Electromagnetic Methods

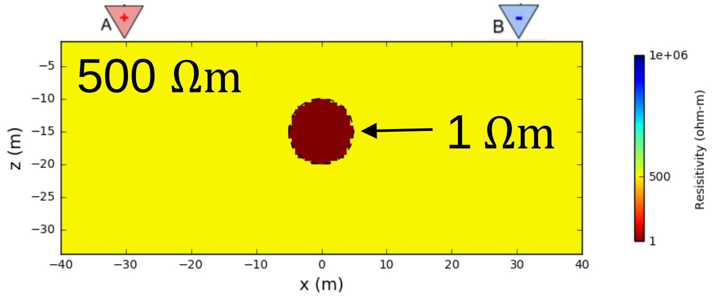
Reading on the GPG:

https://gpg.geosci.xyz/content/electromagnetics/index.html

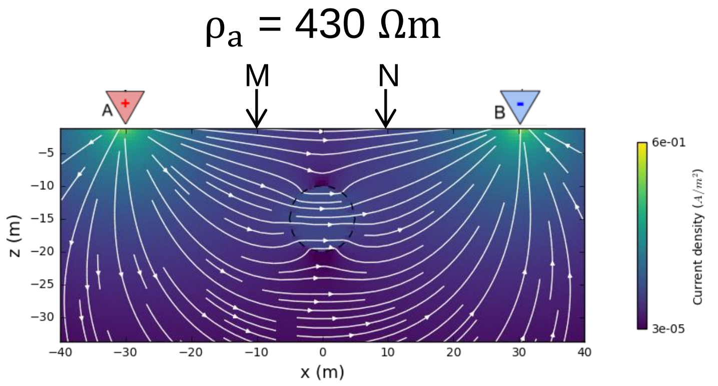
Today’s Topics

- The Problem with DC Resistivity
- Basic EM Experiment
- Physical Properties
- Basic Principles
  - Ampere’s Law
  - Faraday’s Law
  - Ohm’s Law
  - Basic EM Experiment Revisited

The Problem with DCR

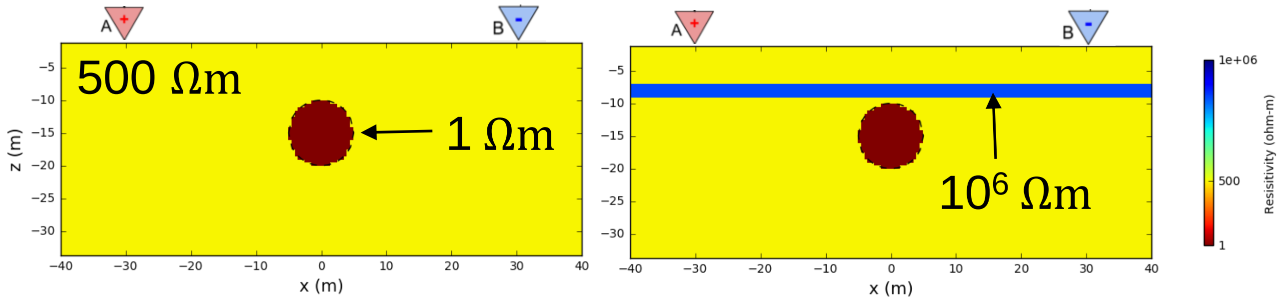


Currents and measured data at MN



The Problem with DCR

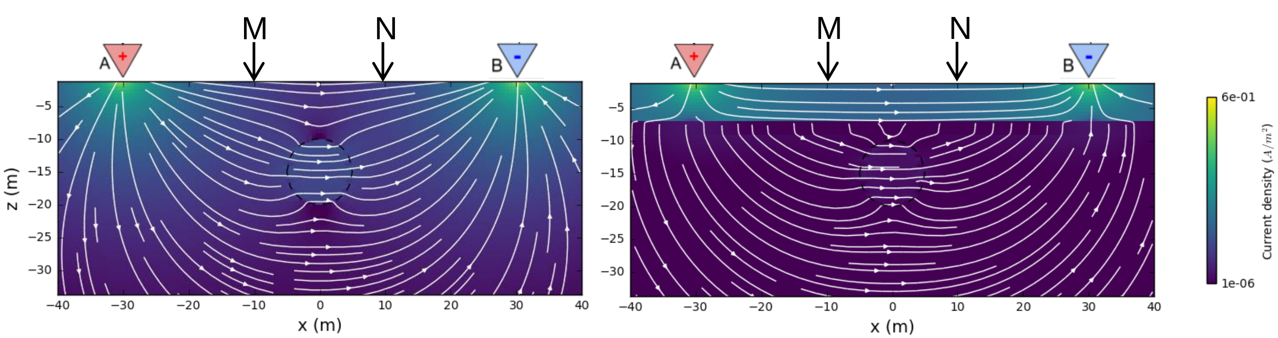
Resistivity models (thin resistive layer)



Currents and measured data at MN

ρ_a = 430 Ωm

ρ_a = 1652 Ωm



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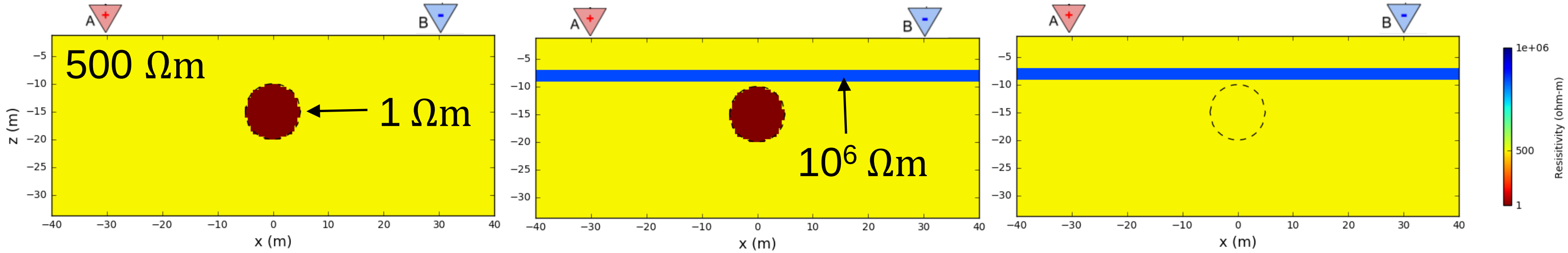

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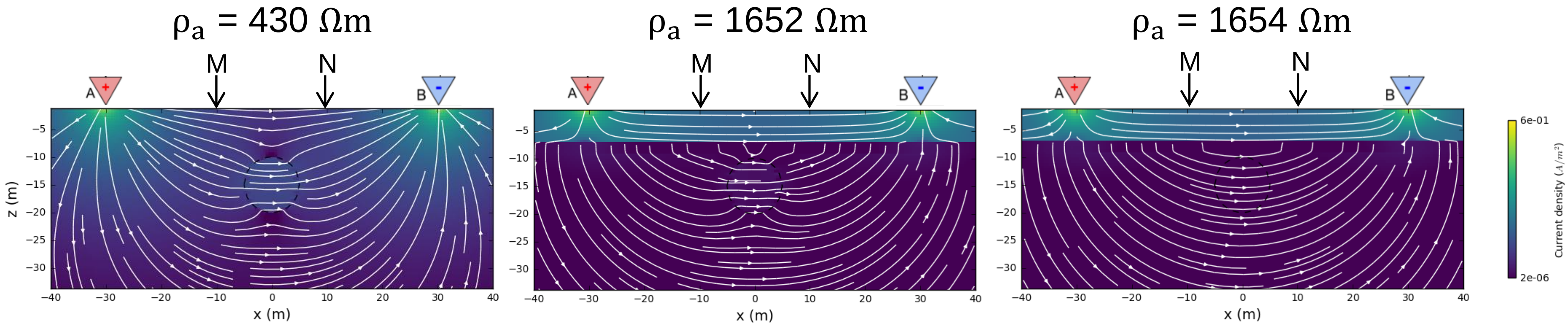
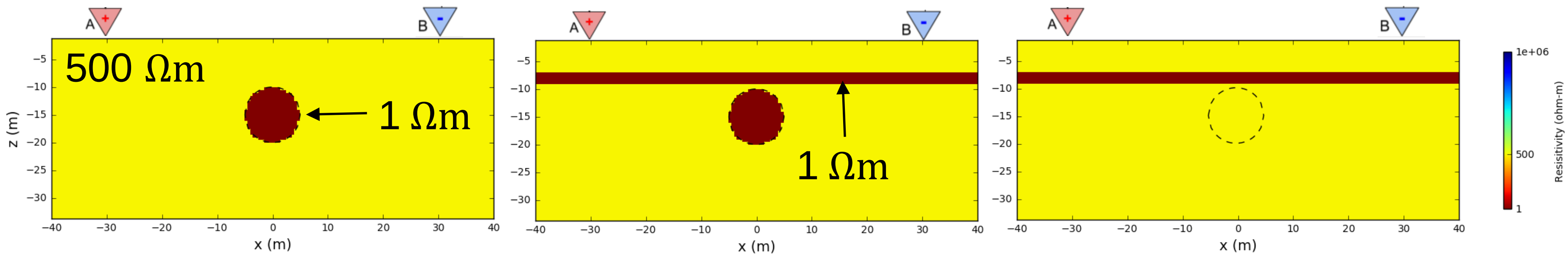
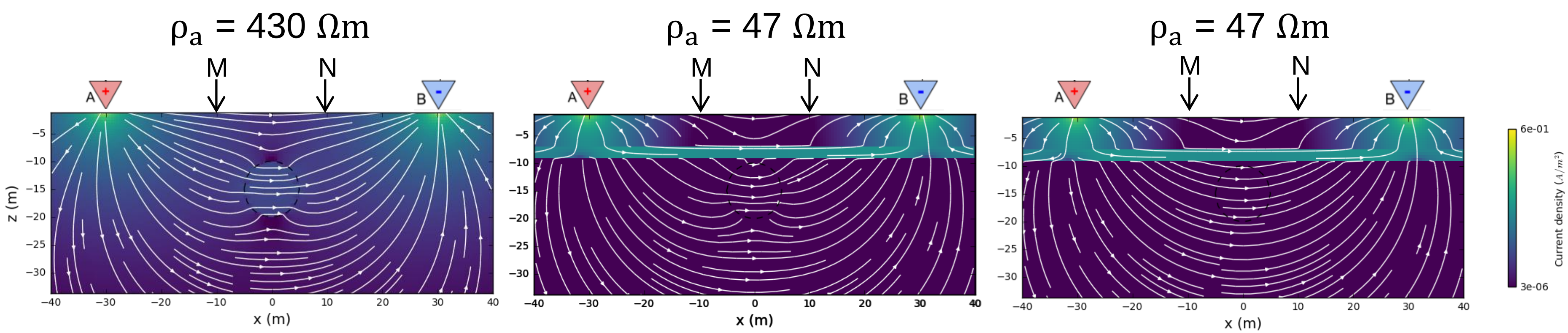


The Problem with DCR

Resistivity models (thin conductive layer)



Currents and measured data at MN



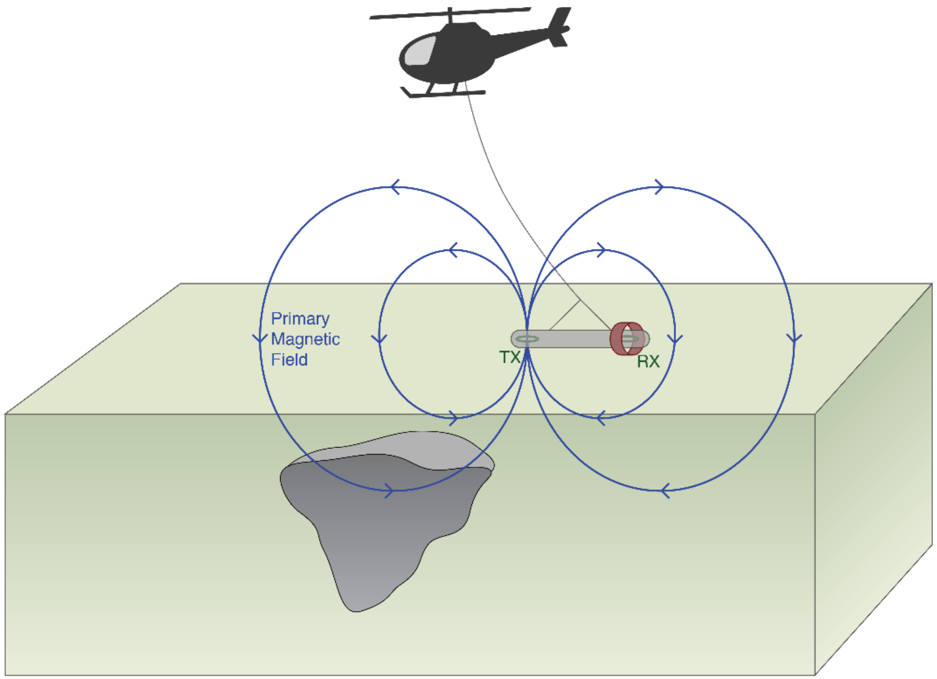
Basic EM Experiment

Reading on the GPG:

https://gpg.geosci.xyz/content/electromagnetics/electromagnetic_introduction.html

Basic Experiment

- **Source (Tx):**
  Current loop makes primary magnetic field

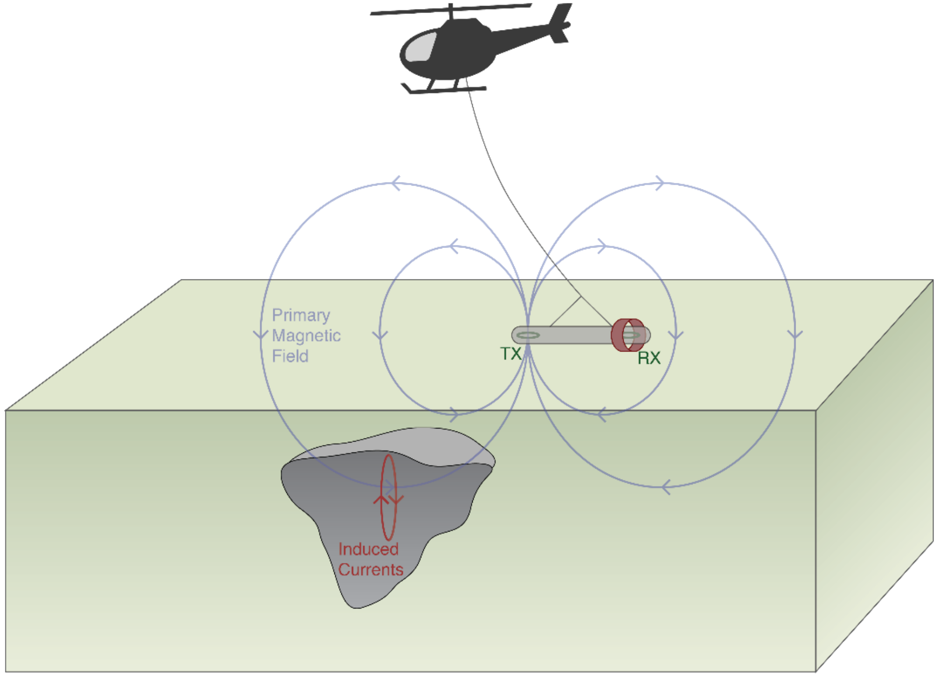


Basic Experiment

- **Source (Tx):**
  Current loop makes primary magnetic field

- **Induction:**
  Time-varying magnetic fields induce electric fields everywhere

  → Large induced currents in conductors



Basic Experiment

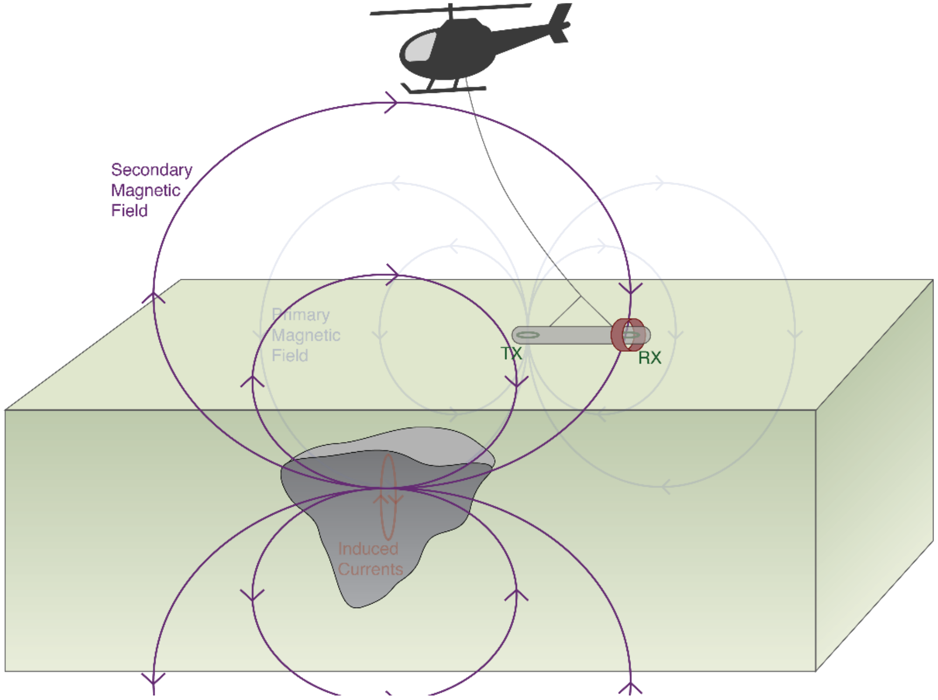
- **Source (Tx):**
  Current loop makes primary magnetic field

- **Induction:**
  Time-varying magnetic fields induce electric fields everywhere

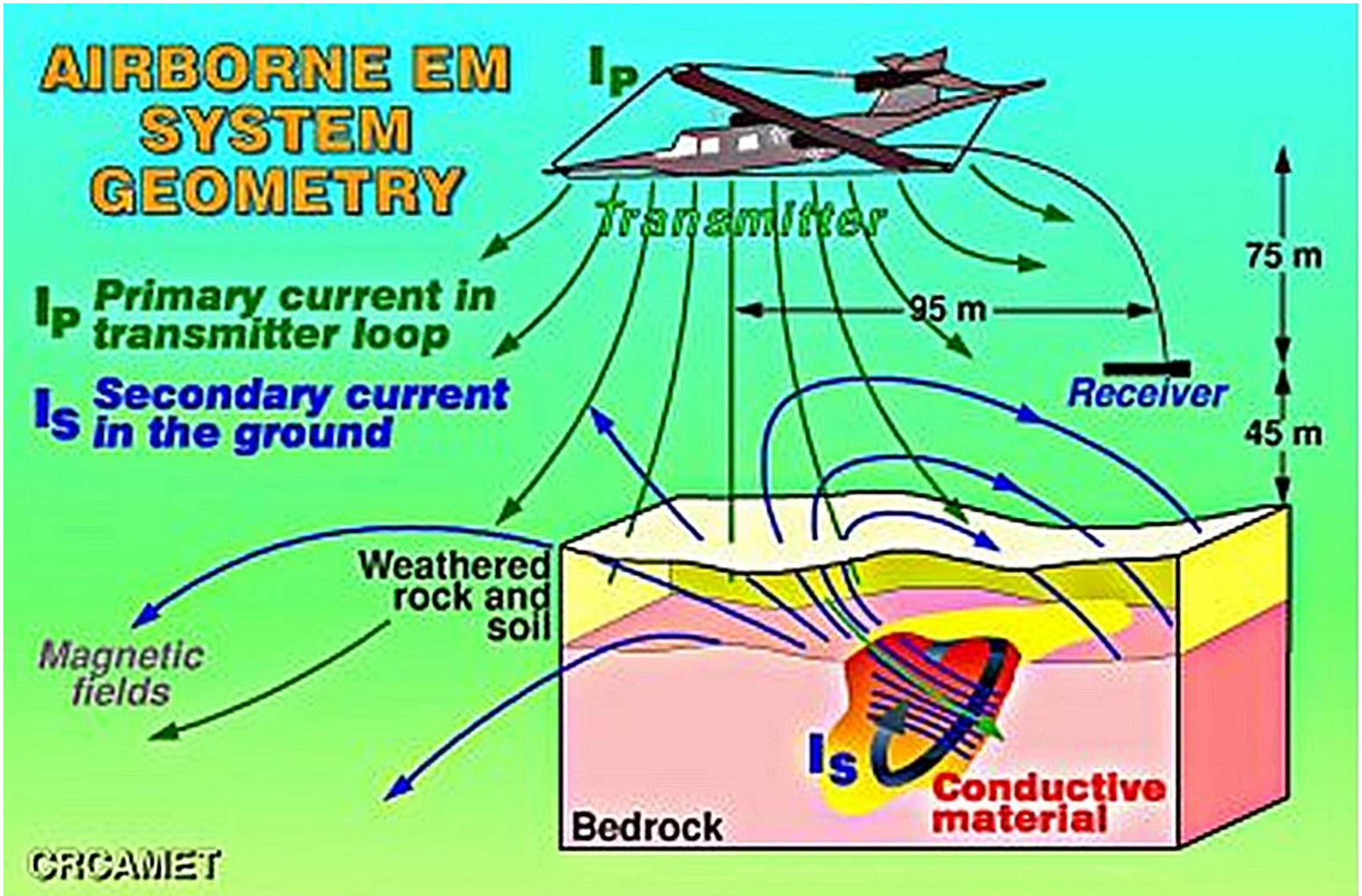
  → Large induced currents in conductors

- **Secondary Fields:**
  Induced currents in conductors produce secondary magnetic fields

- **Receiver (Rx):**
  Measures magnetic fields

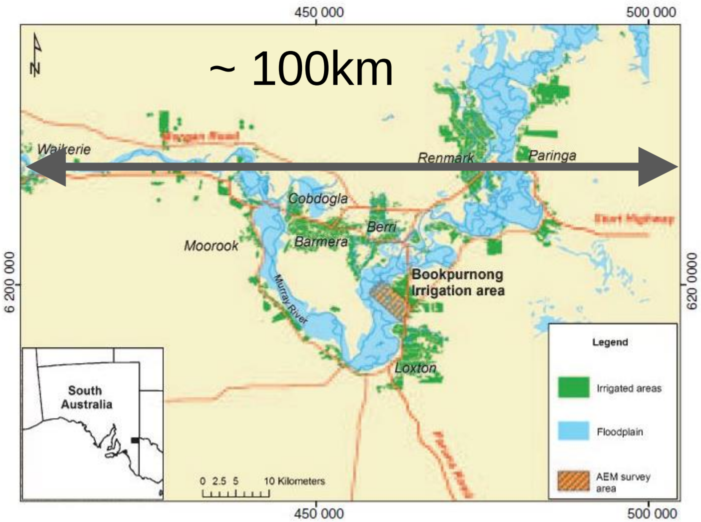


All Together

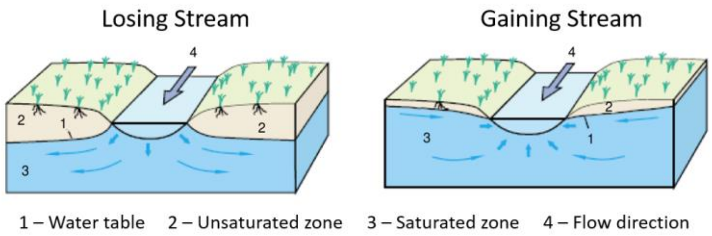


Motivation for Airborne EM

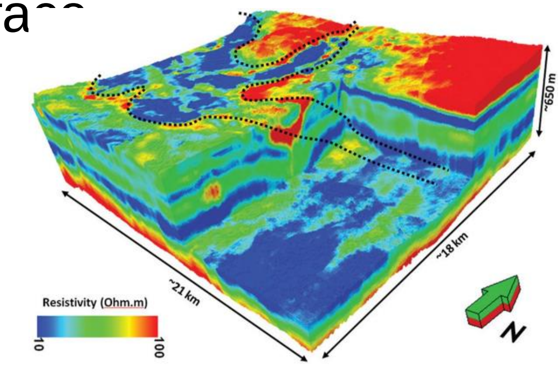
Large areas to be covered



Groundwater



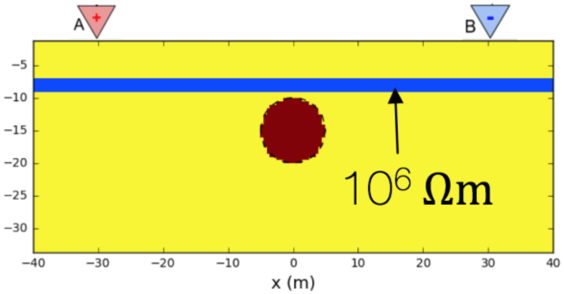
High resolution near surface



Rugged terrain

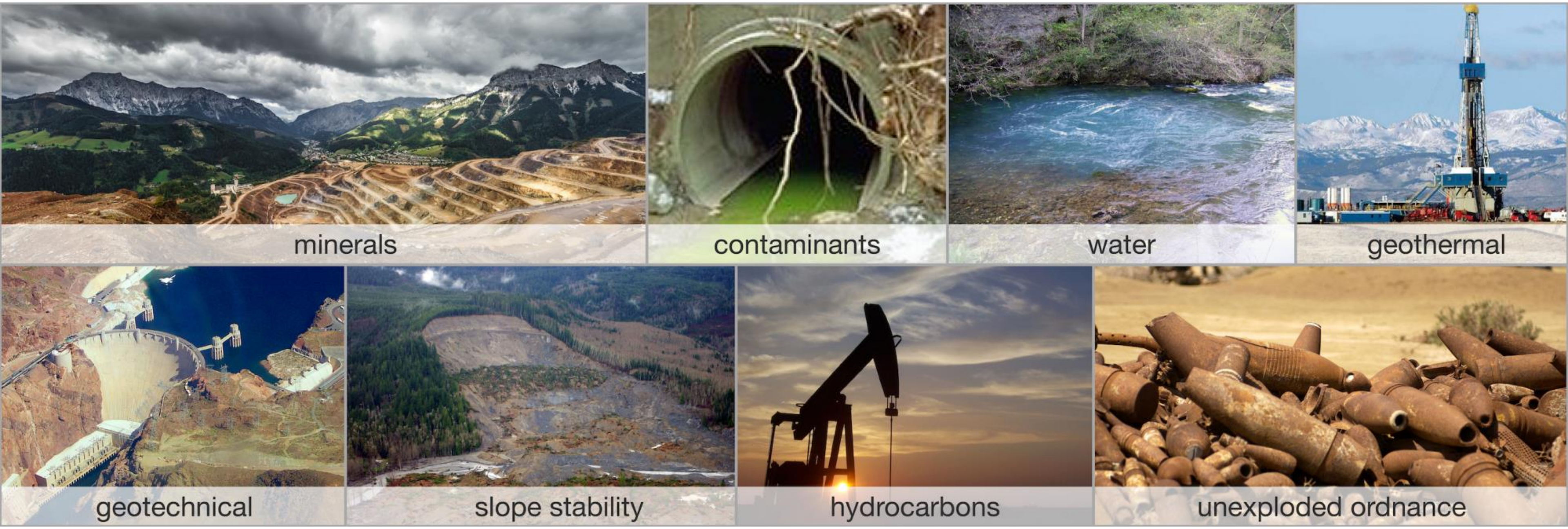
Minerals

Shielding problem



Many applications

Electromagnetics can be used for …

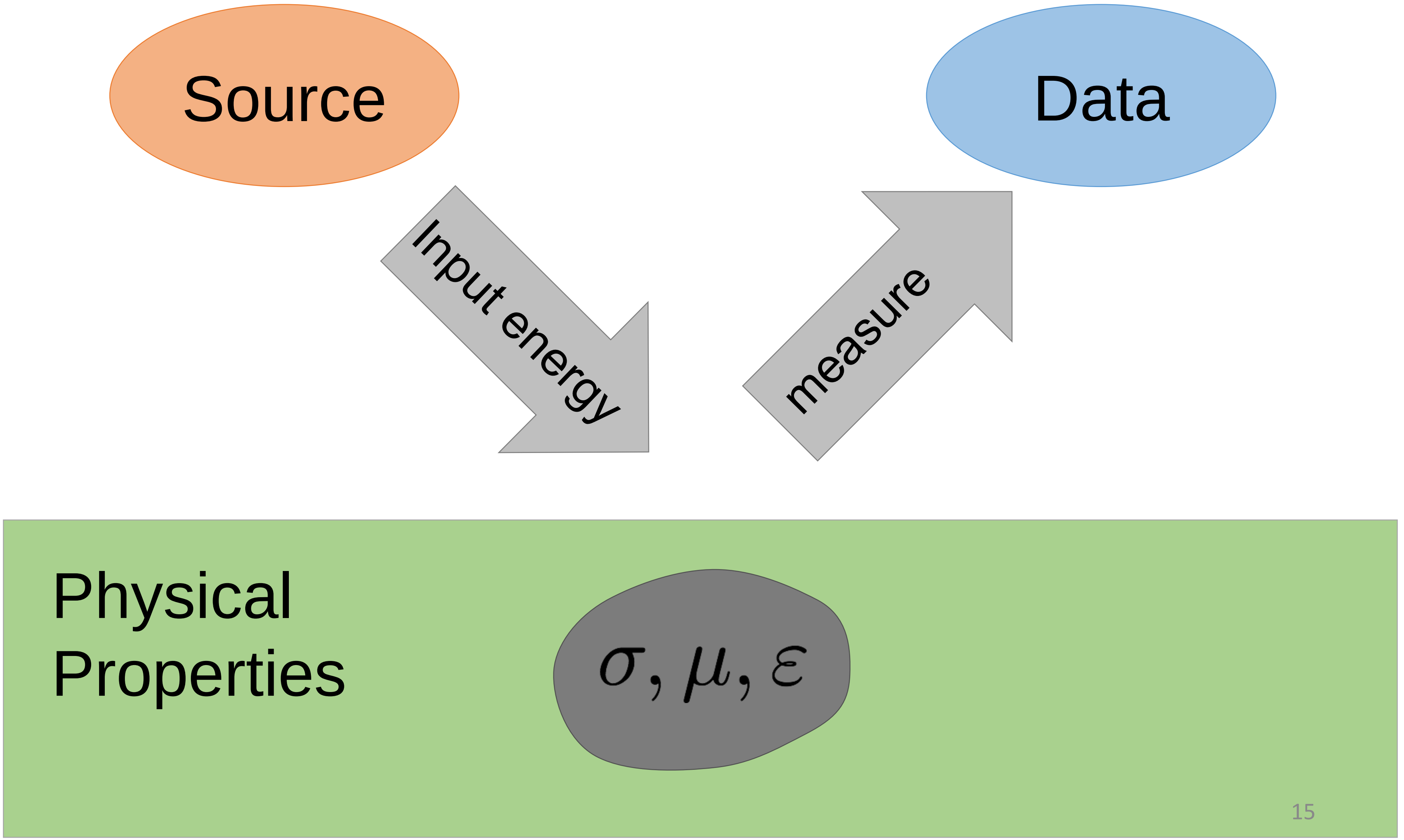


Physical properties

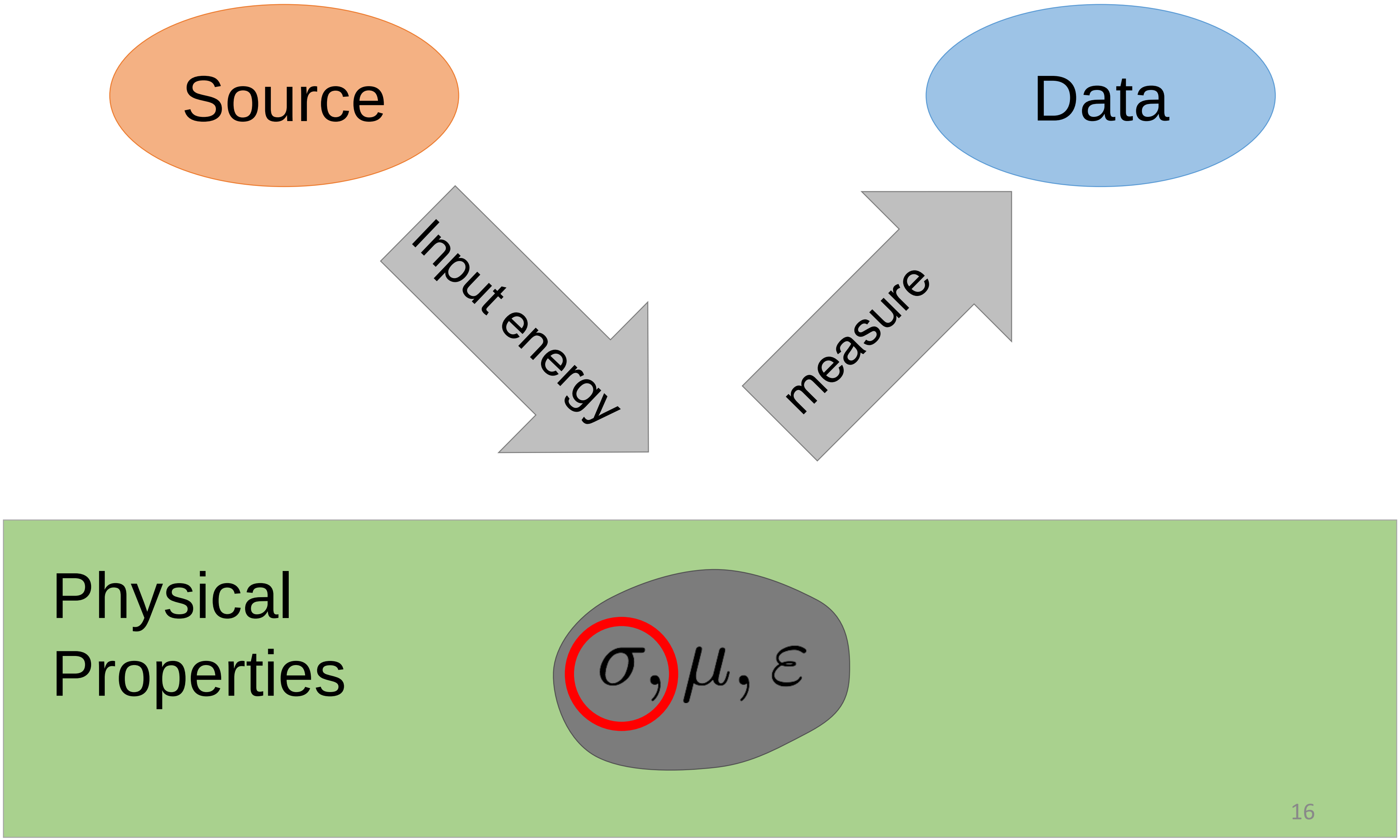
Reading on the GPG:

https://gpg.geosci.xyz/content/electromagnetics/electromagnetic_physical_properties.html

EM Survey & Physical Properties



EM Survey & Physical Properties



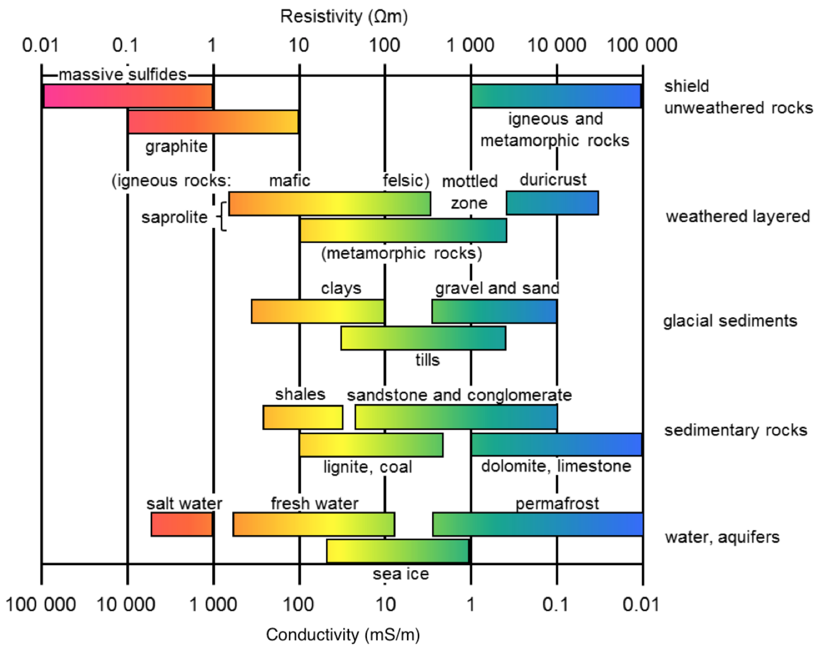
Electrical conductivity

σ: Conductivity [S/m]
ρ: Resistivity [Ωm]

$$\sigma = 1/\rho$$

- Varies over many orders of magnitude

- Depends on many factors:
  - Rock type
  - Porosity
  - Connectivity of pores
  - Nature of the fluid
  - Metallic content of the solid matrix



Basic Principles: Ampere's Law

Reading on the GPG:

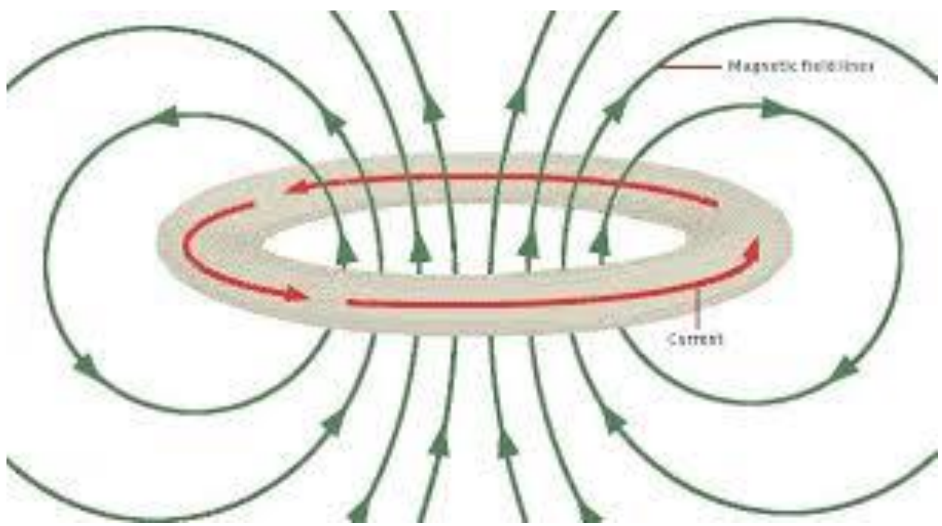
https://gpg.geosci.xyz/content/electromagnetics/electromagnetic_basic_principles.html

Ampere’s Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

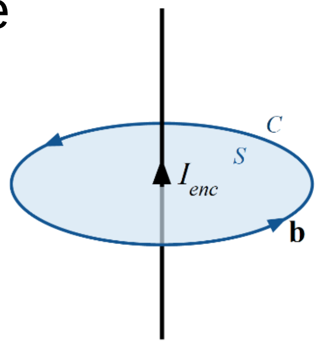
- Relationship between density of electric currents (**J**) and magnetic fields (**H**)

  → Electric current produces magnetic fields



Ampere's Law

Wire



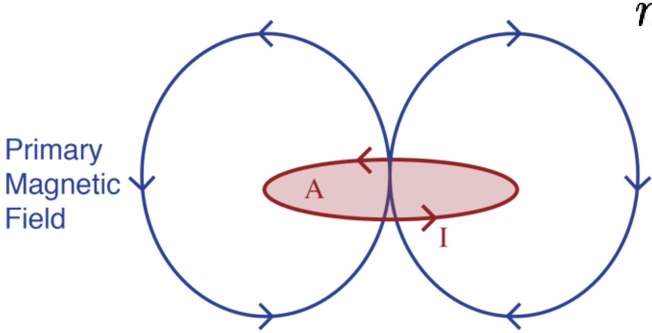
$$\mathbf{B} = \frac{\mu_0 I_{enc}}{2\pi r}\hat{\phi}$$

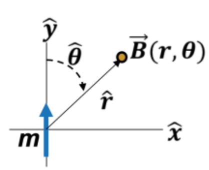
Right hand rule

Current loop

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$$

$$m = IA$$





Wire:

- Right-hand rule
- Magnetic field proportional to:
  - Current
  - $1/r$

Loop:

- Right-hand rule
- Magnetic field proportional to
  - Current X Area
  - $1/r^3$ far enough away (dipole field)

Ampere's Law: Dipolar Field

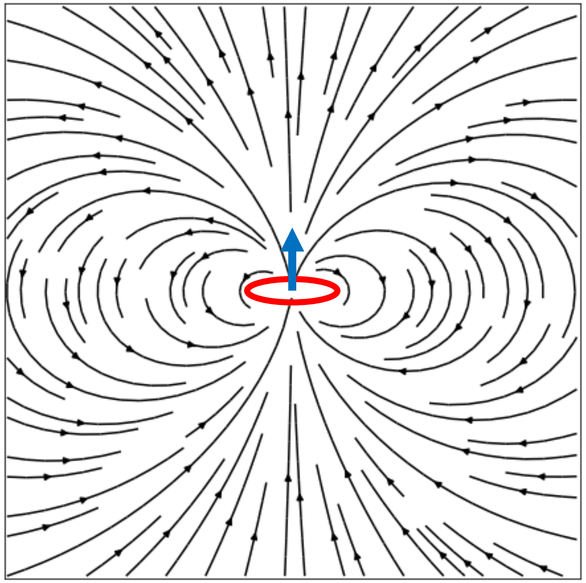
- Dipole moment:

$$m = IA$$

- Field due to a single dipole

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{3\vec{r}(\vec{m}\cdot\vec{r})}{r^5} - \frac{\vec{m}}{r^3}\right)$$

- Here current is CCW so by right-hand rule dipole moment is up



Basic Principles: Faraday’s Law

Reading on the GPG:

https://gpg.geosci.xyz/content/electromagnetics/electromagnetic_basic_principles.html

Faraday’s Law: Differential Form

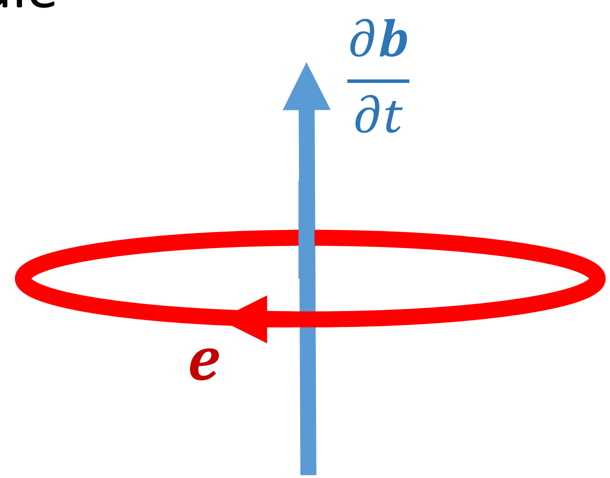
- Time-dependent (or frequency-dependent) magnetic fields produce electric fields

Lenz’ Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$$

- Use “left-hand rule”



Faraday’s Law: Integral Form

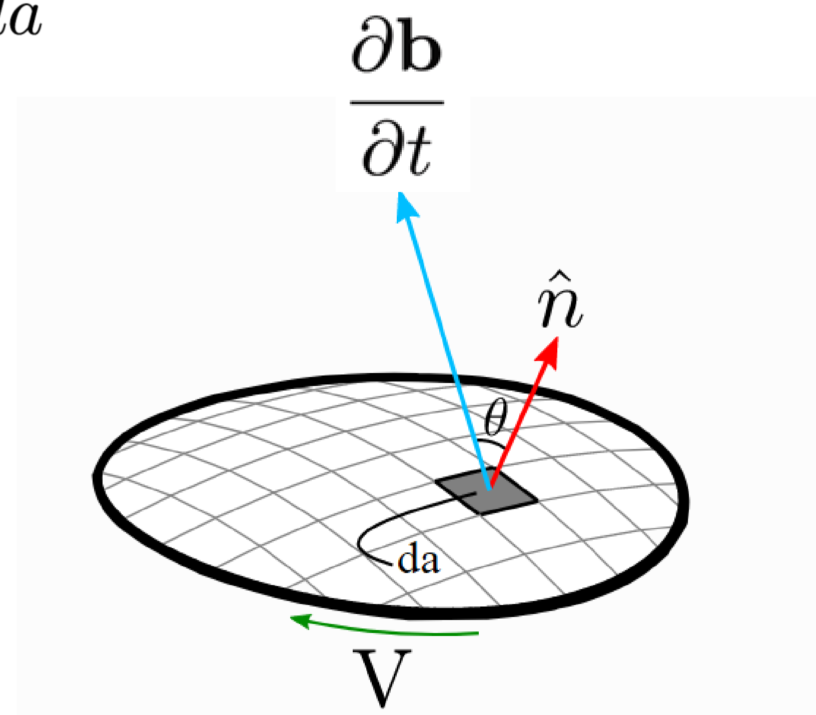
- Time varying magnetic flux induces voltage (EMF) in a current loop

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt}$$

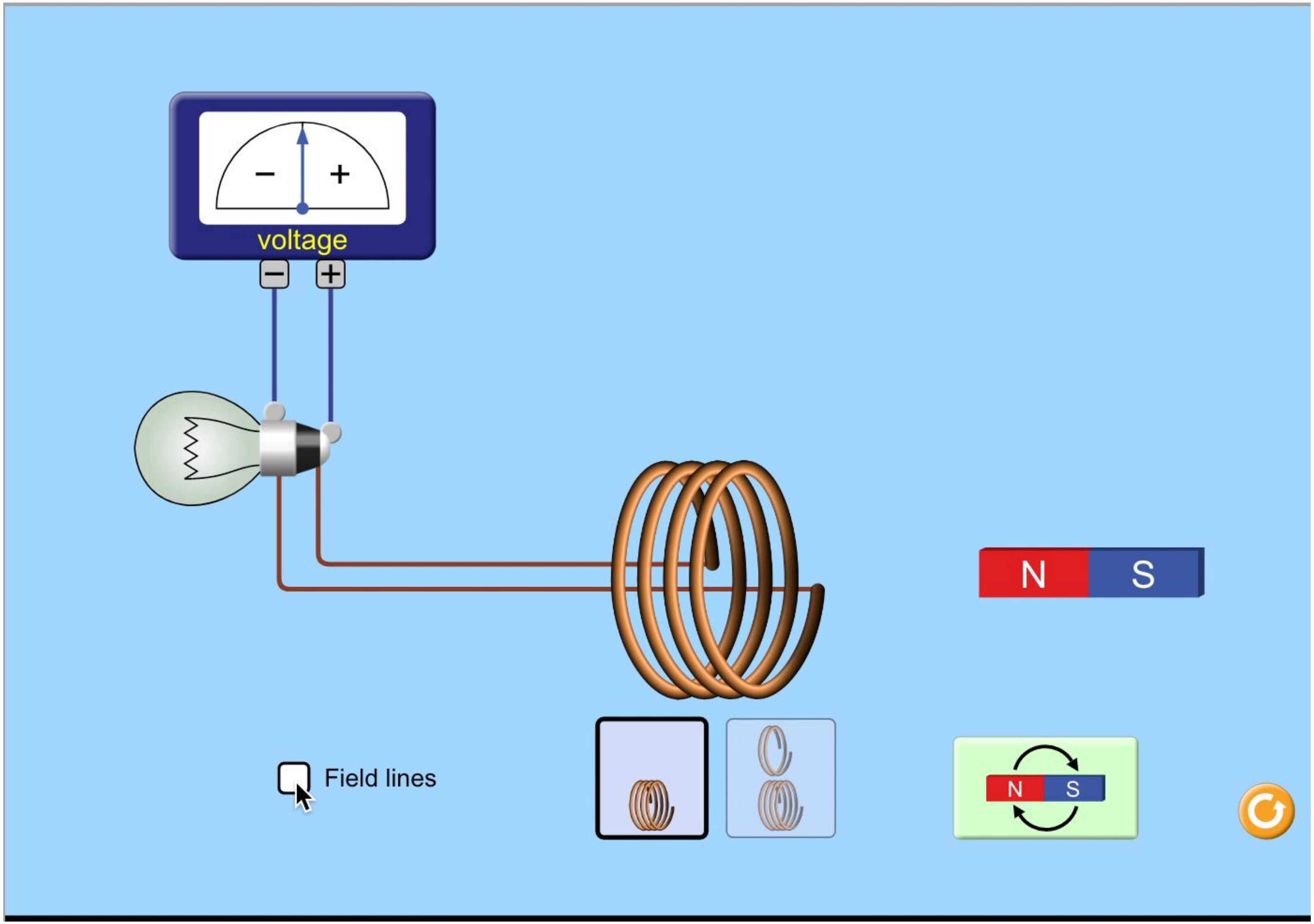
- Magnetic flux is given by:

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \; da$$

- Use left-hand rule



Faraday's Law: Flux from magnet

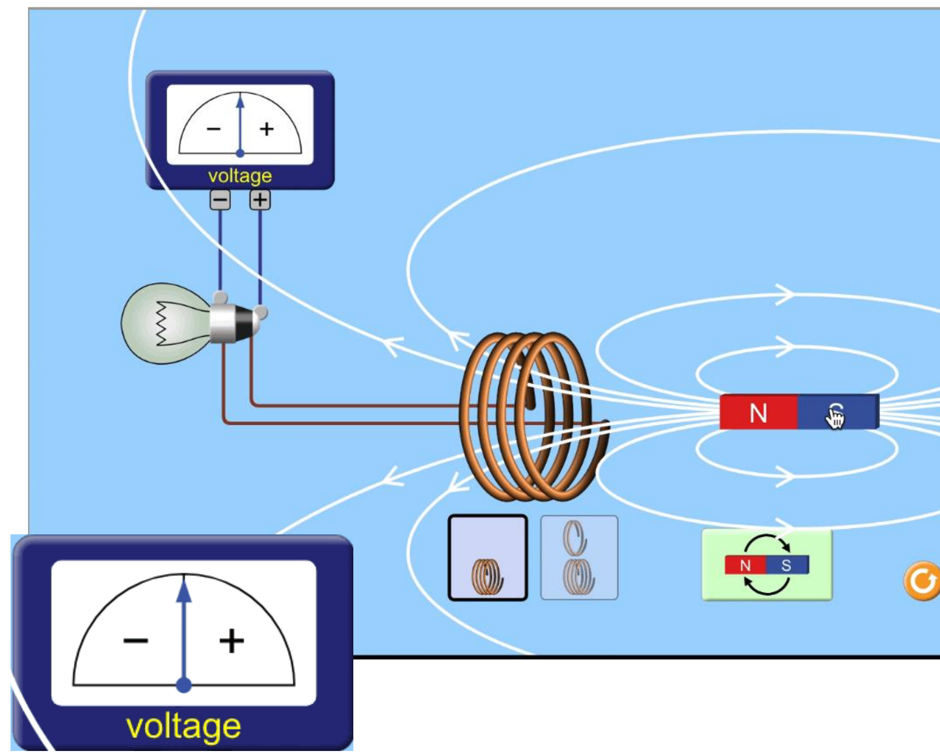


Faraday's Law: Flux from magnet

Magnetic Flux

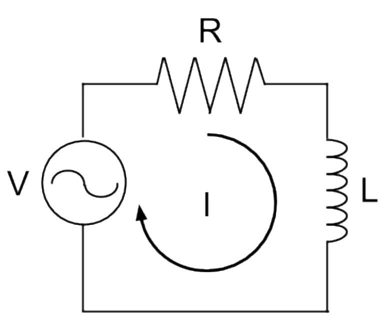
$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

$$\phi_b : \text{constant}$$



Induced EMF

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt} = 0$$



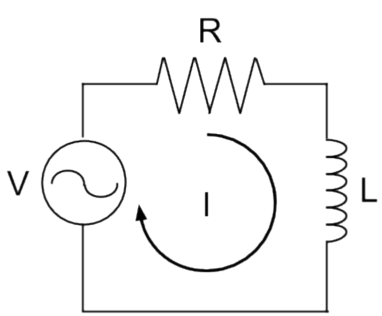
Faraday’s Law: Flux from magnet

Magnetic Flux

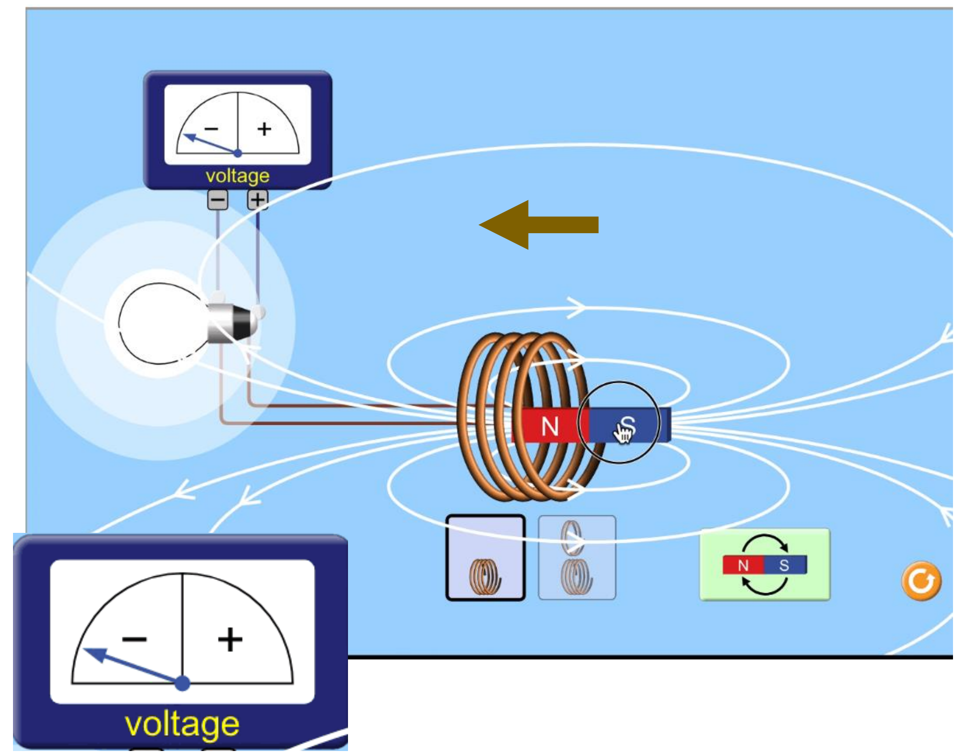
$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \; da$$

Induced EMF

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt} < \mathbf{0}$$



ϕ_b :



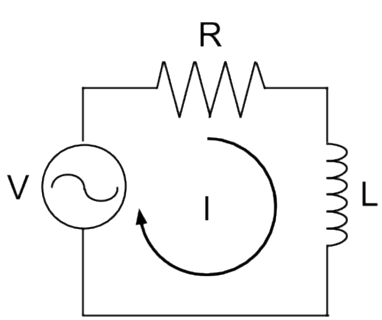
Faraday's Law: Flux from magnet

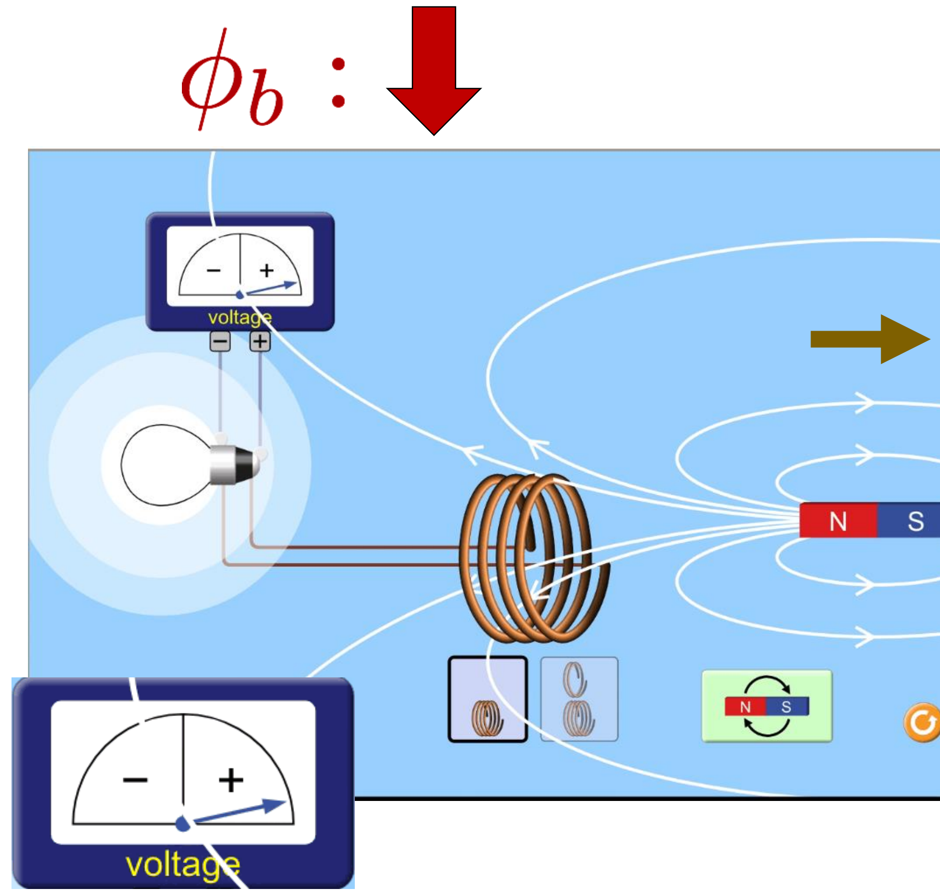
Magnetic Flux

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt} > 0$$





Basic Principles: Ohm's Law

Reading on the GPG:

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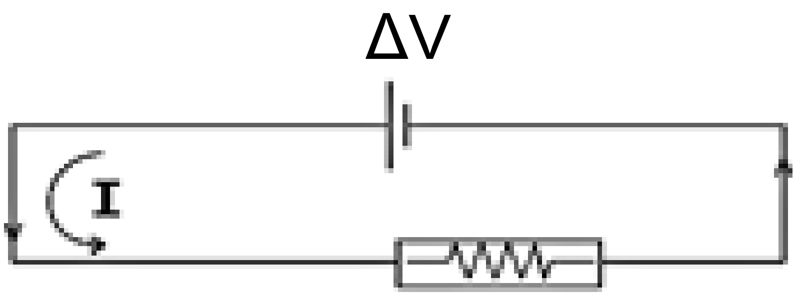
DC resistivity and Ohm's Law

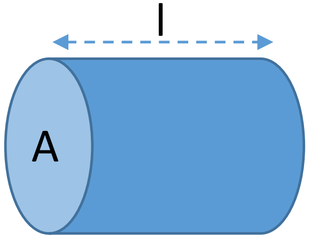
- Electric circuit:



$$\vec{J} = \sigma \vec{E}$$

- Ohm's Law: $$\Delta V = IR$$

- Resistivity: $$\rho = R\frac{A}{l}$$

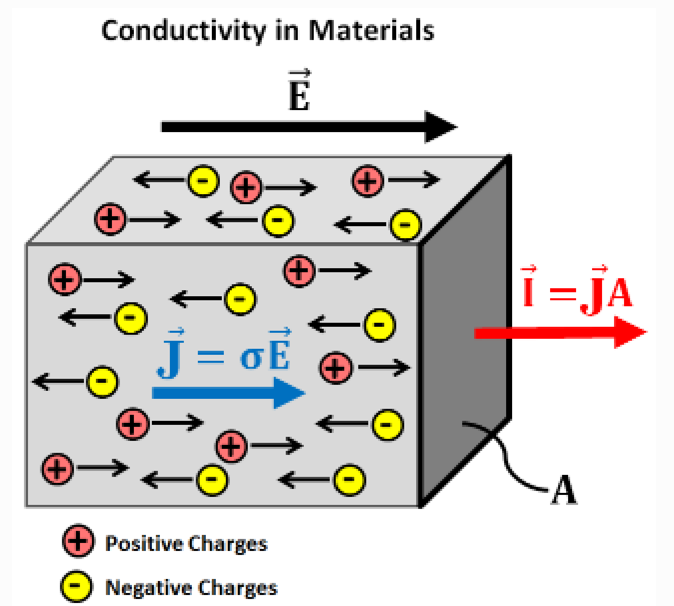


General Ohm's Law

- Relates the electric field to density of electric current in a material
- Electric field and current in same direction
- Electric fields in conductive materials will produce stronger currents
- Electric fields in resistive materials produce very weak currents

$$\vec{J} = \sigma \vec{E}$$

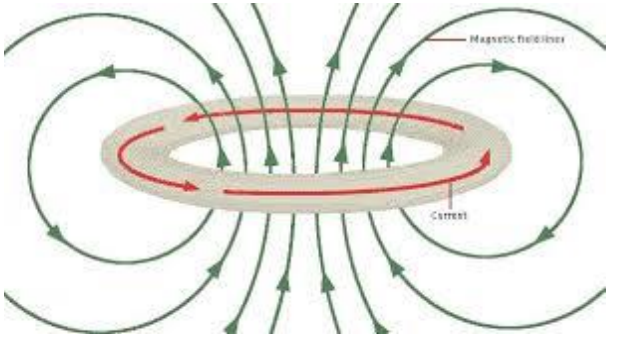
$$\rho = \frac{1}{\sigma}$$



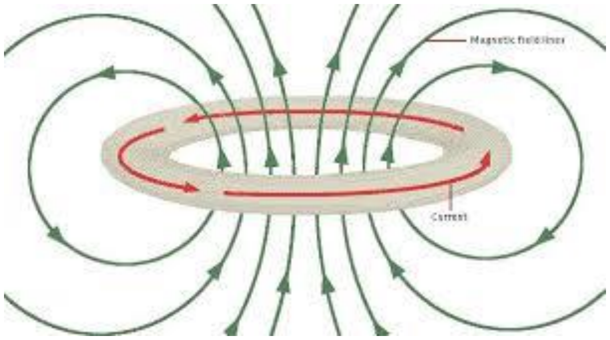
Recap

Ampere's Law: $\nabla \times \mathbf{H} = \mathbf{J}$

- Currents produce magnetic fields
- Right-hand rule



Recap

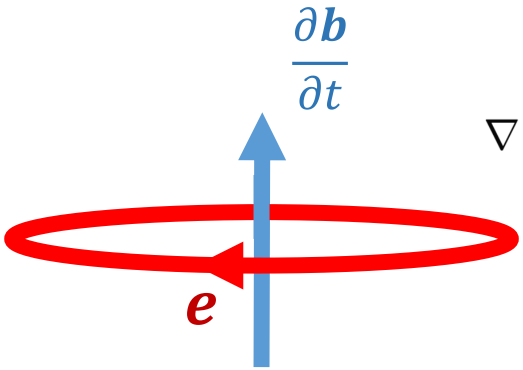


Ampere's Law: $\nabla \times \mathbf{H} = \mathbf{J}$

- Currents produce magnetic fields
- Right-hand rule

Faraday's Law

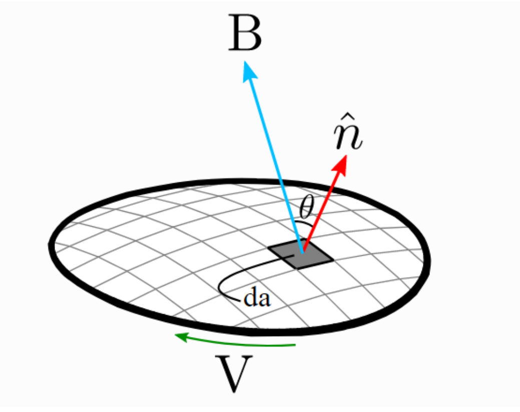
- Time/frequency varying magnetic fields produce electric fields
- Time/frequency varying magnetic flux generates voltage in wire loops
- Left-hand rule



Lenz' Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt}$$



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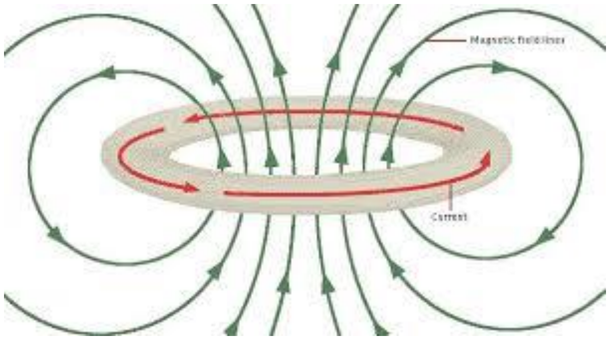

Ampere's Law: $\nabla \times \mathbf{H} = \mathbf{J}$

- Currents produce magnetic fields
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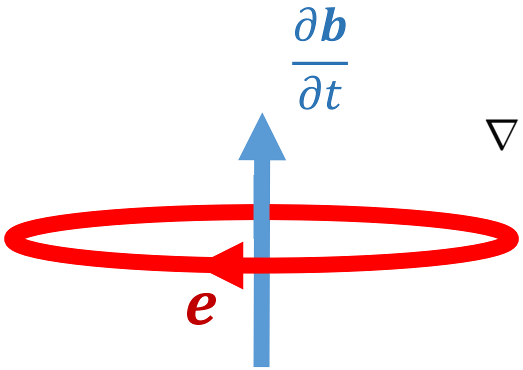

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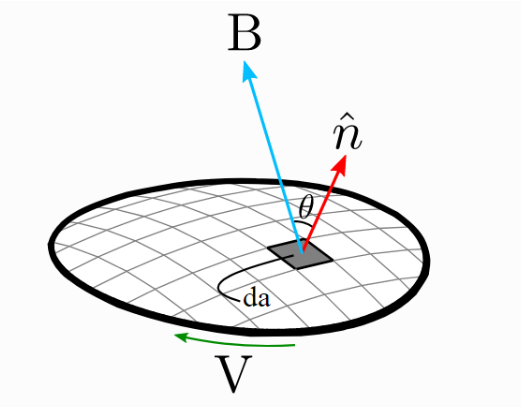

Ohm's Law

- Current resulting from electric fields depends on conductivity/resistivity

$$\vec{J} = \sigma \vec{E}$$

Basic Principles:
EM Experiment Revisited

Reading on the GPG:

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Basic Experiment

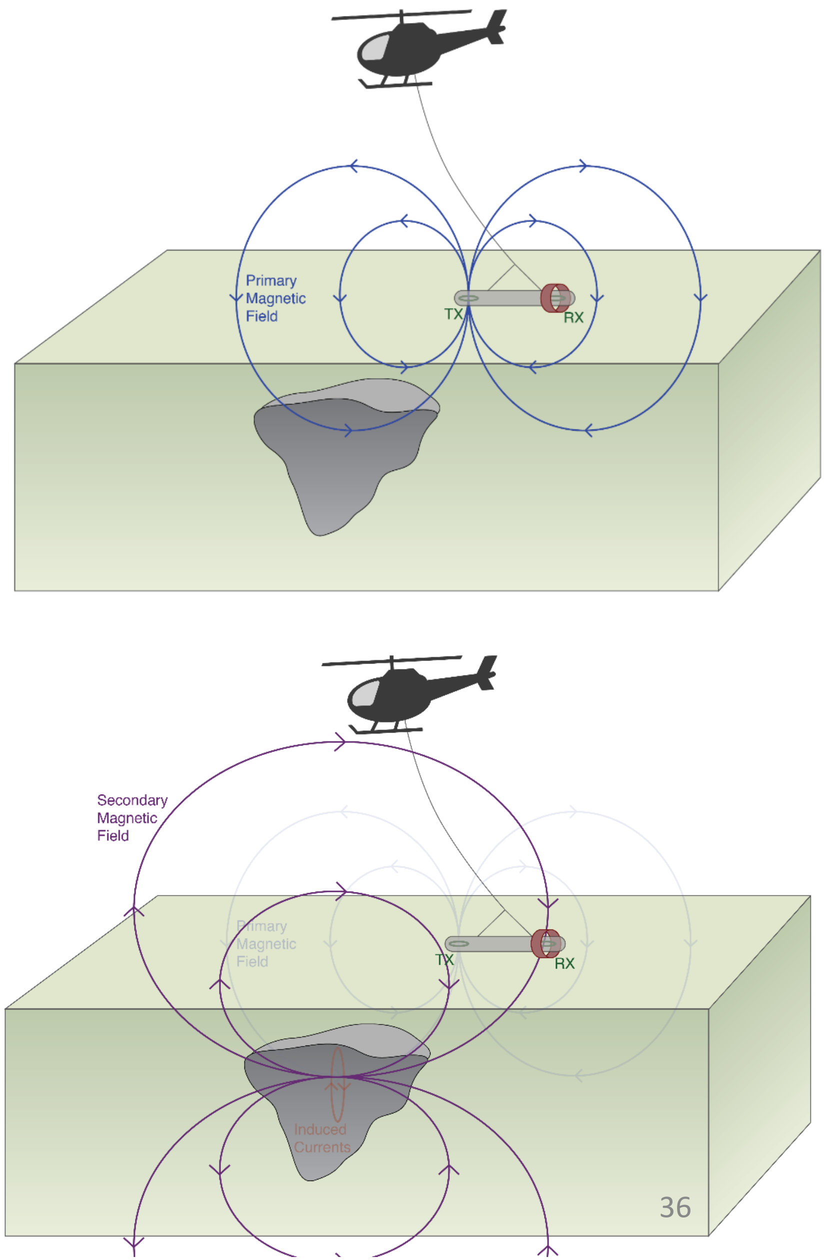
- **Source (Tx):**
  Current loop makes primary magnetic field

- **Induction:**
  Time-varying magnetic fields induce electric fields everywhere

  → Large induced currents in conductors

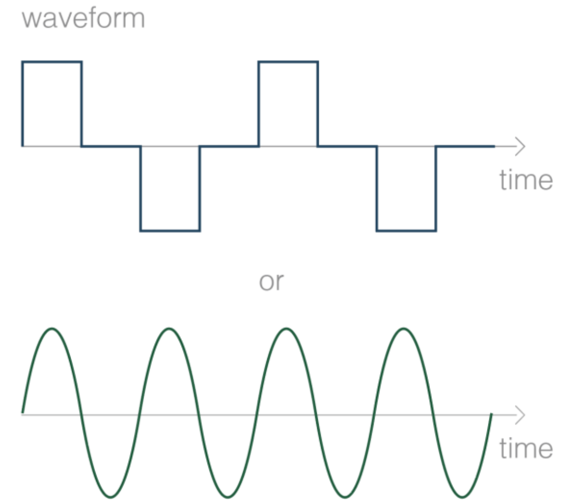
- **Secondary Fields:**
  Induced currents in conductors produce secondary magnetic fields

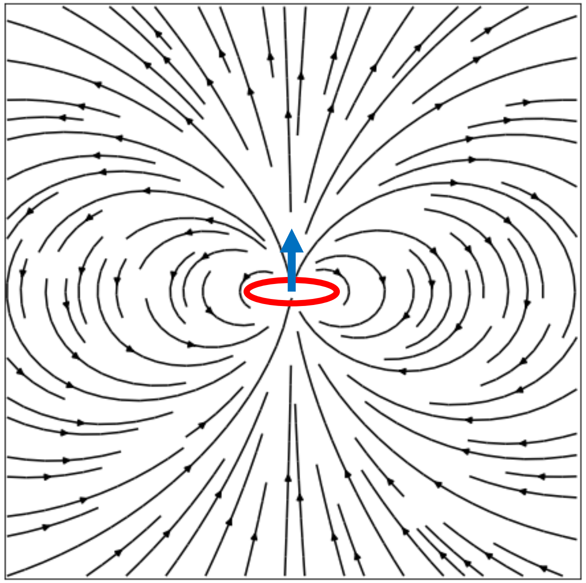
- **Receiver (Rx):**
  Measures magnetic fields

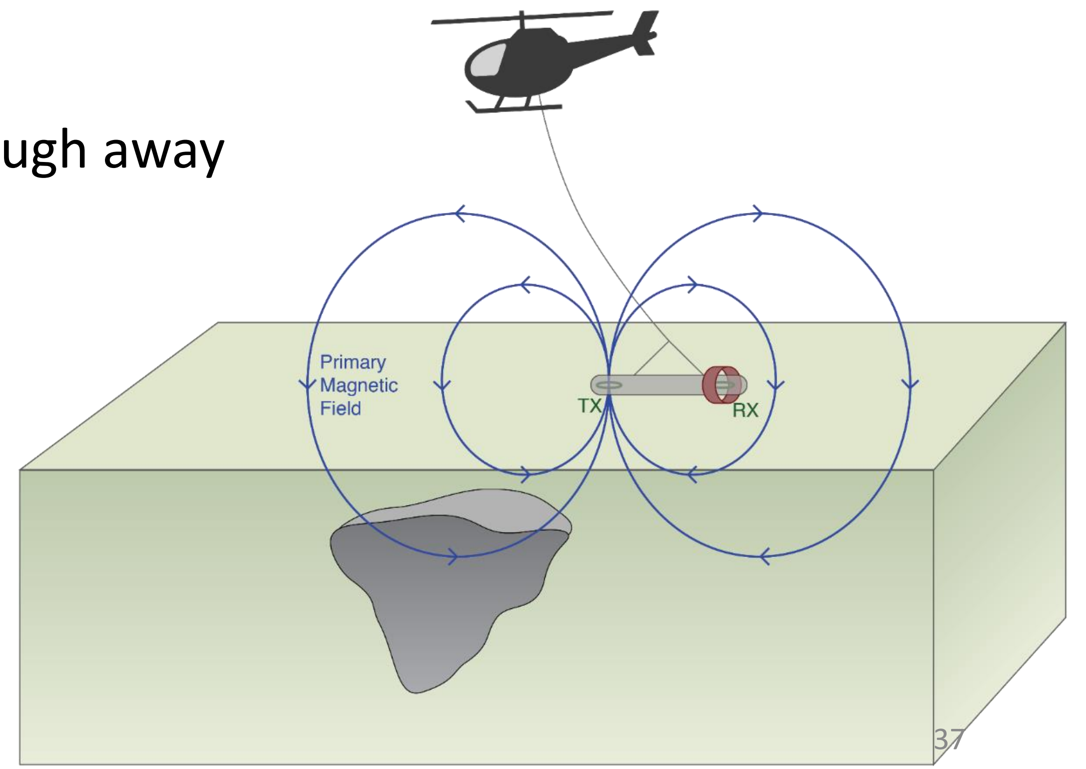


Transmitter

- Transmitter is a current loop
- Currents produce primary magnetic field (Ampere)
- Current and primary field direction related by right hand rule
- Primary field dipolar far enough away

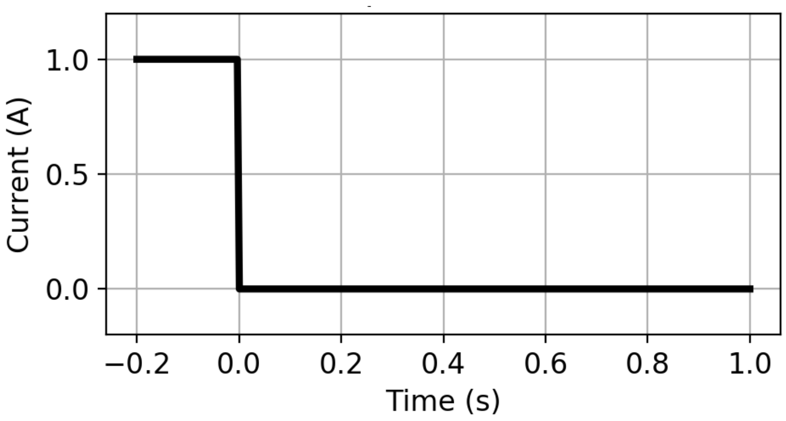




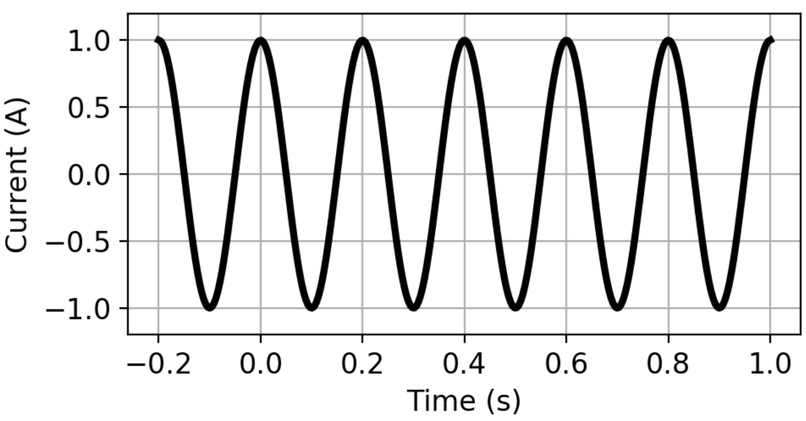


Transmitter Waveforms

Time Domain: Transient Pulse



Frequency Domain: Harmonic

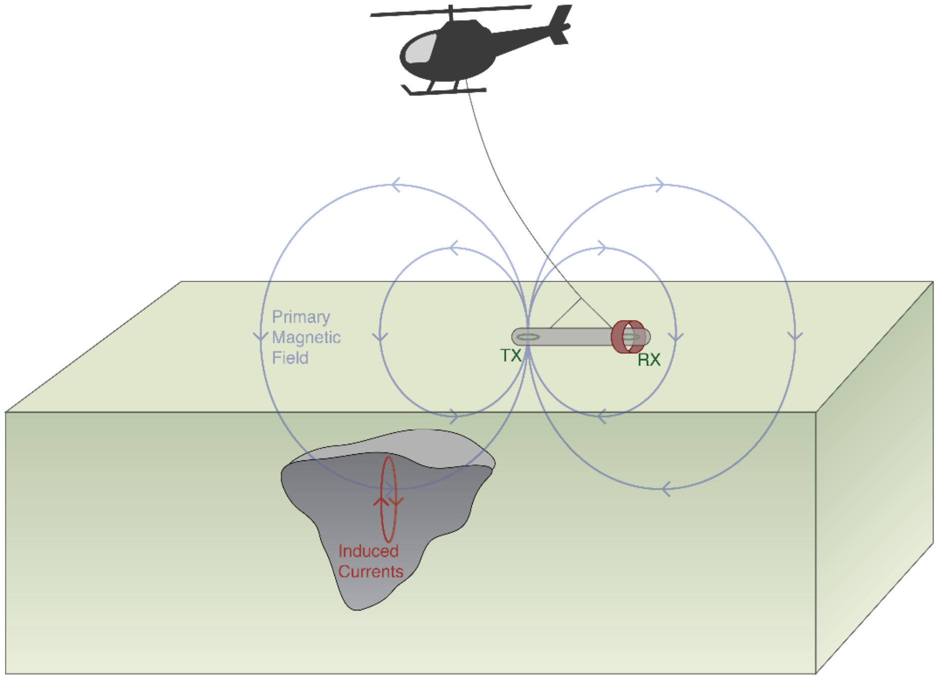


Induction and Induced Currents

- Time-varying/harmonic magnetic fields induce electric fields (Faraday)
- Change in magnetic field and electric field direction related by left-hand rule

Lenz' Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$



Induction and Induced Currents

- Time-varying/harmonic magnetic fields induce electric fields (Faraday)

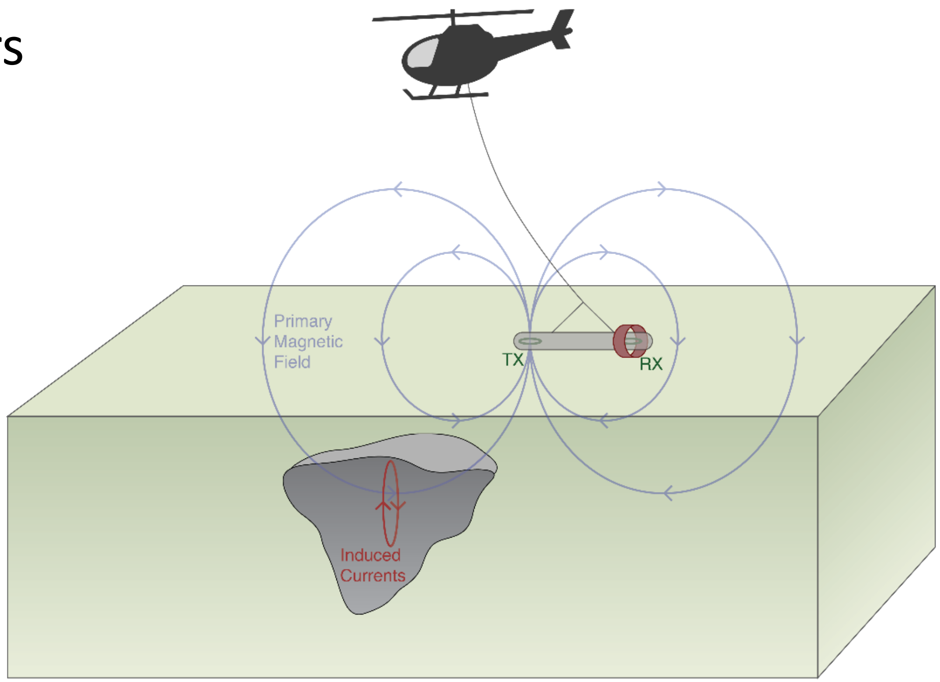
- Change in magnetic field and electric field direction related by left-hand rule

- Induced electric fields (Ohm’s law)
  - Large induced currents in conductors
  - Weak induced currents in resistors

$$\vec{J} = \sigma \vec{E}$$

Lenz’ Law

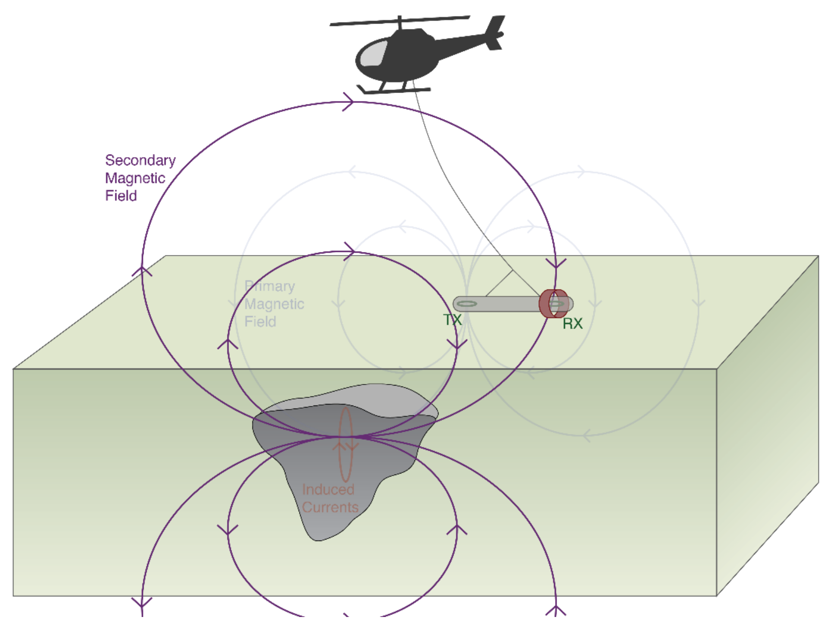
$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$



Secondary Fields

- Induced current produce secondary magnetic field (Ampere)
  - Strong secondary fields from conductors
  - Weak secondary fields from resistors

- Current and secondary field direction related by right hand rule



Receivers

- Secondary fields (and primary fields) are time-varying/harmonic

  → Change in magnetic flux through receiver loop

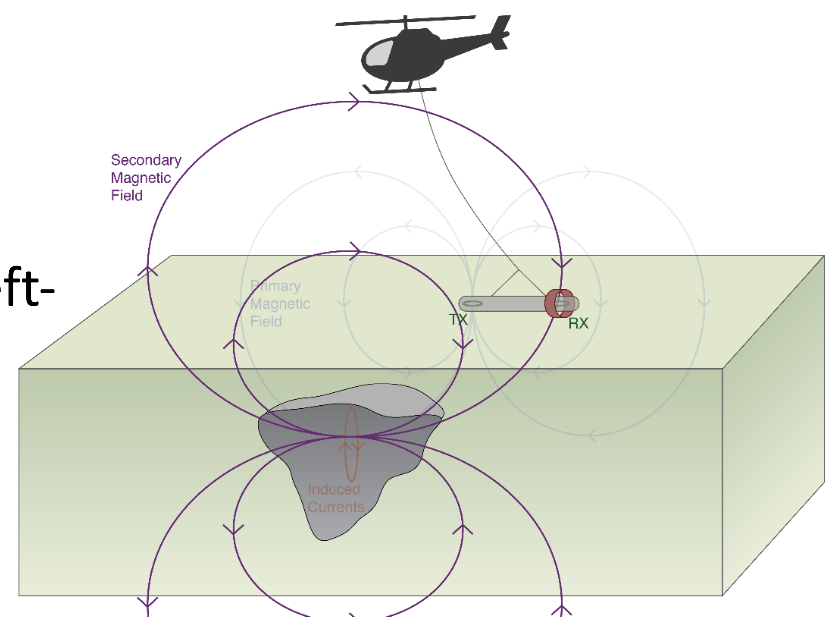
  → Induces voltage in receiver loop (Faraday)

- Only measures component of the field normal to the receiver loop

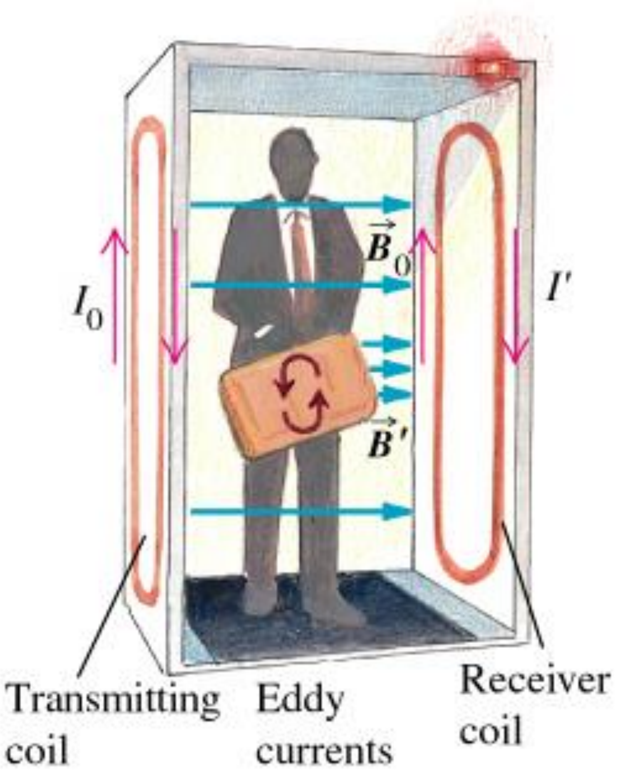
- Voltage and change in flux related by left-hand rule

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

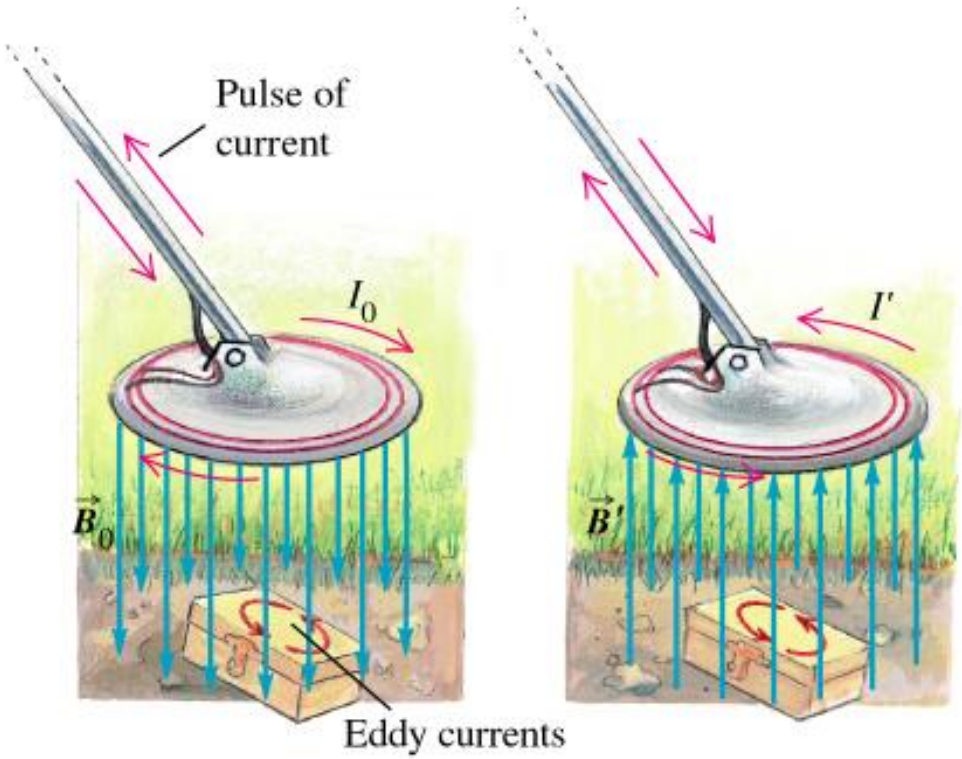
$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt}$$



Other EM Applications



Security scan



Metal detector

Unit Activities

- **Labs: (EM I)**
  - Monday, November 4th
  - Tuesday, November 5th
- **Labs: (EM II)**
  - Monday, November 18th
  - Tuesday, November 19th
- **TBL:**
  - Wednesday, November 15th
- **Quiz:**
  - Wednesday, November 15th