不定积分练习

1. 求不定积分:

(1) $\int \frac{1+\sin x}{1+\cos x}\mathrm{e}^x dx$ $(\mathrm{e}^x \tan\frac{x}{2}+C)$ $\int \mathrm{e}^x(\frac{1-x}{1+x^2})^2 dx$ $(\frac{\mathrm{e}^x}{1+x^2}+C)$

(2) $\int \frac{1}{\sin^6 x+\cos^6 x}dx$ $(\arctan(\frac{\tan 2x}{2})+C)$ $\int \frac{dx}{\sqrt{\sin x\cos^7 x}}(2\sqrt{\tan x}(1+\frac{1}{5}\tan^5 x)+C))$

(3) $\int \frac{\cos x}{2\sin x+\cos x}dx$ $(\frac{2}{5}\ln|2\sin x+\cos x|+\frac{x}{5}+C)$

(4) $\int \frac{x\ln x}{(x^2-1)^{\frac{3}{2}}}dx$ $(-(\frac{\ln x}{\sqrt{x^2-1}}+\arcsin\frac{1}{x})+C)$

(5) $\int \frac{1+x^2}{1+x^2+x^4}dx$ $(\frac{\sqrt{3}}{3}\arctan\frac{x^2-1}{\sqrt{3}x}+C)$ $\int \mathrm{e}^{\sin x}\sin 2x dx$ $(2\mathrm{e}^{\sin x}(\sin x-1)+C)$

(6) $\int \frac{\arctan\sqrt{x}}{\sqrt{x}(1+x)}dx$ $(\arctan^2\sqrt{x}+C)$ $\int x\ln\frac{1+x}{1-x}dx$ $(\frac{x^2-1}{2}\ln\frac{1+x}{1-x}+x+C)$

(7) $\int \frac{\ln[(x+a)^{x+a}(x+b)^{x+b}]}{(x+a)(x+b)}dx$ $(\ln(x+a)\ln(x+b)+C)$

(8) $\int \frac{x+\sin x}{1+\cos x}dx$ $(x\tan\frac{x}{2}+C)$

(9) $\int \mathrm{e}^x\sin^2 x dx$ $(\frac{1}{2}\mathrm{e}^x-\frac{1}{10}(\mathrm{e}^x\cos 2x+2\mathrm{e}^x\sin 2x)+C)$

(10) $\int \frac{1}{\sqrt[3]{(x+1)^2(x-1)^4}}dx$ $(-\frac{3}{2}\sqrt[3]{\frac{x+1}{x-1}}+C)$

(11) $\int \frac{\arctan\mathrm{e}^x}{\mathrm{e}^{2x}}dx$ $(-\frac{1}{2}(\mathrm{e}^{-2x}\arctan\mathrm{e}^x+\mathrm{e}^{-x}+\arctan\mathrm{e}^x)+C)$

(12) $\int \frac{\ln(1+x+x^2)}{(1+x)^2}dx$ $(-\frac{\ln(1+x+x^2)}{1+x}-\frac{1}{2}\ln\frac{(1+x)^2}{1+x+x^2}+\sqrt{3}\arctan\frac{2x+1}{\sqrt{3}}+C)$

(13) $\int \frac{(x+1)\mathrm{e}^x}{(x+2)^2}dx$ $(\frac{\mathrm{e}^x}{x+2}+C)$ $\int \frac{\tan x}{1+\tan x+\tan^2 x}dx$ $(x-\frac{2}{\sqrt{3}}\arctan\frac{2\tan x+1}{\sqrt{3}}+C)$

(14) $\int \frac{x\cos x}{\sin^3 x}dx$ $(-\frac{x}{2\sin^2 x}-\frac{1}{2}\cot x+C)$

(15) $\int \frac{1}{\sin 2x+2\sin x}dx$ $(\frac{1}{8}\ln\frac{1-\cos x}{1+\cos x}+\frac{1}{4(1+\cos x)}+C)$

(16) $\int \frac{1}{\sin x\sqrt{1+\cos x}}dx$ $(\frac{1}{\sqrt{1+\cos x}}-\frac{1}{2\sqrt{2}}\ln\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}-\sqrt{1+\cos x}}+C)$

(17) $\int \frac{ax^2+b}{1+x^2}\arctan x dx$ $(a[x\arctan x-\frac{1}{2}\ln(1+x^2)]-\frac{a-b}{2}(\arctan x)^2+C)$

(18) $\int \sqrt{\frac{x}{1-x\sqrt{x}}}dx$ $(-\frac{4}{3}\sqrt{1-x\sqrt{x}})$

(19) $\int \sqrt{\frac{\mathrm{e}^x-1}{\mathrm{e}^x+1}}dx$ $(\ln(\mathrm{e}^x+\sqrt{\mathrm{e}^{2x}-1})+\arcsin\mathrm{e}^{-x}+C)$

2. 设$f(x)$的一个原函数为$\frac{\sin x}{x}$, 求$\int x^3 f'(x)dx$. $(x^2\cos x-4x\sin x-6\cos x+C)$