DC Resistivity

Reading on the GPG:

https://gpg.geosci.xyz/content/DC_resistivity/index.html

Today’s Topics

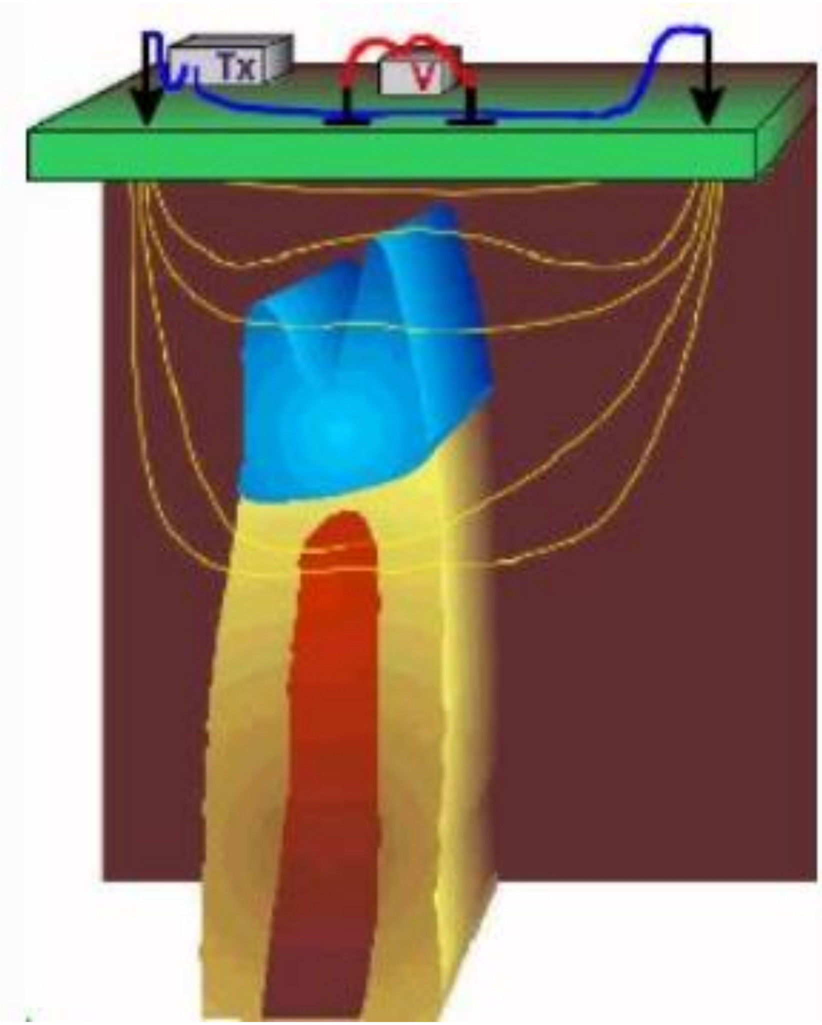
- Introduction to DCR
- Physical Properties: Electrical Conductivity
- Setup: Motivational Problems
- Fundamental Physics:
    - DCR and Ohm’s Law
    - Homogeneous Earth and Apparent Resistivity
    - Currents, Charges, Potentials
    - All together

Introduction to DCR

- DCR is an EM method
- Injects static current into the ground via electrodes
- Path of the current depends on:

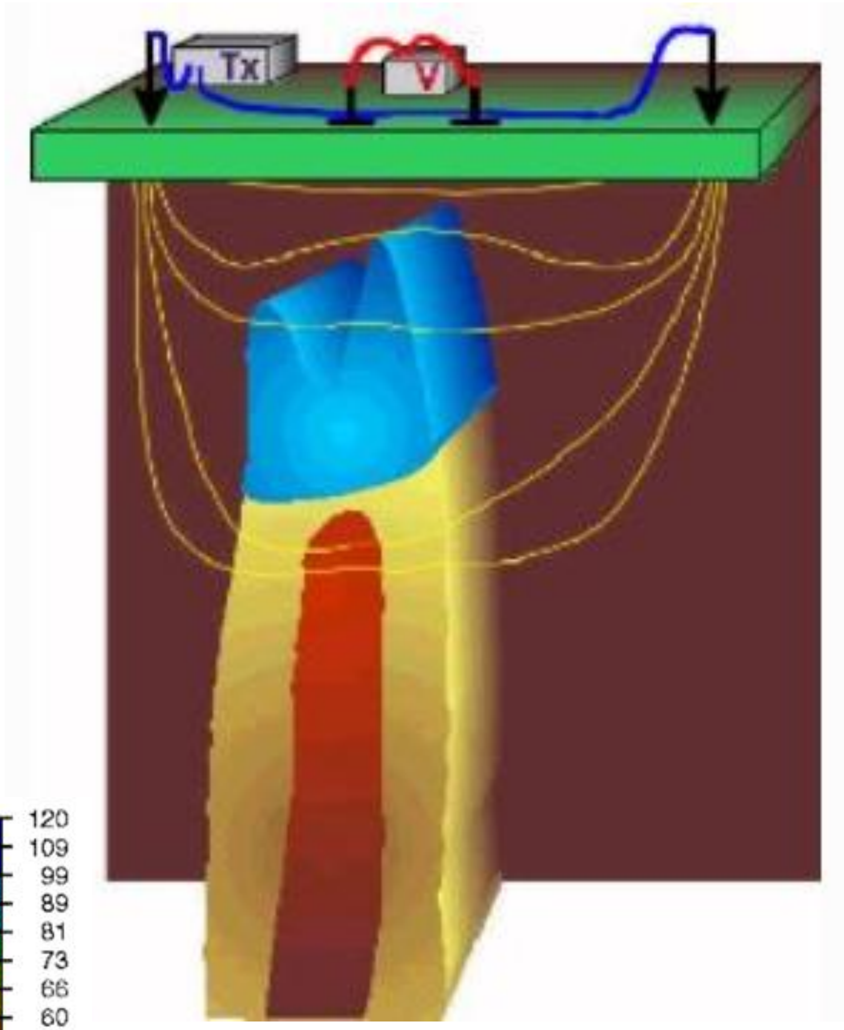
  1) Earth's conductivity (σ) or resistivity (ρ)

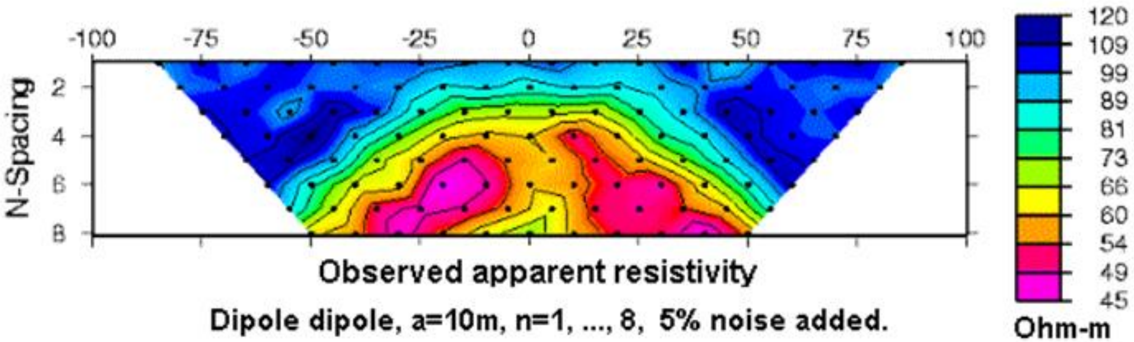
  2) Geometry of electrodes



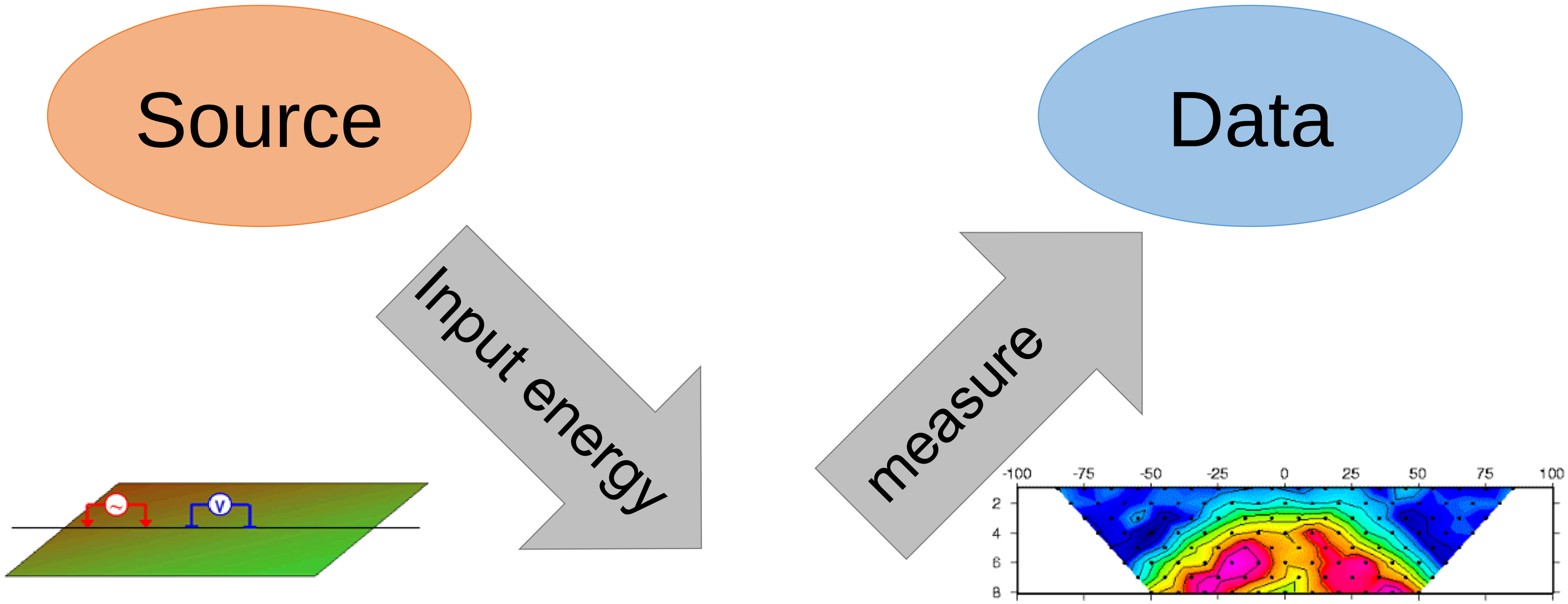
Introduction to DCR

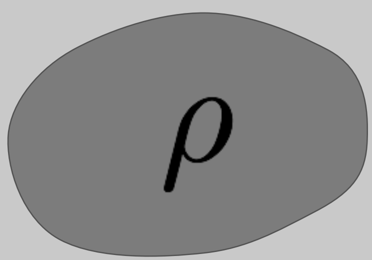
- Measures electric potential (ΔV) across different points via electrodes
- Data represented by a "pseudo-section"
- Infer Earth's conductivity (or resistivity) structure





Introduction to DCR





$$\rho = 1/\sigma$$

ρ : resistivity

σ : electrical conductivity

Physical property

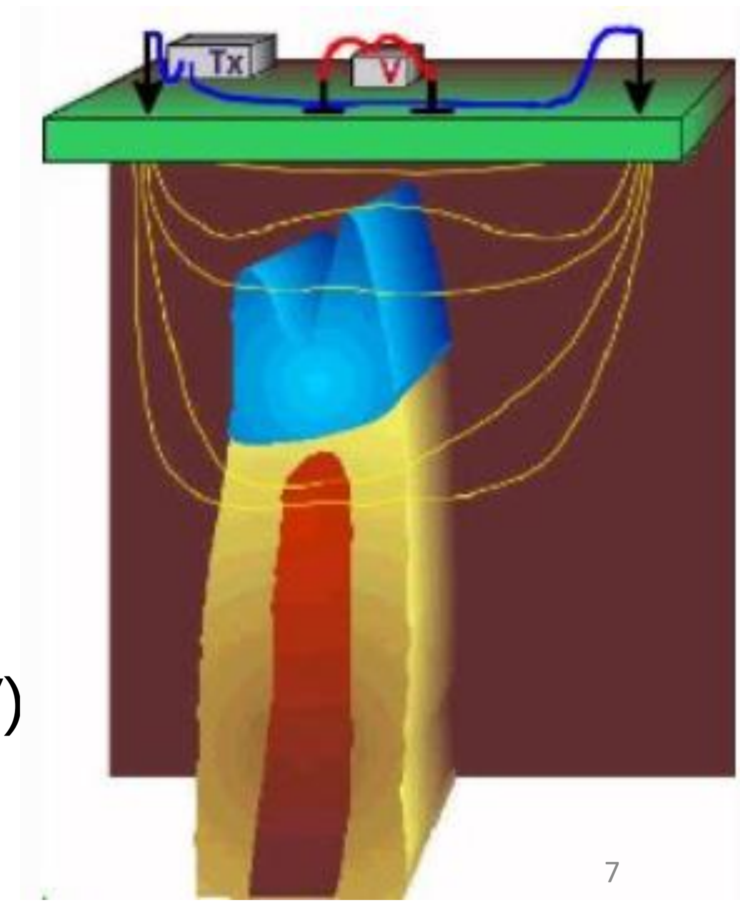
Reading on the GPG:

https://gpg.geosci.xyz/content/DC_resistivity/DC_physical_properties.html

Physical Properties

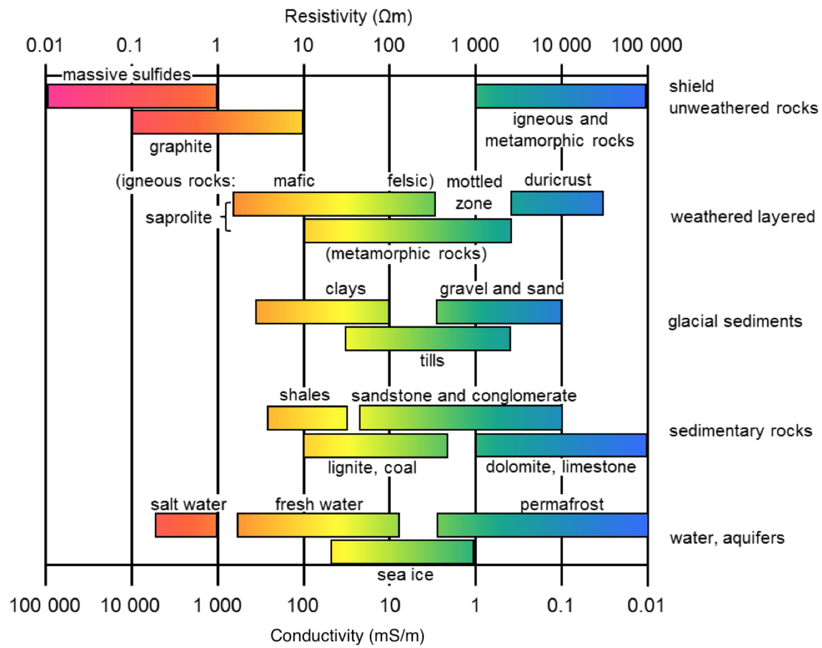
- DCR measurements sensitive to:
  - Electrical resistivity (ρ)
  - Electrical conductivity (σ)
- Units:
  - Electrical resistivity: Ωm
  - Electrical conductivity: S/m
- Distribution of ρ:
  - Impacts current path
  - Impacts measured potentials (V)

$$\rho = \frac{1}{\sigma}$$



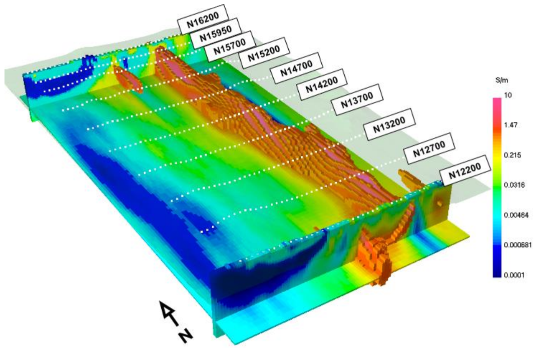
Conductivity/Resistivity of Rocks

- DC resistivity is sensitive to:
  - σ: Conductivity [S/m]
  - ρ: Resistivity [Ωm]
  - $\sigma = 1/\rho$
- Varies over many orders of magnitude
- Depends on many factors:
  - Rock type
  - Porosity
  - Connectivity of pores
  - Nature of the fluid
  - Metallic content of the solid matrix



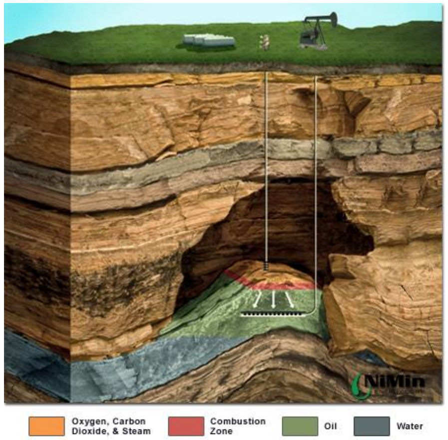
Setup: Motivational Problems

Minerals

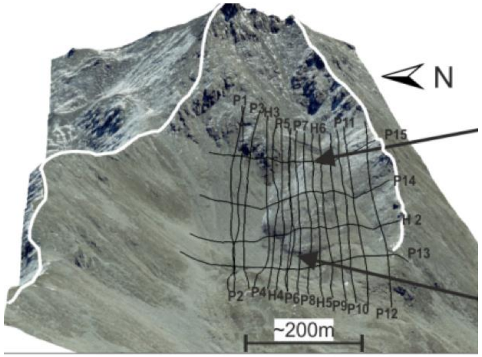


Water inflow in

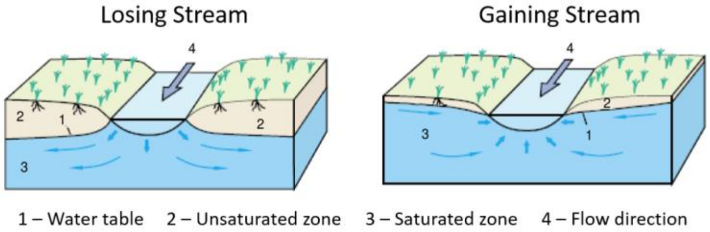
Oil and Gas



Geotechnical



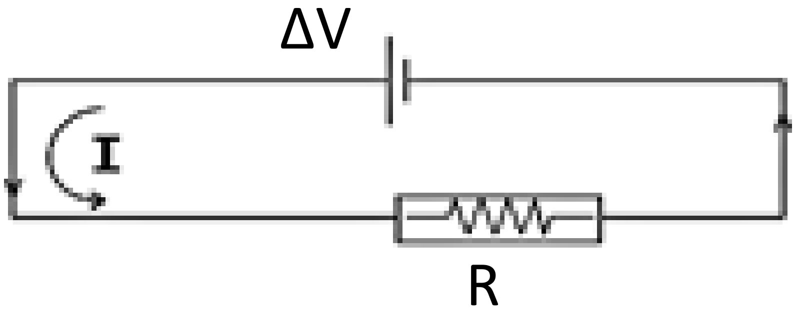
Groundwater



Fundamental Physics: DCR and Ohm's Law

DCR and Ohm's Law

Electrical Circuit

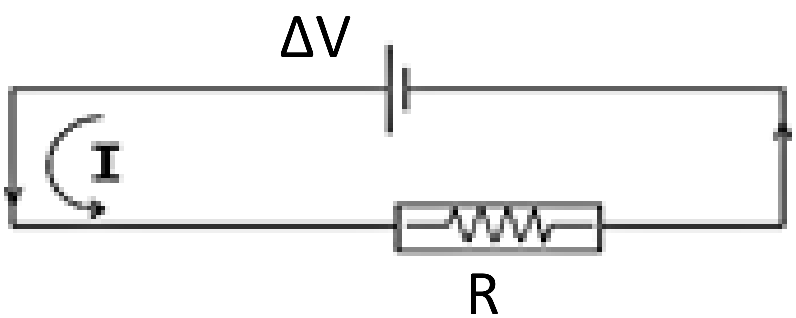


Ohm's Law

$$\Delta V = IR$$

DCR and Ohm's Law

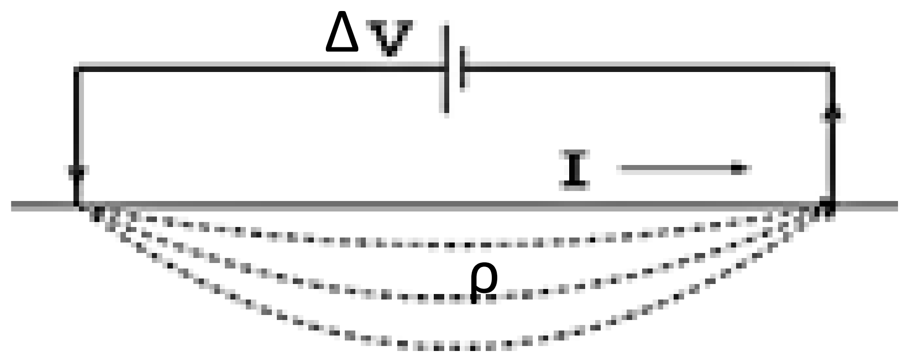
Electrical Circuit



Ohm's Law

$$\Delta V = IR$$

Earth Circuit



For the Earth:

$$\Delta V = I\rho G$$

Depends on:

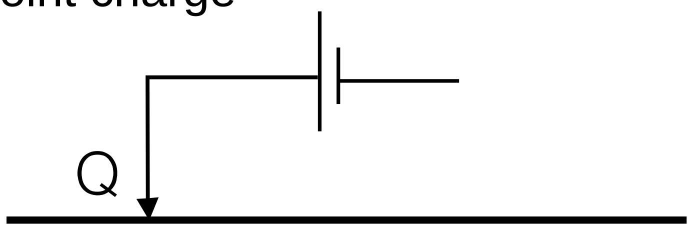
1) Earth's resistivity (ρ)

2) Geometry of electrodes (G)

Fundamental Physics: Homogeneous Earth and Apparent Resistivity

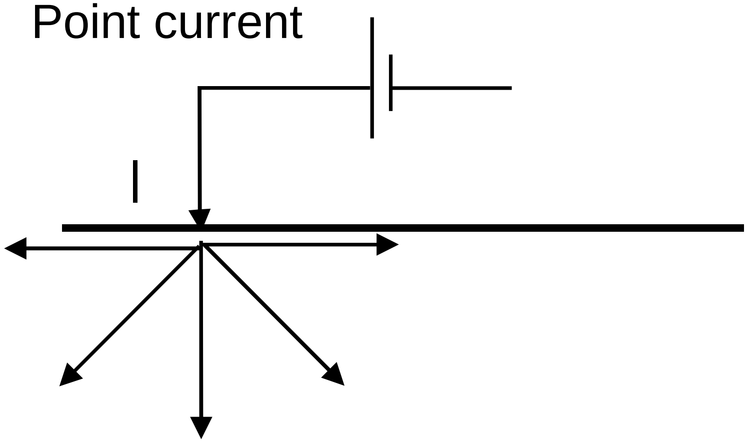
Electric Potential from a Point Source

Point charge



$$V = \frac{Q}{4\pi\epsilon_0 r}$$

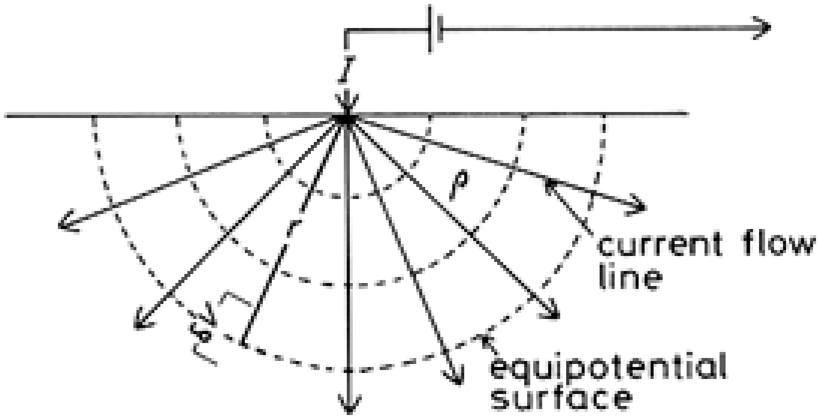
Point current



$$V = \frac{\rho I}{2\pi r}$$

Potential due to a Point Source

Current injected at single point

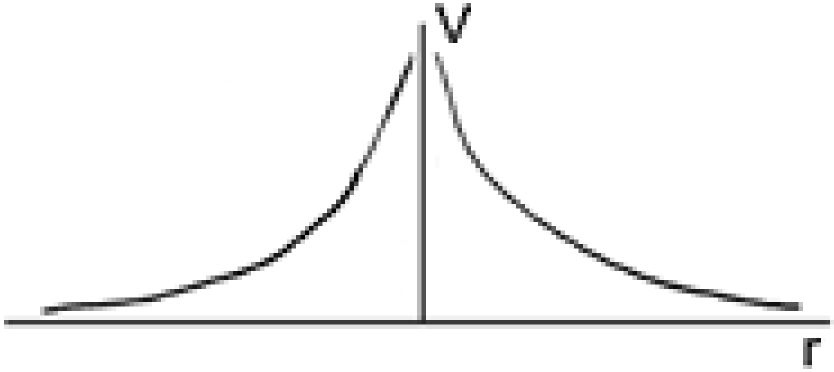


Measured potential

$$V(r) = \frac{I\rho}{2\pi r}$$

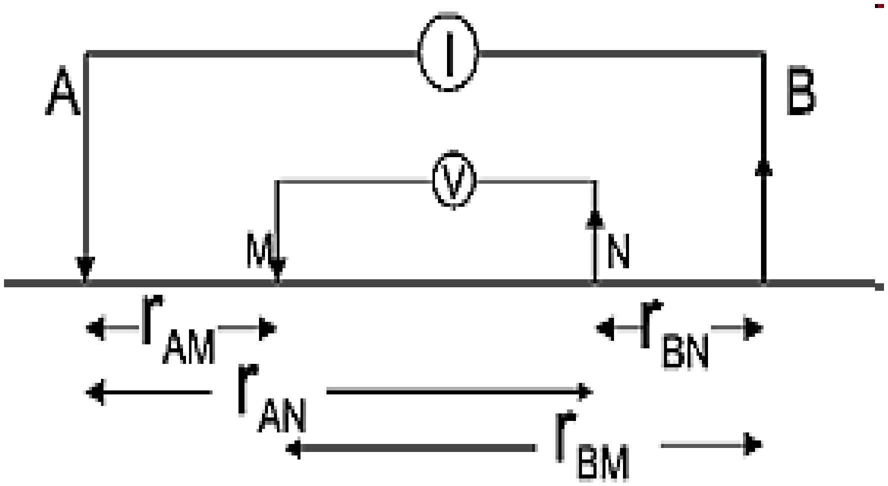
- Current flows radially outward
- Potential decays as 1/r

Potential along surface

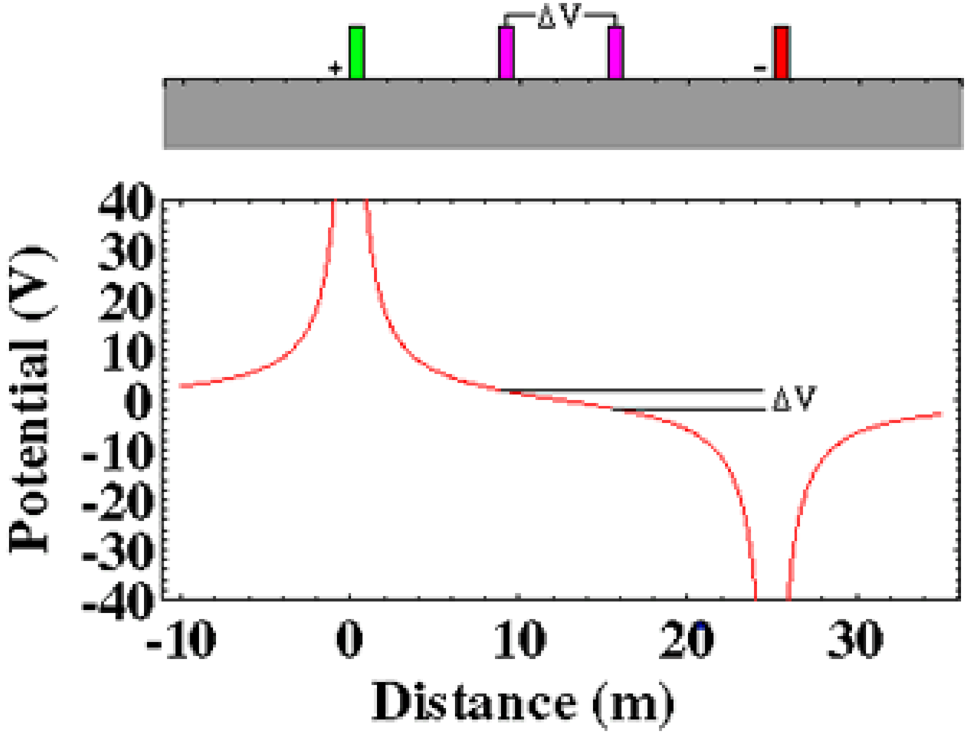


Potential from 4-electrode Array

Electrode Geometry

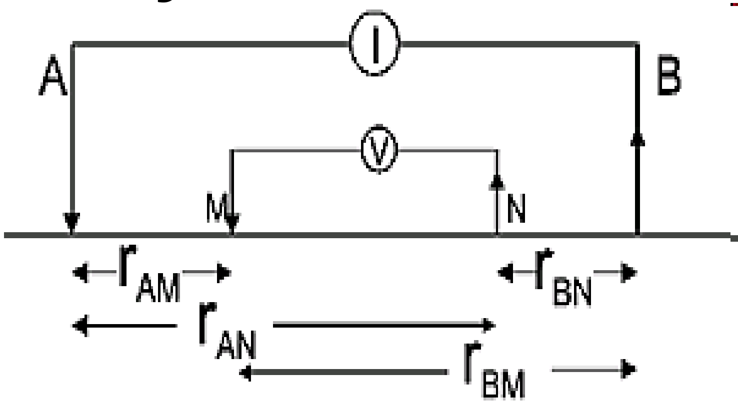


Difference in potential (ΔV)



- Current electrodes (A and B)
- Potential electrodes (M and N)
- Potential difference: $$\Delta V = I\rho G = \frac{I\rho}{2\pi}\left\{\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right\}$$
- Earth resistivity: $$\rho = \frac{\Delta V}{IG}$$

Apparent Resistivity



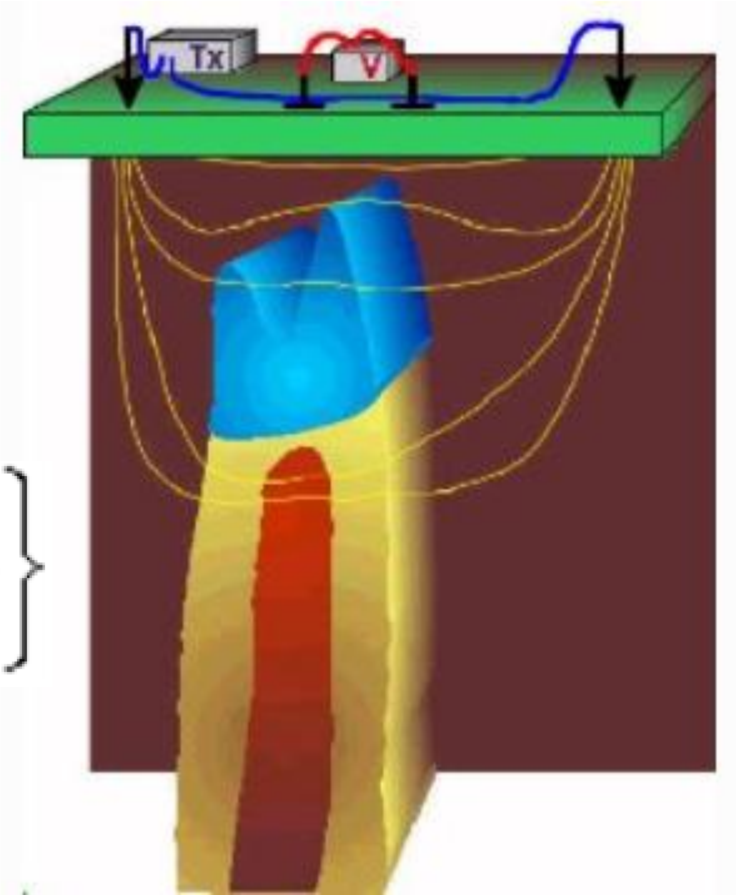
- Earth not homogeneous
- Apparent Resistivity:

  1) Assumes Earth is homogeneous

  2) Indicates regions of high/low resistivity

$$\rho_a = \frac{\Delta V}{IG} \quad \text{where}$$

$$G = \frac{1}{2\pi}\left\{\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right\}$$



Recap: Questions

$$\Delta V = I\rho G = \frac{I\rho}{2\pi}\left\{\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right\}$$

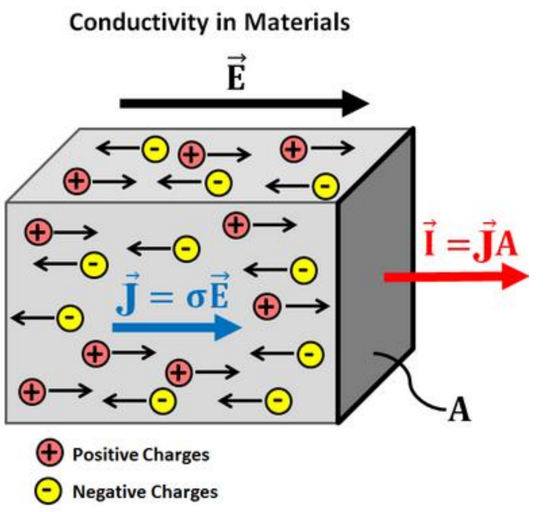
- If the Earth is more conductive, is the measured potential bigger or smaller?
- If electrodes spaced further away, is the measured potential bigger or smaller?

Fundamental Physics: Currents, Charges and Potentials

Maxwell's Equations for DCR

1) Ohm's law in matter:

$$\mathbf{J} = \sigma \mathbf{E}$$



2) Electric Field and Potential:

$$\mathbf{E} = -\nabla \mathbf{V}$$

3) Continuity of Current:

$$\nabla \cdot \mathbf{J} = -\partial Q / \partial t$$

Maxwell's Equations for DCR

Combine equations:

$$\nabla \cdot (\sigma \nabla V) = -\partial Q / \partial t$$

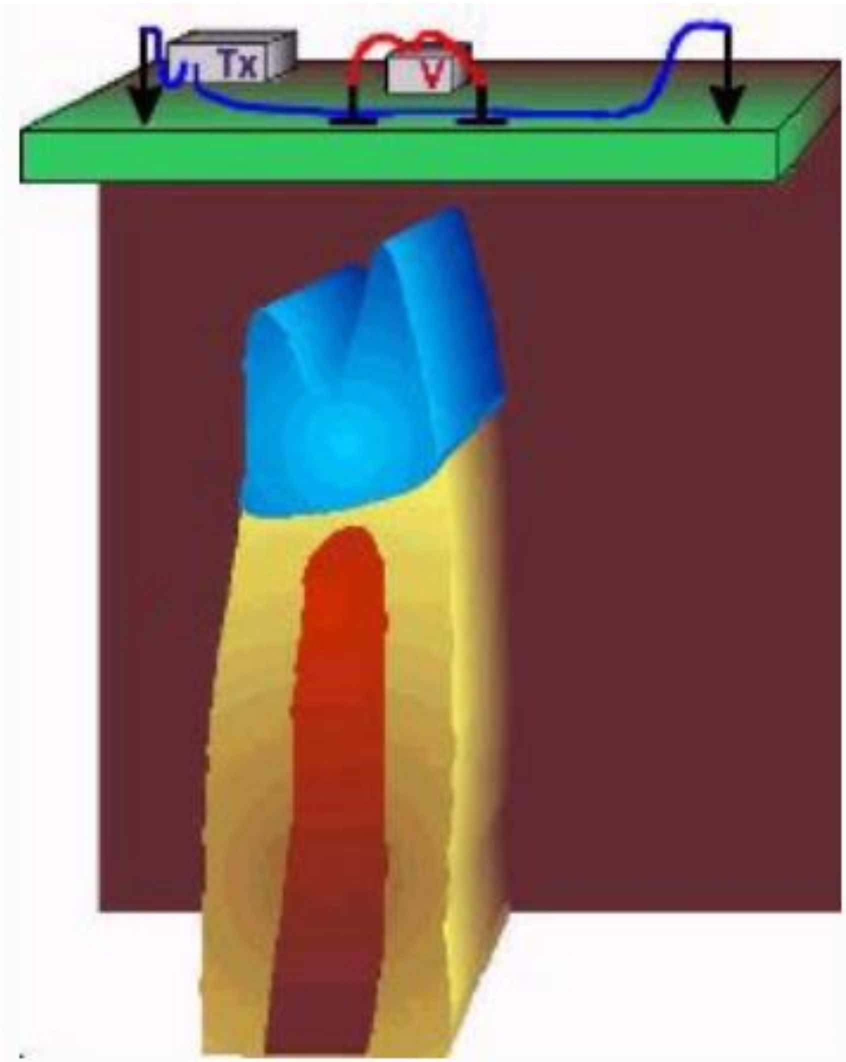
For a current source:

$$\nabla \cdot (\sigma \nabla V) = \begin{cases} -I\delta(r - r_s) & \text{at source} \\ 0 & \text{away from source} \end{cases}$$

For a uniform halfspace:

$$V = \frac{I}{2\pi\sigma}\frac{1}{r} \quad \text{or} \quad \boxed{V = \frac{\rho I}{2\pi r}}$$

Example: Elura Ore Deposit



Physical Properties

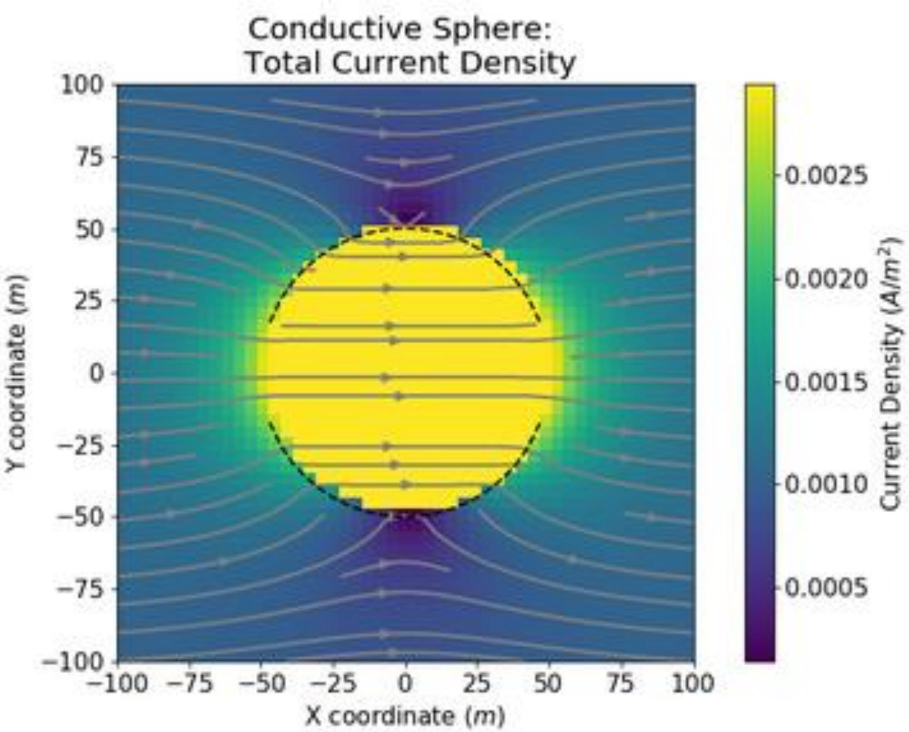
Rock type	Ohm-m
Overburden	**12**
Host rock	**200**
Gossan	**420**
Mineralization (pyritic)	**0.6**
Mineralization (pyrrhotite)	**0.6**

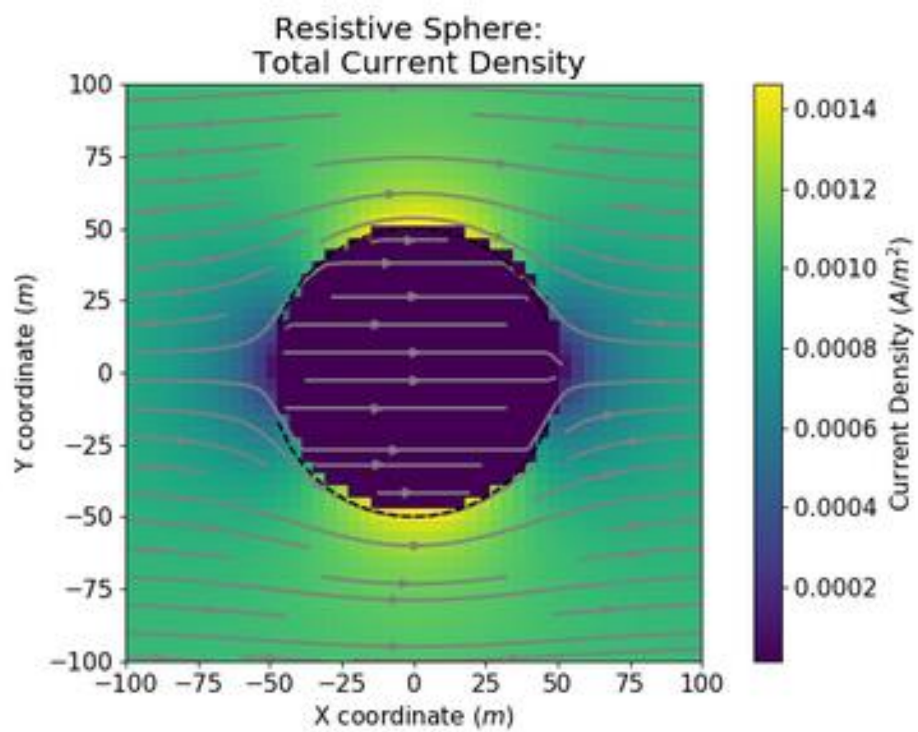
Current Path

- Follows path of least resistance

  → Converges on conductors (left)

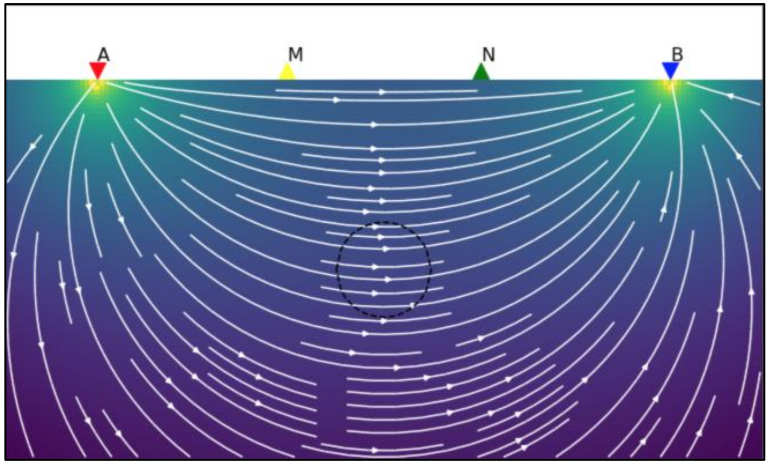
  → Diverges from resistors (right)



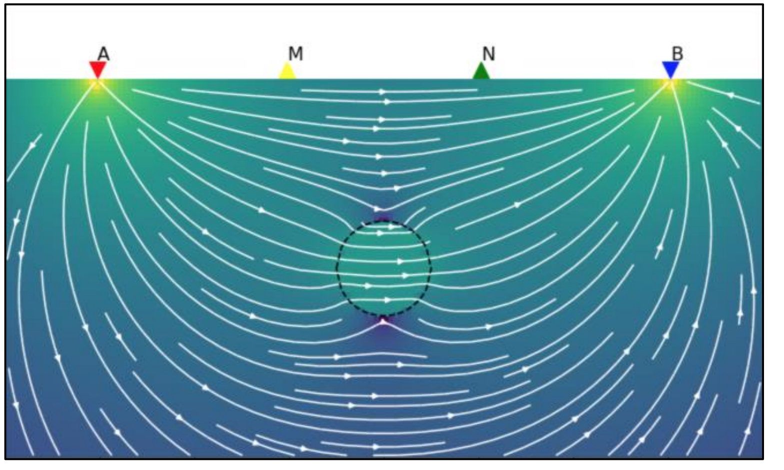


Current Path: Confined Conductor

Homogenous earth

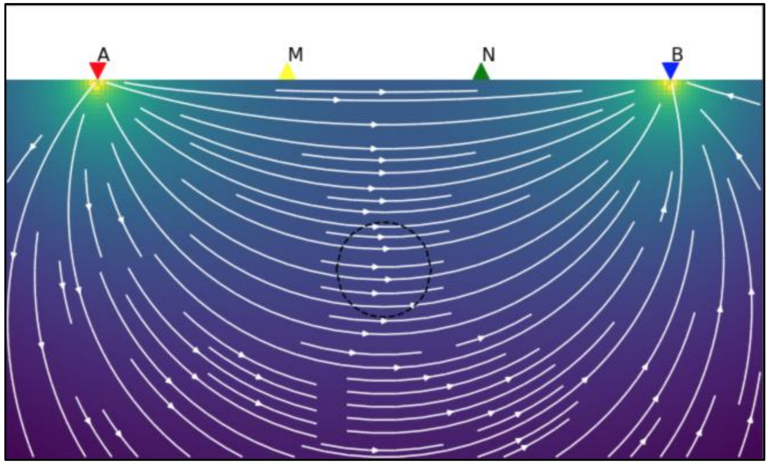


Conductive sphere

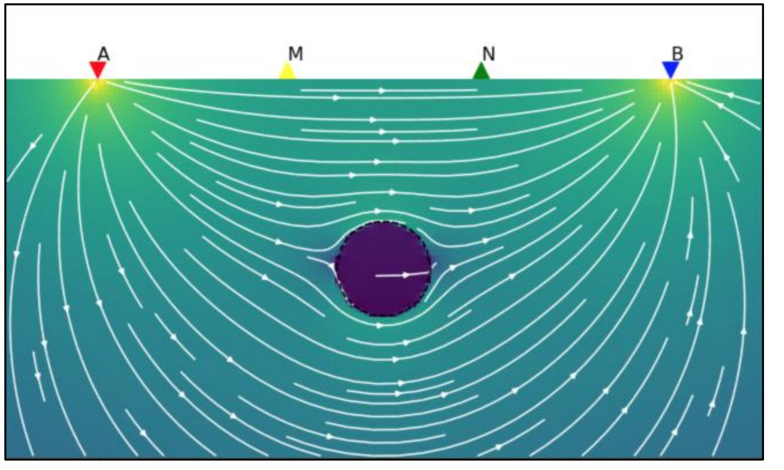


Current Path: Confined Resistor

Homogenous earth

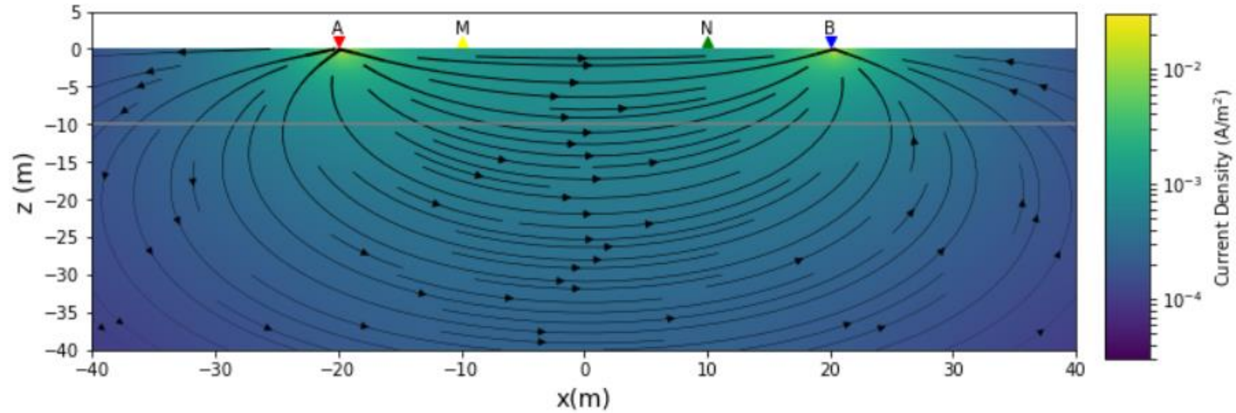


Resistive sphere



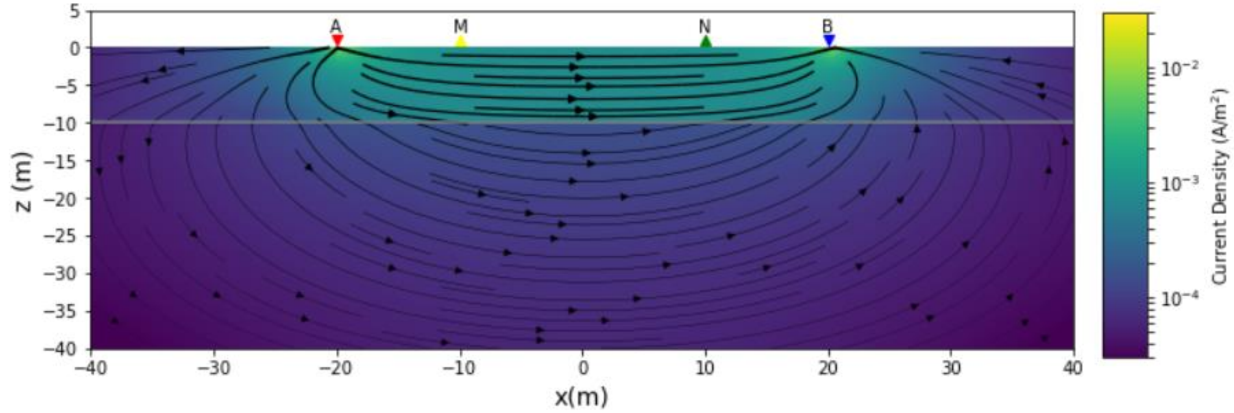
Current Path: 2 Layer Earth

Halfspace



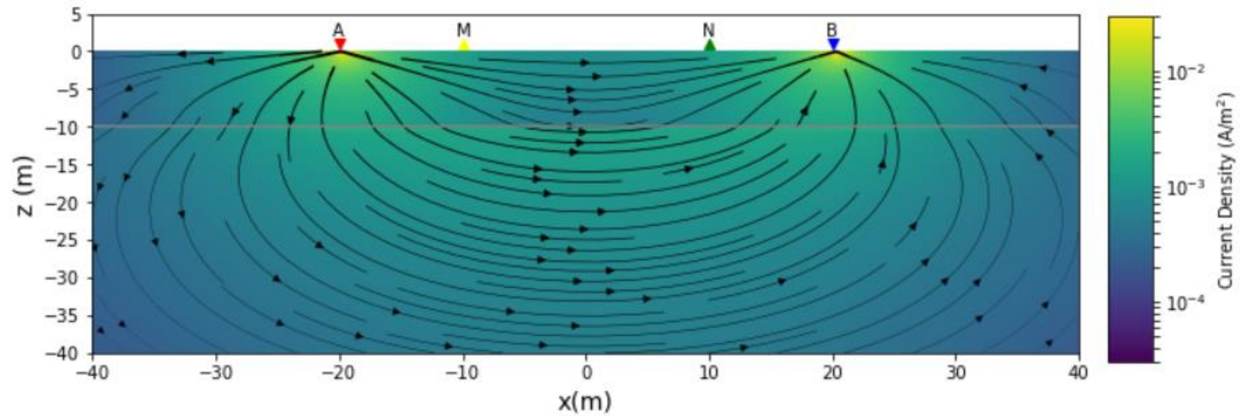
Conductive Overburden

- Most current flows through top layer
- Spreads out in bottom layer

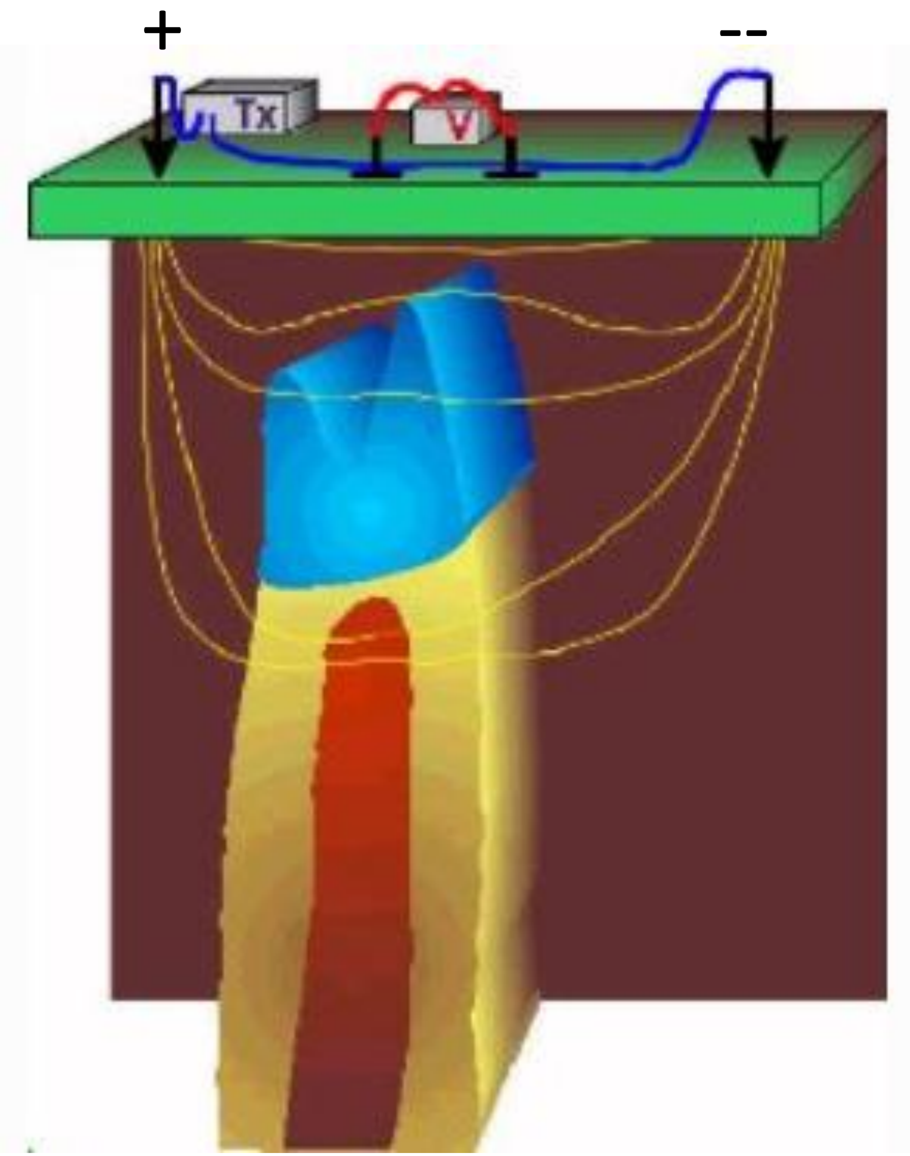


Resistive Overburden

- Current wants to bypass top layer
- Similar current in both layers



Current Path: Elura Ore Deposit



Physical Properties

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Overburden	**12**
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Gossan	**420**
Mineralization (pyritic)	**0.6**
Mineralization (pyrrhotite)	**0.6**

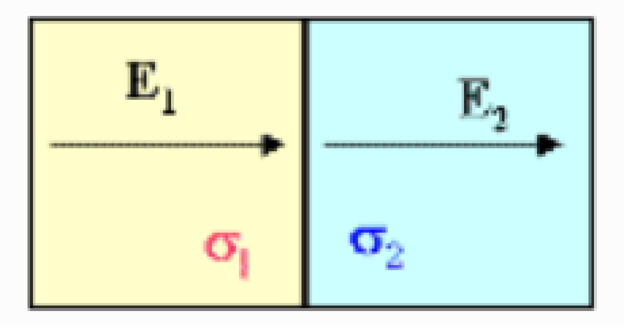
- Gossan (blue) is a resistor. How is the current path affected?
- What is impact of conductive overburden?

Electrical Charges

Away from source: $\nabla \cdot \mathbf{J} = 0$

- Total current entering a volume = Total current leaving a volume
- Normal component of J continuous across boundaries

$$\mathbf{J_n} = \sigma_1 \mathbf{E_{1,n}} = \sigma_2 \mathbf{E_{2,n}}$$



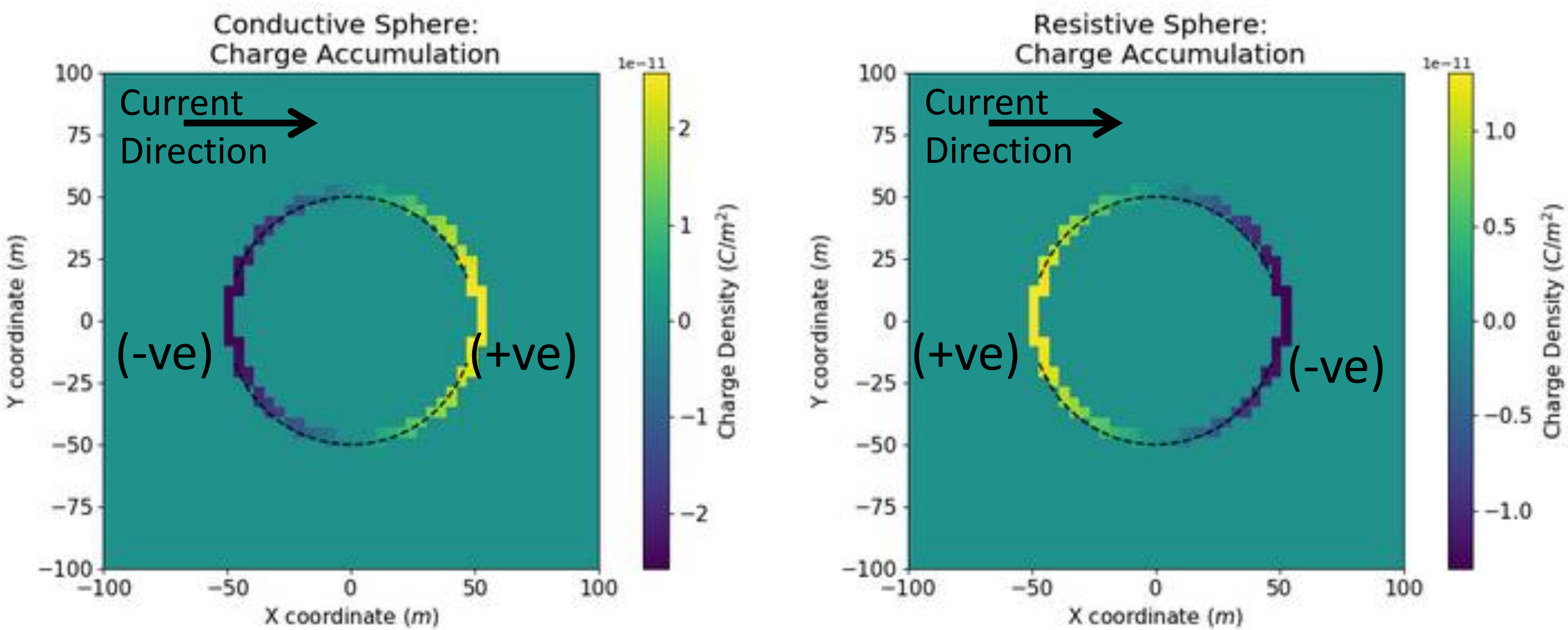
$$\left(\frac{\mathbf{1}}{\sigma_\mathbf{2}} - \frac{\mathbf{1}}{\sigma_\mathbf{1}}\right)\mathbf{J_n} = (\rho_\mathbf{2} - \rho_\mathbf{1})\mathbf{J_n} = \frac{\tau}{\varepsilon_\mathbf{0}}$$

Electrical Charges

- Charges build-up on boundaries

  From resistor into conductor → negative charges build-up

  From conductor into a resistor → positive charges build-up



Electrical Charges: DCR Survey

Normal component of current density is continuous

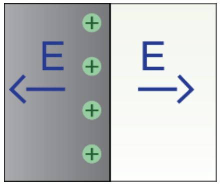
$$J_{1n} = J_{2n}$$

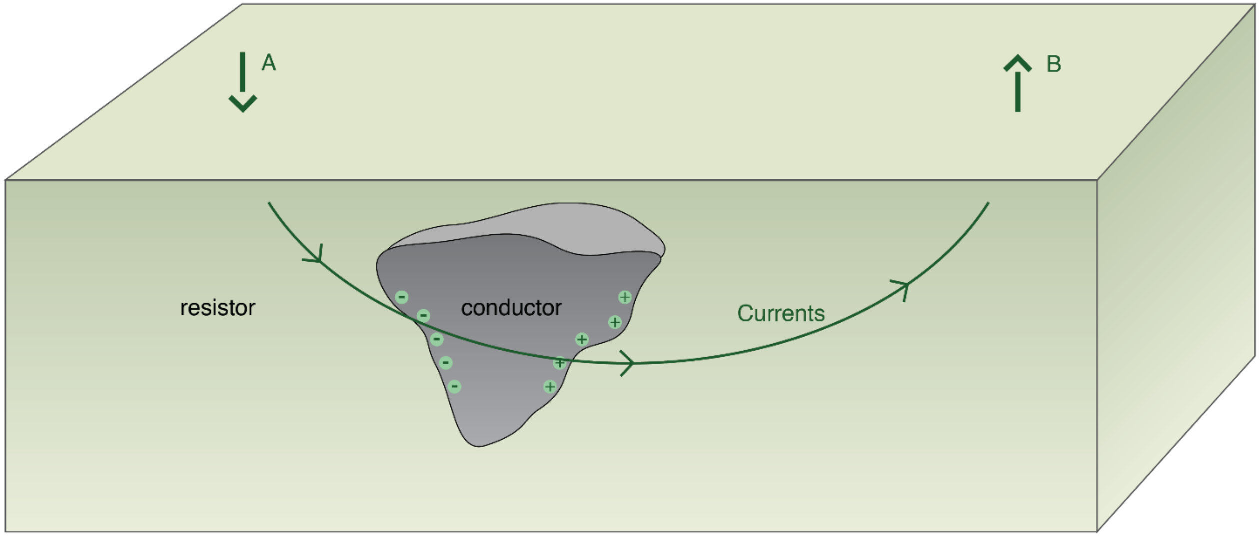
$$\sigma_1 E_{1n} = \sigma_2 E_{2n}$$

Conductivity contrast

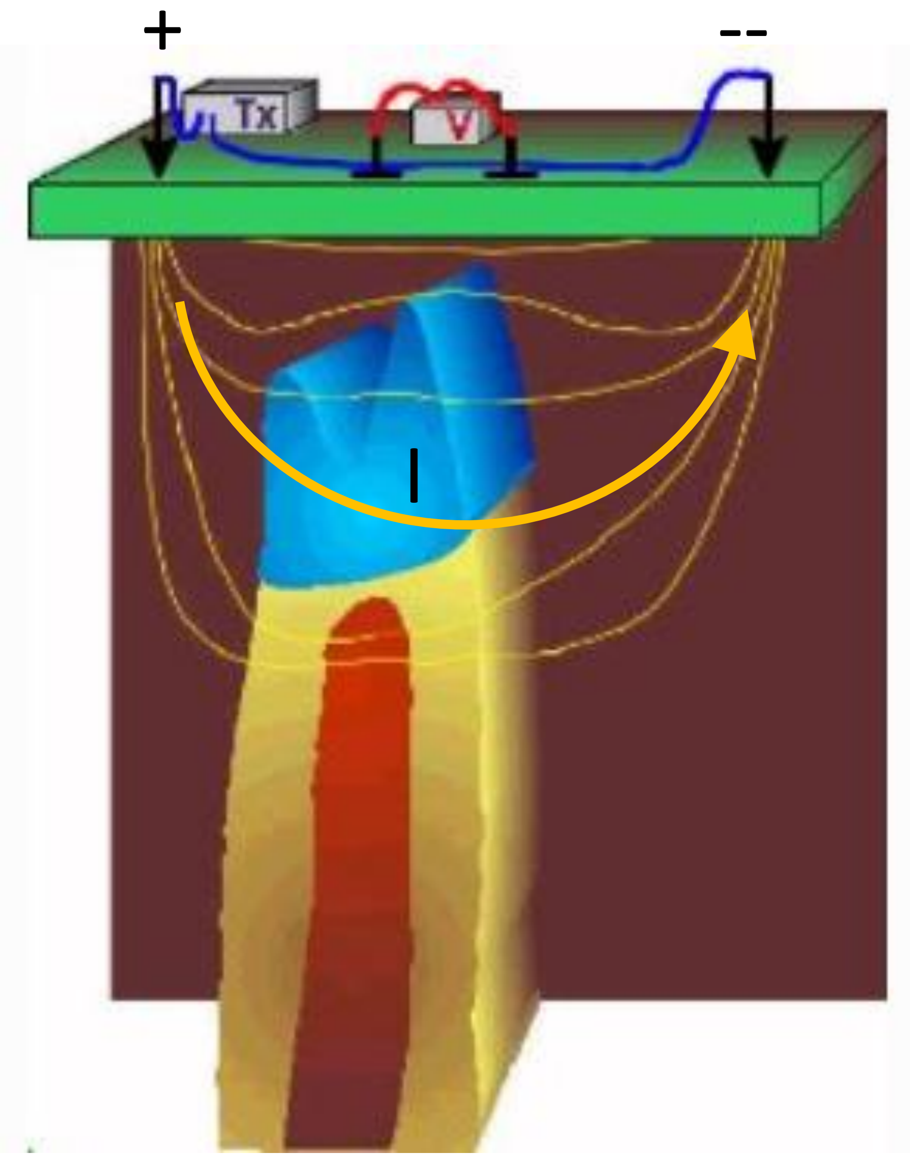
$$\sigma_1 \neq \sigma_2$$

- Electric field discontinuous
- Charge build-up





Electrical Charges: Elura Ore Deposit

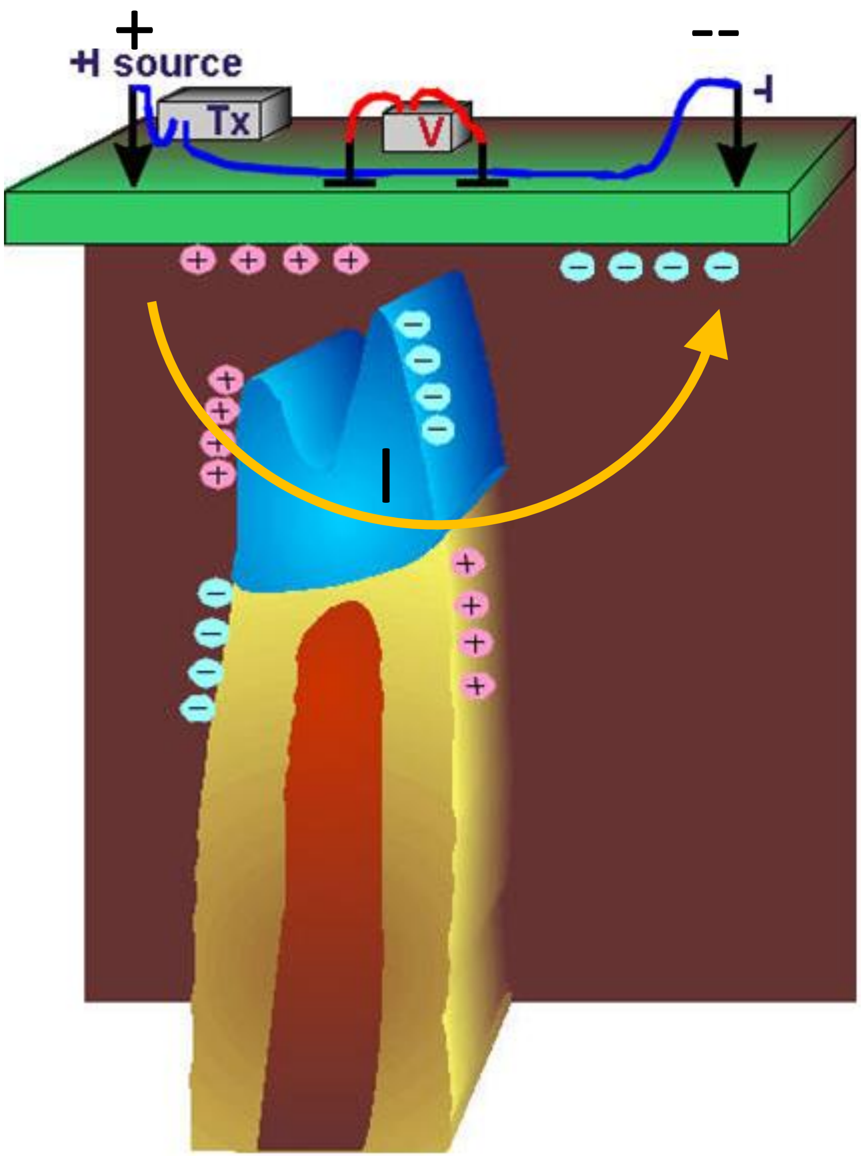


Physical Properties

Rock type	Ohm-m
Overburden	12
Host rock	200
Gossan	420
Mineralization (pyritic)	0.6
Mineralization (pyrrhotite)	0.6

- Where do charges accumulate?

Electrical Charges: Elura Ore Deposit



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Secondary Electric Potential

- Coulomb's Law:

$$\mathbf{V}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i}{\mathbf{r_i}}$$

- Accumulation of charges impacts electric potential

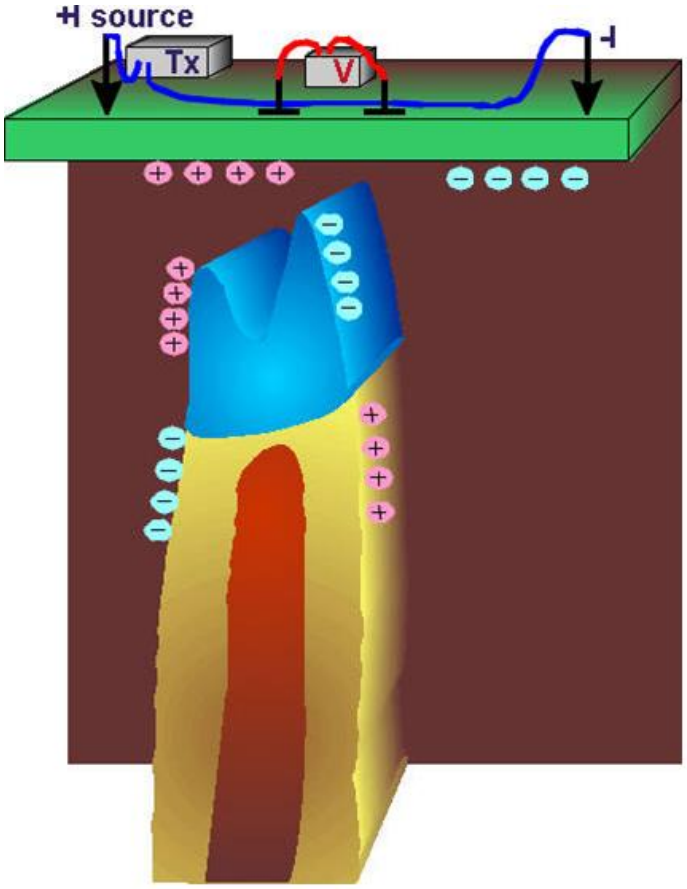
- Secondary potential depends on:

  - Sign of the charges

  - Distribution of charges

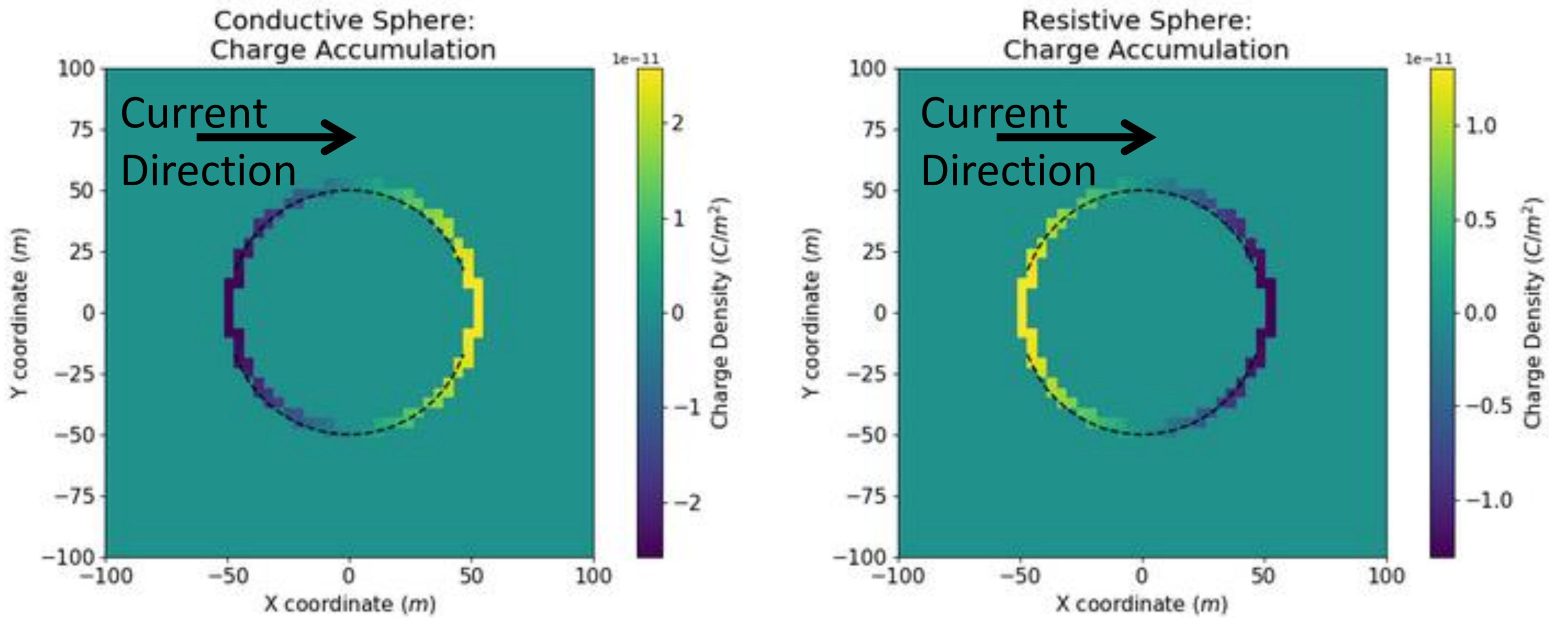
- For a set of potential electrode MN:

$$\mathbf{\Delta V} = \mathbf{V_N} - \mathbf{V_M}$$

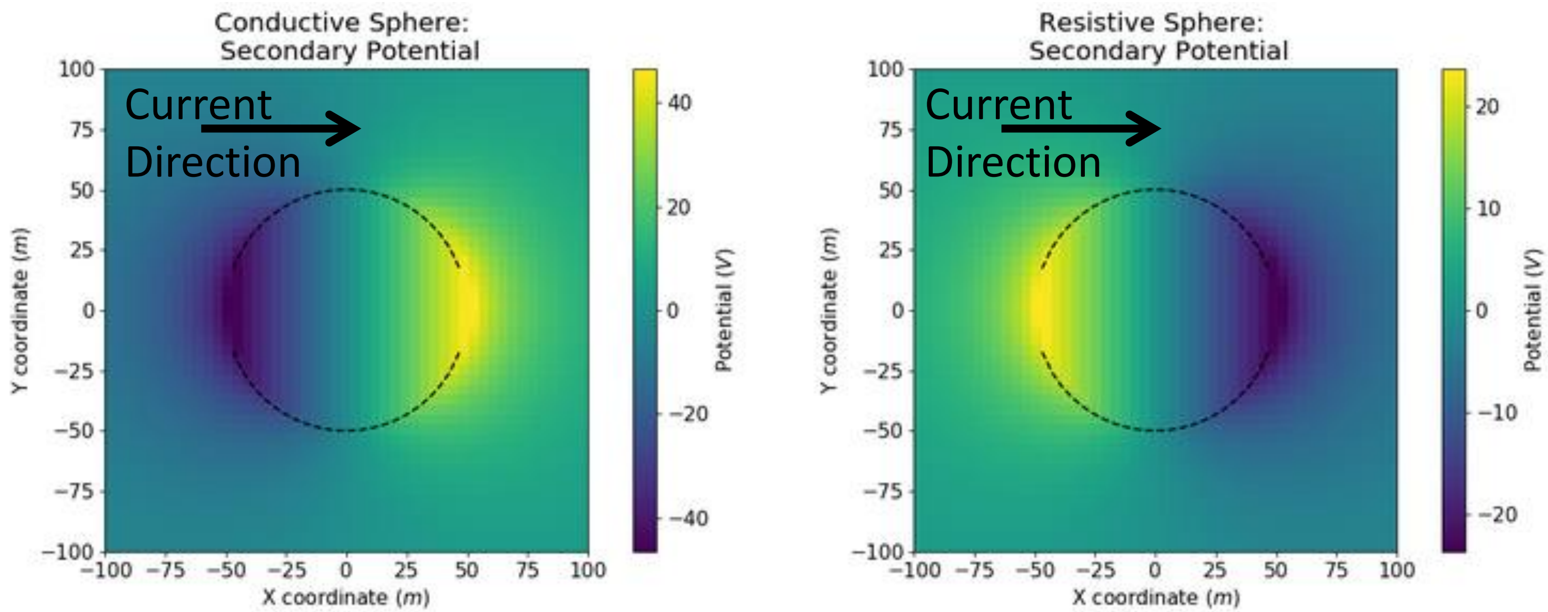


Secondary Electric Potential

- Charges



- Secondary Potential

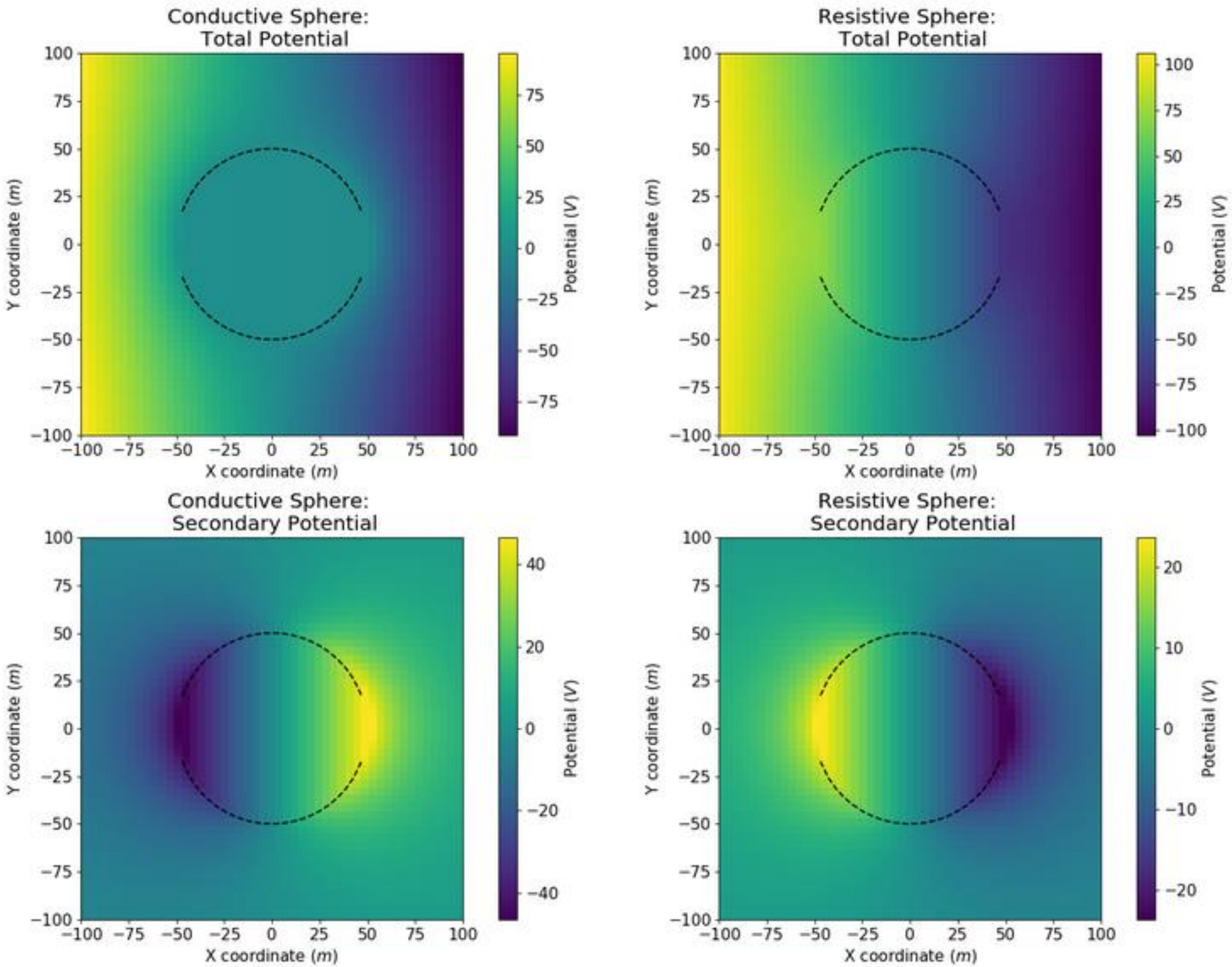


Secondary Potential vs. Total Potential

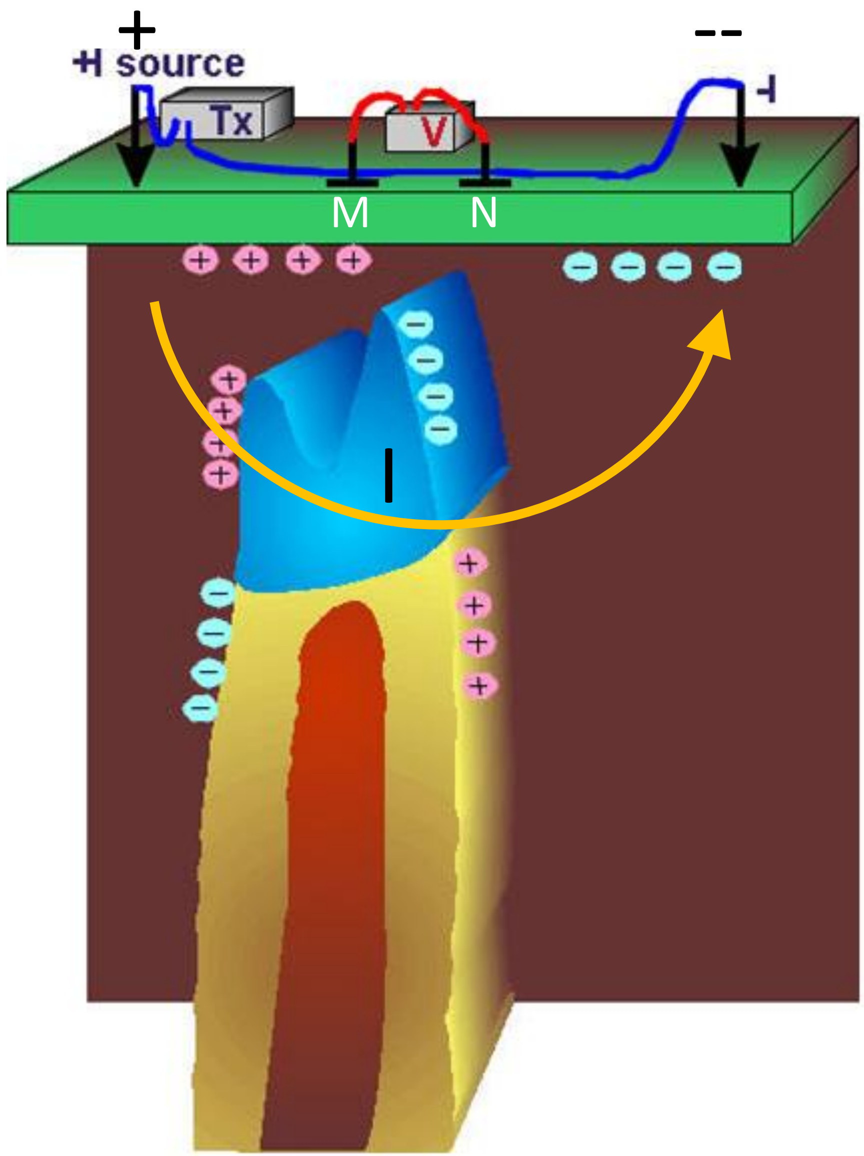
- Total Potential

$$\mathbf{V} = \mathbf{V_0} + \mathbf{V_s}$$

- Secondary Potential



Potentials: Elura Ore Deposit



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- Is anomalous potential +ve or –ve at location N?
- Is $\Delta V = V_N - V_M$ +ve or –ve?

Secondary Potential and Apparent Resistivity

- Apparent resistivity:

$$\rho_a = \frac{\Delta V}{IG} \quad \text{where} \quad G = \frac{1}{2\pi}\left\{\frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}}\right\}$$

- Measured potential difference:

$$\mathbf{\Delta V = \Delta V_0 + \Delta V_s} \quad \text{where} \quad \mathbf{\Delta V_0 = I\rho_0 G}$$

- If $|\mathbf{\Delta V}| < |\mathbf{\Delta V_0}|$

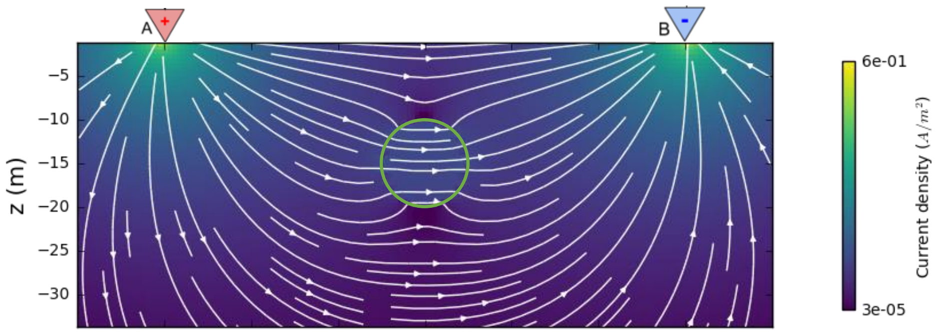
→ Secondary potential opposes primary potential

→ Apparent resistivity smaller than background

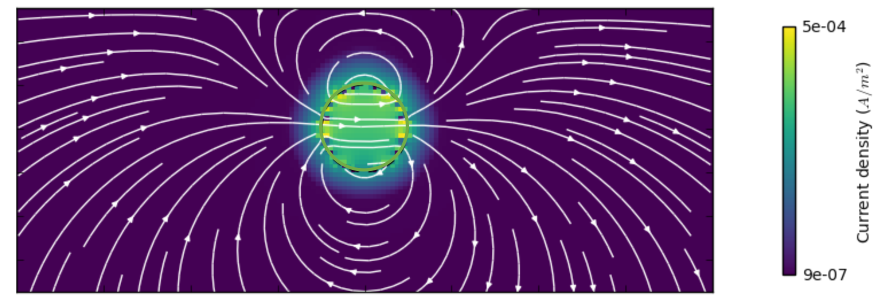
→ Possible conductor below

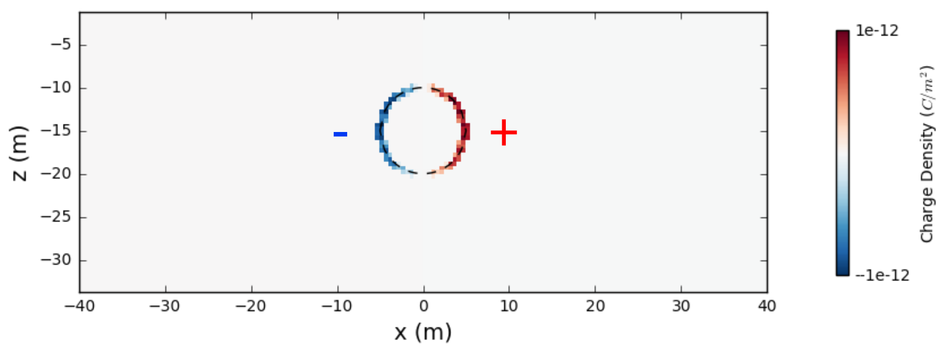
Currents, charges, and potentials

Total currents: J

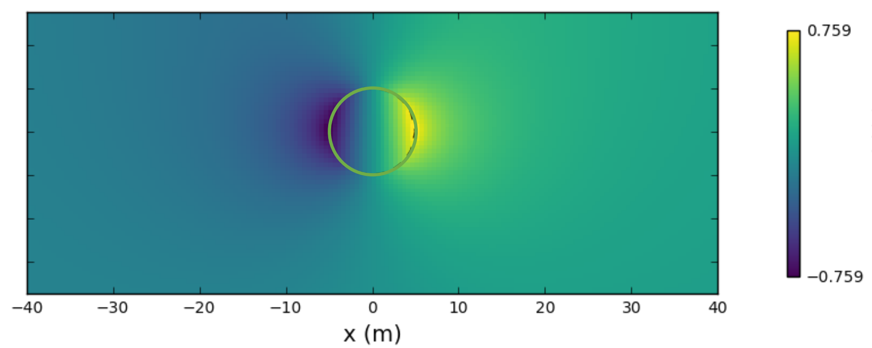


Secondary currents: J_s





Secondary charges: Q_s

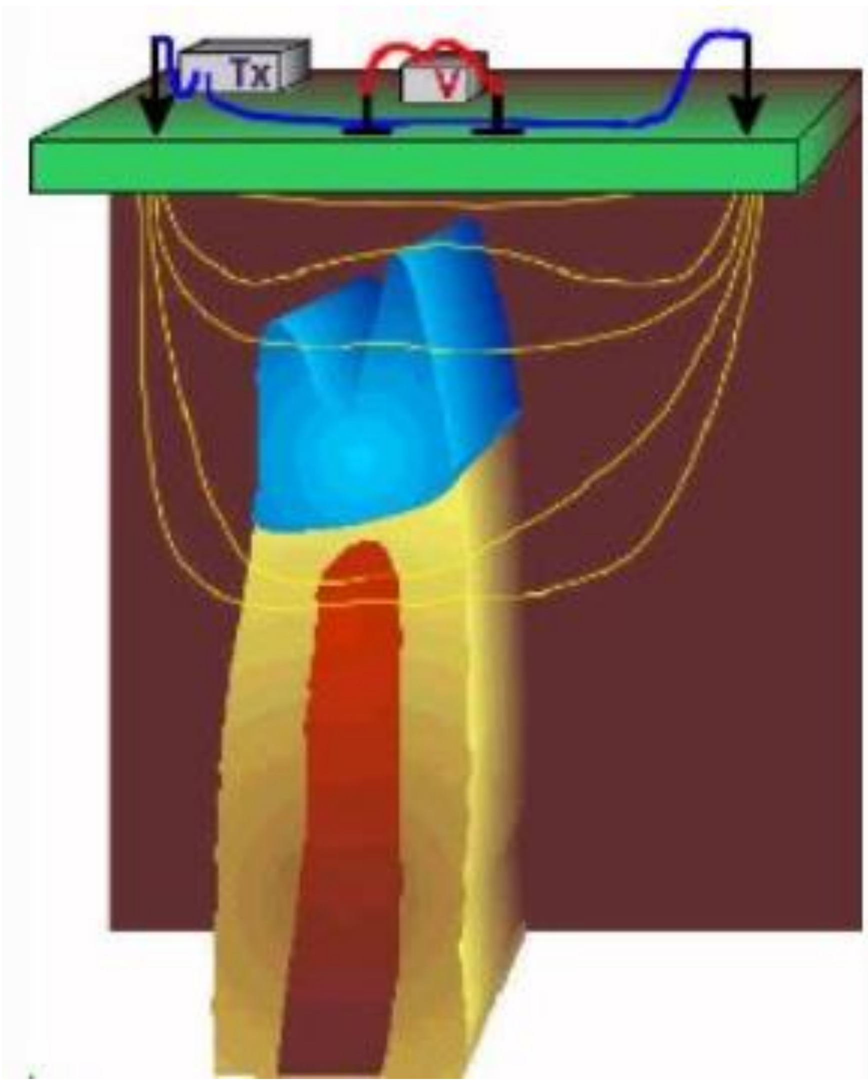


Secondary potential: ϕ_s

Fundamental Physics: All Together Now!

Fundamentals Recap

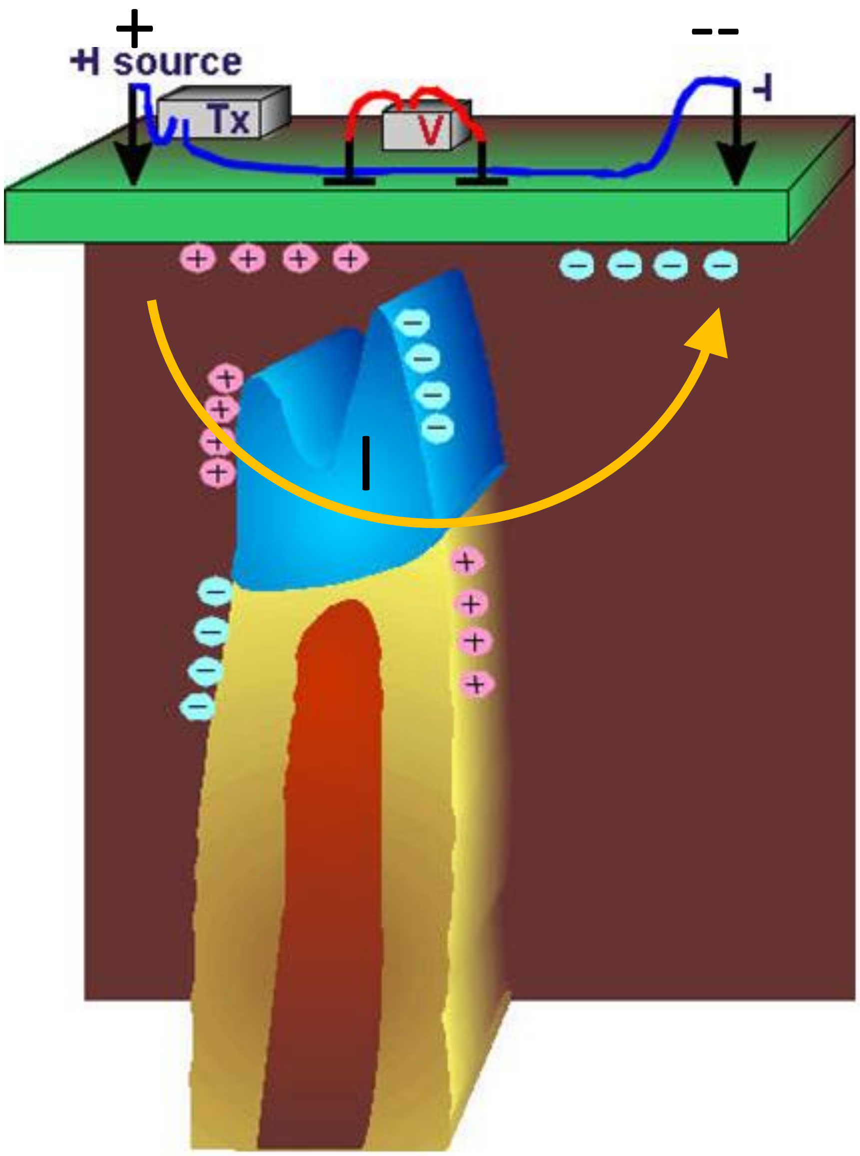
- Current converges on conductors and diverges at resistors
- More current flows deeper if current electrodes are more spaced



Fundamentals Recap

- Current converges on conductors and diverges at resistors
- More current flows deeper if current electrodes are more spaced
- Charges accumulate on boundaries normal to current flow

$$\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)\mathbf{J_n} = (\rho_2 - \rho_1)\mathbf{J_n} = \frac{\tau}{\varepsilon_0}$$



Fundamentals Recap

- Current converges on conductors and diverges at resistors
- More current flows deeper if current electrodes are more spaced
- Charges accumulate on boundaries normal to current flow

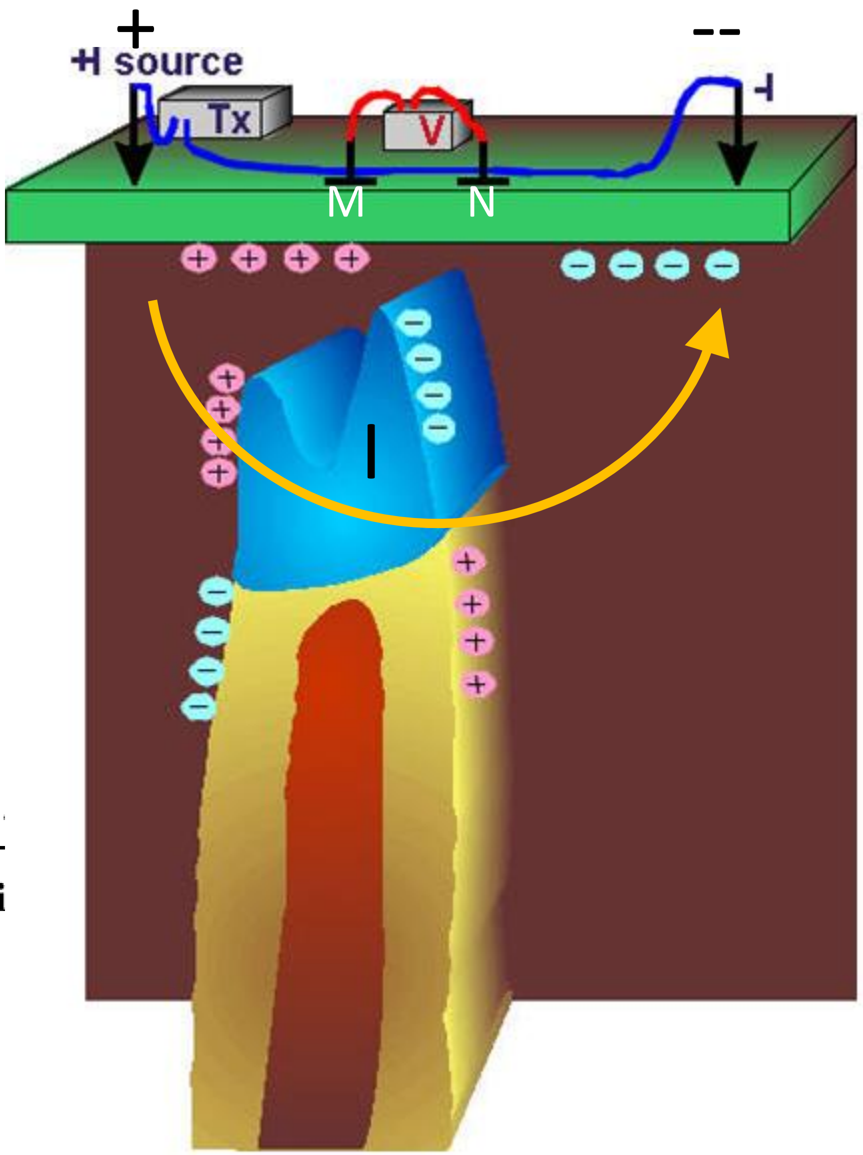
$$\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)\mathbf{J_n} = (\rho_2 - \rho_1)\mathbf{J_n} = \frac{\tau}{\varepsilon_0}$$

- Accumulation of charges changes the secondary potential

$$\mathbf{V}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i}{\mathbf{r_i}}$$

- Differences in potential measured by potential electrodes

$$\mathbf{\Delta V} = \mathbf{V_N} - \mathbf{V_M}$$



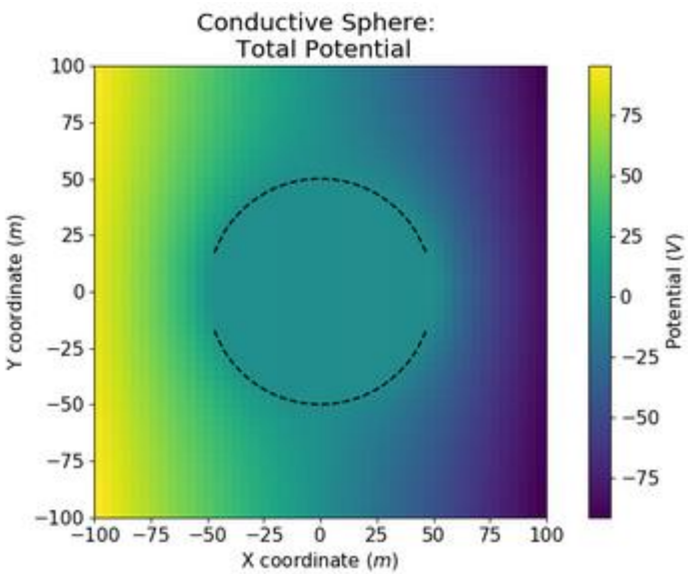
Recap: Questions

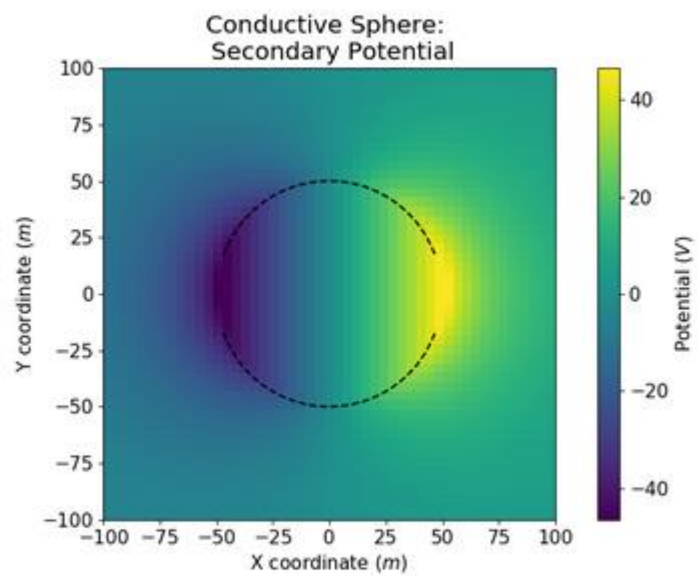
- If current flows from resistive medium (1) into a conductive medium (2), are +ve or –ve charges accumulated?

$$\left(\frac{1}{\sigma_2} - \frac{1}{\sigma_1}\right)\mathbf{J_n} = (\rho_2 - \rho_1)\mathbf{J_n} = \frac{\tau}{\varepsilon_0}$$

- Do potential electrodes measure differences in potential or total potential?

- If the anomalous potential near a body is negative somewhere, can the total potential at that location still be positive?





Unit Activities

- **Labs: (DC)**
  - Monday, October 28th
  - Tuesday, October 29th

- **Quiz:**
  - Friday, November 1st