# The expressive power, satisfiability and path checking problems of MTL and TPTL over data words

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Prof. Manfred Droste

*Candidate*
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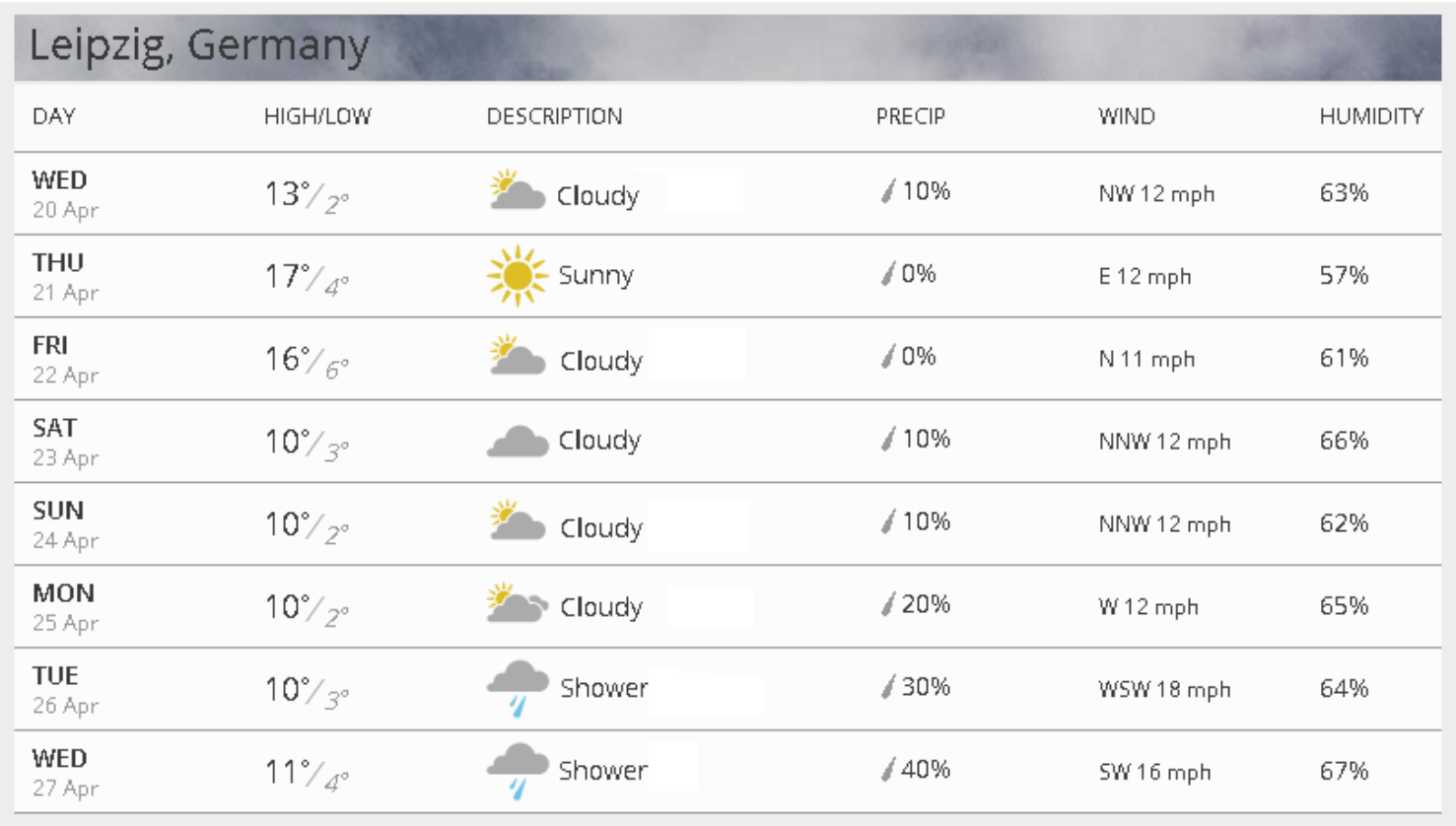
*Universität Leipzig*

19.04.2016



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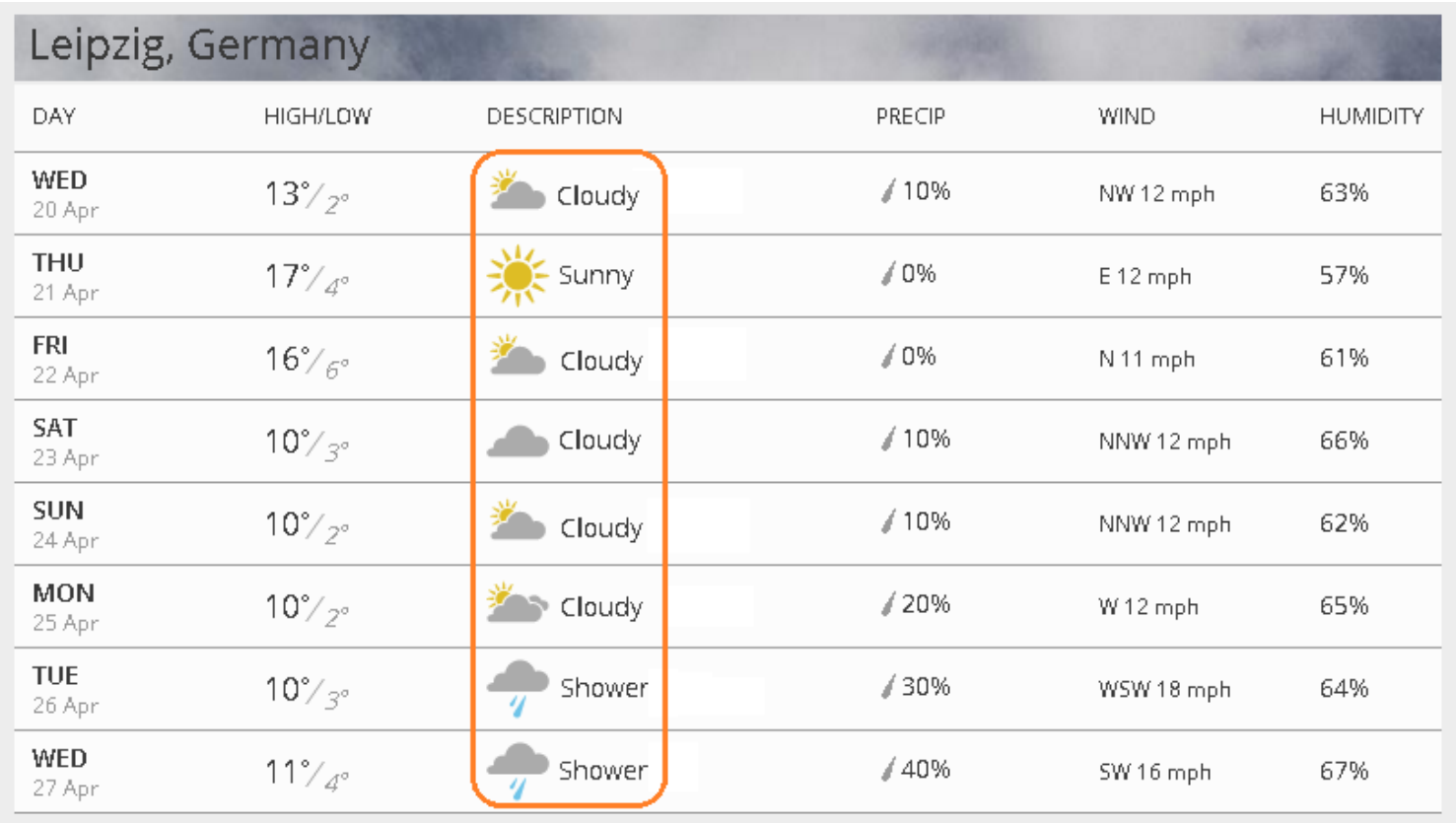

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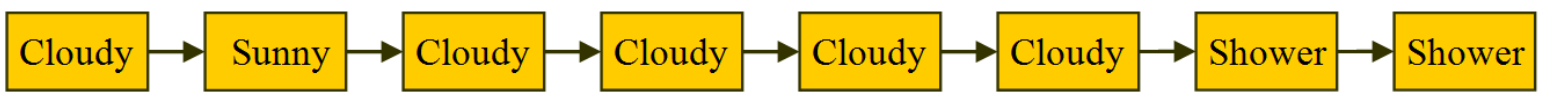

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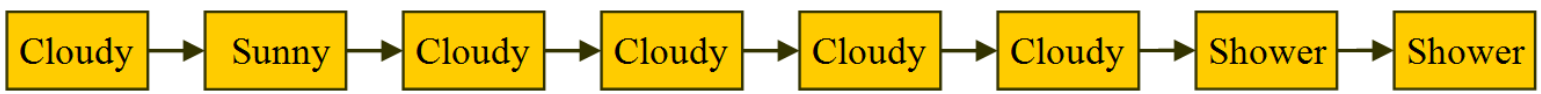


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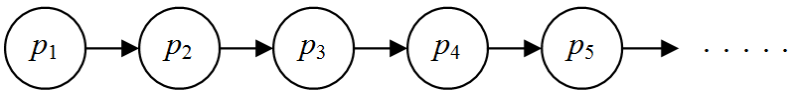


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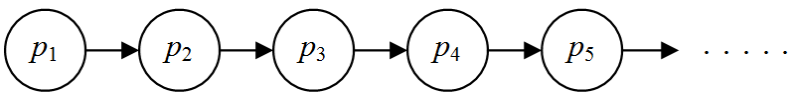


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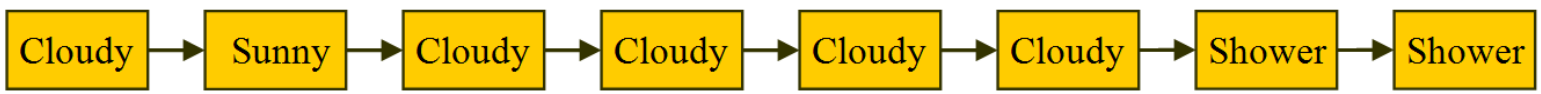


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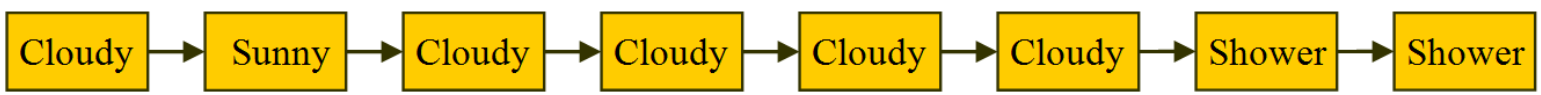

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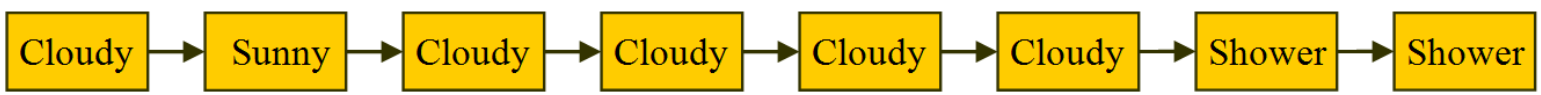

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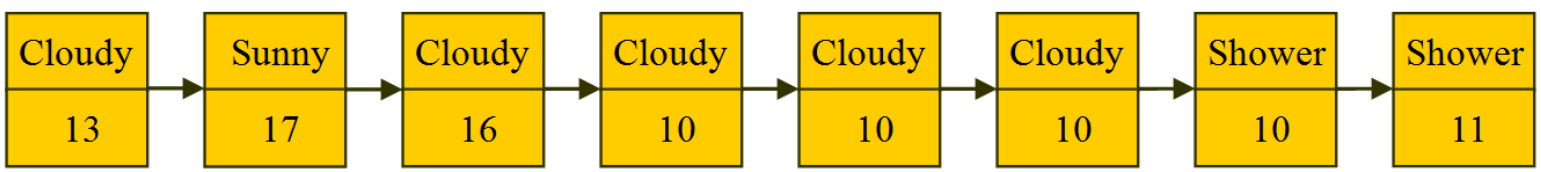

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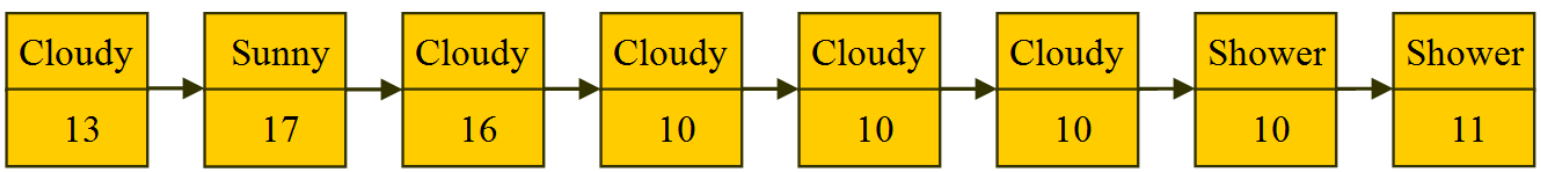


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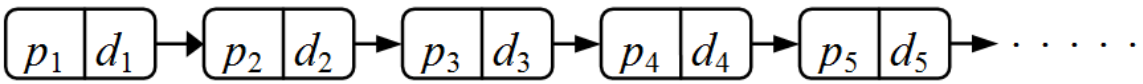


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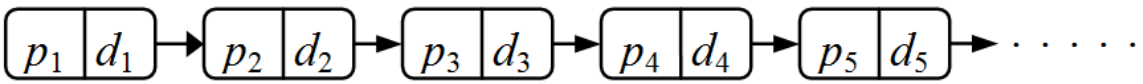


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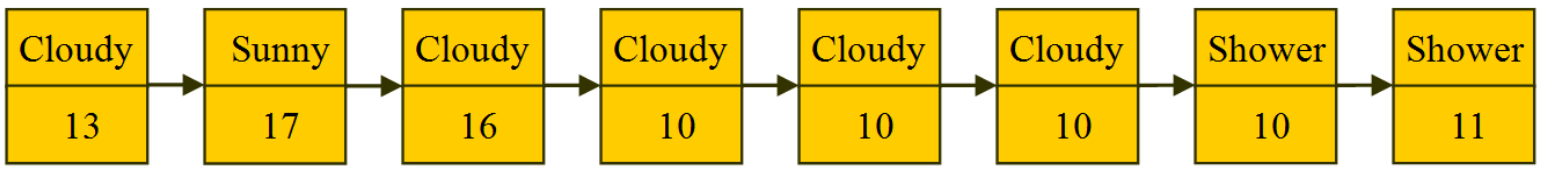

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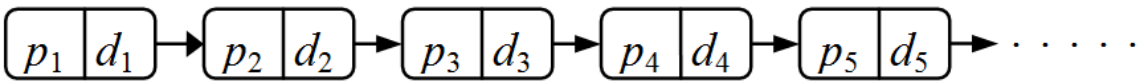


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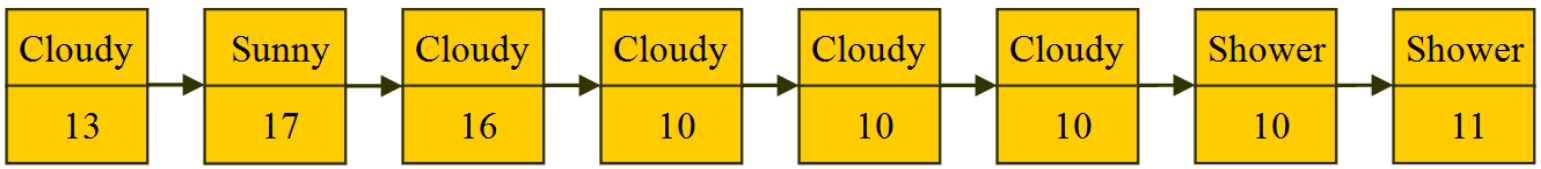


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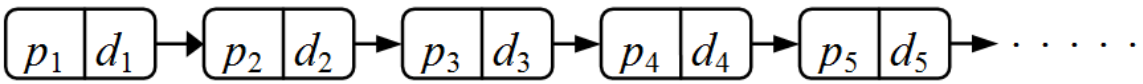


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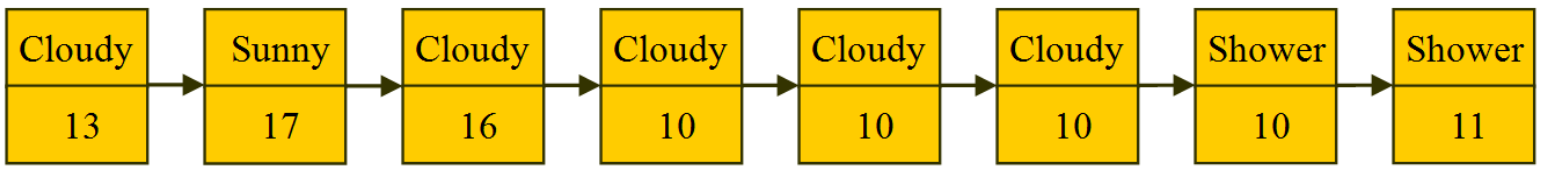


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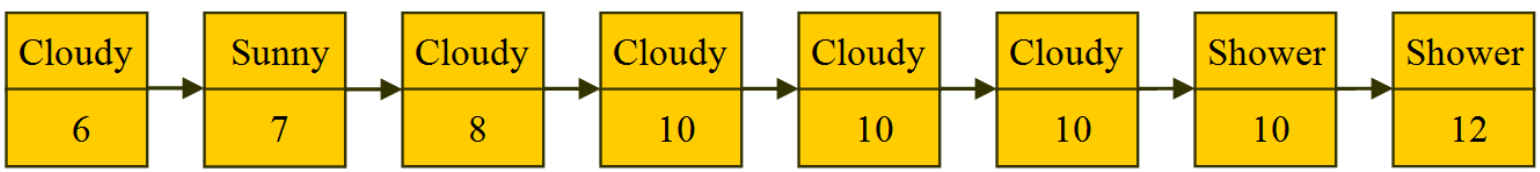
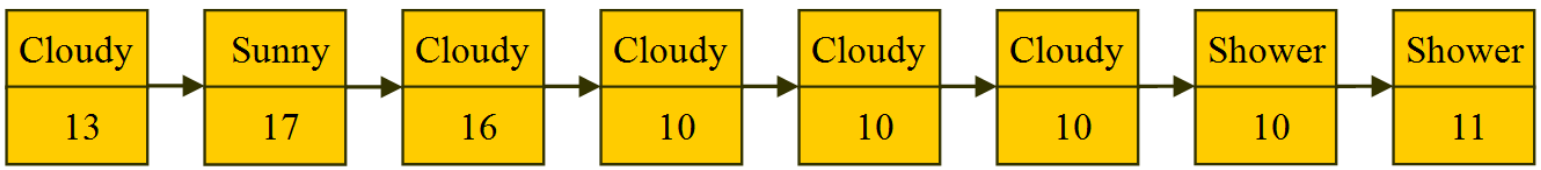



is monotonic.

How to express the following statements in a logic?

- Tomorrow is sunny, and the temperature is 4 degrees higher than that of today.
- It is cloudy until a day with showers.



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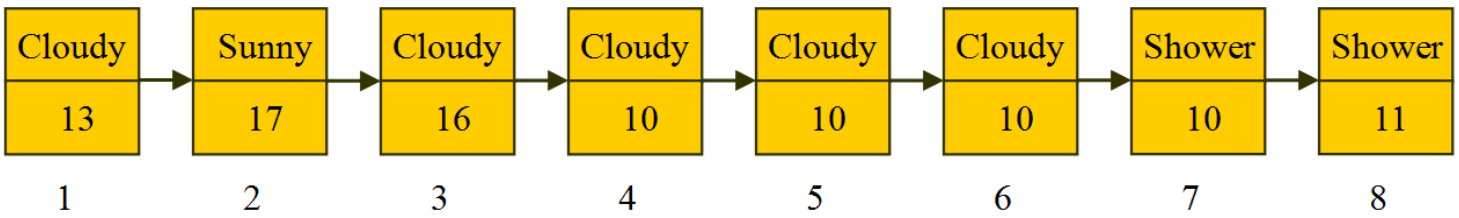

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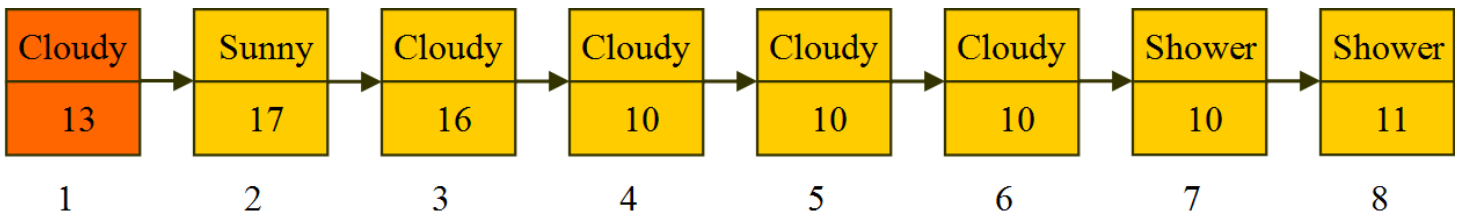


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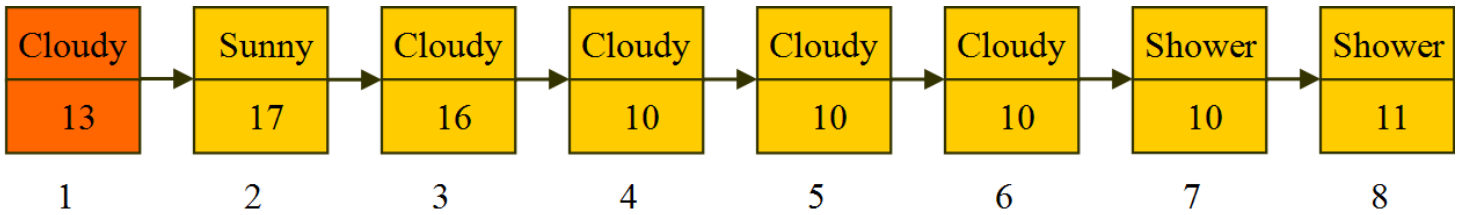


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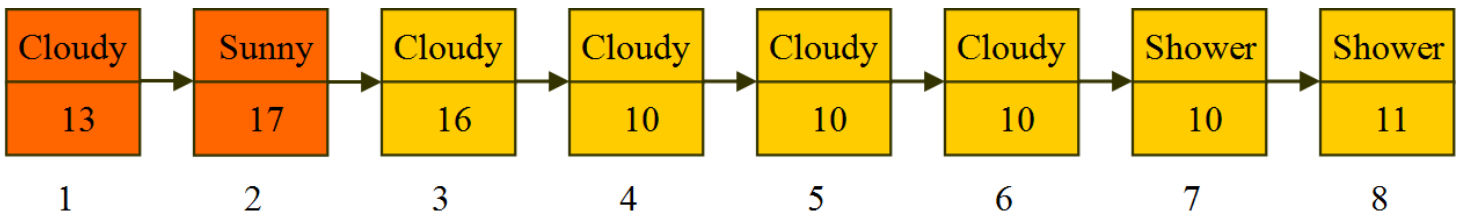


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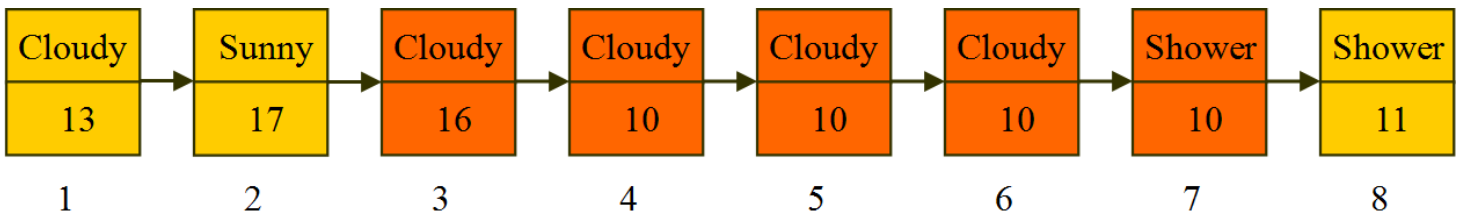


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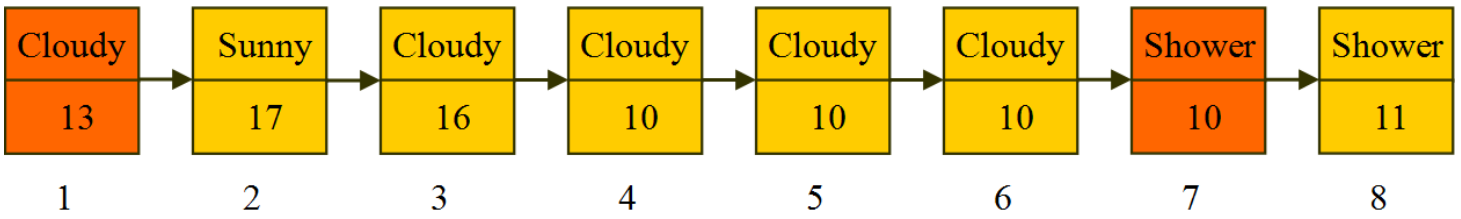


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$(w,1) \models \mathsf{F}\,\text{shower}$ : there is one day in the future with showers.
$(\mathsf{F}\varphi := \mathsf{True}\,\mathsf{U}\,\varphi)$

# Metric Temporal Logic (MTL)

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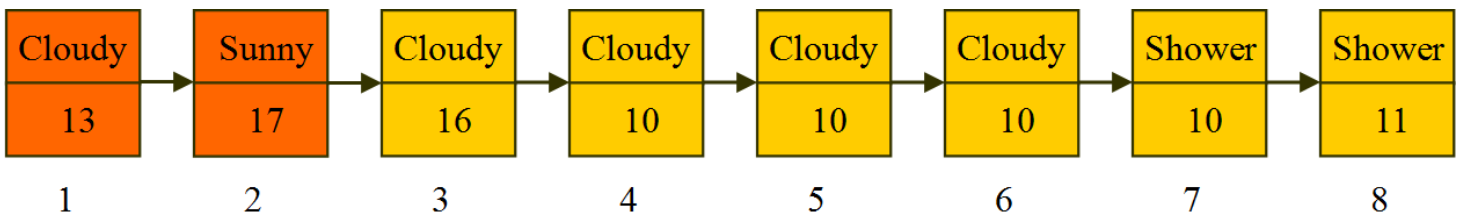


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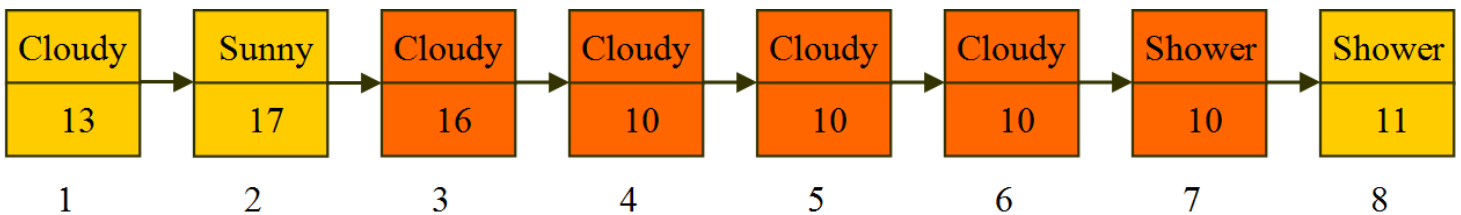


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$(w,3) \models \text{cloudy}\,\mathsf{U}_{[-10,0]}\,\text{shower} : \ldots$ and $(10-16) \in [-10,0]$.

# Timed Propositional Temporal Logic (TPTL)

## Syntax of TPTL

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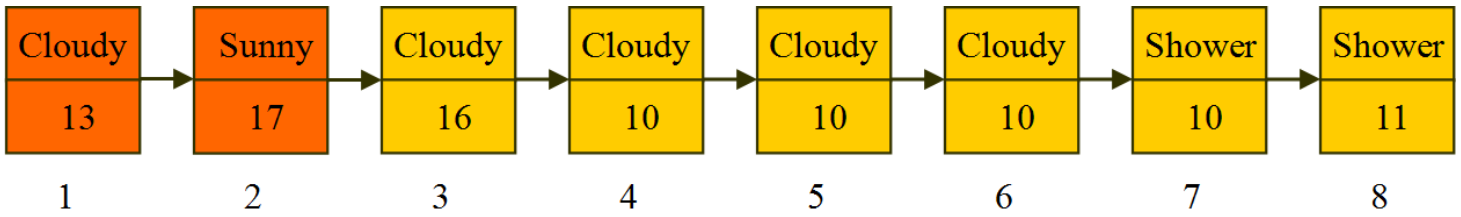


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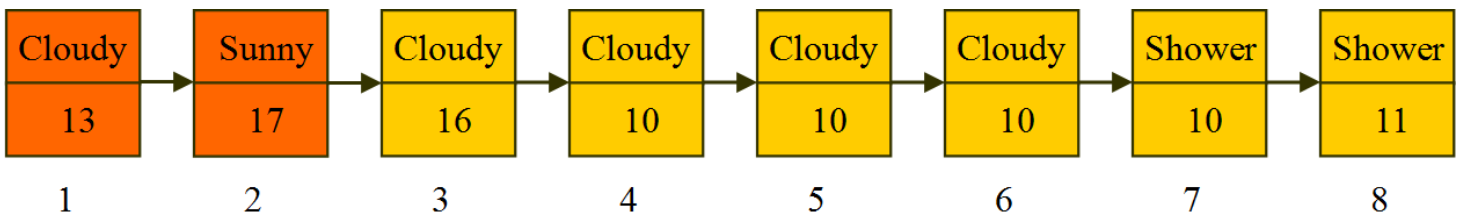


$x.\mathsf{X}(\text{sunny} \wedge x \in [2,6]) \equiv \mathsf{X}_{[2,6]}\,\text{sunny}$

# Timed Propositional Temporal Logic (TPTL)

## Syntax of TPTL

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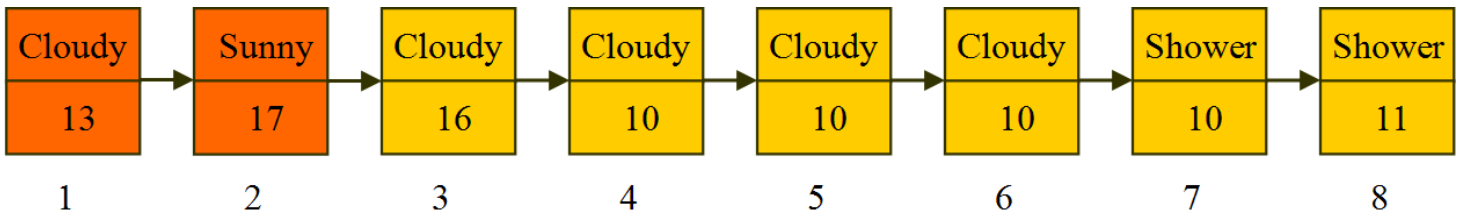


$$(w,1) \models x.\mathsf{X}(\text{sunny} \wedge x \in [2,6])$$

store the current data value 13

check whether $17 - x \in [2,6]$

# Motivation

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  - Satisfiability
  - Complexity of path checking

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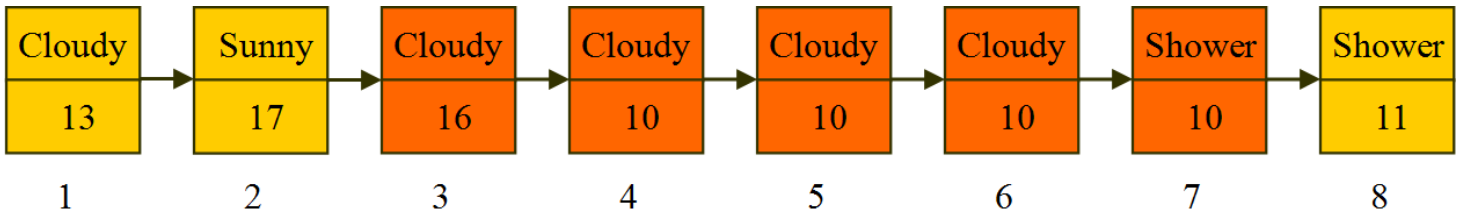
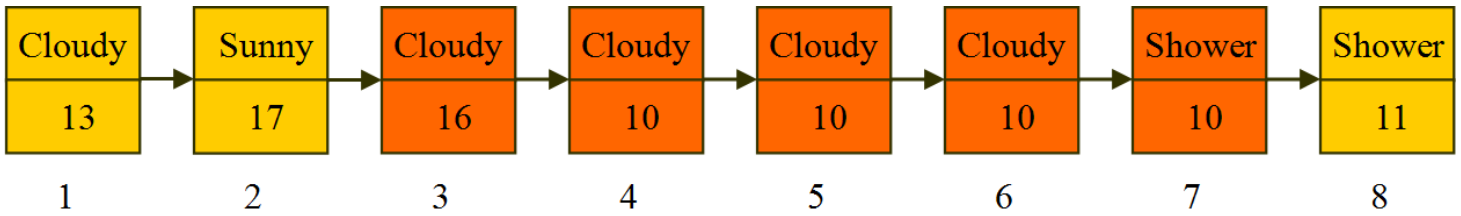



- It is cloudy until a day with showers and the temperature of each of the next few days is 6 degrees lower than that of the initial day.

# What is the expressive power?

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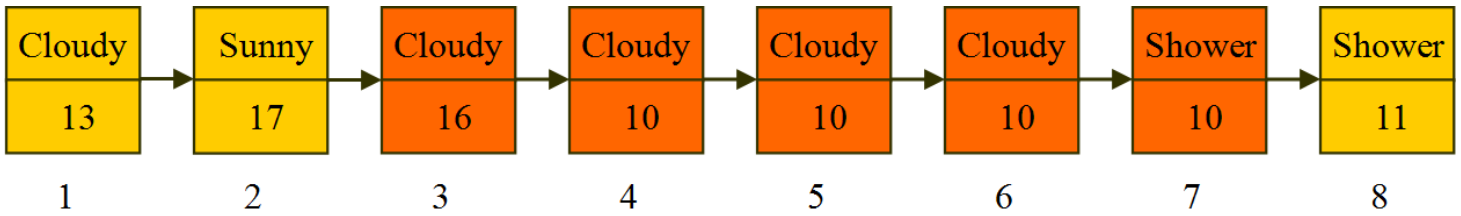


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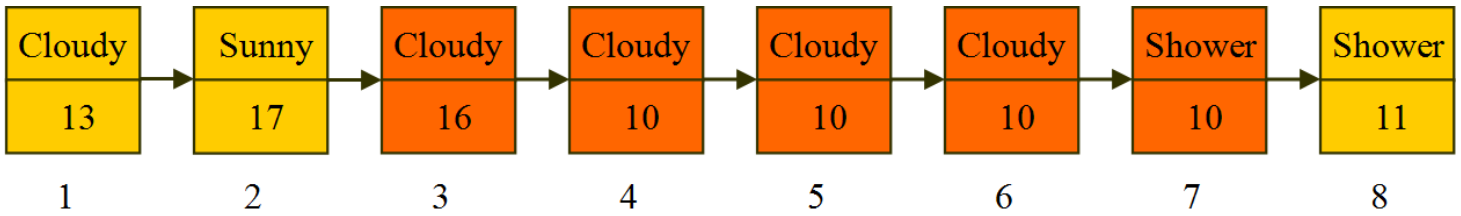


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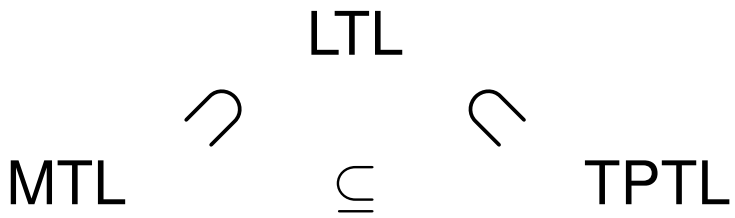
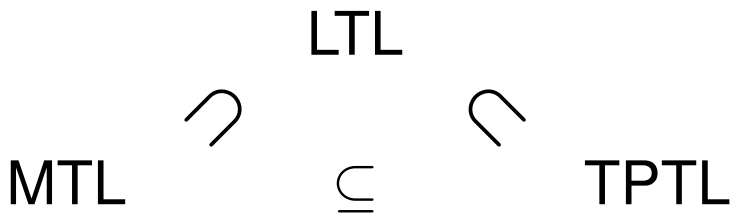
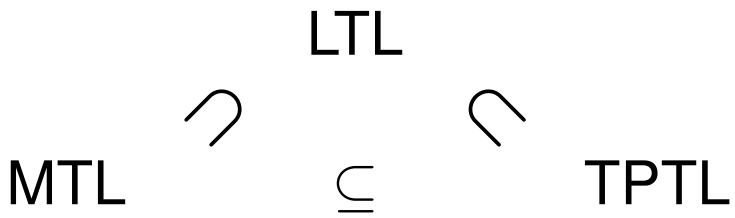
- $\varphi = x.[(\text{cloudy} \wedge x \in [-6,-6])\,\mathsf{U}\,(\text{shower} \wedge x \in [-6,-6])]$
- Is $\varphi$ expressible in MTL?
- Do MTL and TPTL have the same expressive power?


LTL
MTL
⊆
TPTL




- MTL $\equiv$ TPTL on monotonic data words.

[Alur&Henzinger, 1993]

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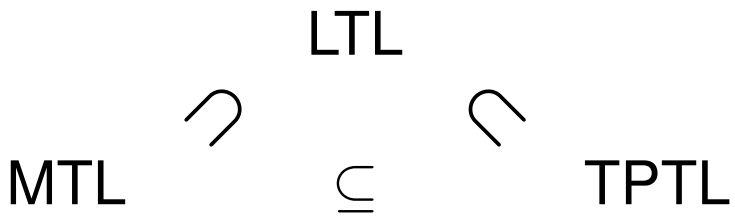


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  - $\mathsf{X}(p \mathsf{U}_{[1,2]} q) \in \mathrm{MTL}_2^{\{1,2\}}$

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Played by: Spoiler • and Duplicator • on $w_0$ and $w_1$.

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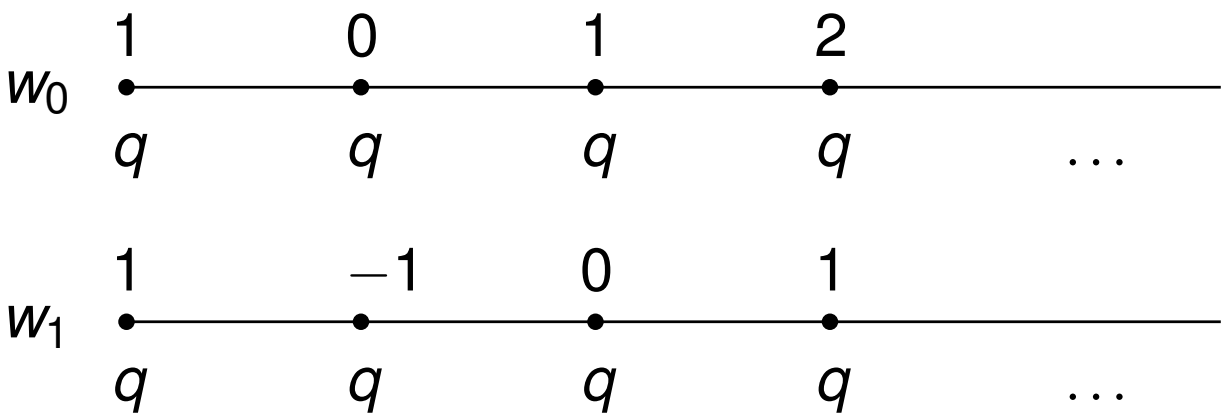

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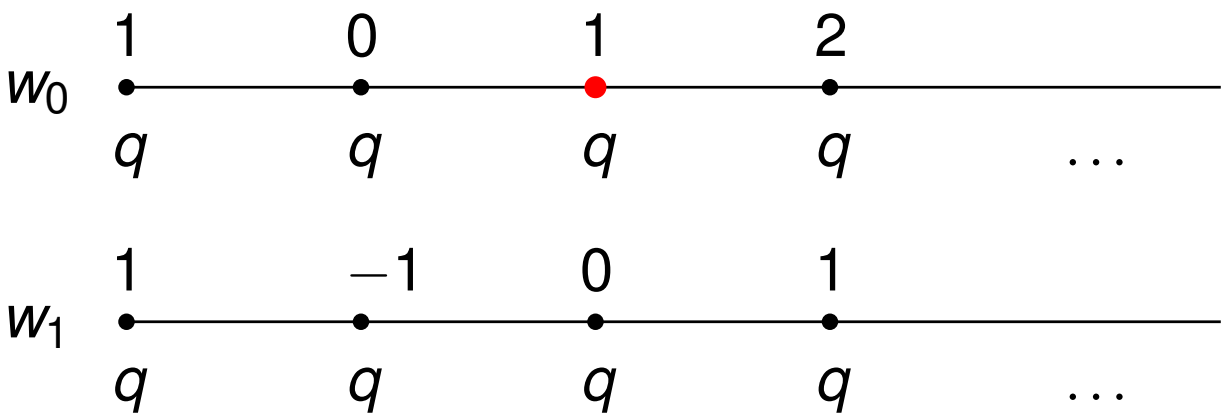

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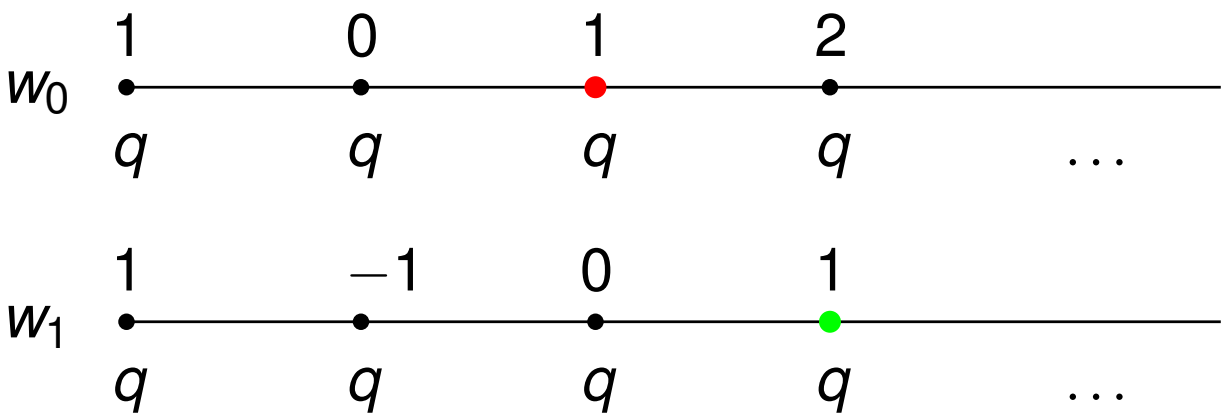

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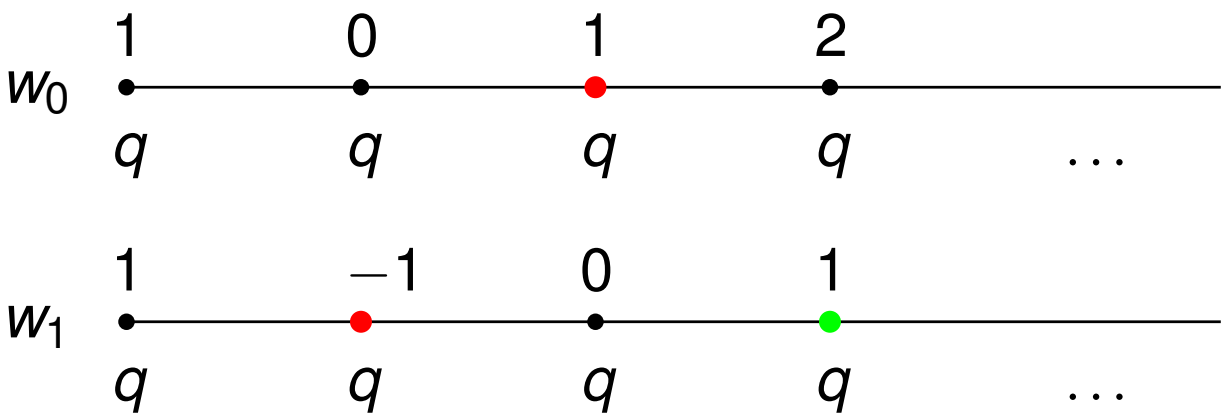

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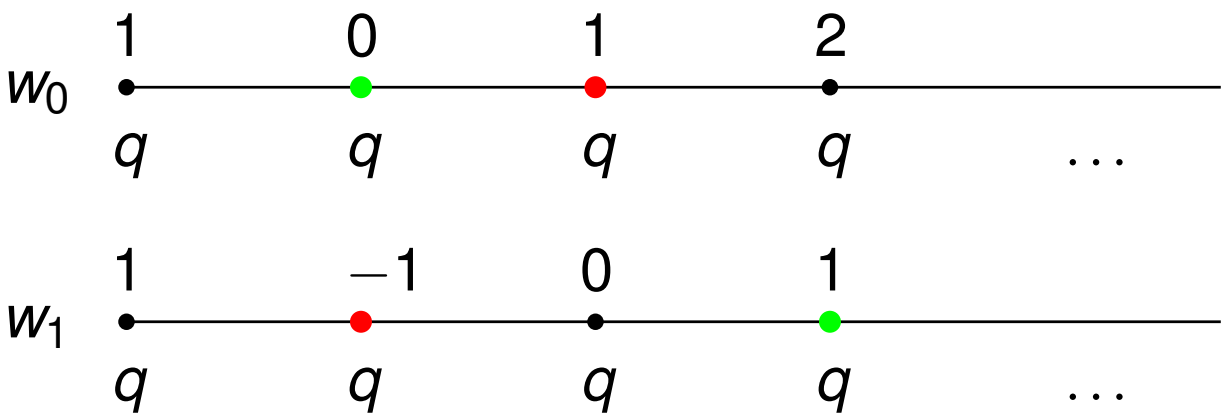

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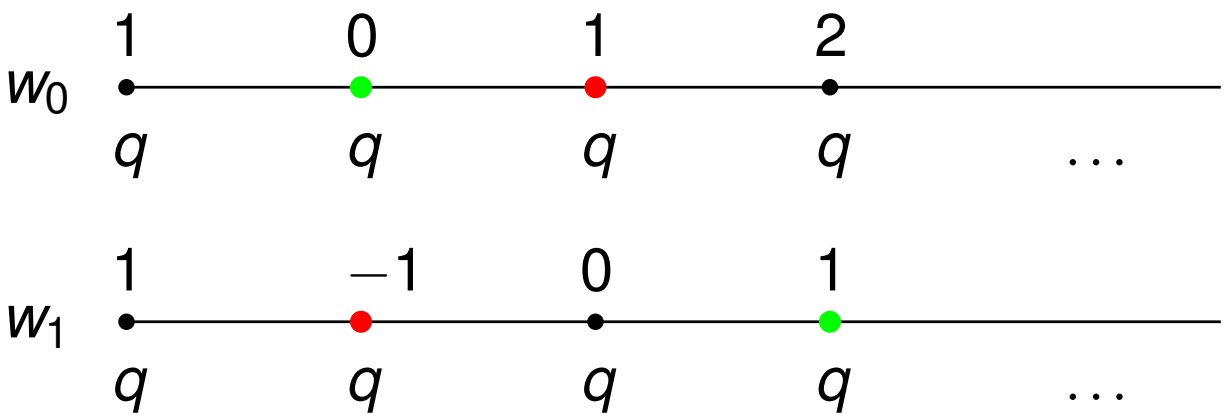


The following are equivalent:

- Duplicator wins the first round on $w_0$ and $w_1$.
- $w_0$ and $w_1$ satisfy the same formulas in $\mathrm{MTL}_1^{\{0,1\}}$.

# Further Applications of the EF Game

- Theorem: The following MTL membership decision problem is undecidable:

  GIVEN: $\varphi \in \text{TPTL}$

  QUESTION: Is $\varphi$ equivalent to an MTL formula?

- Theorem: $\text{MTL} \subsetneq \text{TPTL}^1 \subsetneq \text{TPTL}^2$.

  $\text{TPTL}^k$: at most $k$ many different register variables are allowed in a formula.

# What is the satisfiability?

Satisfiability Problem (SAT)

INPUT: a formula $\varphi$

QUESTION: Is there a data word $w$ s.t. $w \models \varphi$?

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Infinitary SAT of TPTL is undecidable ($\Sigma^1_1$-complete).

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Also holds for the fragments of MTL and TPTL[1] where

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Proof idea:

- Finitary SAT: reduction from the halting problem of nondeterministic 2-counter machines.
- Infinitary SAT: reduction from the recurring problem of nondeterministic 2-counter machines.

Theorem: Infinitary SAT and finitary SAT coincide for positive TPTL and MTL, the SAT problem is $\Sigma_1^0$-complete.

"Positive": negation $\neg$ only applied to propositions.

$$p \,\mathsf{U}\, \neg q \text{ (positive)} \qquad \neg(p \,\mathsf{U}\, q) \text{ (not positive)}$$

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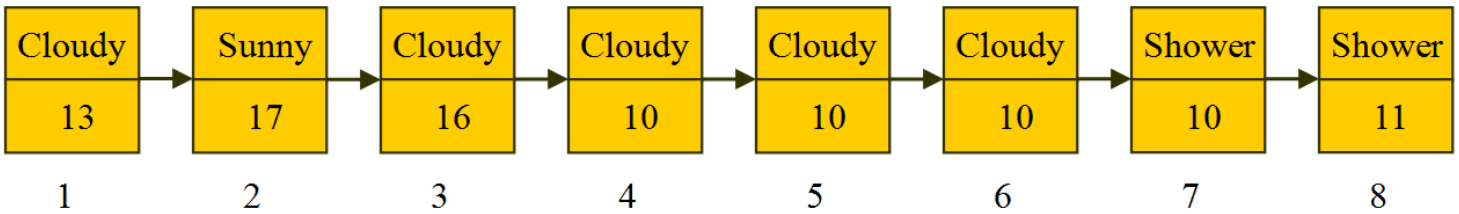
Theorem: SAT of positive $\mathrm{TPTL}_{\{\mathsf{F},\mathsf{X}\}}$ is NP-complete.

$\mathrm{TPTL}_{\{\mathsf{F},\mathsf{X}\}}$: the fragment of TPTL where only $\mathsf{F}, \mathsf{X}$ are allowed.

# What is path checking?



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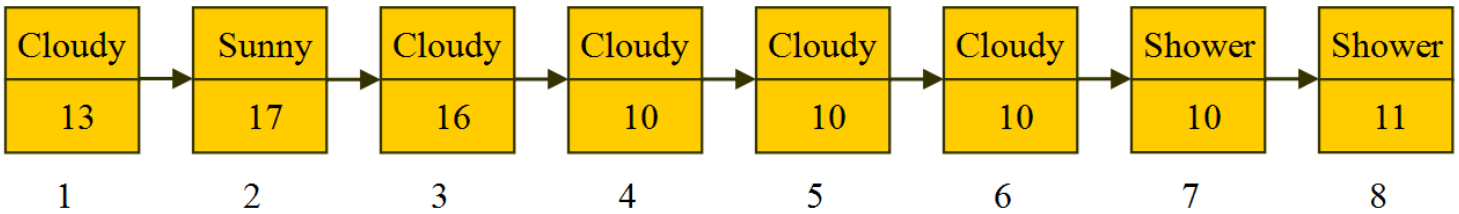


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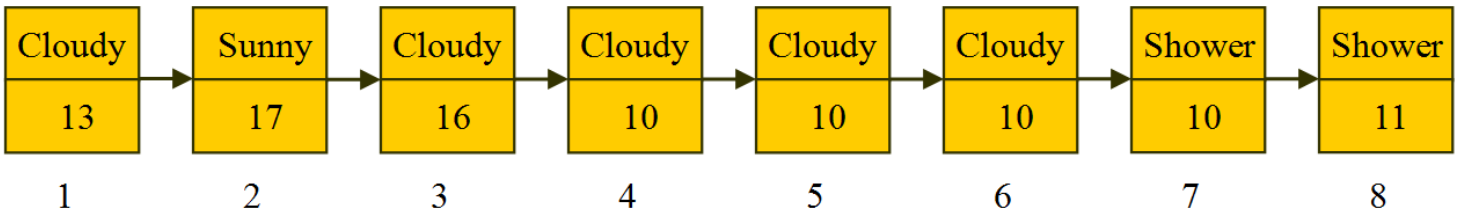


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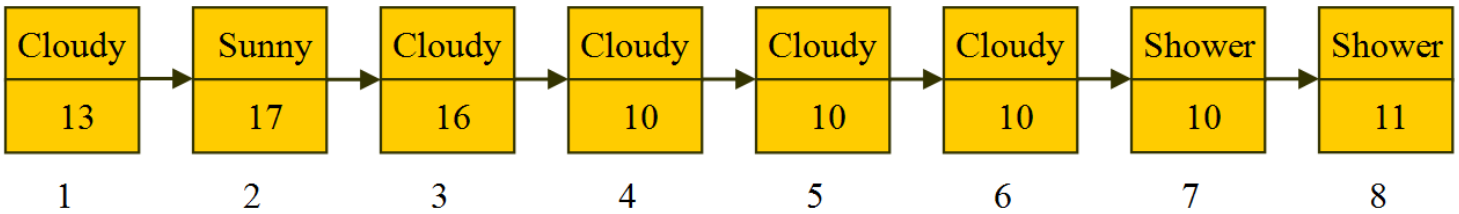


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Path checking Problem

INPUT: data word $w$ and formula $\varphi$

QUESTION: whether $w \models \varphi$?

- Path checking for Freeze LTL over deterministic one-counter machine is PSPACE-complete.

[Demri, Lazic & Sangnier, 2008]

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- Path checking for MTL and TPTL over ***all*** data words?

**Theorem**:
Path checking for MTL and TPTL[1] are P-complete.

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Proof idea:

Upper bound: Path checking for TPTL$^1$ is in P.

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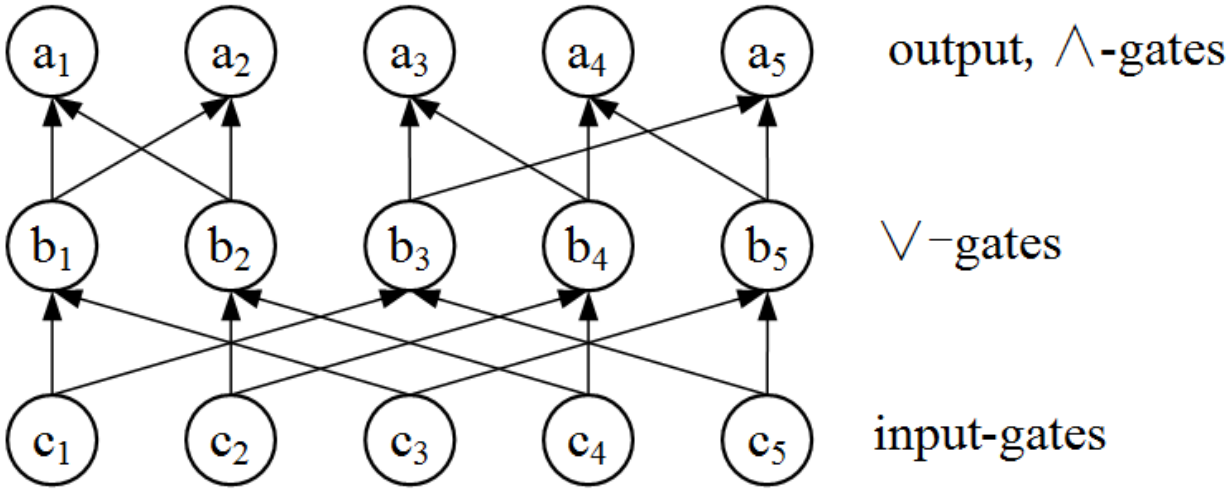
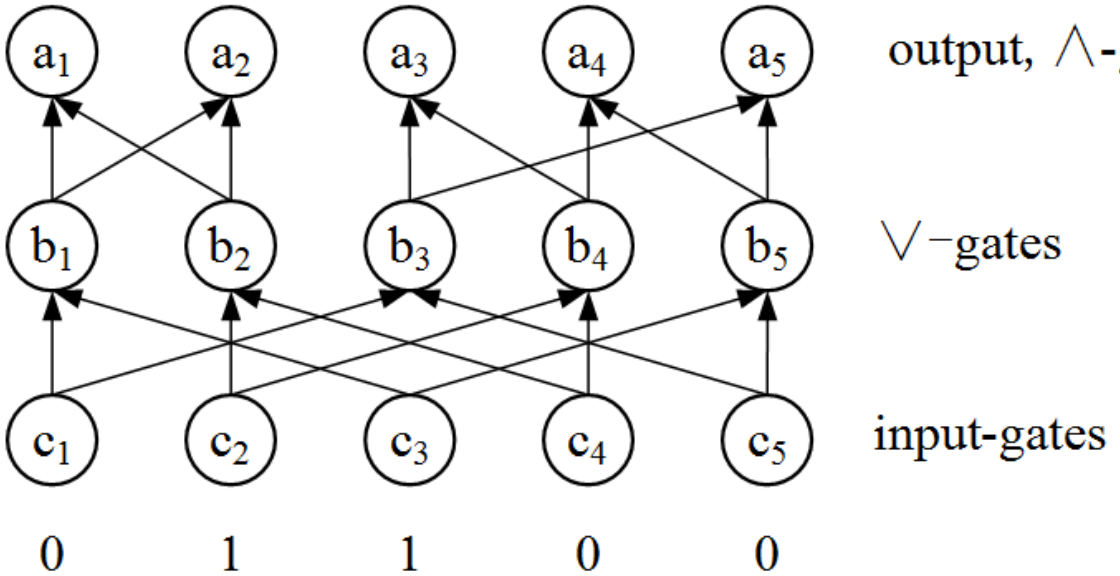
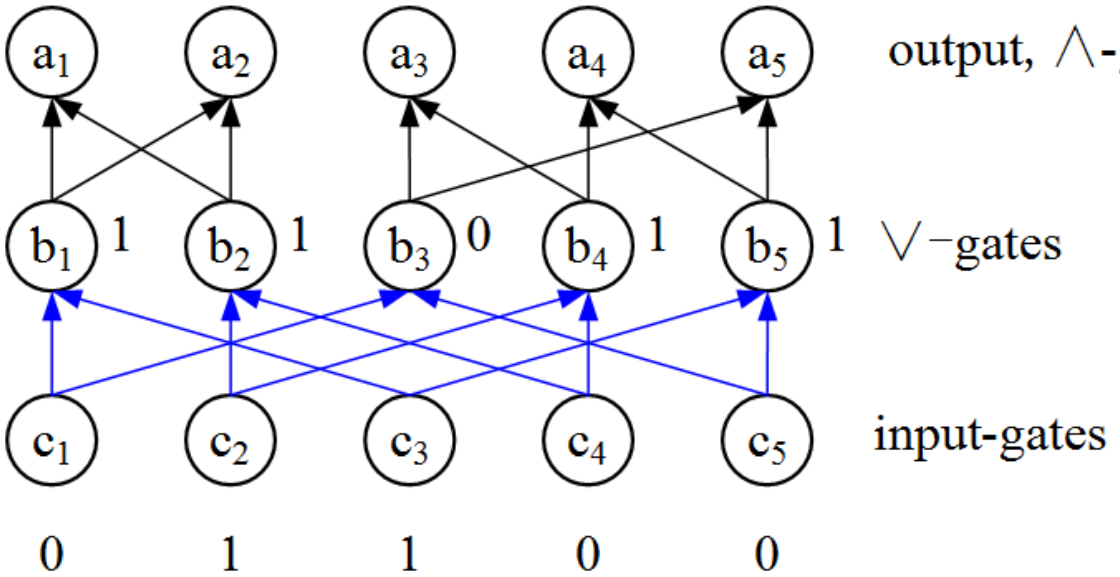
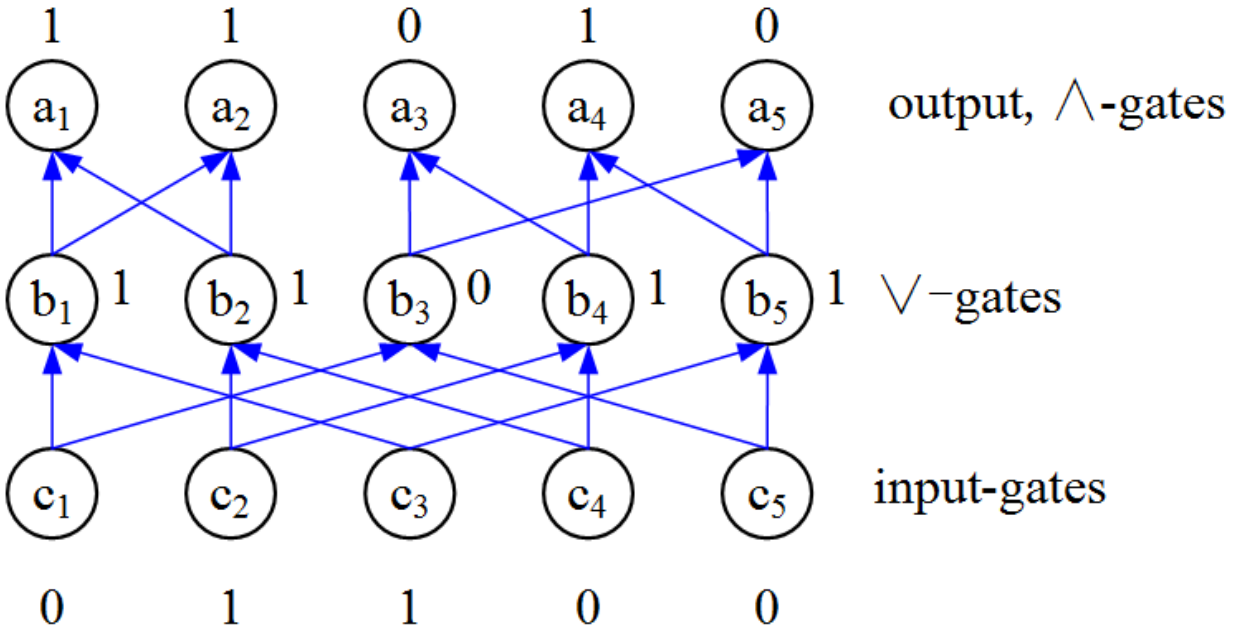
Lower bound: Path checking for MTL is P-hard.
Use a reduction from the circuit value problem for SAM2-circuits, which is P-complete.


$a_1$ $a_2$ $a_3$ $a_4$ $a_5$
$b_1$ $b_2$ $b_3$ $b_4$ $b_5$
$c_1$ $c_2$ $c_3$ $c_4$ $c_5$
output, $\bigwedge$-gates
$\bigvee$−gates
input-gates


a1
a2
a3
a4
a5
b1
b2
b3
b4
b5
c1
c2
c3
c4
c5
0
1
1
0
0
output, ∧-gates
∨−gates
input-gates


a1
a2
a3
a4
a5
output, ∧-gates
b1
1
b2
1
b3
0
b4
1
b5
1
∨−gates
c1
c2
c3
c4
c5
input-gates
0
1
1
0
0


1 1 0 1 0
a1 a2 a3 a4 a5
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c1 c2 c3 c4 c5
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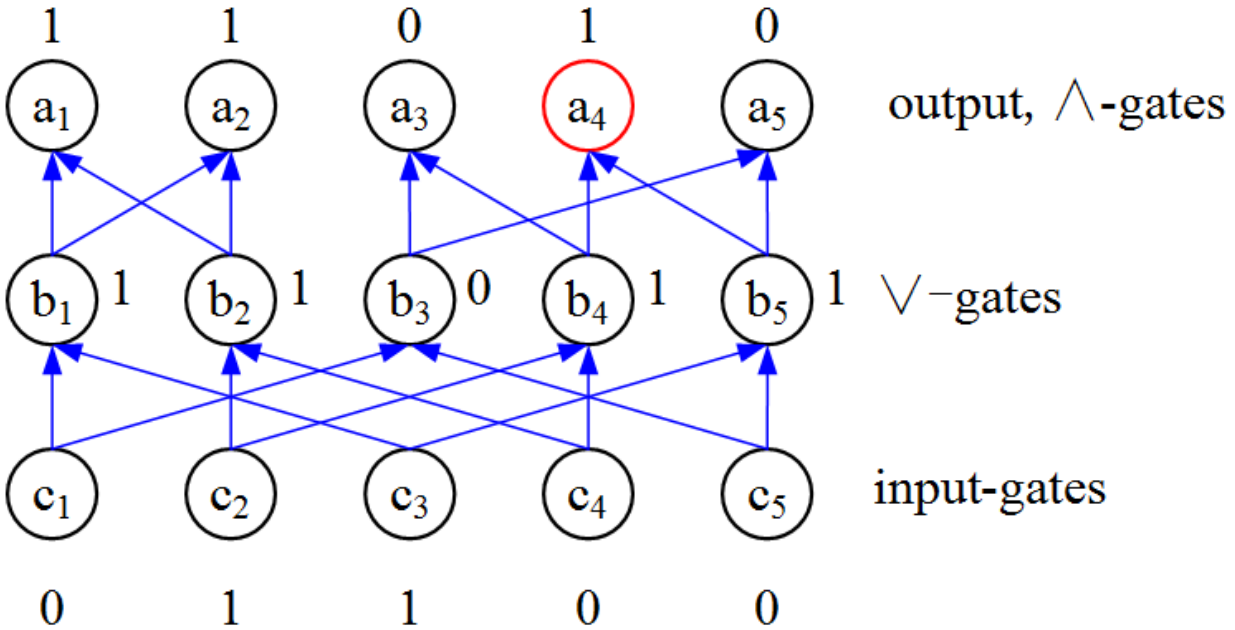
0 1 1 0 0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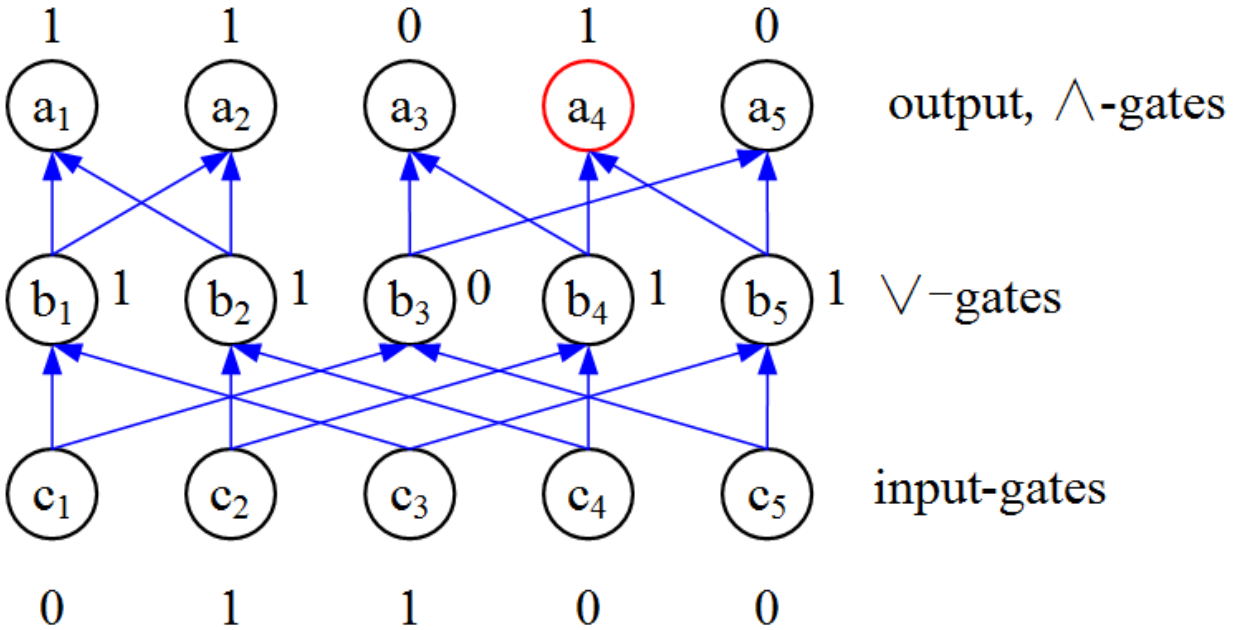


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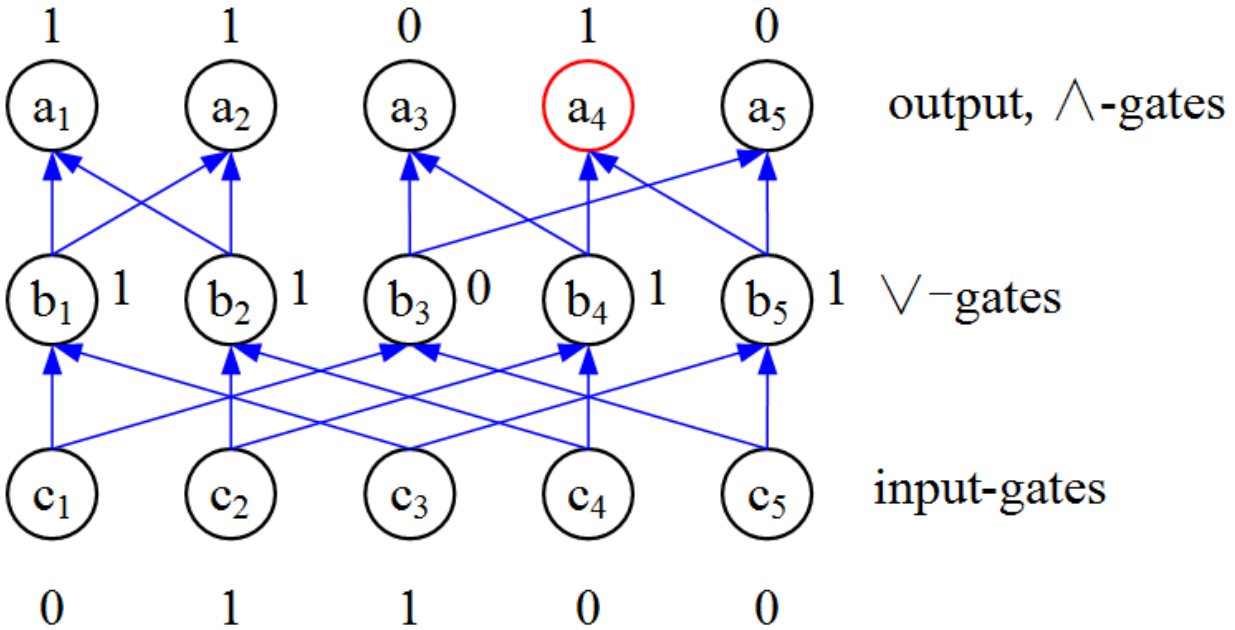
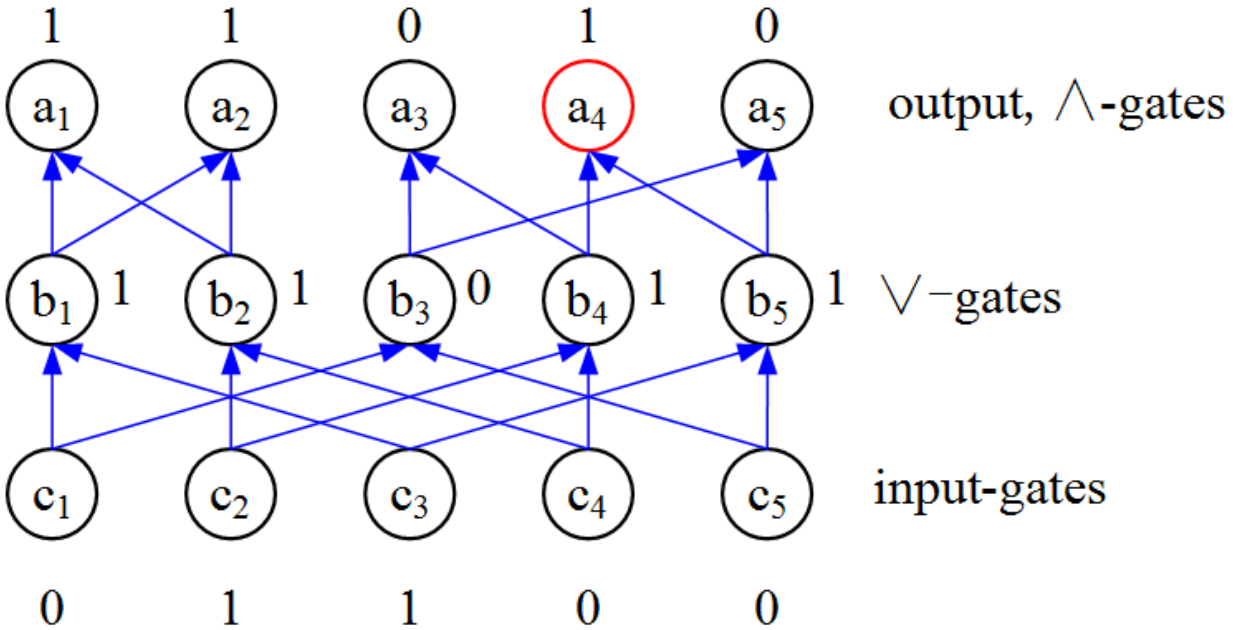



- $w \models \varphi$ if and only if the output of $a_4$ is 1.
- It is essential that $w$ is ***not*** monotonic.




- $w \models \varphi$ if and only if the output of $a_4$ is 1.
- It is essential that $w$ is ***not*** monotonic.
- If $\varphi$ is a TPTL$^1$ formula, then $w$ can be monotonic.

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We use an alternating polynomial time algorithm (APTIME) to decide this problem, and APTIME=PSPACE.

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  - Path checking for deterministic 1-counter machines.

# Future work

- $\mathrm{TPTL}^1 \subsetneq \mathrm{TPTL}^2 \subsetneq \mathrm{TPTL}^3 \subsetneq \mathrm{TPTL}^4 \cdots$ is strict?
- Some decidable fragment of MTL and TPTL (e.g., is $\mathrm{MTL}_{\{F\}}$ decidable)?
- Over multi-data word, continuous semantics, ...