup's population share  $p_j$ :

$$MDBW = \sum_j p_j |y_j - y_{ref}|$$

$y_{ref}$  refers to the subgroup with the highest estimate in the case of favourable indicators and to the subgroup with the lowest estimate in the case of adverse indicators.

Note that the 95% confidence intervals calculated for MDBW are simulation-based estimates.

#### Interpretation

MDBW takes only positive values, with larger values indicating higher levels of inequality. MDBW is zero if there is no inequality.

#### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. MDBW quantifies the level of inequality in each year. Figure b shows that absolute subnational regional inequality, as measured by the MDBW, reduced from 43.4 percentage points in 2005 to 22.4 percentage points in 2010.



Figure a. Births attended by skilled health personnel disaggregated by subnational region

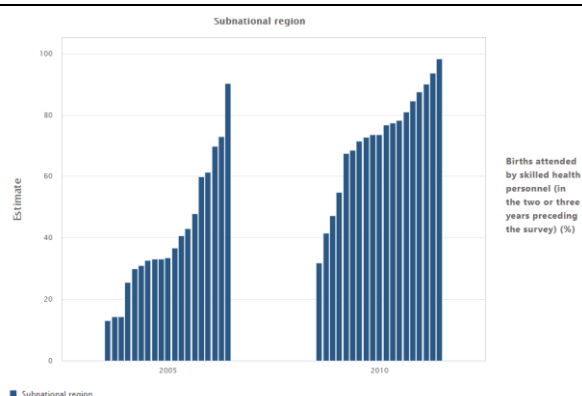
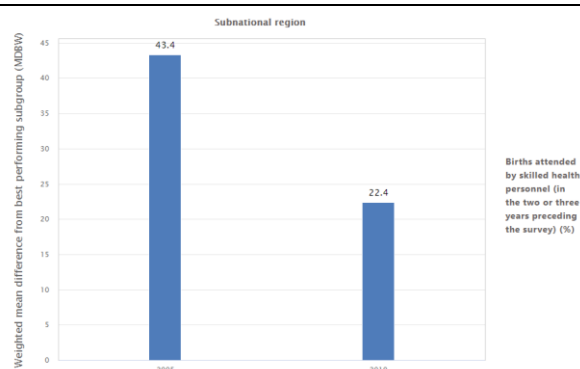


Figure b. Subnational inequality in births attended by skilled health personnel: mean difference from best performing subgroup (weighted) (MDBW)



### MEAN DIFFERENCE FROM BEST PERFORMING SUBGROUP (WEIGHTED) (MDBW)

Shows the weighted mean difference between each population subgroup and a reference subgroup.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.10 Mean difference from mean (unweighted) (MDMU)

### Definition

MDMU shows the unweighted mean difference between each subgroup and the setting average.

MDMU is an absolute measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. MDMU is missing if at least one subgroup estimate or subgroup population share is missing.<sup>5</sup>

### Calculation

MDMU is calculated as the average of absolute differences between the subgroup estimates  $y_j$  and the setting average  $\mu$ , divided by the number of subgroups  $n$ :

<sup>5</sup> While MDMU is an unweighted measure, the setting average is calculated as the weighted average of subgroup estimates. Subgroups are weighted by their population share. Therefore, if any subgroup population share is missing, the setting average, and hence MDMU, cannot be calculated.

$$MDMU = \frac{1}{n} * \sum_j |y_j - \mu|$$

Note that the 95% confidence intervals calculated for MDMU are simulation-based estimates.

## Interpretation

MDMU takes only positive values, with larger values indicating higher levels of inequality. MDMU is zero if there is no inequality.

## Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. MDMU quantifies the level of inequality in each year. Figure b shows that absolute subnational regional inequality, as measured by the MDMU, reduced from 18.4 percentage points in 2005 to 12.6 percentage points in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

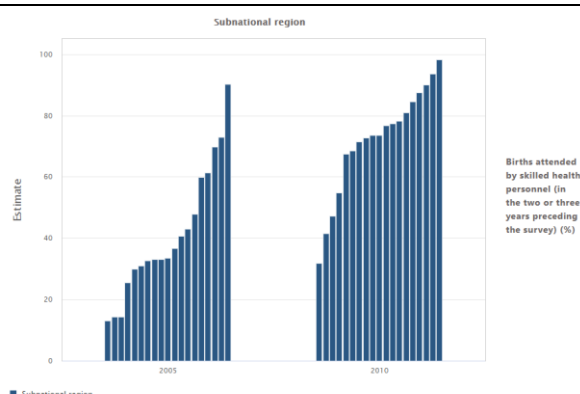
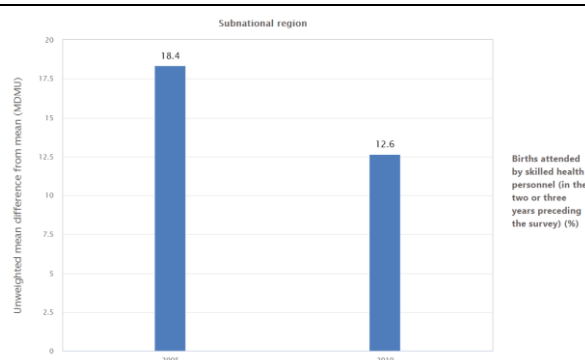


Figure b. Subnational regional inequality in births attended by skilled health personnel: mean difference from mean (unweighted) (MDMU)



## MEAN DIFFERENCE FROM MEAN (UNWEIGHTED) (MDMU)

Shows the unweighted mean difference between each population subgroup and the setting average.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Does not take into account the **population size** of subgroups (unweighted measure)

## 3.11 Mean difference from mean (weighted) (MDMW)

### Definition

MDMW shows the weighted mean difference between each population subgroup and the setting average.

MDMW is an absolute measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. MDMW is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

MDMW is calculated as the weighted average of absolute differences between the subgroup estimates  $y_j$  and the setting average  $\mu$ . Absolute differences are weighted by each subgroup's population share  $p_j$ :

$$MDMW = \sum_j p_j |y_j - \mu|$$

Note that the 95% confidence intervals calculated for MDMW are simulation-based estimates.

### Interpretation

MDMW takes only positive values, with larger values indicating higher levels of inequality. MDMW is zero if there is no inequality.

### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. MDMW quantifies the level of inequality in each year. Figure b shows that absolute subnational regional inequality, as measured by the MDMW, reduced from 17.1 percentage points in 2005 to 10.5 percentage points in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

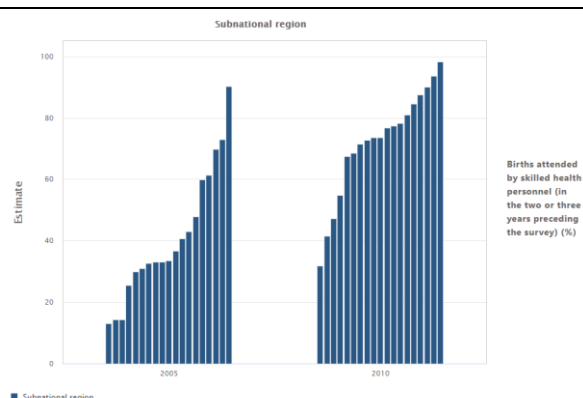
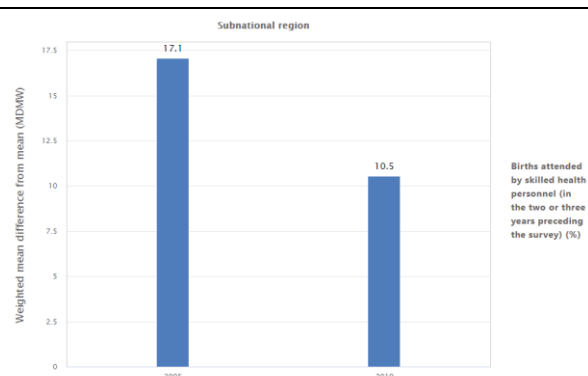


Figure b. Subnational inequality in births attended by skilled health personnel: mean difference from mean (weighted) (MDMW)





## MEAN DIFFERENCE FROM MEAN (WEIGHTED) (MDMW)

Shows the weighted mean difference between each population subgroup and the setting average.

Takes only positive values, with larger values indicating higher levels of inequality. Takes the value zero if there is no inequality

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.12 Mean log deviation (MLD)

### Definition

MLD is a relative measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. MLD is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

MLD is calculated as the sum of products between the negative natural logarithm of the share of the indicator of each subgroup ( $-\ln\left(\frac{y_j}{\mu}\right)$ ) and the population share of each subgroup ( $p_j$ ). MLD may be more easily readable when multiplied by 1000:

$$\text{MLD} = \sum_j p_j \left( -\ln\left(\frac{y_j}{\mu}\right) \right) * 1000$$

where  $y_j$  indicates the estimate for subgroup  $j$ ,  $p_j$  the population share of subgroup  $j$  and  $\mu$  the setting average.

### Interpretation

If there is no inequality, MLD takes the value zero. Greater absolute values indicate higher levels of inequality. MLD is more sensitive to differences further from the setting average (by the use of the logarithm).

### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. MLD quantifies the level of inequality in each year. Figure b shows that relative subnational regional inequality, as measured by the MLD, reduced from 101.0 in 2005 to 23.2 in 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

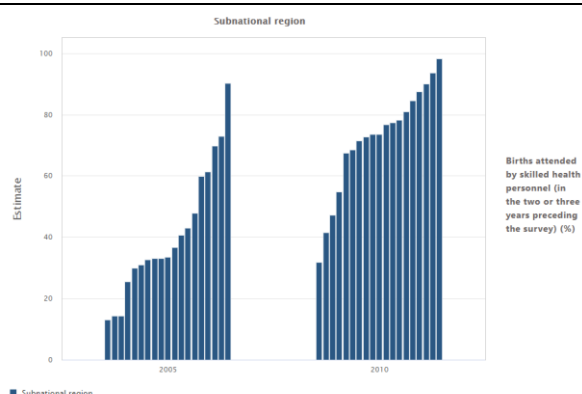
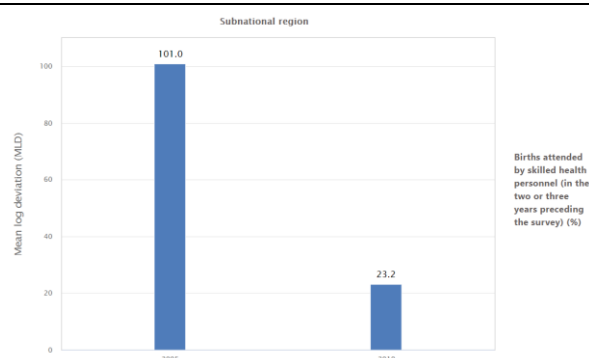


Figure b. Subnational inequality in births attended by skilled health personnel: mean log deviation (MLD)



### MEAN LOG DEVIATION (MLD)

Measures the sum of products between the negative natural logarithm of the share of the indicator of each subgroup and the population share of each subgroup.

Takes the value zero if there is no inequality. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.13 Population attributable fraction (PAF)

### Definition

PAF shows the potential for improvement in setting average of an indicator, in relative terms, that could be achieved if all population subgroups had the same level of the indicator as a reference group.

PAF is a relative measure of inequality that takes into account all population subgroups. It is calculated for all inequality dimensions, provided that all subgroup estimates and subgroup population shares are available.

### Calculation

PAF is calculated by dividing the population attributable risk (PAR) by the setting average  $\mu$  and multiplying the fraction by 100:

$$PAF = \frac{PAR}{\mu} * 100$$

## Interpretation

PAF takes positive values for favourable indicators and negative values for adverse indicators. The larger the absolute value of PAF, the larger the level of inequality. PAF is zero if no further improvement can be achieved, i.e. if all subgroups have reached the same level of the indicator as the reference subgroup.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. PAF measures the potential improvement in overall coverage of skilled birth attendance that could be achieved if all quintiles had the same level of coverage as quintile 5, i.e. if there was no economic-related inequality. Figure b shows that setting average could have been 97.3% higher in 2005 and 28.3% higher in 2010 if there had been no economic-related inequality. PAF decreased between 2005 and 2010 indicating a decrease in relative economic-related inequality.

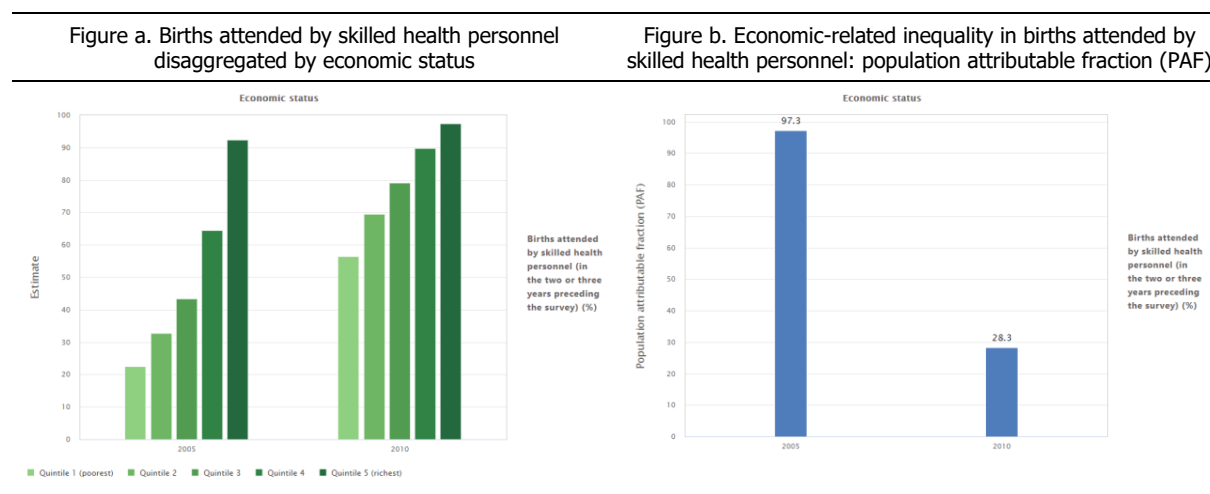


Figure c shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. PAF measures the potential improvement in overall coverage of skilled birth attendance that could be achieved if all regions had the same level of coverage as the best performing region, i.e. if there was no subnational regional inequality. Figure d shows that setting average could have been 92.6% higher in 2005 and 29.6% higher in 2010 if there had been no subnational regional inequality. PAF decreased between 2005 and 2010 indicating a decrease in relative subnational regional inequality.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

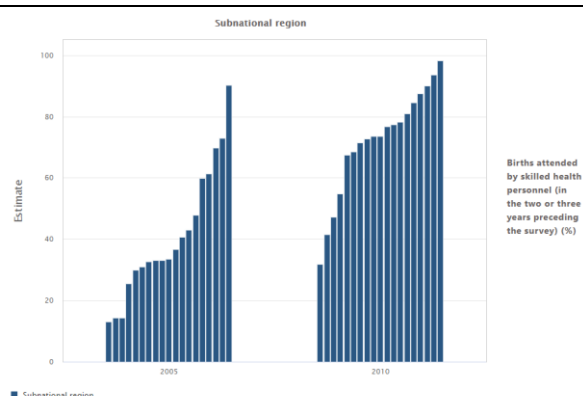
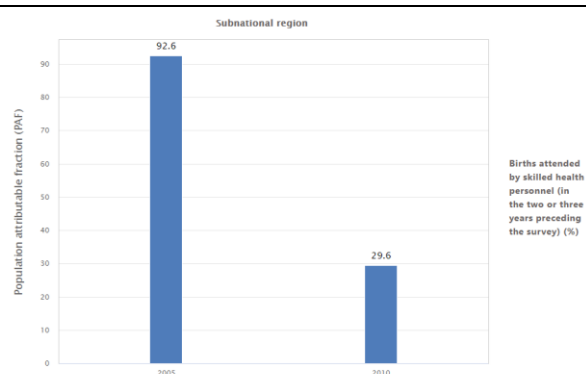


Figure b. Subnational inequality in births attended by skilled health personnel: population attributable fraction (PAF)



## POPULATION ATTRIBUTABLE FRACTION (PAF)

Shows the potential for improvement in setting average, in relative terms, that could be achieved if all population subgroups had the same level of the indicator as a reference group.

Takes the value zero if there is no inequality / no further improvement can be achieved. Takes positive values for favourable indicators and negative values for adverse indicators. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **all inequality dimensions**
- ✓ Takes into account **all population subgroups**
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.14 Population attributable risk (PAR)

### Definition

PAR shows the potential for improvement in setting average that could be achieved if all population subgroups had the same level of the indicator as a reference group.

PAR is an absolute measure of inequality that takes into account all population subgroups. It is calculated for all inequality dimensions, provided that all subgroup estimates and subgroup population shares are available.

### Calculation

PAR is calculated as the difference between the estimate for the reference subgroup  $y_{ref}$  and the setting average  $\mu$ :

$$PAR = y_{ref} - \mu$$

Note that the reference subgroup  $y_{ref}$  depends on the characteristics of the inequality dimension and indicator type, for which PAR is calculated. Table 2 provides an overview of the calculation of PAR.

**Table 2** Calculation of the Population Attributable Risk (PAR)

Dimension type	Reference subgroup selected?	Indicator type	
		Favourable indicator	Adverse indicator
Binary dimension	Yes	Reference group – $\mu$	Reference group – $\mu$
	No	Highest – $\mu$	Lowest – $\mu$
Ordered dimension	N/A	Most-advantaged – $\mu$	Most-advantaged – $\mu$
Non-ordered dimension	Yes	Reference group – $\mu$	Reference group – $\mu$
	No	Highest – $\mu$	Lowest – $\mu$

## Interpretation

PAR takes positive values for favourable indicators and negative values for adverse indicators. The larger the absolute value of PAR, the higher the level of inequality. PAR is zero if no further improvement can be achieved, i.e. if all subgroups have reached the same level of the indicator as the reference subgroup.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. PAR measures the potential improvement in setting coverage of skilled birth attendance that could be achieved if all quintiles had the same level of coverage as quintile 5, i.e. if there was no economic-related inequality. Figure b shows that setting average could have been 45.6 percentage points higher in 2005 and 21.4 percentage points higher in 2010 if there had been no economic-related inequality. PAR decreased between 2005 and 2010 indicating a decrease in absolute economic-related inequality.

Figure a. Births attended by skilled health personnel disaggregated by economic status

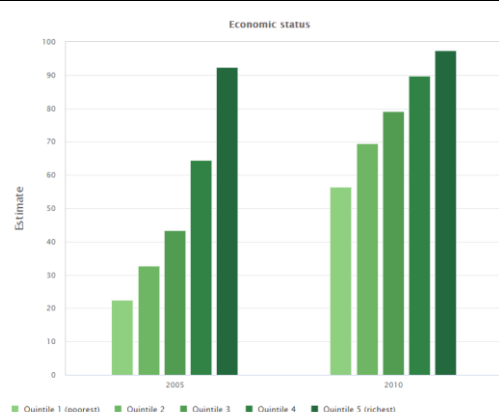


Figure b. Economic-related inequality in births attended by skilled health personnel: population attributable risk (PAR)

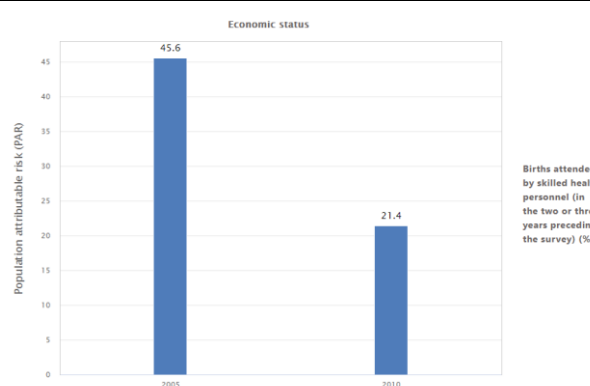


Figure c shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. PAR measures the potential improvement in setting coverage of skilled birth attendance that could be achieved if all regions had the same level of coverage as the best performing region, i.e. if there was



no subnational regional inequality. Figure d shows that setting average could have been 43.4 percentage points higher in 2005 and 22.4 percentage points higher in 2010 if there had been no subnational regional inequality. PAR decreased between 2005 and 2010 indicating a decrease in absolute subnational regional inequality.

Figure c. Births attended by skilled health personnel disaggregated by subnational region

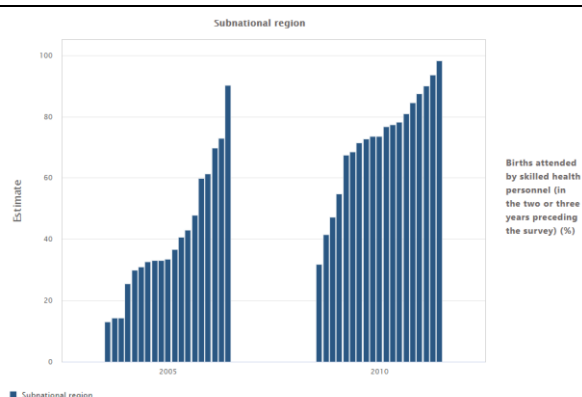
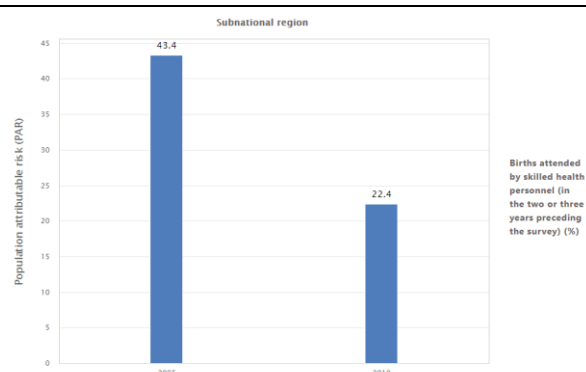


Figure d. Subnational regional inequality in births attended by skilled health personnel: population attributable risk (PAR)



## POPULATION ATTRIBUTABLE RISK (PAR)

Shows the potential for improvement in setting average that could be achieved if all population subgroups had the same level of the indicator as a reference group.

Takes the value zero if there is no inequality / no further improvement can be achieved. Takes positive values for favourable indicators and negative values for adverse indicators. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **all inequality dimensions**
- ✓ Takes into account **all population subgroups**
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.15 Ratio (R)

### Definition

R is a relative measure of inequality that shows the ratio of two population subgroups. It is calculated for all inequality dimensions, provided that subgroup estimates are available for the two subgroups used in the calculation of R.

### Calculation

R is calculated as the ratio of two subgroups:

$$R = y_{high} / y_{low}$$

Note that the selection of  $y_{high}$  and  $y_{low}$  depends on the characteristics of the inequality dimension and the type of indicator, for which R is calculated. Table 3 provides an overview of the calculation of R.

**Table 3** Calculation of the Ratio (R)

Dimension type	Reference subgroup selected?	Indicator type	
		Favourable indicator	Adverse indicator
Binary dimension	Yes	Reference group / Other group	Other group / Reference group
	No	Highest / Lowest	Highest / Lowest
Ordered dimension	N/A	Most-advantaged / Most-disadvantaged	Most-disadvantaged / Most-advantaged
Non-ordered dimension	Yes	Reference group / Other group (that maximizes the ratio)	Other group (that maximizes the ratio) / Reference group
	No	Highest / Lowest	Highest / Lowest

R is calculated for all dimensions of inequality. In the case of binary and non-ordered dimensions, R is missing if at least one subgroup estimate is missing. In the case of ordered dimensions, R is missing if the estimates for the most-advantaged and/or most-disadvantaged subgroup are missing.

### Interpretation

If there is no inequality, R takes the value one. R takes only positive values. The further the value of R from one, the higher the level of inequality.

Note that R is displayed on a logarithmic scale. R values are intrinsically asymmetric: a ratio of one (no inequality) is halfway between a ratio of 0.5 (the denominator subgroup having half the value of the numerator subgroup) and a ratio of 2.0 (the denominator subgroup having double the value of the numerator subgroup). On a regular axis, R values would be concentrated at the lower end of the scale, with a few very large outlier values at the upper end of the scale. On a logarithmic axis, these values are equally spaced, making them easier to read and interpret.

### Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. The ratio quantifies the level of inequality in each year. Figure b shows that the ratio of quintile 5 to quintile 1 reduced from 4.1 in 2005 to 1.7 in 2010. In 2005, coverage in quintile 5 was about four times higher than in quintile 1, while in 2010, coverage in quintile 5 was less than two times higher than in quintile 1. Relative economic-related inequality decreased between 2005 and 2010.

Figure a. Births attended by skilled health personnel disaggregated by economic status

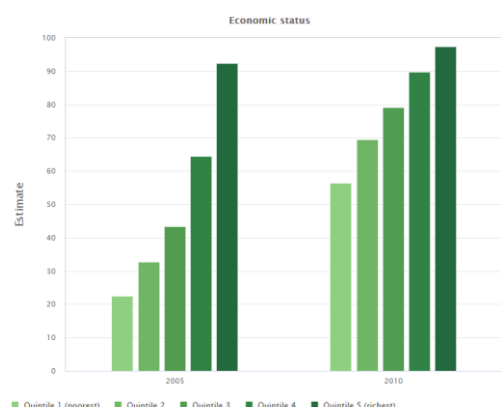


Figure b. Economic-related inequality in births attended by skilled health personnel: ratio (R)

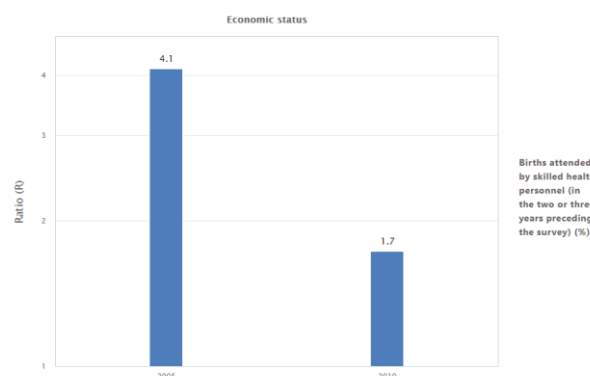


Figure c shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. The ratio quantifies the level of inequality in each year. Figure d shows that the ratio of the best to the worst performing region reduced from 6.9 in 2005 to 3.1 in 2010. In 2005, coverage in the best performing region was almost seven times higher than in the worst performing region, while in 2010, coverage in the best performing region was about three times higher than in the worst performing region. Relative economic-related inequality decreased between 2005 and 2010.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

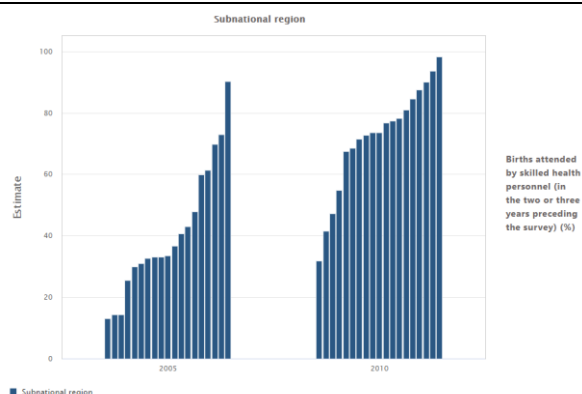
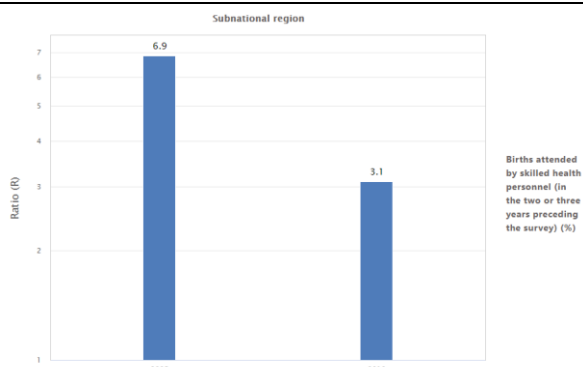


Figure b. Subnational inequality in births attended by skilled health personnel: ratio (R)



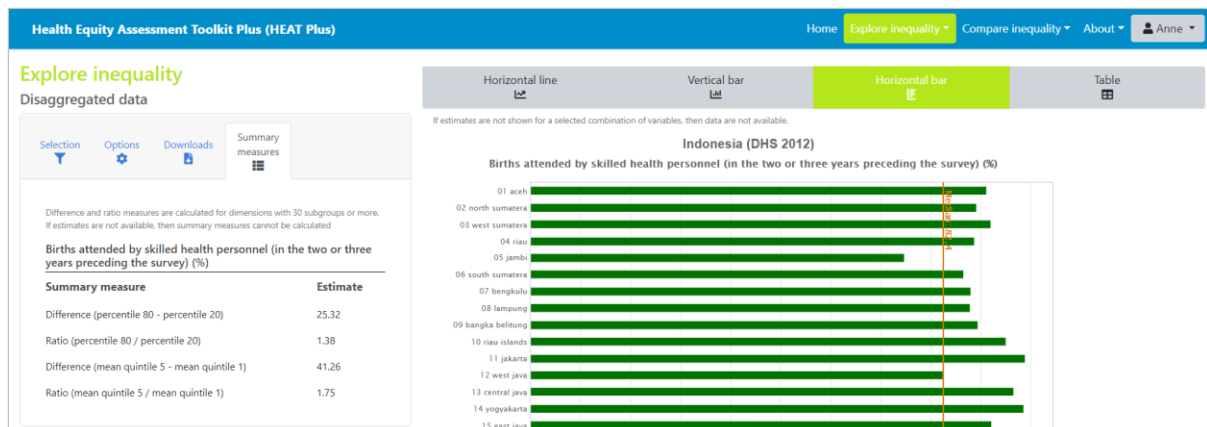
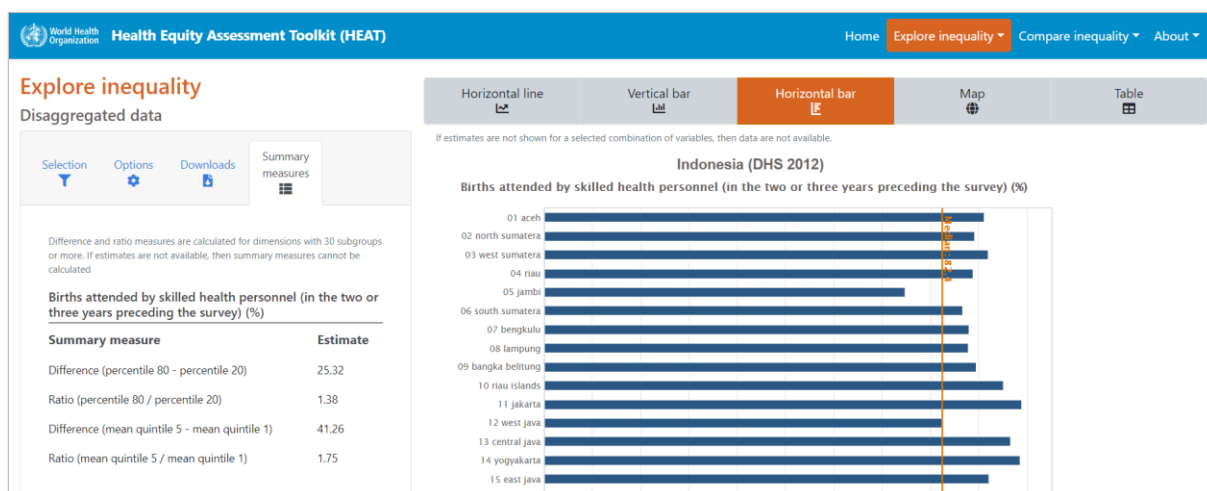
## Other ratio measures

In addition to the ratio measure described above, variations of the ratio are calculated for non-ordered inequality dimensions with many subgroups, such as subnational region. The following ratio measures are calculated for

- Dimensions with more than 30 subgroups:
  - **Ratio of percentile 80 to percentile 20.** The ratio of percentile 80 to percentile 20 is calculated by identifying the subgroups that correspond to percentiles 80 and 20 and dividing the estimate for percentile 80 by the estimate for percentile 20:  $R_{p80p20} = y_{p80}/y_{p20}$

- **Ratio of mean estimates in quintile 5 to quintile 1.** The ratio of mean estimates in quintile 5 and quintile 1 is calculated by dividing subgroups into quintiles, determining the mean estimate for each quintile and dividing the mean estimate in quintile 5 by the mean estimate in quintile 1:  $R_{q5q1} = y_{q5}/y_{q1}$
- Dimensions with more than 60 subgroups:
  - **Ratio of percentile 90 to percentile 10.** The ratio of percentile 90 to percentile 10 is calculated by identifying the subgroups that correspond to percentiles 90 and 10 and dividing the estimate for percentile 90 by the estimate for percentile 10:  $R_{p90p10} = y_{p90}/y_{p10}$
  - **Ratio of mean estimates in decile 10 to decile 1.** The ratio of mean estimates in decile 10 to decile 1 is calculated by dividing subgroups into deciles, determining the mean estimate for each decile and dividing the mean estimate in decile 10 by the mean estimate in decile 1:  $R_{d10d1} = y_{d10}/y_{d1}$
- Dimensions with more than 100 subgroups:
  - **Ratio of percentile 95 to percentile 5.** The ratio of percentile 95 to percentile 5 is calculated by identifying the subgroups that correspond to percentiles 95 and 5 and dividing the estimate for percentile 95 by the estimate for percentile 5:  $R_{p95p5} = y_{p95}/y_{p5}$
  - **Ratio of mean estimates in the top 5% to the bottom 5%.** The ratio of mean estimates in the top 5% to the bottom 5% is calculated by dividing subgroups into vigintiles, determining the mean estimate for each vigintile and dividing the mean estimate in the top 5% by the mean estimate in the bottom 5%:  $R_{v20v1} = y_{v20}/y_{v1}$

For dimensions with many subgroups, these measures may be a more accurate reflection of the level of inequality than measuring the ratio of the maximum and minimum values using the (range) ratio, as they avoid using possible outlier values. They are displayed in the 'Summary measures' tab of the selection menu for horizontal bar graphs showing disaggregated data under the 'Explore inequality' component of the tool.



## RATIO (R)

Measures the ratio of two population subgroups.

Takes the value one if there is no inequality. Takes only positive values (larger or smaller than one). The further the value from one, the higher the level of inequality. Variations of the ratio are calculated for non-ordered inequality dimensions with many subgroups. These measures avoid using possible outlier values.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **all inequality dimensions**
- ✓ Takes into account **two population subgroups** (simple measure)
- ✓ Does not take into account the **population size** of subgroups (unweighted measure)

## 3.16 Relative concentration index (RCI)

### Definition

RCI shows the gradient across population subgroups, on a relative scale. It indicates the extent to which an indicator is concentrated among disadvantaged or advantaged subgroups.

RCI is a relative measure of inequality that takes into account all population subgroups. It is calculated for ordered dimensions with more than two subgroups, such as economic status. Subgroups are weighted according to their population share. RCI is missing if at least one subgroup estimate or subgroup population share is missing.

## Calculation

RCI is calculated by dividing the absolute concentration index (ACI) by the setting average  $\mu$  and multiplying the fraction by 100:

$$RCI = \frac{ACI}{\mu} * 100$$

## Interpretation

RCI is bounded between -100 and +100 and takes the value zero if there is no inequality. Positive values indicate a concentration of the indicator among the advantaged, while negative values indicate a concentration of the indicator among the disadvantaged. The greater the absolute value of RCI, the higher the level of inequality.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. RCI quantifies the level of inequality in each year. Figure b shows that relative economic-related inequality, as measured by the RCI, reduced from 28.2 in 2005 to 11.1 in 2010.

Figure a. Births attended by skilled health personnel disaggregated by economic status

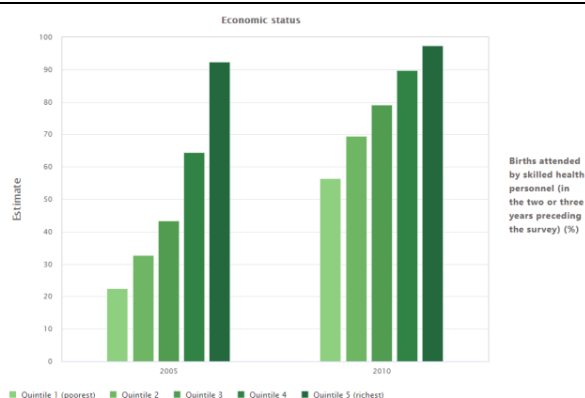
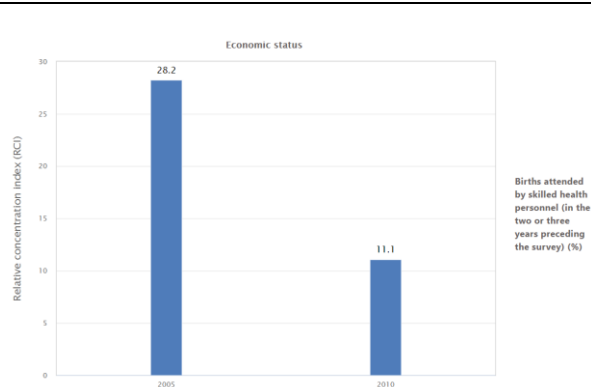


Figure b. Economic-related inequality in births attended by skilled health personnel: relative concentration index (RCI)





## RELATIVE CONCENTRATION INDEX (RCI)

Measures the extent to which an indicator is concentrated among disadvantaged or advantaged population subgroups, in relative terms.

Takes the value zero if there is no inequality. Takes values between -100 and +100. Positive values indicate a concentration among advantaged, negative values among disadvantaged subgroups. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **ordered inequality dimensions**, such as economic status (ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.17 Relative index of inequality (RII)

### Definition

RII represents the ratio of estimated values of an indicator of the most-advantaged to the most-disadvantaged (or vice versa for adverse indicators), while taking into account all the other subgroups – using an appropriate regression model.

RII is a relative measure of inequality that takes into account all population subgroups. It is calculated for ordered dimensions with more than two subgroups, such as economic status. Subgroups are weighted according to their population share. RII is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

To calculate RII, a weighted sample of the whole population is ranked from the most-disadvantaged subgroup (at rank 0) to the most-advantaged subgroup (at rank 1). This ranking is weighted, accounting for the proportional distribution of the population within each subgroup. The population of each subgroup is then considered in terms of its range in the cumulative population distribution, and the midpoint of this range. According to the definition currently used in HEAT, the indicator of interest is then regressed against this midpoint value using a generalized linear model with logit link, and the predicted values of the indicator are calculated for the two extremes (rank 1 and rank 0).

For **favourable indicators**, the ratio of the estimated values at rank 1 ( $v_1$ ) to rank 0 ( $v_0$ ) (covering the entire distribution) generates the RII value:

$$RII = v_1/v_0$$

For **adverse indicators**, the calculation is reversed and the RII value is calculated as the ratio of the estimated values at rank 0 ( $v_0$ ) to rank 1 ( $v_1$ ) (covering the entire distribution):

$$RII = v_0/v_1$$

### Interpretation

If there is no inequality, RII takes the value one. RII takes only positive values. The further the value of RII from one, the higher the level of inequality. For favourable indicators, values larger than one

indicate a concentration of the indicator among the advantaged and values smaller than one indicate a concentration of the indicator among the disadvantaged. For adverse indicators, values larger than one indicate a concentration of the indicator among the disadvantaged and values smaller than one indicate a concentration of the indicator among the advantaged.

Note that RII is displayed on a logarithmic scale. RII values are intrinsically asymmetric: a ratio of one (no inequality) is halfway between a ratio of 0.5 (the denominator subgroup having half the value of the numerator subgroup) and a ratio of 2.0 (the denominator subgroup having double the value of the numerator subgroup). On a regular axis, RII values would be concentrated at the lower end of the scale, with a few very large outlier values at the upper end of the scale. On a logarithmic axis, these values are equally spaced, making them easier to read and interpret.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. RII quantifies the level of inequality in each year. Figure b shows that relative economic-related inequality, as measured by the RII, reduced from 7.7 in 2005 to 2.2 in 2010. In 2005, coverage in quintile 5 was nearly 8 times higher than in quintile 1, but this reduced to coverage in quintile 5 being just over twice as high than in quintile 1 in 2010.

Figure a. Births attended by skilled health personnel disaggregated by economic status

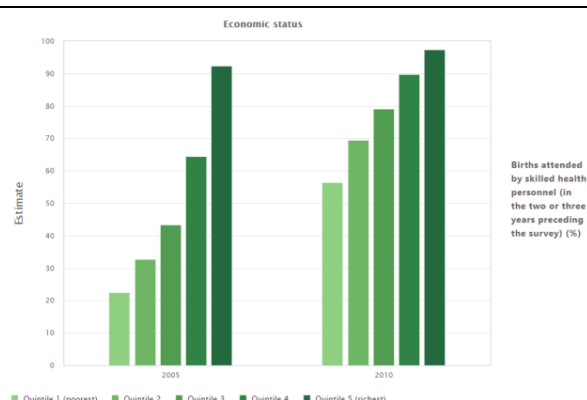
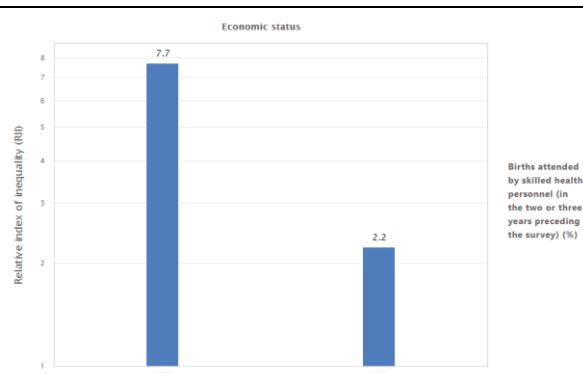


Figure b. Economic-related inequality in births attended by skilled health personnel: relative index of inequality (RII)







## RELATIVE INDEX OF INEQUALITY (RII)

Represents the ratio of estimated values of an indicator of the most-advantaged to the most-disadvantaged (or vice versa for adverse indicators), while taking into account all other subgroups.

Takes the value one if there is no inequality. Takes only positive values (larger or smaller than one). The further the value from one, the higher the level of inequality. For favourable indicators, values larger than one indicate a concentration among the advantaged and values smaller than one among the disadvantaged. For adverse indicators, it's the other way around: positive values indicate a concentration among the disadvantaged and negative values among the advantaged.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **ordered inequality dimensions**, such as economic status (ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.18 Slope index of inequality (SII)

### Definition

SII represents the difference in estimated values of an indicator between the most-advantaged and most-disadvantaged (or vice versa for adverse indicators), while taking into consideration all the other subgroups – using an appropriate regression model.

SII is an absolute measure of inequality that takes into account all population subgroups. It is calculated for ordered dimensions with more than two subgroups, such as economic status. Subgroups are weighted according to their population share. SII is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

To calculate SII, a weighted sample of the whole population is ranked from the most-disadvantaged subgroup (at rank 0) to the most-advantaged subgroup (at rank 1). This ranking is weighted, accounting for the proportional distribution of the population within each subgroup. The population of each subgroup is then considered in terms of its range in the cumulative population distribution, and the midpoint of this range. According to the definition currently used in HEAT, the indicator of interest is then regressed against this midpoint value using a generalized linear model with logit link, and the predicted values of the indicator are calculated for the two extremes (rank 1 and rank 0).

For **favourable indicators**, the difference between the estimated values at rank 1 ( $v_1$ ) and rank 0 ( $v_0$ ) (covering the entire distribution) generates the SII value:

$$SII = v_1 - v_0$$

For **adverse indicators**, the calculation is reversed and the SII value is calculated as the difference between the estimated values at rank 0 ( $v_0$ ) and rank 1 ( $v_1$ ) (covering the entire distribution):

$$SII = v_0 - v_1$$

## Interpretation

If there is no inequality, SII takes the value zero. Greater absolute values indicate higher levels of inequality. For favourable indicators, positive values indicate a concentration of the indicator among the advantaged and negative values indicate a concentration of the indicator among the disadvantaged. For adverse indicators, positive values indicate a concentration of the indicator among the disadvantaged and negative values indicate a concentration of the indicator among the advantaged.

## Example

Figure a shows data on skilled birth attendance disaggregated by economic status for two years (2005 and 2010). For each year, there are five bars – one for each wealth quintile. The graph shows that, overall, coverage increased in all quintiles and inequality between quintiles reduced over time. SII quantifies the level of inequality in each year. Figure b shows that absolute economic-related inequality, as measured by the SII, reduced from 74.3 percentage points in 2005 to 52.6 percentage points in 2010.

Figure a. Births attended by skilled health personnel disaggregated by economic status

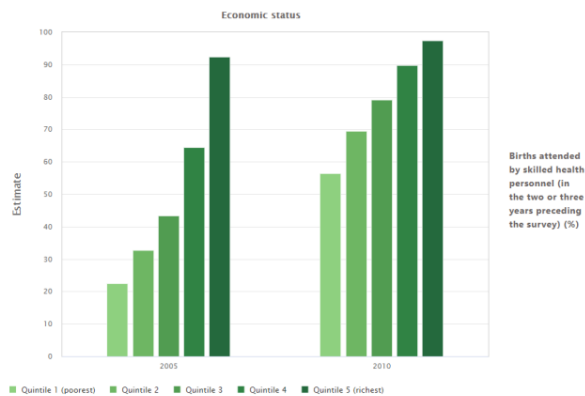
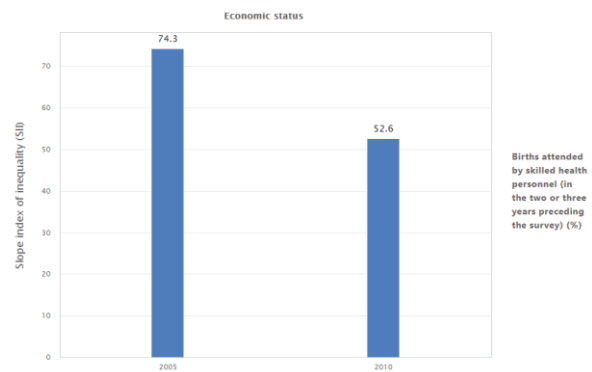


Figure b. Economic-related inequality in births attended by skilled health personnel: slope index of inequality (SII)





## SLOPE INDEX OF INEQUALITY (SII)

Represents the difference in estimated values of an indicator between the most-advantaged and the most-disadvantaged (or vice versa for adverse indicators), while taking into account all other subgroups.

Takes the value zero if there is no inequality. For favourable indicators, positive values indicate a concentration among the advantaged and negative values among the disadvantaged. For adverse indicators, it's the other way around: positive values indicate a concentration among the disadvantaged and negative values among the advantaged. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **absolute inequality** (absolute measure)
- ✓ Suitable for **ordered inequality dimensions**, such as economic status (ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## 3.19 Theil index (TI)

### Definition

TI is a relative measure of inequality that takes into account all population subgroups. It is calculated for non-ordered dimensions with more than two subgroups, such as subnational region. Subgroups are weighted according to their population share. TI is missing if at least one subgroup estimate or subgroup population share is missing.

### Calculation

TI is calculated as the sum of products of the natural logarithm of the share of the indicator of each subgroup ( $\ln \frac{y_j}{\mu}$ ), the share of the indicator of each subgroup ( $\frac{y_j}{\mu}$ ) and the population share of each subgroup ( $p_j$ ). TI may be more easily interpreted when multiplied by 1000:

$$TI = \sum_j p_j \frac{y_j}{\mu} \ln \frac{y_j}{\mu} * 1000$$

where  $y_j$  indicates the estimate for subgroup  $j$ ,  $p_j$  the population share of subgroup  $j$  and  $\mu$  the setting average.

### Interpretation

If there is no inequality, TI takes the value zero. Greater absolute values indicate higher levels of inequality. TI is more sensitive to differences further from the setting average (by the use of the logarithm).

### Example

Figure a shows data on skilled birth attendance disaggregated by subnational region for two years (2005 and 2010). For each year, there are multiple bars – one for each region. The graph shows that, overall, coverage increased in all regions and inequality between regions reduced over time. TI

quantifies the level of inequality in each year. Figure b shows that relative subnational regional inequality, as measured by the TI, reduced from 94.6 in 2005 to 20.5 in 2010. In 2010, the level of inequality was one fifth of the level in 2005.

Figure a. Births attended by skilled health personnel disaggregated by subnational region

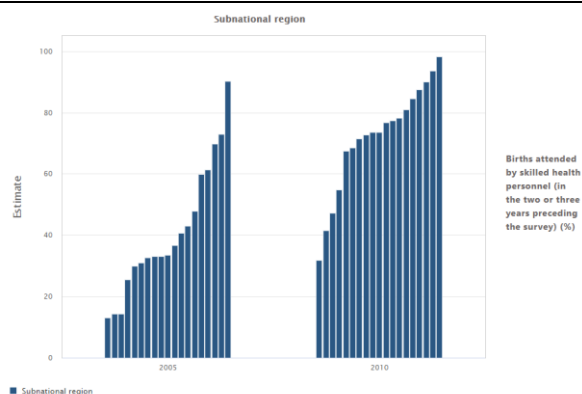
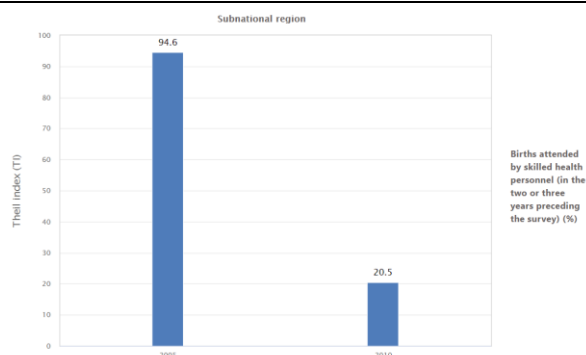


Figure b. Subnational inequality in births attended by skilled health personnel: theil index (TI)



## THEIL INDEX (TI)

Measures the sum of products of the natural logarithm of the share of the indicator of each subgroup, the share of the indicator of each subgroup and the population share of each subgroup

Takes the value zero if there is no inequality. The larger the absolute value, the higher the level of inequality.

- ✓ Measures **relative inequality** (relative measure)
- ✓ Suitable for **non-ordered inequality dimensions**, such as subnational region (non-ordered measure)
- ✓ Takes into account **all population subgroups** (complex measure)
- ✓ Takes into account the **population size** of subgroups (weighted measure)

## Annex

## Annex 1 Summary measures: overview

Summary measure (abbreviation)	Formula	Absolute/Relative	Simple/Complex	Weighted/Unweighted	Ordered/Non-ordered	Unit	Value of no inequality	Interpretation
Absolute concentration index (ACI)	$ACI = \sum_j p_j(2X_j - 1)y_j$	Absolute	Complex	Weighted	Ordered	Unit of indicator	Zero	Positive (negative) values indicate a concentration of the indicator among the advantaged (disadvantaged). The larger the absolute value of ACI, the higher the level of inequality.
Between-group standard deviation (BGSD)	$BGSD = \sqrt{\sum_j p_j(y_j - \mu)^2}$	Absolute	Complex	Weighted	Non-ordered	Unit of indicator	Zero	BGSD takes only positive values with larger values indicating higher levels of inequality.
Between-group variance (BGV)	$BGV = \sum_j p_j(y_j - \mu)^2$	Absolute	Complex	Weighted	Non-ordered	Squared unit of indicator	Zero	BGV takes only positive values with larger values indicating higher levels of inequality.
Coefficient of variation (COV)	$COV = \frac{BGSD}{\mu} * 100$	Relative	Complex	Weighted	Non-ordered	Unit of indicator	Zero	COV takes only positive values with larger values indicating higher levels of inequality.
Difference (D)	$D = y_{high} - y_{low}$	Absolute	Simple	Unweighted	Ordered/Non-ordered	Unit of indicator	Zero	The larger the absolute value of D, the higher the level of inequality.
Index of disparity (unweighted) (IDIS)	$IDIS = \frac{1}{n} \sum_j  y_j - \mu  * 100$	Relative	Complex	Unweighted	Non-ordered	No unit	Zero	IDIS takes only positive values with larger values indicating higher levels of inequality.
Index of disparity (weighted) (IDISW)	$IDISW = \frac{\sum_j p_j  y_j - \mu }{\mu} * 100$	Relative	Complex	Weighted	Non-ordered	No unit	Zero	IDISW takes only positive values with larger values indicating higher levels of inequality.
Mean difference from best performing subgroup (unweighted) (MDBU)	$MDBU = \frac{1}{n} * \sum_j  y_j - y_{ref} $	Absolute	Complex	Unweighted	Non-ordered	Unit of indicator	Zero	MDBU takes only positive values with larger values indicating higher levels of inequality.
Mean difference from best performing subgroup (weighted) (MDBW)	$MDBW = \sum_j p_j  y_j - y_{ref} $	Absolute	Complex	Weighted	Non-ordered	Unit of indicator	Zero	MDBW takes only positive values with larger values indicating higher levels of inequality.
Mean difference from mean (unweighted) (MDMU)	$MDMU = \frac{1}{n} * \sum_j  y_j - \mu $	Absolute	Complex	Unweighted	Non-ordered	Unit of indicator	Zero	MDMU takes only positive values with larger values indicating higher levels of inequality.
Mean difference from mean (weighted) (MDMW)	$MDMW = \sum_j p_j  y_j - \mu $	Absolute	Complex	Weighted	Non-ordered	Unit of indicator	Zero	MDMW takes only positive values with larger values indicating higher levels of inequality.
Mean log deviation (MLD)	$MLD = \sum_j p_j (-\ln(\frac{y_j}{\mu})) * 1000$	Relative	Complex	Weighted	Non-ordered	No unit	Zero	The larger the absolute value of MLD, the higher the level of inequality.
Population attributable fraction (PAF)	$PAF = \frac{PAR}{\mu} * 100$	Relative	Complex	Weighted	Ordered/Non-ordered	No unit	Zero	PAF takes only positive values for favourable indicators and only negative values for adverse indicators. The larger the absolute value of PAF, the larger the level of inequality.
Population attributable risk (PAR)	$PAR = y_{ref} - \mu$	Absolute	Complex	Weighted	Ordered/Non-ordered	Unit of indicator	Zero	PAR takes only positive values for favourable indicators and only negative values for adverse indicators. The larger the absolute value, the higher the level of inequality.
Slope index of inequality (SII)	$SII = v_1 - v_0$ for favourable indicators; $SII = v_0 - v_1$ for adverse indicators	Absolute	Complex	Weighted	Ordered	Unit of indicator	Zero	For favourable (adverse) indicators, positive values indicate a concentration among the advantaged (disadvantaged) and negative values indicate a concentration among the disadvantaged (advantaged). The larger the absolute value of SII, the higher the level of inequality.
Ratio (R)	$R = y_{high}/y_{low}$	Relative	Simple	Unweighted	Ordered/Non-ordered	No unit	One	R takes only positive values. The further the value of R from 1, the higher the level of inequality.
Relative concentration index (RCI)	$RCI = \frac{ACI}{\mu} * 100$	Relative	Complex	Weighted	Ordered	No unit	Zero	RCI is bounded between -100 and +100. Positive (negative) values indicate a concentration of the indicator among the advantaged (disadvantaged). The larger the absolute value of RCI, the higher the level of inequality.
Relative index of inequality (RII)	$RII = v_1/v_0$ for favourable indicators; $RII = v_0/v_1$ for adverse indicators	Relative	Complex	Weighted	Ordered	No unit	One	RII takes only positive values. For favourable (adverse) indicators, values > 1 indicate a concentration among the advantaged (disadvantaged) and values < 1 values indicate a concentration among the disadvantaged (advantaged). The further the value of RII from 1, the higher the level of inequality.

## HEAT Technical Notes

Summary measure (abbreviation)	Formula	Absolute/ Relative	Simple/ Complex	Weighted/ Unweighted	Ordered/ Non-ordered	Unit	Value of no inequality	Interpretation
Theil index (TI)	$TI = \sum_j p_j \frac{y_j}{\mu} \ln \frac{y_j}{\mu} * 1000$	Relative	Complex	Weighted	Non-ordered	No unit	Zero	The larger the absolute value of TI, the greater the level of inequality.

$y_j$  = Estimate for subgroup j.

$y_{high}$  = Estimate for subgroup high.

$y_{low}$  = Estimate for subgroup low.

$y_{ref}$  = Estimate for reference subgroup.

$p_j$  = Population share for subgroup j.

$X_j = \sum_j p_j - 0.5p_j$  = Relative rank of subgroup j.

$\mu$  = Setting average.

$v_0$  = Predicted value of the hypothetical person at the bottom of the social-group distribution (rank 0).

$v_1$  = Predicted value of the hypothetical person at the top of the social-group distribution (rank 1).

$n$  = Number of subgroups.

