Formula Sheet

Constants

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \qquad \varepsilon_0 = 8.85 \times 10^{-12} F/m \qquad c = 3 \times 10^8 \text{ m/s}$$

Magnetics

magnetization	$\mathbf{M} = \kappa \mathbf{H}$
magnetic permeability	$\mathbf{B} = \mu \mathbf{H} = \mu_0 (1 + \kappa) \mathbf{H}$
dipole moment	$\mathbf{m} = \mathbf{M} \times Vol.$
depth half-width relationship	monopole: $z \sim \frac{1}{2} x_{1/2}$, dipole: $z \sim x_{1/2}$
magnetic charge density	$\tau = \vec{M} \cdot \hat{n}$
magnetic field from a dipole	$\mathbf{B} = \dfrac{\mu_0 \mathbf{m}}{4\pi r^3} [2cos(\theta)\hat{\mathbf{r}} + sin(\theta)\hat{\boldsymbol{\theta}}]$

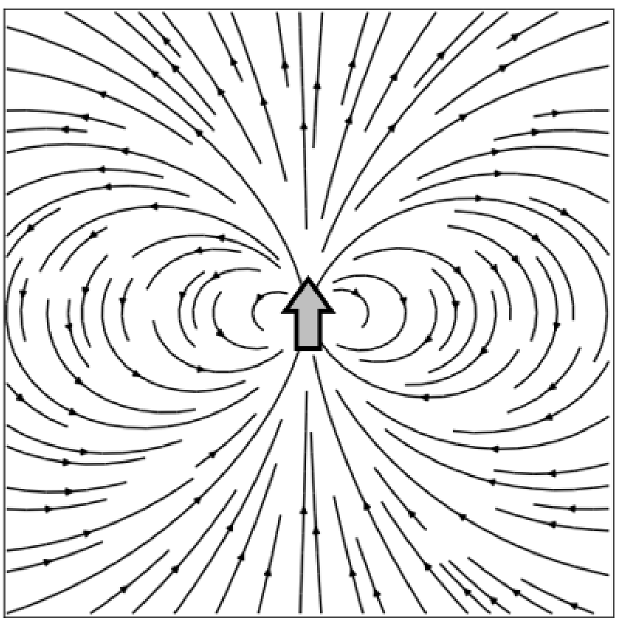

Figure 1: Magnetic field due to vertical magnetic dipole

Seismic

velocities	$v_p = \sqrt{\dfrac{K + 4/3\mu}{\rho}}$ $\qquad v_s = \sqrt{\dfrac{\mu}{\rho}}$
general	$Z = \rho v$ $\qquad R = \dfrac{Z_2 - Z_1}{Z_2 + Z_1}$ $\qquad T = \dfrac{2Z_1}{Z_2 + Z_1}$
general	$d = vt$ $\qquad \lambda = vT = \dfrac{v}{f}$
Vertical resolution	$L = \dfrac{\lambda}{4}$
Refraction arrivals	$t = \dfrac{x}{v_2} + 2z\dfrac{\sqrt{v_2^2 - v_1^2}}{v_1 v_2} = \dfrac{x}{v_2} + t_i$
Cross-over distance	$x_{cross} = \left(\dfrac{v_1 v_2}{v_2 - v_1}\right) t_i = 2z\sqrt{\dfrac{v_2 + v_1}{v_2 - v_1}}$
Refraction Angles	$\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$
Refraction Angles (for three layers)	$\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2} = \dfrac{\sin\theta_3}{v_3}$
Reflection hyperbola	$t(x)^2 = t_0^2 + \dfrac{x^2}{v^2}$ $\qquad$ x=distance from Tx to Rx

GPR

Reflection coefficient:	$R = \dfrac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$
Transmission coefficient:	$T = \dfrac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$
Pulse length (Δt) and central frequency (f_c) of wavelet:	$\Delta t = \dfrac{1}{f_c}$
GPR signal velocity:	$v \approx \dfrac{c}{\sqrt{\varepsilon_r}}$
GPR wavelength:	$\lambda = \dfrac{V}{f_c}$
Vertical resolution limit:	$L > \dfrac{\lambda}{4} = \dfrac{V}{4f_c}$
Horizontal resolution limit:	$L > \sqrt{\dfrac{Vd}{2f_c}}$
Refraction Angles	$\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2}$

Skin depth (quasi-static) $\delta = 503\sqrt{\frac{1}{\sigma f}}$

Skin depth (wave regime) $\delta = \frac{0.0053\sqrt{\varepsilon_r}}{\sigma}$

Velocity of light $c = 0.3m/ns$ or $3 \times 10^8 m/s$

DC

Resistance & resistivity $R = \frac{\rho L}{A}$

Electric Potential for a homogeneous earth:

Single Electrode $V = \frac{\rho_0 I}{2\pi r}$

Four Electrode Array $\Delta V = \frac{\rho_0 I}{2\pi}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)$

Apparent Resistivity:

Single Electrode $V = \frac{\rho_a I}{2\pi r}$

Four Electrode Array $\rho_a = \frac{2\pi\Delta V}{I}\left(\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right)^{-1}$

Wenner Array $\rho_a = \frac{2\pi a\Delta V}{I}$

IP

Intrinsic Chargeability $\eta = \frac{V_s}{V_0}$

IP datum (sample a chanel) $d^{IP} = \frac{V_s(t)}{V_0}$

EM

skin depth	$\delta = 500\sqrt{\frac{\rho}{f}}$
angular frequency	$\omega = 2\pi f$
apparent conductivity for EM31 ($s \ll \delta$)	$\sigma_a = \frac{4}{\omega \mu_0 s^2} Im\left(\frac{H_s}{H_p}\right)$
expansion of $H_s \cos(\omega t - \psi)$	$H_s \cos(\omega t - \psi) = H_s[\cos(\omega t)\cos(\psi) + \sin(\omega t)\sin(\psi)]$
phase lag	$\psi = \frac{\pi}{2} + tan^{-1}\left(\frac{\omega L}{R}\right)$
induction number	$\alpha = \frac{\omega L}{R}$

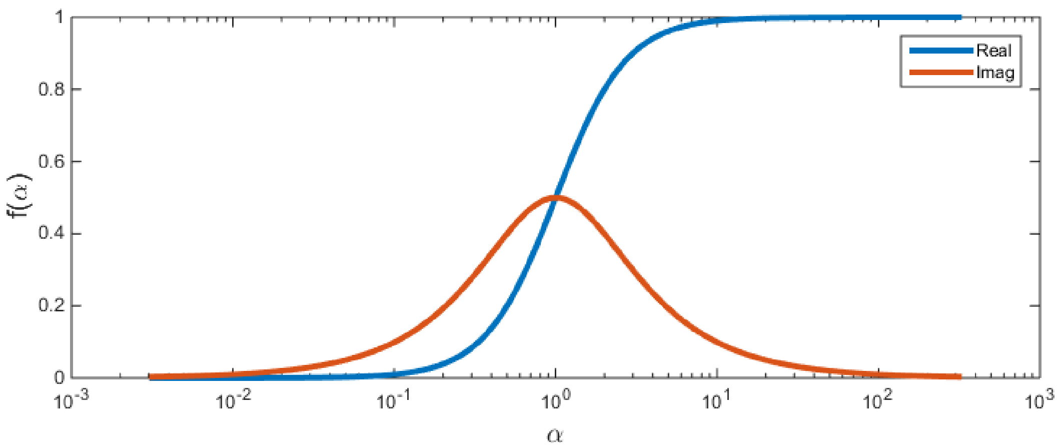

Figure 2: Frequency EM: Response function curve