Lab 10: DC Resistivity

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Overview

DC resistivity surveys are widely used in site-characterization, mineral exploration and geotechnical studies. The purpose of this lab is three-fold:

- Provide an understanding about how currents flow in the ground, the electrical potentials that would be measured on the surface, and the secondary fields that result because of charges that occur on a buried structure
- Show how potentials can be converted into an apparent resistivity and how these data can be plotted as a pseudosection
- Provide an opportunity to interpret data on a pseudosection by using an interactive app.

Instructions

- Use the Jupyter notebook to help you answer the questions in this lab

Resources

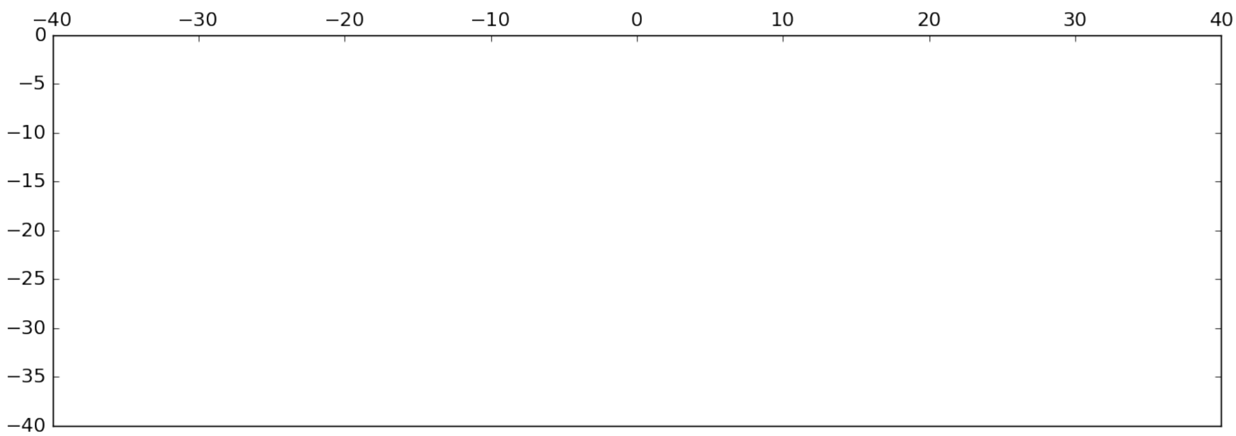
- GPG:DC Resistivity
- DC Binder

1 Understanding currents, fields, charges, and potentials

Use the cylinder model app to develop a basic understanding of currents, potentials, primary fields, secondary fields, and charges.

Q1. Set the cylinder and background resistivities to $\sim$ 50 ohm-m (ensuring that the cylinder and background conductivities are the same) so that we are working with a homogeneous halfspace. Place the A and B current electrodes at -30m and 30m so that current is being injected at -30m and taken out at 30m.

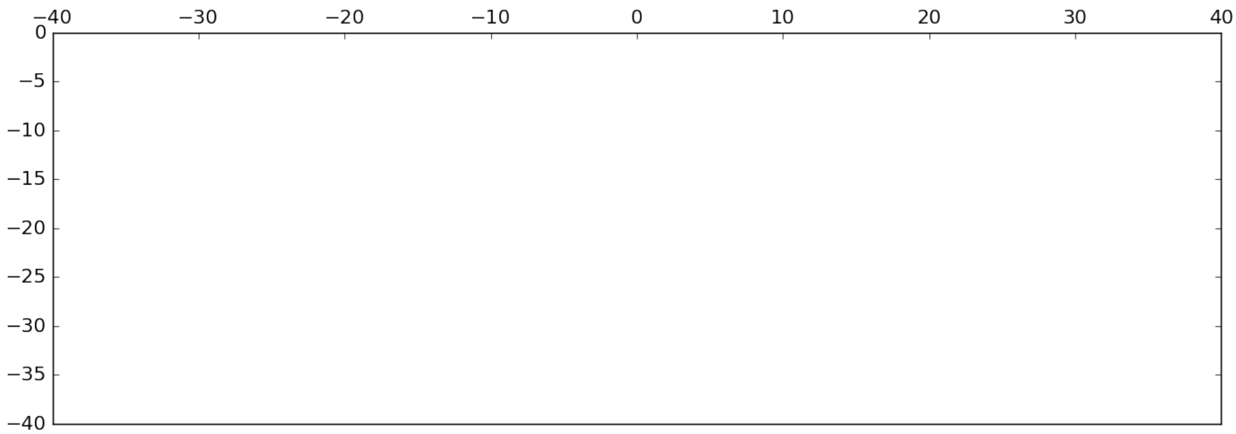
a. Bring up the image that shows the Primary Potentials: The potentials are in volts and the scale is linear. In the space below provide a labelled sketch of the lines of equi-potential. In your sketch, include the locations of the A and B electrodes.



Now use the app to visualize the electric field. The units for electric field is V/m and the scale is $\log_{10}$.

b. The electric field is the negative gradient of the potential. On your diagram above, sketch arrows that show the direction of the electric field. Are the electric fields near the current electrodes larger or smaller than the electric fields at depth? Why?

c. The current density is connected to the electric field via Ohm's Law $\mathbf{J} = \sigma \mathbf{E}$. In the space below, sketch the flow lines of the current as it travels from the A to the B electrode.



Now use the app to visualize the current density. The units for Current Density are A/m^2 and the scale is $\log_{10}$. How does your sketch match with the computed image?

The effects of buried conductive or resistive cylinders

Conductive Cylinder

Change the resistivity of the cylinder to $\sim$ 10 ohm-m so that it is more conductive than the background.

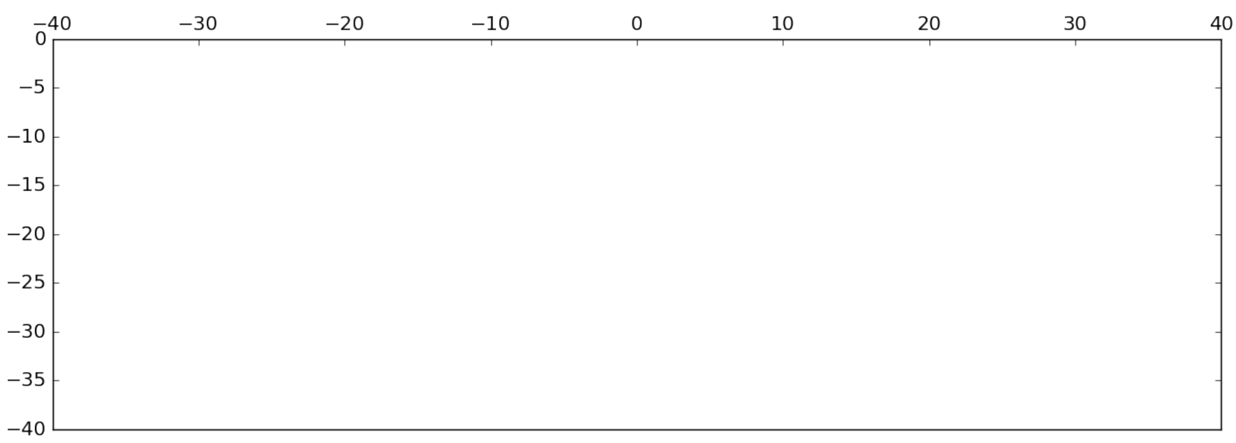
Q2. How has the total potential changed compared to the halfspace? The effects are subtle but see what you can infer (careful: the colorbar changes).

Q3. View the current density. What are the main changes compared to the halfspace? Does the current appear to be channeled into the cylinder or flowing around it?

Q4. Look carefully at the boundary of the cylinder. Are the currents on the outside of the cylinder flowing in the same direction as the currents inside? Any change in direction is attributed to charges that are built up on the boundary. Remember that those charges are needed to keep the normal component of the current continuous and they arise whenever there is a change in conductivity across a boundary. Use the app to visualize the charge density. Which side has negative charges and which has positive?

Q5. As a first approximation the charges look like an electric dipole. (Remember GPR) Suppose you had two charges [+Q, -Q] separated by distance equal to the radius of the cylinder.

> **a.** In the plot below sketch the electric potential you would expect to see. (Hint: This will be a contour diagram. To help with the construction, first think about the potential for an individual charge). This is called the secondary electric potential. It is associated only with the body. The total potential you measure is the sum of the primary and secondary potentials (Note that in the app: the button labeled "Primary" should be labeled "Total").
>
> **b.** The electric field is the negative gradient of the potential. Add the electric field lines for your dipole to the plot. Because of Ohm's law, the currents will follow these field lines. Compare your sketch with what you observe in the app.



Q6. Having obtained the charge density, we can use Coulomb's law to compute the potential anywhere (see the GPG: Charge Distribution). Suppose you are at the location x=-30m. Would you expect the total electric potential to be higher or lower than the potential you would have measured if the earth had been a halfspace? What about x=30m? Defend your answer.

Resistive Cylinder

Now change the resistivity of the cylinder to $\sim 10^3$ ohm-m so that it is more resistive than the background.

Q7. Using the app, describe how the presence of a resistive cylinder changes the distribution of total potentials.

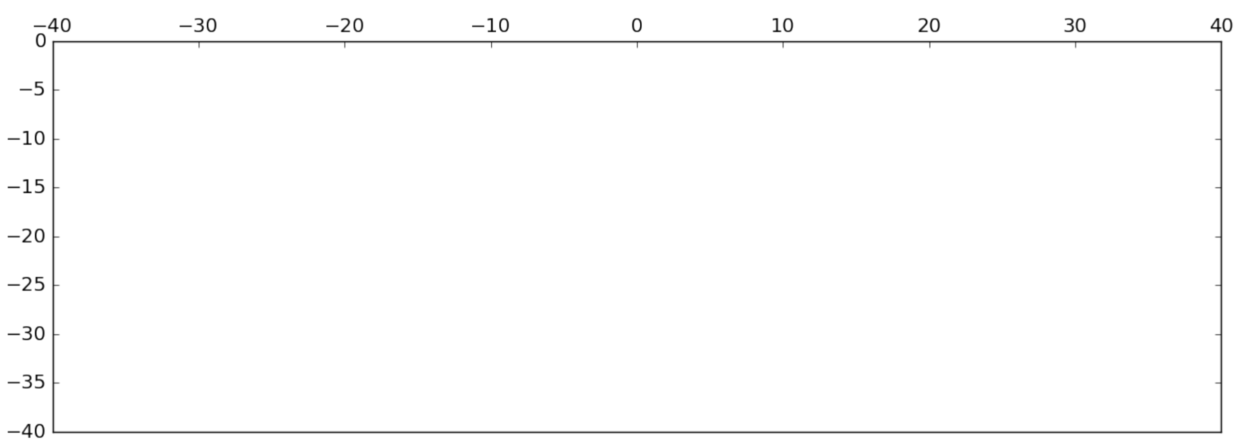
Q8. Describe how the total currents interact with the resistive cylinder.

Q9. Briefly describe the observed patterns in charge density. Where are the positive and negative charges accumulating?

Q10. As in Question 5, a first approximation the charges look like an electric dipole. Suppose you had two charges [+Q, -Q] separated by distance equal to the radius of the cylinder.

a. In the plot below sketch the electric potential you would expect to see. This is called the secondary electric potential. It is associated only with the body. The total potential you measure is the sum of the primary and secondary potentials (Note that in the app: the button labeled "Primary" should be labeled "Total").

b. The electric field is the negative gradient of the potential. Add the electric field lines for your dipole to the plot. Because of Ohm's law, the currents will follow these field lines. Compare your sketch with what you observe in the app.



Q11. Suppose you are at the location $x = -30$m. Would you expect the total electric potential to be higher or lower than the potential you would have measured if the earth had been a halfspace? What about $x = 30$m? Defend your answer.

2 Potential Differences and Apparent Resistivities

Having used the first app to gain a general understanding of the physics behind a simple DC resistivity survey we will now understand what values are actually measured in the field and how these measurements can be processed, plotted, inverted, and interpreted. The physical principles and intuition are best built up by looking at a layered earth. It suffices to consider a 2-layered earth in which the thickness of a top layer and resistivities of the top layer and underlying halfspace are variable.

The widget in the notebook displays the surface potentials for a scenario much like the one you investigated with the cylinder model where we had a cylinder embedded in a halfspace of differing conductivity. In practice we cannot measure the potentials everywhere, we are limited to those locations where we place potential electrodes. For each source (current electrode pair A,B) many potential differences are measure between different M and N electrodes to characterize the overall distribution of potential differences.

Q12. Start with a 500 Ωm uniform halfspace and make some choices for locations of A, B, M, N. Record the electrode locations and the measured potential differences for each in table 1. Also, note the characteristics of the surface potential plot and where your M and N electrodes are with respect to A and B. Choose what you consider to be very different geometries (for instance having both M and N on one side of the current dipoles.)

A	B	M	N	ΔV_{MN}

Table 1: Potential difference measurements over a uniform halfspace.

The potential (V) due to a positive monopole current at the surface of a homogeneous halfspace of resistivity (ρ) is given by the equation,

$$V = \frac{\rho I}{2\pi r} \tag{1}$$

where I is the current and r is the distance between the source and measurement location.

For dipole current sources with A (+I, current source) and B (-I, current sink) electrodes the potential at any point, P, is equal to the sum of the potentials from each current electrode, $V_P = V_A + V_B$. The measured potentials at the M and N potential electrodes can then be defined in the following manner:

$$V_M = \frac{\rho I}{2\pi}\left[\frac{1}{AM} - \frac{1}{MB}\right] \tag{2}$$

$$V_N = \frac{\rho I}{2\pi}\left[\frac{1}{AN} - \frac{1}{NB}\right] \tag{3}$$

where AM, MB, AN, and NB are the distances between the corresponding electrodes.

The potential difference ΔV_{MN} in a dipole-dipole survey can therefore be expressed as follows,

$$\Delta V_{MN} = V_M - V_N = \rho I \underbrace{\frac{1}{2\pi}\left[\frac{1}{AM} - \frac{1}{MB} - \frac{1}{AN} + \frac{1}{NB}\right]}_{G} \tag{4}$$

and the resistivity of the halfspace ρ is equal to,

$$\rho = \frac{\Delta V_{MN}}{IG} \tag{5}$$

In this equation G is often referred to as the geometric factor. Although the ground is not typically a homogeneous halfspace, Eq. 5 is still used to compute an apparent resistivity (ρ_a). The apparent resistivity is the resistivity of a uniform halfspace which best reproduces the measured potential difference. In the apparent resistivity formula the current I is the same as that used in the field survey. In a pole-dipole or dipole-pole type survey the B or N electrodes are placed very far away (mathematically $\to \infty$) so that they do not influence the current or potential distribution. In doing so the components of the geometric factor are simplified since any factors which are associated with the pole electrode go to 0. More information can be found in the DC resistivity portion of the GPG (Measurements: potential difference).

Q13. Compute the apparent resistivities (ρ_a) for the ΔV_{MN} measurements recorded in table 1. Fill in table 2. In the modelling we assume the a unit current of 1 A is injected. How do these ρ_a values compare with one another and how do they compare with the true halfspace resistivity?

AM	MB	AN	NB	ΔV_{MN}	ρ_a

Table 2: Calculated apparent resistivities for uniform halfspace.

Q14. Now change the earth model to have a 5m thick upper layer with a resistivity of 100 Ohm-m. Let the lower layer have a resistivity of 1000 Ohm-m.

a. Look at the current densities. What is the major difference between these current densities and those you had for a halfspace. Where are the majority of currents travelling?

b. Specify electrode locations (A,B,M,N) to be (-25, 25, -10, 10) m. Record the potentials at M and N (they are shown on the app), and use the equations above to compute G, the geometric factor, ΔV_{MN} and ρ_a. Fill in table 3. Is your value the same as that provided in the app (See upper right hand corner of the figure)? How does your apparent resistivity value compare to the true resistivities of the layered earth model?

V_M	V_N	G	ΔV_{MN}	ρ_a

Table 3: Measurements from the layered Earth model with (A,B,M,N) electrodes placed at (-25, 25, -10, 10) m.

c. As you did for the uniform halfspace select several different locations for A,B,M,N to make potential measurements and record your results in table 4. Use the same electrode locations as you did in table 1 so that it is easy to compare with the halfspace values.

A	B	M	N	ΔV_{MN}	ρ_a

Table 4: Surface measurements for a 2-layered earth model.

d. Sounding Mode. Figure 1 shows the electrode layout for a Wenner Sounding. The basic principle is that the depth of investigation is related to the scale size. When "a" is small the survey is sensitive to the near surface resistivity, but information about greater depths can be obtained by increasing "a". A Wenner Sounding can be carried out by fixing the centre of the array at one location and expanding about that point. Fill in table 5 using "a" values $a = (2, 4, 8, 16, 26)$ centred about 0m and sketch how the apparent resistivities vary as a function of "a". Can you explain why the results are, or not, reasonable?

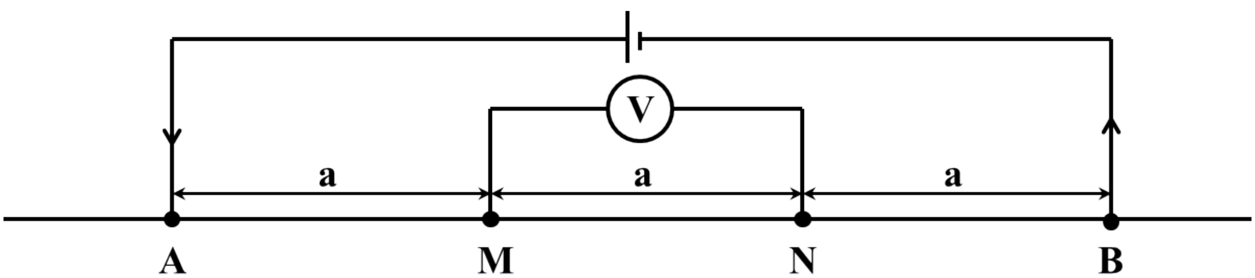


Figure 1: Skematic of a Wenner array.

a	A	B	M	N	ΔV_{MN}	ρ_a
2						
4						
8						
16						
26						

Table 5: Wenner sounding measurements.

3 Pseudo-Sections

Plotting DC resistivity data in a meaningful way is not as simple as plotting many other types of data since many electrode locations are involved with each measurement. Varying electrode separations further complicates the situation since the depth of investigation is tied to the separation distance. To account for these factors 2D profiles are often plotted as pseudo-sections by extending 45° lines downwards from the A-B and M-N midpoints and plotting the corresponding ΔV_{MN} or ρ_a value at the intersection of these lines as shown in the pseudo-section portion of the notebook. For pole-dipole or dipole-pole surveys the 45° line is simply extended from the location of the pole. By using this method of plotting, the long offset electrodes plot deeper than those with short offsets. This provides a rough idea of the region sampled by each data point, but the vertical axis of a pseudo-section is not a true depth.

Using the slider in the pseudo-section portion of the notebook step through the different transmitters to see how the points in the pseudo-section are built up. While pseudo-sections are often useful for identifying outliers or noisy data they should never be directly interpreted since the shape and location of anomalies can be highly distorted based on the survey geometry.

Q15. Using the pseudo-section portions of the notebook create pseudo-sections of the apparent resistivities for dipole-dipole, pole-dipole, and dipole-pole surveys over the conductive cylinder. Describe some of the differences between each of ρ_a pseudo-sections. How well do the locations of the conductive anomalies in the pseudo-sections correlate with the true location of the conductive cylinder?

4 Parametric Inversion

Using the final portion of the notebook you will now preform a simple parametric inversion to recover the size, location, and conductivity of the cylinder which best fits the observed data. Instead of using a numerical optimization algorithm to find the best fitting model you will preform this inversion by hand. Pseudo-sections of the observed and predicted apparent resistivities along with the normalized data misfit are provided to help you assess how well a given model fits the observed data.

Q16. Slowly change each of the model parameters to get a feel for how they change the predicted data. Describe your procedure and what criteria you used for deciding upon a "best fitting" solution.

a. Once you have found your "preferred"? solution, change each of the parameters to get some idea of how well constrained each parameter is. Record the range of values for each parameter that still provides an acceptable misfit in table 6.

Parameter	Range
rhohalf	
rhosph	
xc	
zc	
r	

Table 6: Range of suitable inversion parameters.

b. In many inverse problems a fundamental non-uniqueness exists in which there are many possible models which fit the data. In these situations additional constraints are needed to help determine which solution is optimal or the most geologically reasonable. A common example of this non-uniqueness which often occurs is a trade-off between the resistivity contrast of the body and its size/volume. Do you think that this trade-off holds in this case?