



Analysis of complex time series using refined composite multiscale entropy



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ABSTRACT

Multiscale entropy (MSE) is an effective algorithm for measuring the complexity of a time series that has been applied in many fields successfully. However, MSE may yield an inaccurate estimation of entropy or induce undefined entropy because the coarse-graining procedure reduces the length of a time series considerably at large scales. Composite multiscale entropy (CMSE) was recently proposed to improve the accuracy of MSE, but it does not resolve undefined entropy. Here we propose a refined composite multiscale entropy (RCMSE) to improve CMSE. For short time series analyses, we demonstrate that RCMSE increases the accuracy of entropy estimation and reduces the probability of inducing undefined entropy.

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1. Introduction

Numerous techniques have been developed to measure the complexity of time series generated from nonlinear dynamical systems. Pincus proposed a family of statistics, called approximate entropy (ApEn), to measure the regularity of a time series [1]. In general, a lower value of ApEn reflects a higher degree of regularity, whereas a less predictable time series is often characterized by higher ApEn values. ApEn has been widely applied to biomedical signal analysis [2]. However, because of the effect of bias, ApEn strongly depends on the data length and lacks relative consistency in some cases. To remove the deficiencies, a modification of ApEn known as sample entropy (SampEn) was proposed by Richman et al. [3]. However, the results of both ApEn and SampEn algorithms for estimating entropy are not always associated with complexity. For example, the SampEn of white noise is higher than that of $1/f$ noise, which is considered by most scientists to be the most complex and results in contradictory findings when entropy-based algorithms are applied to real-world datasets obtained in health and disease states [4]. Therefore, Costa et al. [5] proposed the multiscale entropy (MSE) algorithm to calculate SampEn over a range of scales to represent the complexity of a time series. The MSE algorithm resolves the contradiction between the lower entropy and higher complexity of $1/f$ noise compared with white noise. Costa et al. applied the MSE algorithm to differentiate heart

rate time series among healthy young, healthy elderly, and congestive heart failure subjects. In recent years, the MSE algorithm has received a considerable amount of attention and has been successfully applied to analyze time series generated from various dynamical systems including physiological signals [6–8] and vibrational signals [9,10].

In the MSE algorithm, to quantify regularity at different time scales, the original time series is coarse-grained and the SampEn for each coarse-grained time series is subsequently calculated. The coarse-grained time series for a scale factor τ are obtained by calculating the arithmetic mean of τ neighboring values without overlapping. This coarse-graining procedure is similar to moving-averaging and the decimation of the original time series. The decimation procedure shortens the length of the coarse-grained time series by a factor of τ . For an N -points time series, the length of the coarse-grained time series at a scale factor of τ is equal to N/τ . Therefore, the coarse-grained time series at large scales may not be adequately long to obtain an accurate SampEn. Furthermore, in some cases, the SampEn is undefined because no template vectors are matched to one another. Inaccurate or undefined SampEns lead to the reduction of reliability in distinguishing time series generated by different systems. The problems in accuracy and validity are challenges of the MSE algorithm at large scale factors.

One approach to addressing the aforementioned concerns is to modify the definition of the SampEn algorithm. Based on the concept of fuzzy sets, a fuzzy entropy (FuzzyEn) algorithm has been proposed to solve the problems of SampEn [11–13]. In FuzzyEn,

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shapes are constructed by removing the local baselines of vectors, and an exponential function is used to obtain a fuzzy measurement of the similarity of two shapes. Because of these modifications, FuzzyEn is more accurate than SampEn; the validity of FuzzyEn for a short time series is also guaranteed [12]. Zhang et al. [14] proposed an improved multiscale entropy (IMSE) algorithm, which extended the FuzzyEn to the same multiscale framework as that of MSE, and then used it to identify the bearing health condition of rotary machine. Despite the advantage mentioned previously, the computational cost of IMSE is extremely high because exponential calculation is necessary in the computation of the shapes' similarities. In addition, the physical meaning of IMSE is not consistent with that of MSE because the entropy value calculated using FuzzyEn is quite different from that calculated using SampEn. Recently, Wu et al. [15] proposed the composite multiscale entropy (CMSE) algorithm to address the accuracy concern of the MSE algorithm. In the CMSE algorithm, at a scale factor of τ , the sample entropies of all coarse-grained time series, corresponding to different starting points of the coarse-graining process, are calculated and the CMSE value is defined as the means of τ entropy values. Compared with the MSE method, the CMSE is used to estimate entropy more accurately but increases the probability of inducing undefined entropy. In this paper, we modify the CMSE algorithm slightly and propose a refined composite multiscale entropy (RCMSE) algorithm. Two synthetic noise signals were used to evaluate the performance of the RCMSE algorithm. Simulation results reveal that the overall performance of the RCMSE algorithm is superior to those of the MSE and CMSE algorithms.

The remainder of this paper is organized as follows. Section 2 provides a review of the SampEn, MSE, and CMSE algorithms. The proposed RCMSE algorithm is also introduced in this section. In Section 3, several experiments that demonstrate the effectiveness of the proposed RCMSE algorithm are described. A conclusion is provided in Section 4.

2. Methods

In this section, the theoretical backgrounds of the SampEn, MSE, and CMSE algorithms are briefly reviewed. In addition, the concept of the RCMSE algorithm is introduced.

2.1. Sample entropy

Let $\mathbf{x} = \{x_1 \ x_2 \ \dots \ x_N\}$ represent a time series of length N . The SampEn algorithm can be summarized as follows [3].

1. Construct template vectors with dimension m by using Eq. (1):

$$\mathbf{x}_i^m = \{x_i \ x_{i+1} \ \dots \ x_{i+m-1}\}, \quad 1 \leq i \leq N - m \quad (1)$$

2. A match occurs when the distance between two template vectors $(\mathbf{x}_i^m, \mathbf{x}_j^m)$ is smaller than a predefined tolerance r . The distance between the two vectors is calculated by using the infinity norm:

$$d_{ij}^m = \|\mathbf{x}_i^m - \mathbf{x}_j^m\|_\infty, \quad 1 \leq i, j \leq N - m, \quad j \neq i. \quad (2)$$

3. We call $(\mathbf{x}_i^m, \mathbf{x}_j^m)$ an m -dimensional matched vector pair if d_{ij}^m is less than or equal to a tolerance r . Let n^m represent the total number of m -dimensional matched vector pairs.
4. Steps 1–3 are repeated for $m = m + 1$, and n^{m+1} is obtained to represent the total number of $(m + 1)$ -dimensional matched vector pairs.
5. The SampEn is defined as the logarithm of the ratio of n^{m+1} to n^m ; that is,

$$\text{SampEn}(\mathbf{x}, m, r) = -\ln \frac{n^{m+1}}{n^m} \quad (3)$$

For a short time series, the SampEn algorithm may cause the following problems: 1) the SampEn often yields an inaccurate estimation; and 2) n^m or n^{m+1} in Eq. (3) may be zero, thus inducing an undefined SampEn. To obtain a reasonable SampEn, the length of the time series is suggested to be in the range of 10^m to 30^m [16]. Regarding $m = 2$, Costa et al. [7] suggested that the time series be longer than 750 data points.

2.2. Multiscale entropy and composite multiscale entropy

Costa et al. [4,5] have indicated that it is generally difficult to distinguish the inter-beat interval time series of different diseased and healthy states if only a single-scale SampEn is used. Therefore, they proposed the concept of MSE to solve this difficulty. The MSE algorithm consists of two procedures: 1) a coarse-graining procedure, which is used to obtain the representations of the original time series on different time scales; and 2) the SampEn, which is used to quantify the regularities of the coarse-grained time series. To obtain the coarse-grained time series at a scale factor of τ , the original time series is divided into non-overlapping windows of length τ and the data points inside each window are averaged. As shown in Fig. 1, τ coarse-grained time series are divided from the original time series for a scale factor of τ . The k -th coarse-grained time series $\mathbf{y}_k^{(\tau)} = \{y_{k,1}^{(\tau)} \ y_{k,2}^{(\tau)} \ \dots \ y_{k,p}^{(\tau)}\}$ of \mathbf{x} is defined as follows:

$$y_{k,j}^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+k}^{j\tau+k-1} x_i, \quad 1 \leq j \leq \frac{N}{\tau}, \quad 1 \leq k \leq \tau \quad (4)$$

In the conventional MSE algorithm proposed by Costa et al. [5], the MSE at a scale factor of τ is defined as the SampEn of the first coarse-grained time series; that is,

$$\text{MSE}(\mathbf{x}, \tau, m, r) = \text{SampEn}(\mathbf{y}_1^{(\tau)}, m, r) \quad (5)$$

The length of the time series is reduced by a factor of τ when the coarse-graining procedure described in Eq. (4) is applied to the original time series \mathbf{x} . For a time series with 1000 data points, the coarse-grained time series at a scale factor of 20 contains only 50 data points, which violates the length requirements suggested by [7] and [16]. Therefore, for analyzing short time series, the MSE algorithm often induces inaccurate SampEns at large time scales. Wu et al. [15] proposed the CMSE algorithm to improve the accuracy of the MSE. In the CMSE algorithm, at a scale factor of τ , the SampEns of all coarse-grained time series are calculated and the CMSE value is defined as the means of τ SampEns; that is,

$$\begin{aligned} \text{CMSE}(\mathbf{x}, \tau, m, r) &= \frac{1}{\tau} \sum_{k=1}^{\tau} \text{SampEn}(\mathbf{y}_k^{(\tau)}, m, r) \\ &= \frac{1}{\tau} \sum_{k=1}^{\tau} \left(-\ln \frac{n_{k,\tau}^{m+1}}{n_{k,\tau}^m} \right) \end{aligned} \quad (6)$$

where $n_{k,\tau}^m$ represents the total number of m -dimensional matched vector pairs and is constructed from the k -th coarse-grained time series at a scale factor of τ . As reported in [15], at long scales, the entropies calculated using the MSE and CMSE algorithms are nearly equivalent statistically at each scale, but the CMSE can provide more accurate estimations than the MSE can.

2.3. Refined composite multiscale entropy

In the CMSE algorithm, the logarithms of the ratio of $n_{k,\tau}^{m+1}$ to $n_{k,\tau}^m$ for all τ coarse-grained series are first examined, and then the average of these logarithms is defined as the entropy value. The CMSE value is undefined when one of the values of $n_{k,\tau}^{m+1}$ or

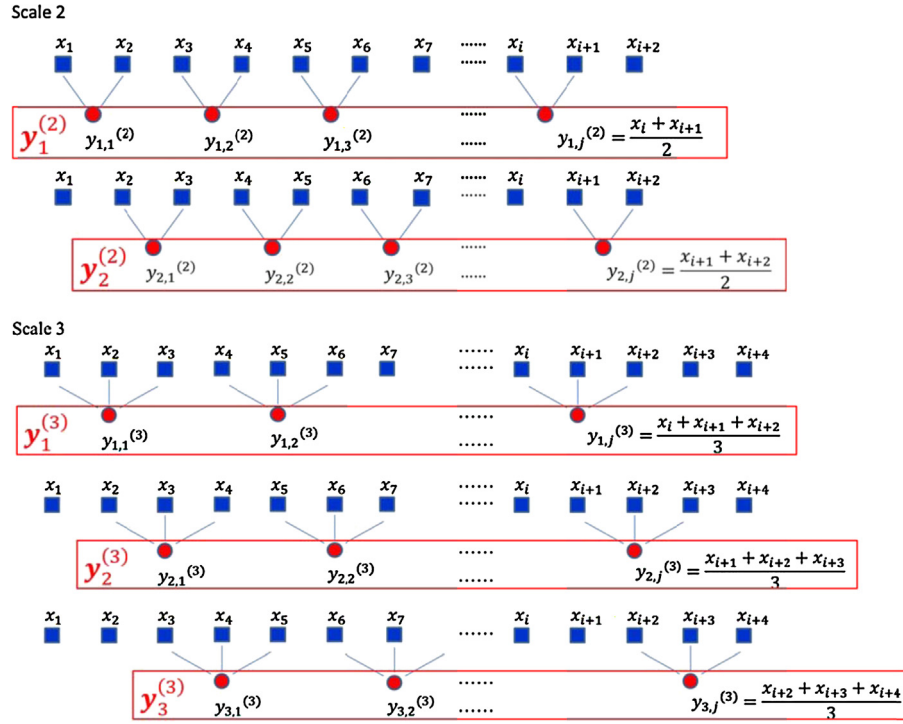


Fig. 1. Schematic illustration of the coarse-graining procedure. Modified from [15].

$n_{k,\tau}^m$ is zero. Therefore, compared with the MSE, the probability of inducing undefined entropy is increased when the CMSE is applied to analyze a short time series. This deficiency of the CMSE algorithm limits its applications in short time series analysis. In this paper, we propose the RCMSE algorithm to address the validity issue. The RCMSE algorithm consists of the following procedures:

- 1) The coarse-graining procedure described in (4) is used to obtain the coarse-grained time series on different time scales.
- 2) At a scale factor of τ , the number of matched vector pairs, $n_{k,\tau}^{m+1}$ and $n_{k,\tau}^m$, is calculated for all τ coarse-grained series.
- 3) Let $\bar{n}_{k,\tau}^m$ ($\bar{n}_{k,\tau}^{m+1}$) represent the mean of $n_{k,\tau}^m$ ($n_{k,\tau}^{m+1}$) for $1 \leq k \leq \tau$. The RCMSE value at a scale factor of τ is defined as the logarithm of the ratio of $\bar{n}_{k,\tau}^{m+1}$ to $\bar{n}_{k,\tau}^m$. In other words, the RCMSE at a scale factor of τ is provided as Eq. (7)

$$RCMSE(\mathbf{x}, \tau, m, r) = -\ln\left(\frac{\bar{n}_{k,\tau}^{m+1}}{\bar{n}_{k,\tau}^m}\right) \quad (7)$$

$$\text{where } \bar{n}_{k,\tau}^{m+1} = \frac{1}{\tau} \sum_{k=1}^{\tau} n_{k,\tau}^{m+1} \text{ and } \bar{n}_{k,\tau}^m = \frac{1}{\tau} \sum_{k=1}^{\tau} n_{k,\tau}^m.$$

Eq. (7) can be simplified as

$$\begin{aligned} RCMSE(\mathbf{x}, \tau, m, r) &= -\ln\left(\frac{\bar{n}_{k,\tau}^{m+1}}{\bar{n}_{k,\tau}^m}\right) - \ln\left(\frac{\frac{1}{\tau} \sum_{k=1}^{\tau} n_{k,\tau}^{m+1}}{\frac{1}{\tau} \sum_{k=1}^{\tau} n_{k,\tau}^m}\right) \\ &= -\ln\left(\frac{\sum_{k=1}^{\tau} n_{k,\tau}^{m+1}}{\sum_{k=1}^{\tau} n_{k,\tau}^m}\right) \end{aligned} \quad (8)$$

Based on Eq. (8), we can conclude that the RCMSE value is undefined only when all $n_{k,\tau}^{m+1}$ or $n_{k,\tau}^m$ are zeros. Therefore, compared with the CMSE algorithm, the RCMSE algorithm reduces the probability of inducing undefined entropy.

3. Experiments

To evaluate the effectiveness of the RCMSE, two types of synthetic noise signals, white and $1/f$ noises, were applied to com-

Table 1

The probabilities of inducing undefined entropy when the MSE, CMSE, and RCMSE are applied to analyze the white and $1/f$ noises.

Scale factor	RCMSE		MSE		CMSE	
	white noise	$1/f$ noise	white noise	$1/f$ noise	white noise	$1/f$ noise
1–8	0	0	0	0	0	0
9	0	0	0	0	0	0.005
10	0	0	0	0	0	0.005
11	0	0	0	0	0	0.015
12	0	0	0	0.005	0	0.015
13	0	0	0	0.005	0	0.025
14	0	0	0	0.01	0	0.155
15	0	0	0	0.01	0	0.210
16	0	0	0	0.02	0	0.310
17	0	0	0	0.05	0	0.450
18	0	0	0	0.06	0	0.575
19	0	0	0	0.075	0	0.660
20	0	0	0	0.075	0	0.690

pare the effectiveness of the MSE and CMSE algorithms. Validity, accuracy, and computational cost were considered in the numerical experiments. In the study described in this section, we calculated the MSE, CMSE, and RCMSE values from Scale 1 to Scale 20 ($\tau = 1$ to 20), and the SampEn was calculated with $m = 2$ and $r = 0.15\sigma$, where σ denotes the standard deviation of the original time series. All the experiments were performed on an Intel(R) Core(TM) i7-2600 CPU with 3.4 GHz and 8 GB of RAM using Matlab 7.8 (R2009a) in a Windows 7 system.

3.1. Comparison of validity

Table 1 shows the probabilities of inducing undefined entropy when the MSE, CMSE, and RCMSE algorithms are applied to analyze white and $1/f$ noises with 1000 data points. The numerical results were quantified by calculating 200 independent noise samples. For the MSE and CMSE algorithms, the probability of inducing undefined entropy was zero when they were applied to analyze white noise. In other words, for all 200 white noise samples,

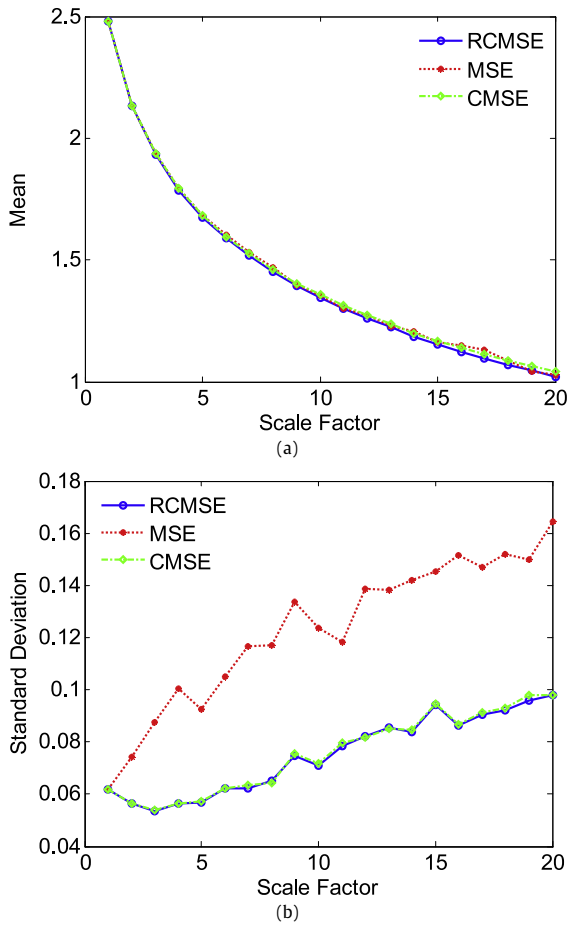


Fig. 2. (a) Mean of 200 analysis results of white noise. (b) Standard deviation of 200 analysis results of white noise.

these two algorithms were used to calculate the entropy values of Scales 1 to 20 successfully. However, at a scale factor of 12, the probability of inducing undefined entropy was 0.005 when the MSE was applied to analyze $1/f$ noise. In other words, 1 of 200 $1/f$ noise samples failed to be analyzed at Scale 12 when the MSE algorithm was used. Based on Table 1, we obtained the following results:

- 1) As reported in [5], the MSE value of a $1/f$ noise was greater than that of a white noise when the time scale was larger than 5. According to Table 1, the probability of inducing undefined entropy was zero when the MSE and CMSE were used to analyze white noise, but this was not the case when they were used to analyze $1/f$ noise. The validities of the MSE and CMSE decreased as the entropy of the time series increased.
- 2) In the MSE and CMSE algorithms, the length of the coarse-grained time series decreased as the time scale increased. According to Table 1, the probability of inducing undefined entropy increased as the time scale increased. Therefore, the shorter the time series, the lower the validities of the MSE and CMSE algorithms are.
- 3) In the MSE algorithm, the entropy is undefined when $n_{1,\tau}^m$ or $n_{1,\tau}^{m+1}$ is zero. However, in the CMSE algorithm, the entropy is undefined if one of the values of $n_{k,\tau}^{m+1}$ or $n_{k,\tau}^m$ ($1 \leq k \leq \tau$) is zero. Theoretically, the validity performance of the CMSE should be worse than that of the MSE. The results shown in Table 1 are consistent with this theoretical inference.
- 4) For both white and $1/f$ noises, the probability of inducing undefined entropy was zero when the RCMSE algorithm was

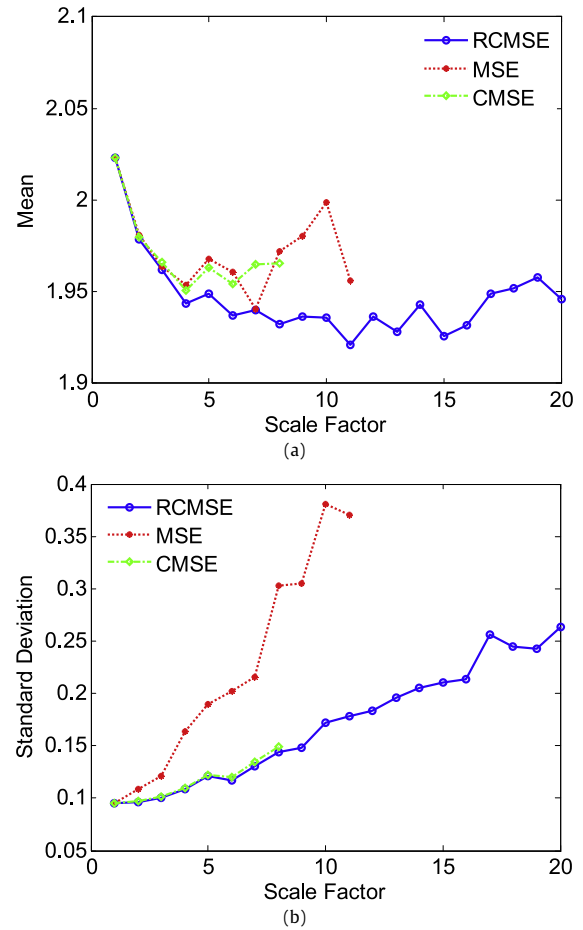


Fig. 3. (a) Mean of 200 analysis results of $1/f$ noise. (b) Standard deviation of 200 analysis results of $1/f$ noise.

used. Thus, regarding validity, the RCMSE algorithm is superior to both MSE and CMSE algorithms.

3.2. Comparison of accuracy

In this subsection, we verify the accuracy of the MSE, CMSE, and RCMSE algorithms. Two hundred independent noise samples were used in each simulation, and each noise sample contained 1000 data points. Figs. 2 and 3 present the means and standard deviations (SDs) of entropies. Regarding white noise, the means of the entropy values obtained using the MSE, CMSE, and RCMSE were nearly equal (see Fig. 2a), but the SD of the MSE was greater than those of the CMSE and RCMSE (see Fig. 2b). In the case of $1/f$ noise, the entropies of some samples obtained using MSE and CMSE were undefined when the time scales were larger than 9 and 12, respectively, whereas the entropies of all samples were well defined when the RCMSE algorithm was used. Fig. 3b indicates that the SDs of the entropy value obtained using RCMSE were all lower than those obtained using the MSE and CMSE algorithms. This result indicates that the entropies obtained using the RCMSE algorithm were more consistent than those obtained using the CMSE algorithm. According to Fig. 3a, the means of the entropy values obtained using MSE and CMSE were nearly equal but greater than that obtained using RCMSE. The following question then arose: “Which estimation is statistically close to the real entropy?” To answer this question, the entropies of $1/f$ noise at a time scale of 20 ($\tau = 20$) were calculated using several data lengths ($N = 1000$; 2000; 3000; 5000; 10,000; 20,000; and 30,000). The means and SDs of the entropies are shown in Table 2. When the original

Table 2

Means and standard deviations of the RCMSE, MSE, and CMSE of $1/f$ noise at scale factor of 20.

Data length	RCMSE	MSE	CMSE
1000	1.946 ± 0.264	–	–
2000	1.879 ± 0.147	1.931 ± 0.315	1.924 ± 0.155
3000	1.859 ± 0.113	1.895 ± 0.211	1.878 ± 0.115
5000	1.836 ± 0.088	1.843 ± 0.140	1.842 ± 0.089
10,000	1.800 ± 0.071	1.799 ± 0.086	1.802 ± 0.071
20,000	1.773 ± 0.061	1.773 ± 0.065	1.773 ± 0.061
30,000	1.752 ± 0.065	1.751 ± 0.068	1.752 ± 0.065

Table 3

The execution times of the MSE, CMSE, and RCMSE algorithms used to analyze the $1/f$ noise.

Data length	RCMSE	MSE	CMSE	Ratio of RCMSE to MSE
1000	0.1433	0.0324	0.1434	4.4228
2000	0.3168	0.0798	0.3171	3.9699
3000	0.5276	0.1428	0.5281	3.6947
5000	1.0619	0.3157	1.0641	3.3636
10,000	2.9375	1.0195	2.9485	2.8813
20,000	9.1494	3.5995	9.2196	2.5419
30,000	18.7077	7.7255	18.8491	2.4216

time series was sufficiently long ($N \geq 20,000$), the means of the entropies obtained using these three algorithms were nearly equivalent, but they were quite different when the length of the original time series was short. Consider the difference between the entropy of $1/f$ noise with 2000 data points and the entropy of $1/f$ noise with 30,000 data points. The result obtained using the RCMSE algorithm was the lowest among these three algorithms. In other words, the RCMSE outperformed the other two algorithms regarding independence of the data length when the time series was characterized. Based on the previous discussions, we conclude that the RCMSE algorithm is the most reliable among these algorithms.

3.3. Comparison of computational cost

The computational complexity of the SampEn algorithm is $O(N^2)$ because two loops are required. Therefore, at a scale factor of τ , the execution time required to calculate the SampEn of the coarse-grained time series is $O((N/\tau)^2)$. To simplify the problem, the following is assumed:

$$O((N/\tau)^2) = k(N/\tau)^2 \quad \text{for all time scales.} \quad (9)$$

Eq. (9) is true when the time series N is sufficiently long. Therefore, the execution time for the MSE algorithm is given by the following equation

$$T(s) = \sum_{\tau=1}^s k(N/\tau)^2. \quad (10)$$

In the CMSE and RCMSE algorithms, at a scale factor of τ , τ coarse-grained time series must be calculated. Therefore, the execution times for these two algorithms are equal and given by the following equation

$$T^*(s) = \sum_{\tau=1}^s k\tau(N/\tau)^2 \quad (11)$$

The ratio of $T^*(s)$ to $T(s)$ is given as follows

$$\frac{T^*(s)}{T(s)} = \frac{\sum_{\tau=1}^s \tau(N/\tau)^2}{\sum_{\tau=1}^s (N/\tau)^2} \quad (12)$$

According to Eq. (12), in the case of $s = 20$, the ratio of $T^*(20)$ to $T(20)$ is equal to 2.254.

The execution times of the MSE, CMSE, and RCMSE algorithms used to analyze the $1/f$ noise with several data lengths are shown in Table 3. In case of $N = 30,000$, the computational cost of RCMSE is about 2.4216 times higher than that of MSE. The numerical result is close to the theoretical result obtained by Eq. (12). In addition, the computational cost of the CMSE is the highest among the three algorithms. According to Eqs. (6) and (8), for a scale factor of τ , the logarithm function should be calculated τ times in the CMSE algorithm, but only one logarithm calculation is required in the RCMSE algorithm. Therefore, the RCMSE algorithm should be slightly faster than the CMSE algorithm. The results presented in Table 3 are consistent with this inference.

4. Conclusion

The MSE algorithm is a popular method for characterizing the complexity of time series. However, the MSE algorithm may encounter problems in validity and accuracy when it is applied to short time series analyses. In the real world, recordings of experimental data, such as physiological time series, are frequently short. Therefore, it is necessary to develop a more reliable algorithm to characterize the complexity of short time series. The CMSE algorithm is superior to the MSE algorithm in accuracy but it deteriorates the validity. In this paper, we proposed the RCMSE algorithm as an improvement of the CMSE algorithm. Simulation results reveal that the RCMSE algorithm outperforms the CMSE algorithm in validity, accuracy of entropy estimation, independence of the data length, and computational efficiency. For analyzing short time series, the RCMSE algorithm can be used to increase the accuracy of entropy estimation and reduce the probability of inducing undefined entropy compared with the MSE algorithm. However, the computational cost of the RCMSE is higher than that of the MSE. In addition, for long time series analysis, the improvements in accuracy and validity are not significant when using the RCMSE. Therefore, for long time series analysis, a hybrid method can be used to balance the trade-off between accuracy and computational cost. In the hybrid method, the small-scale and large-scale analyses are performed using the MSE and RCMSE, respectively. As reported in [7], the SampEn is significantly independent of data length when the number of data points is greater than 750. Therefore, in the hybrid method, the RCMSE algorithm is suggested for use when the coarse-grained time series, N/τ , is shorter than 750.

Conflict of interest

The authors declare no conflict of interest.

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