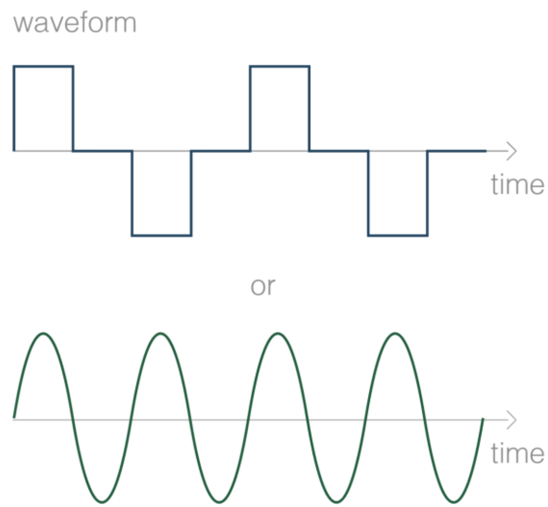
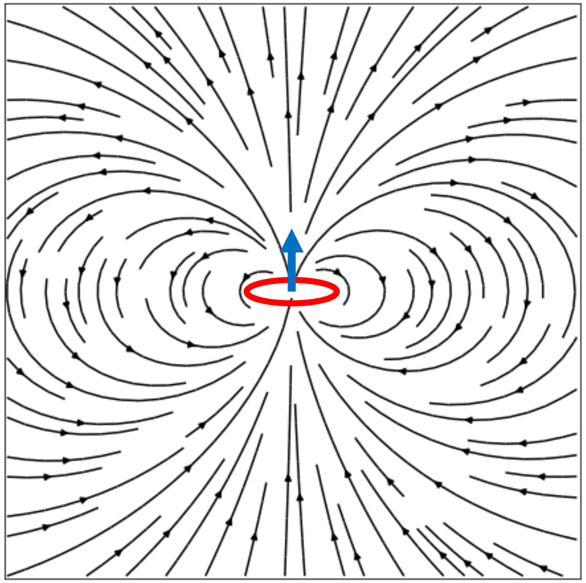
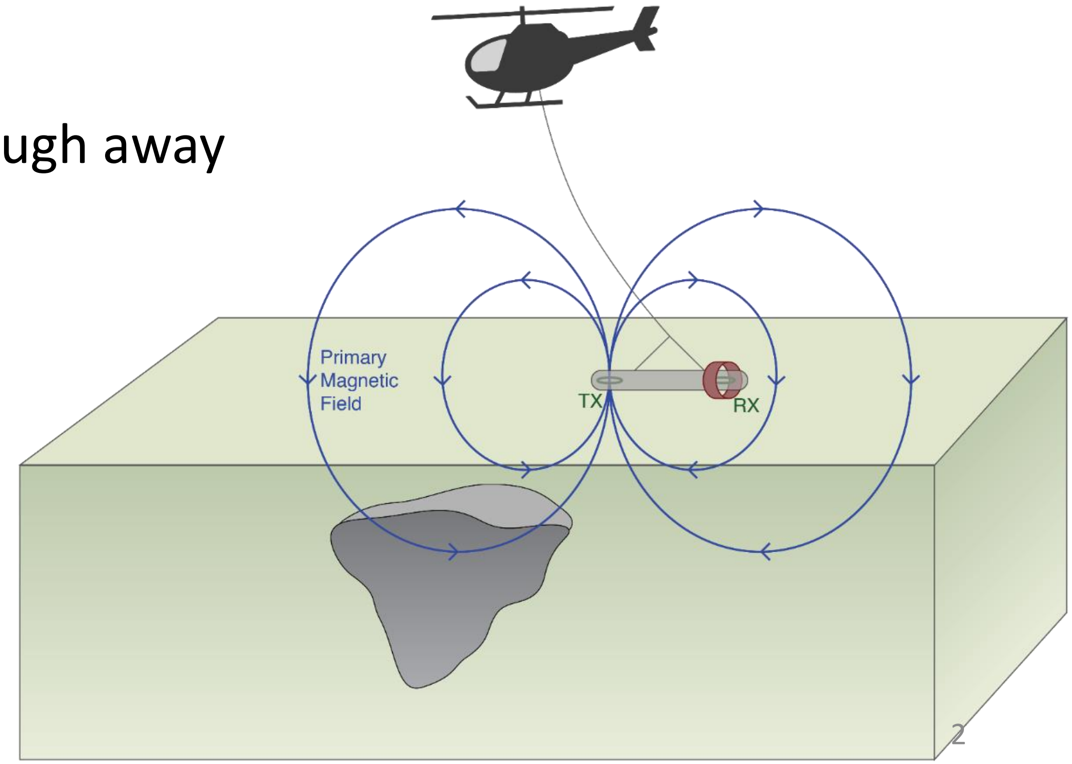
From last time

Transmitter

- Transmitter is a current loop
- Currents produce primary magnetic field (Ampere)
- Current and primary field direction related by right hand rule
- Primary field dipolar far enough away





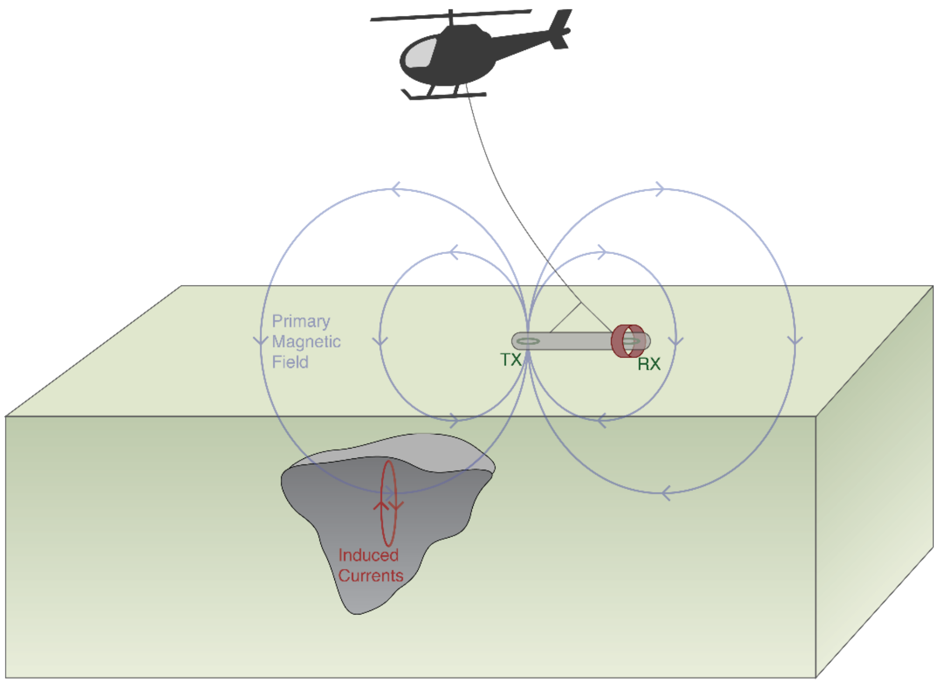


Induction and Induced Currents

- Time-varying/harmonic magnetic fields induce electric fields (Faraday)
- Change in magnetic field and electric field direction related by left-hand rule

Lenz' Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$



Induction and Induced Currents

- Time-varying/harmonic magnetic fields induce electric fields (Faraday)

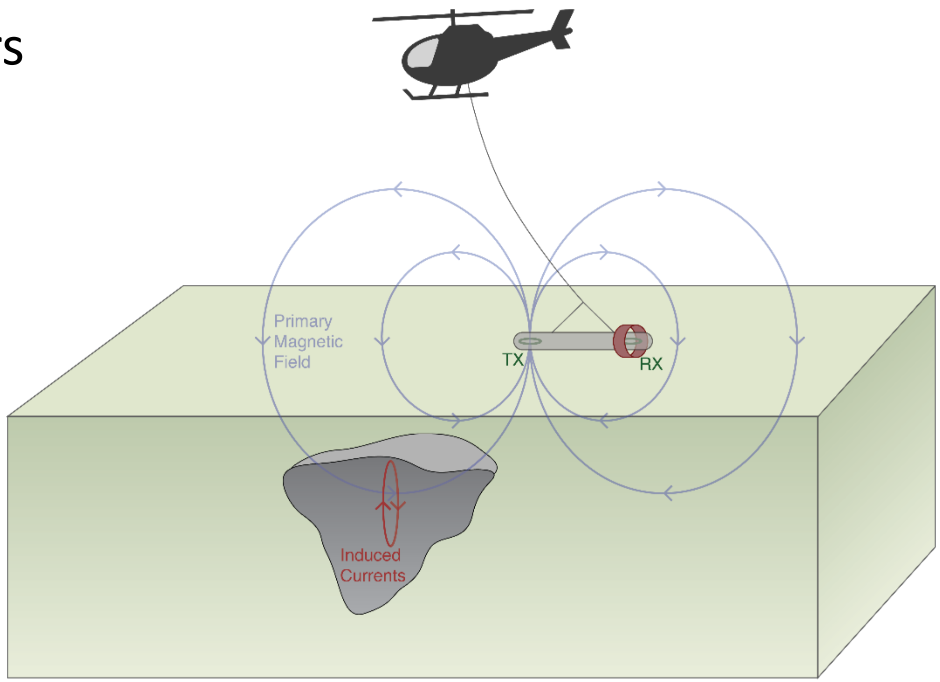
- Change in magnetic field and electric field direction related by left-hand rule

- Induced electric fields (Ohm's law)
  - Large induced currents in conductors
  - Weak induced currents in resistors

$$\vec{J} = \sigma \vec{E}$$

Lenz' Law

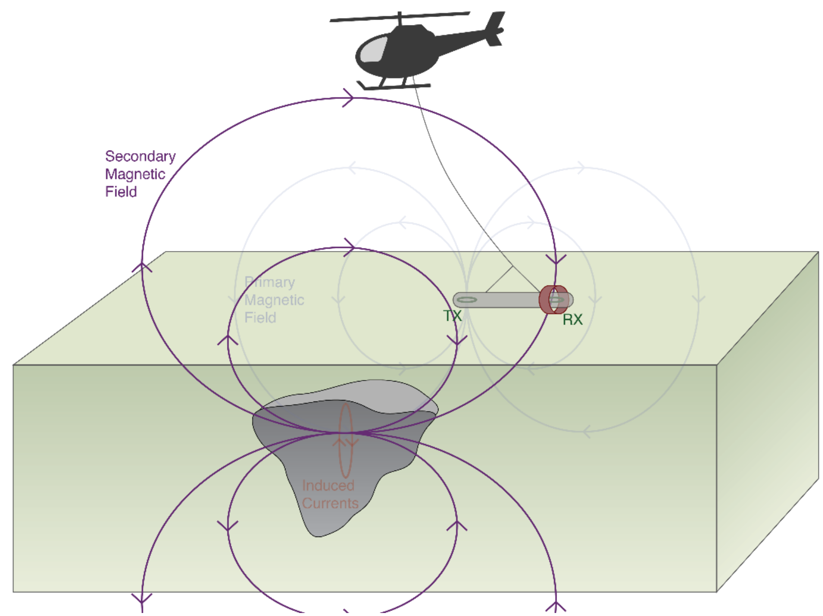
$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$



Secondary Fields

- Induced current produce secondary magnetic field (Ampere)
  - Strong secondary fields from conductors
  - Weak secondary fields from resistors

- Current and secondary field direction related by right hand rule



Receivers

- Secondary fields (and primary fields) are time-varying/harmonic

  → Change in magnetic flux through receiver loop

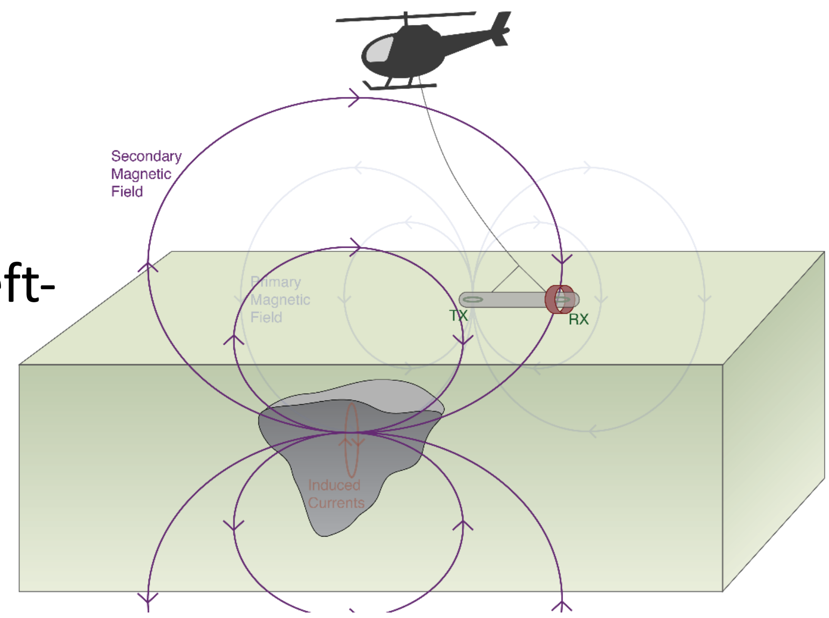
  → Induces voltage in receiver loop (Faraday)

- Only measures component of the field normal to the receiver loop

- Voltage and change in flux related by left-hand rule

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \; da$$

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt}$$

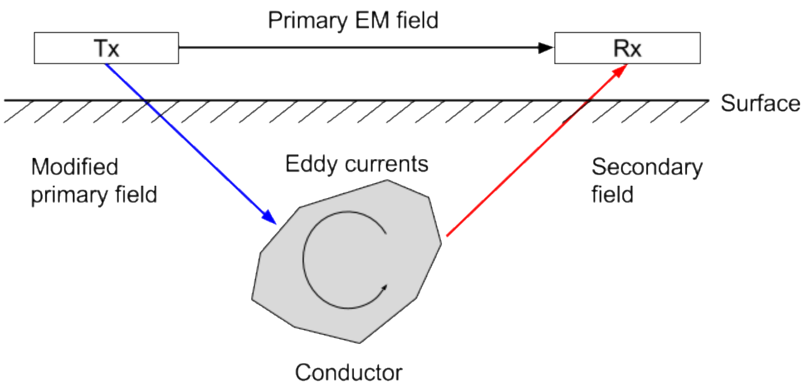


Today's Topics

- Circuit Model for EM
  - Motivation
  - 2 Loop Model for Induction
  - Coupling
  - EM Response at Receiver
  - Airborne FEM Example

A Circuit Model for EM: Motivation

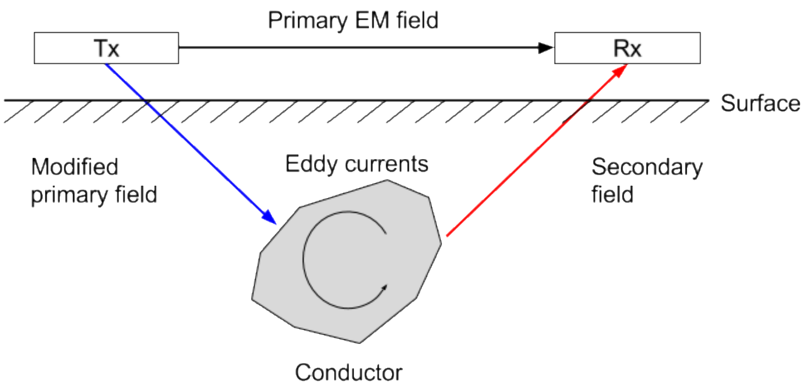
Motivation

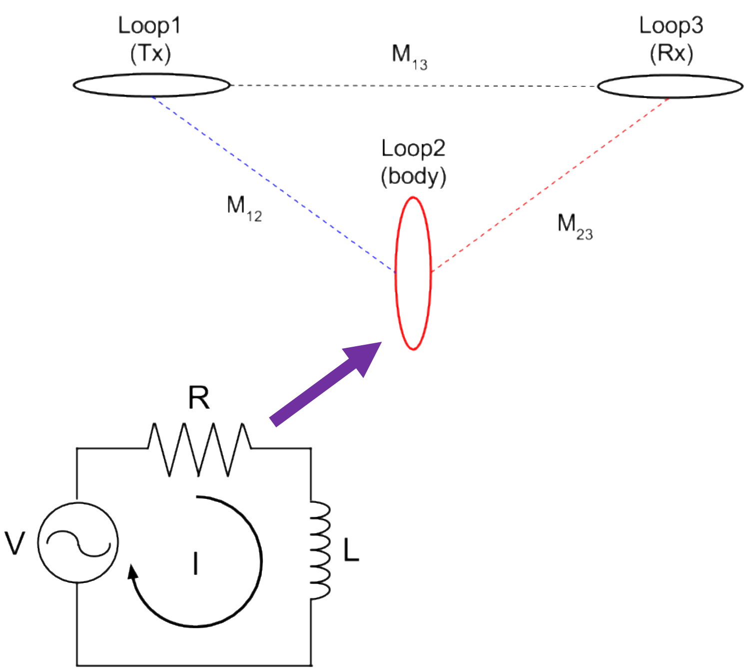


How is the excitation of the target and the data impacted by:

- The transmitter current
- The conductivity of the target
- The dimension and orientation of the target

Motivation

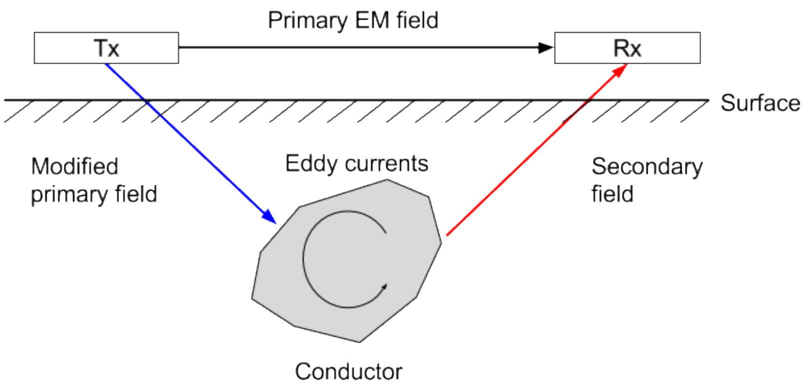


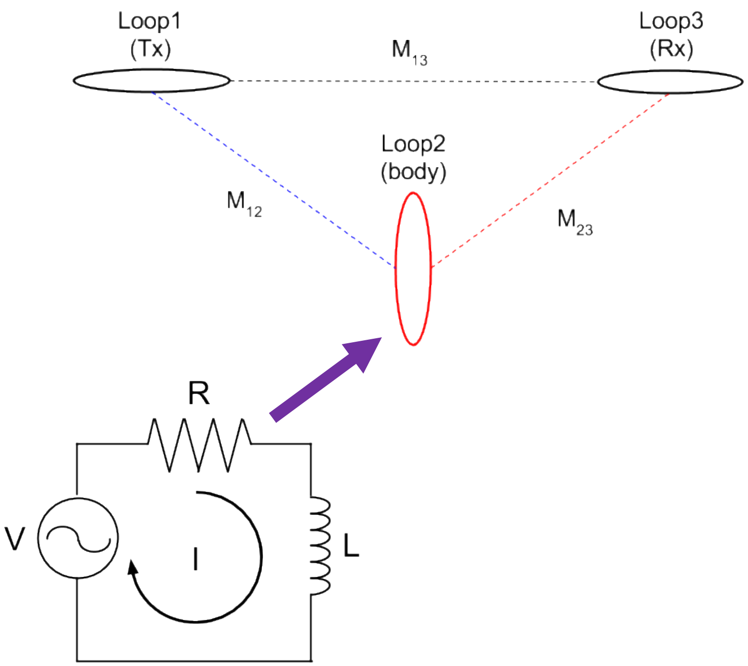


The 3 loop model:

- The target is approximated by an LR circuit
- **R** is the resistance of the circuit
- **L** is the inductance of the circuit
- For more conductive targets → L/R is bigger

Motivation





The 3 loop model:

- Primary field induces EMF in loop 2
- The EMF produces induced currents in loop 2
- Induced current produces secondary magnetic field
- Secondary field measured by receiver coil

A Circuit Model for EM:
2 Loop Model for Induction

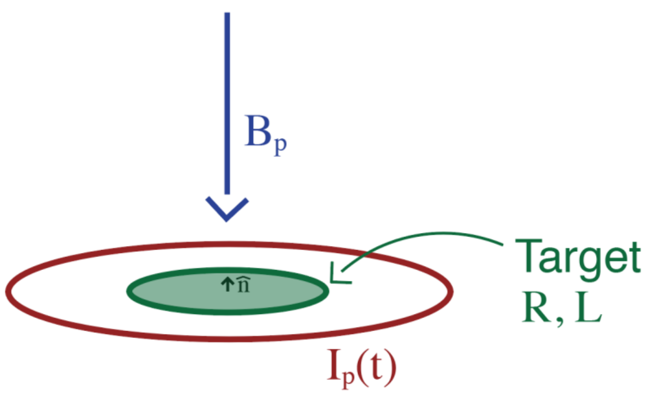
Two Loop EM App

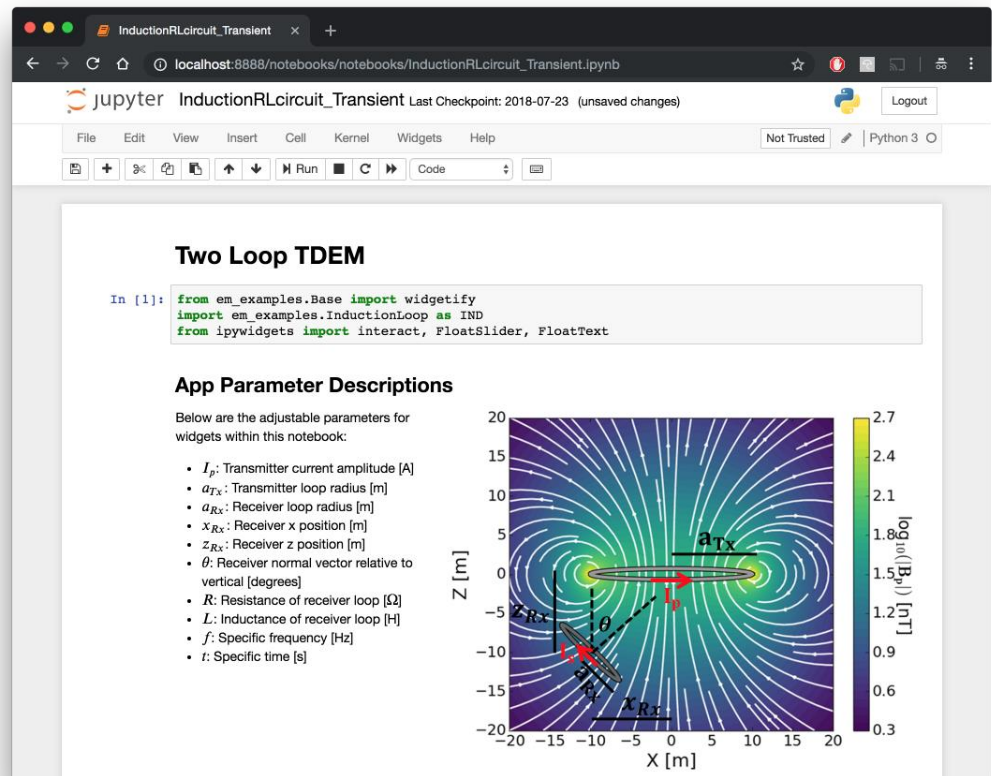
- **How does the magnetic field from a current loop induce currents in another loop?**

Lenz' Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$\mathbf{j} = \sigma \mathbf{e}$$





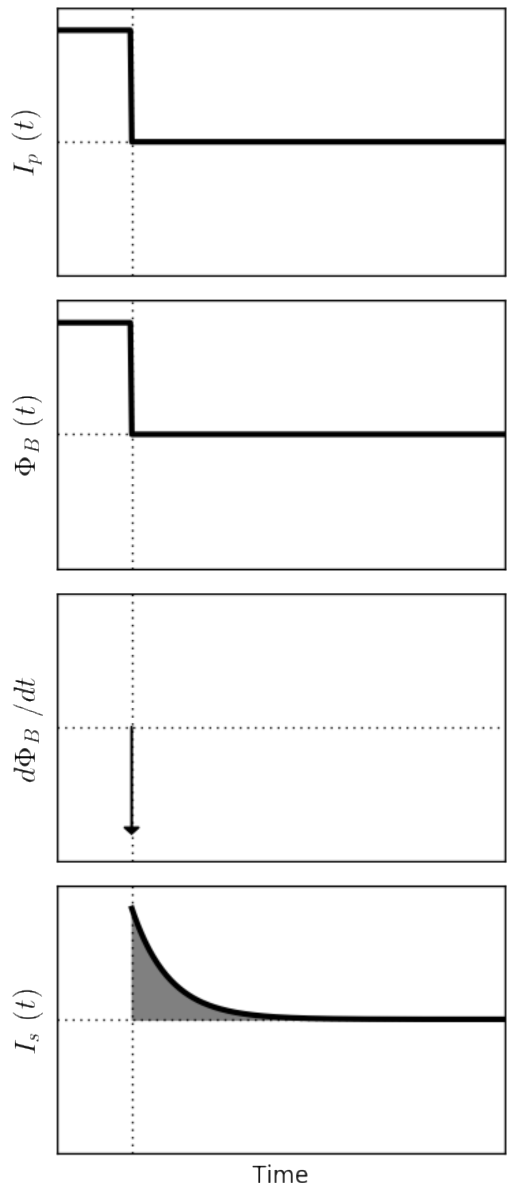
Two Coil Example: Transient

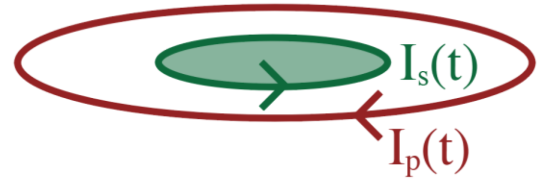
Primary currents

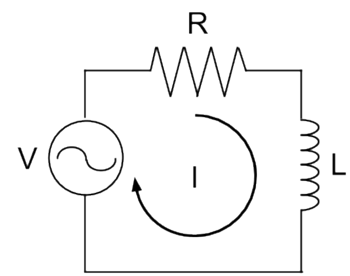
Magnetic flux

Time-variation of magnetic flux

Secondary currents





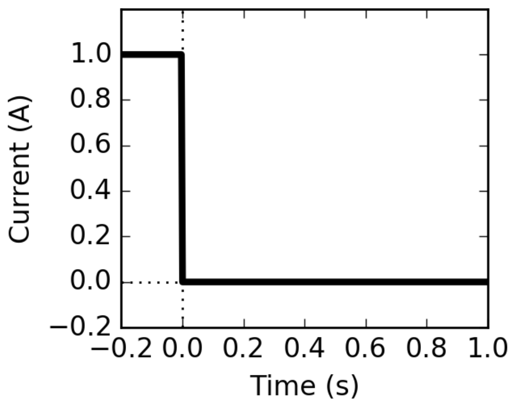


$$I_s(t) = I_s e^{-t/\tau}$$

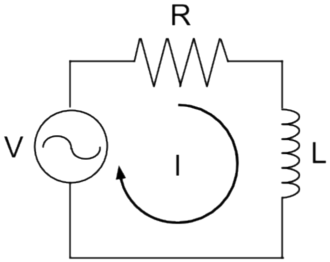
$$\tau = L/R$$

Response Function: Transient

Step-off current in Tx

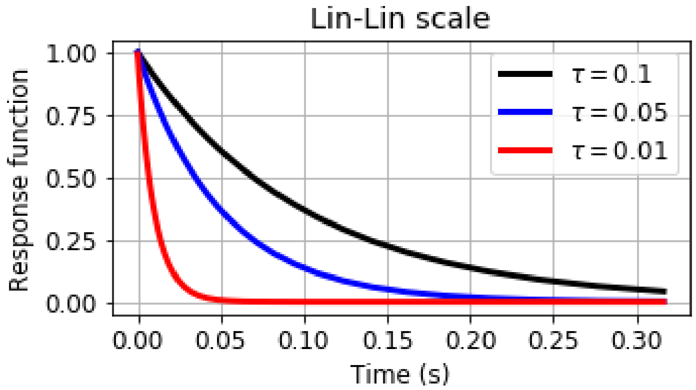


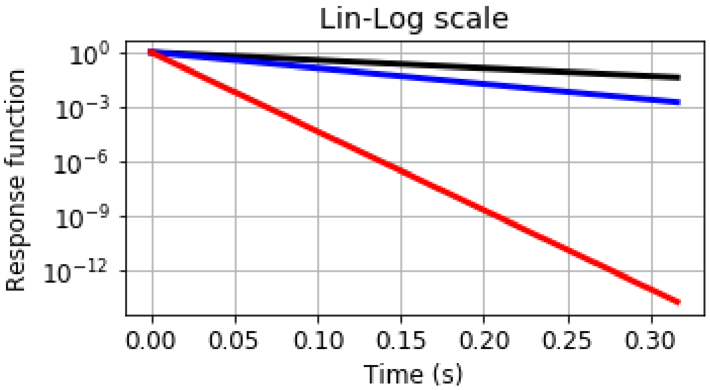
Time constant



$$\tau = L/R$$

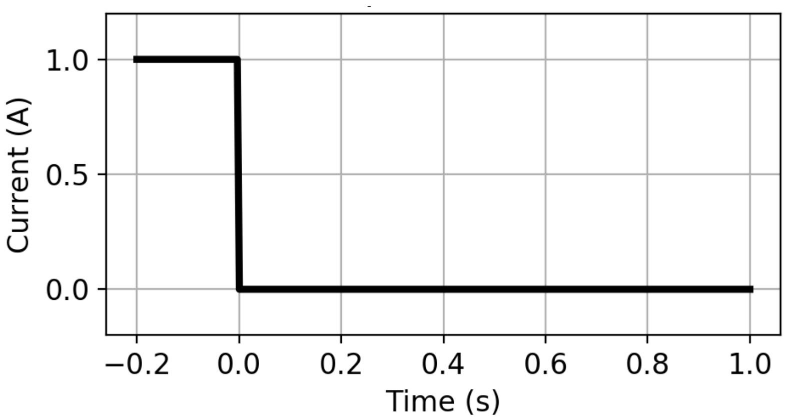
Response function: $q(t) = e^{-t/\tau}$



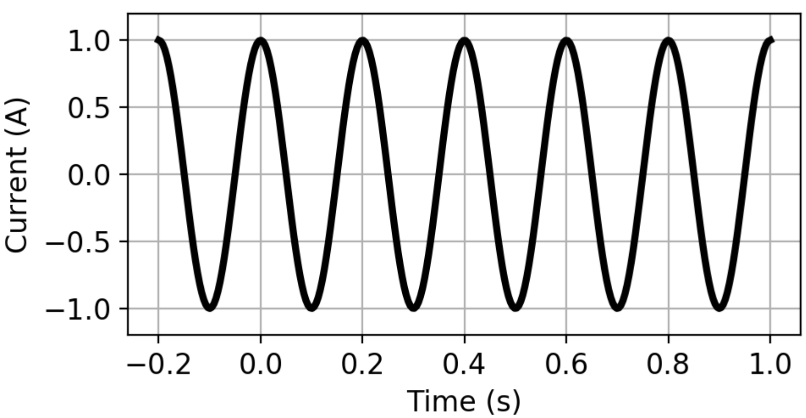


Transient and Harmonic Signals

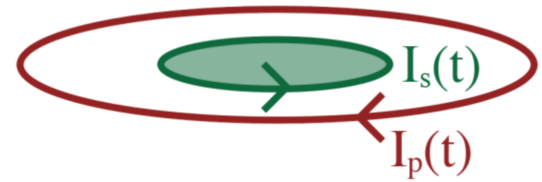
We have seen a transient pulse…



What happens when he have a harmonic?



Two Coil Example: Harmonic

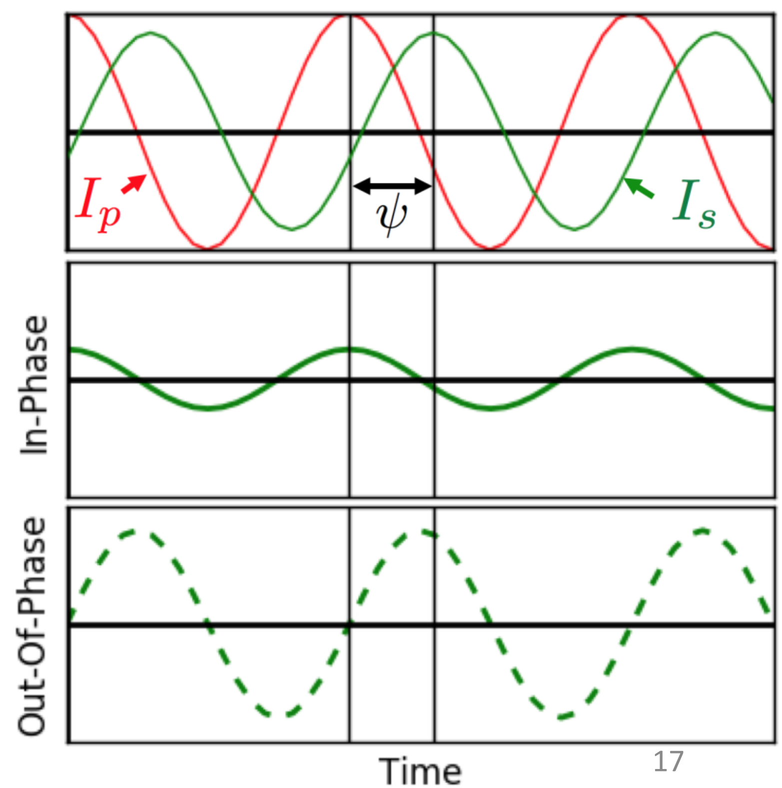


Induced Currents

$$I_p(t) = I_p \cos \omega t$$

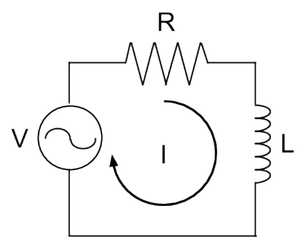
$$I_s(t) = I_s \cos(\omega t - \psi)$$

$$= \underbrace{I_s \cos \psi}_{\substack{\text{In-Phase} \\ \text{Real}}} \cos \omega t + \underbrace{I_s \sin \psi}_{\substack{\text{Out-of-Phase} \\ \text{Quadrature} \\ \text{Imaginary}}} \sin \omega t$$

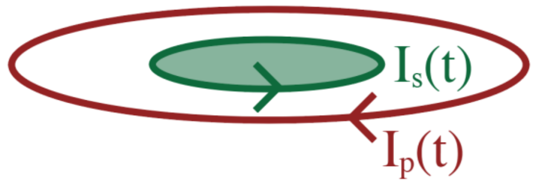


Phase Lag

$$\psi = \frac{\pi}{2} + \tan^{-1}\left(\frac{\omega L}{R}\right)$$



Two Coil Example: Harmonic



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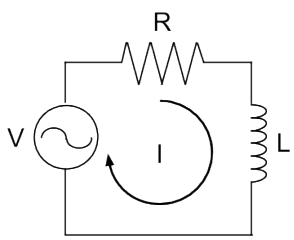
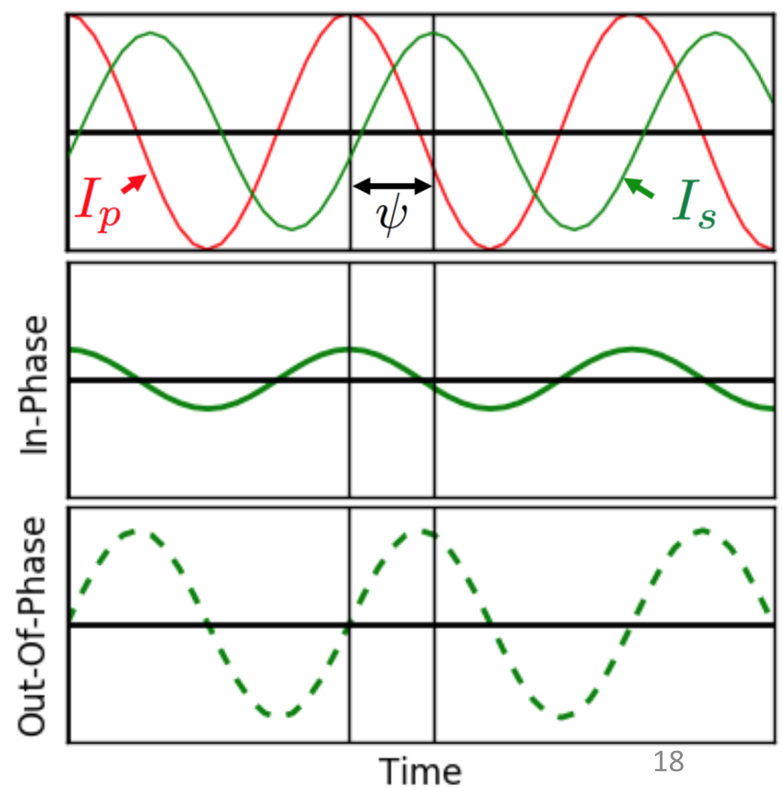
Phase Lag

$$\psi = \frac{\pi}{2} + \tan^{-1} \underbrace{\left(\frac{\omega L}{R} \right)}_{\alpha}$$

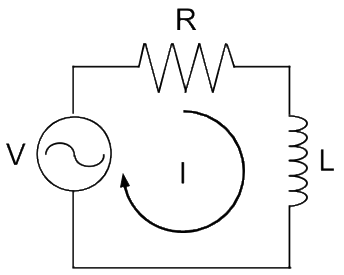
Induction number



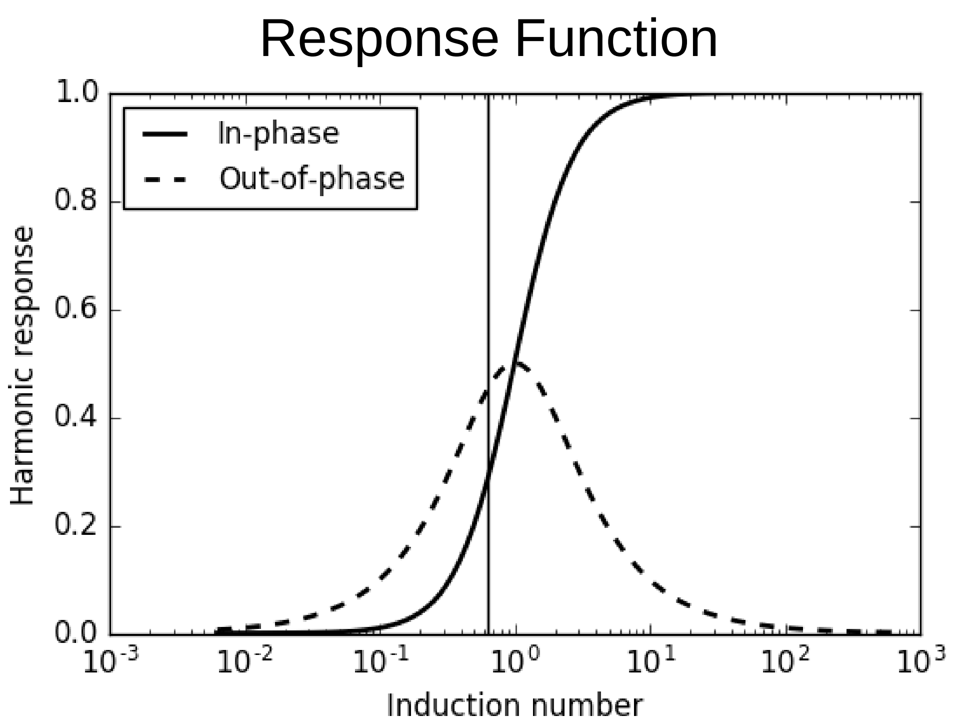


Response Function: Harmonic

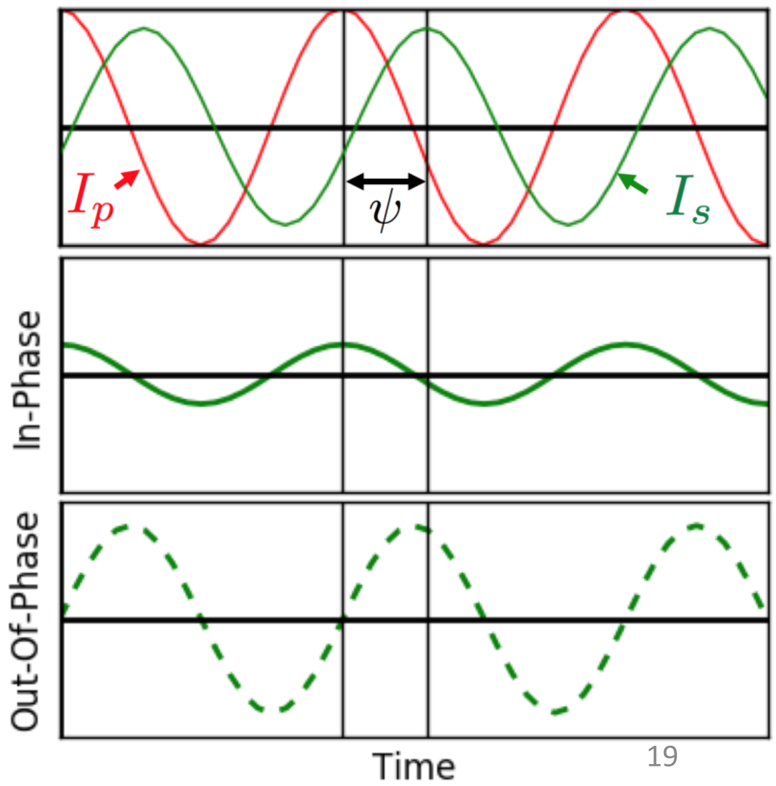
- Quantifies how a target/loop responds to a time harmonic magnetic field



- Partitions real and imaginary parts

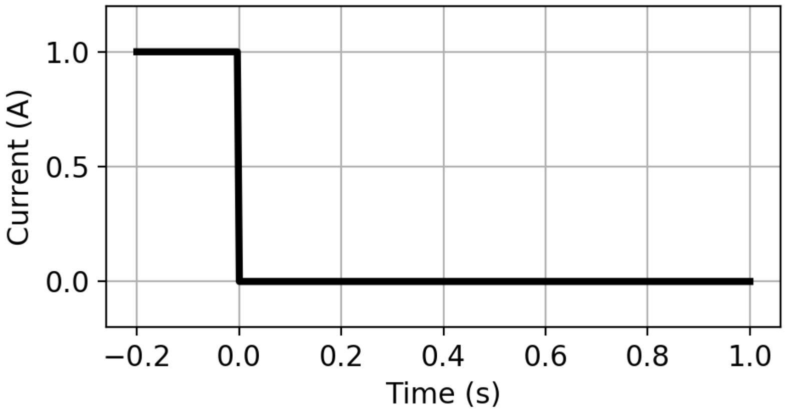


$$\alpha = \frac{\omega L}{R}$$

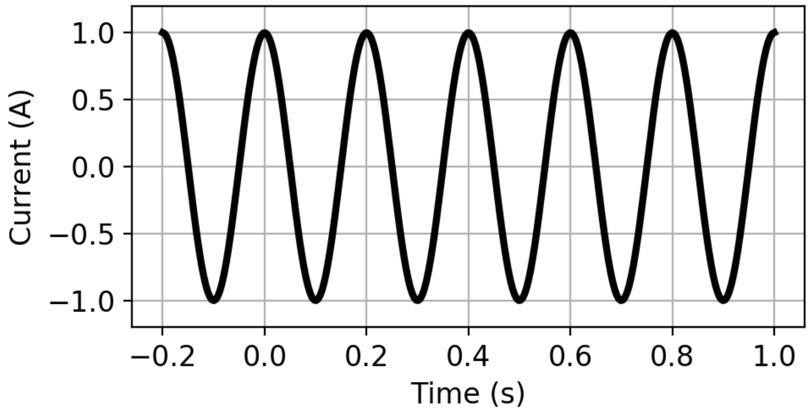


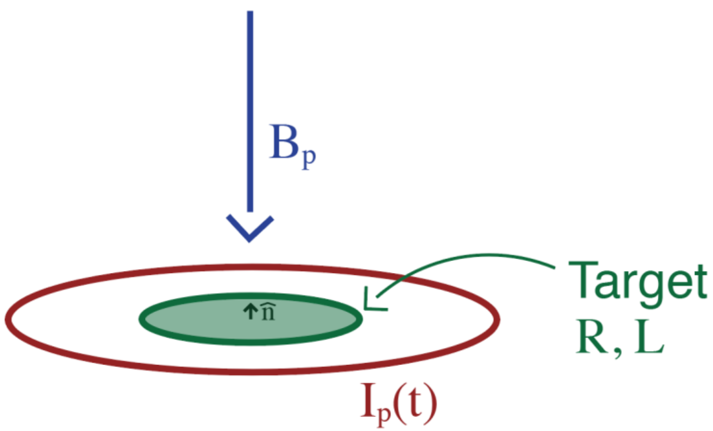
Transient vs Harmonic Response

Step-off



Harmonic





In both:

- Induce currents

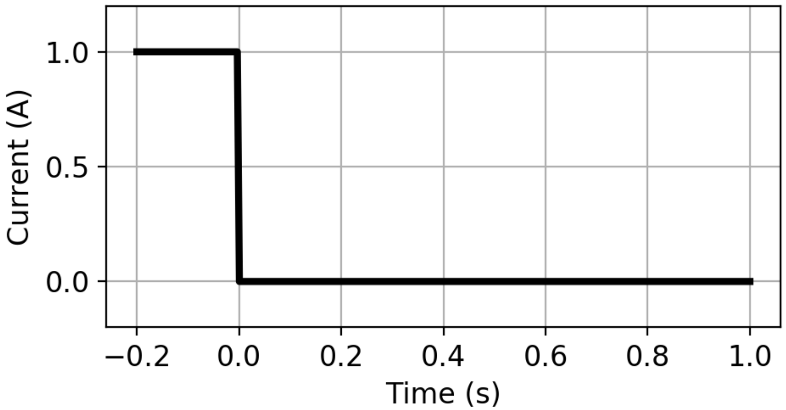
$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

- Generate secondary magnetic fields

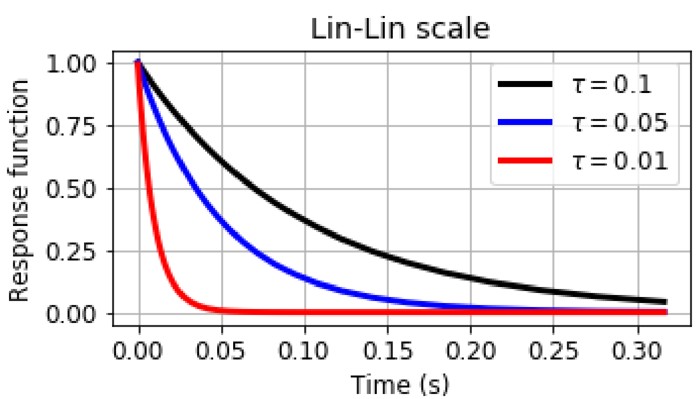
$$\nabla \times \mathbf{h} = \mathbf{j}$$

Transient vs Harmonic Response

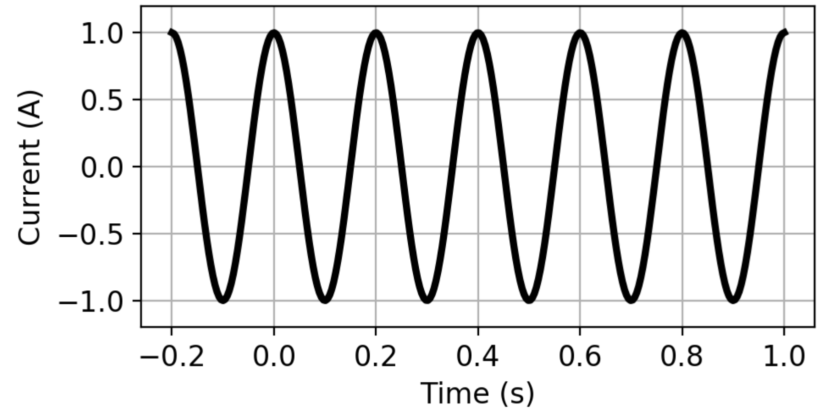
Step-off



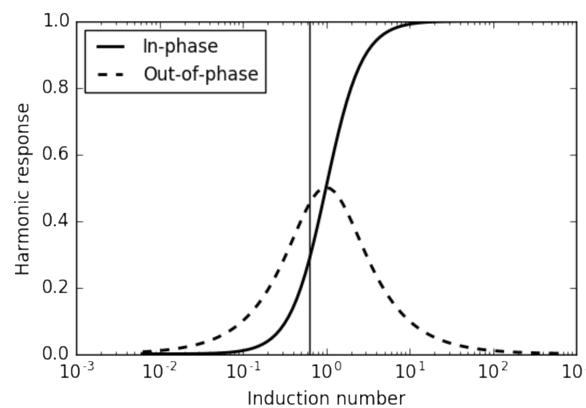
$$q(t) = e^{-t/\tau}$$



Harmonic



$$\alpha = \frac{\omega L}{R}$$



A Circuit Model for EM: Coupling

Coupling

- Orientation of the target/loop relative to the primary field
- Transmitter: Primary

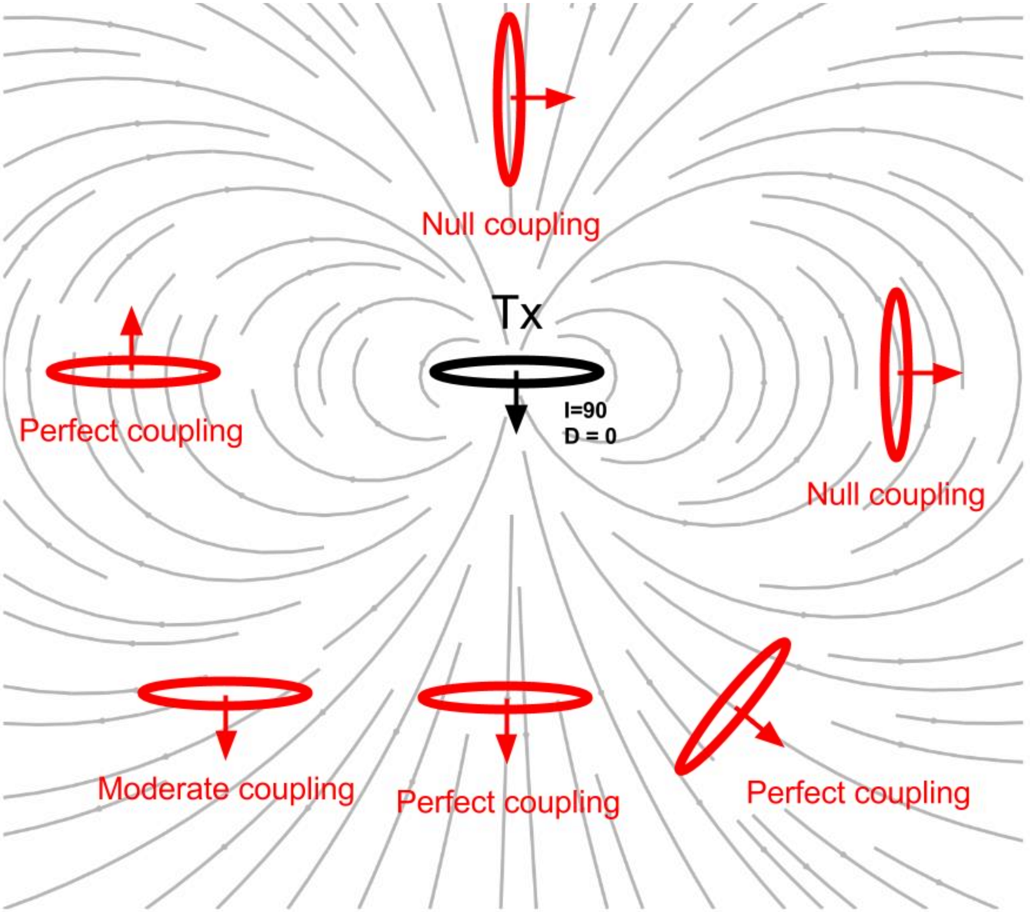
$$I_p(t) = I_p \cos(\omega t)$$

$$\mathbf{B}_p(t) \sim I_p cos(\omega t)$$

- Target: Secondary

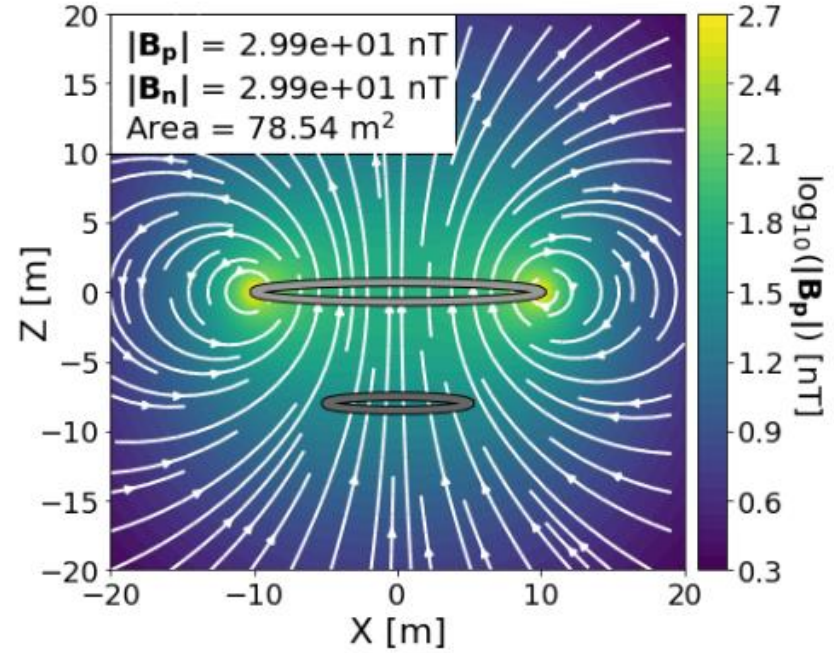
$$EMF = -\frac{\partial \phi_{\mathbf{B}}}{\partial t}$$
$$= -\frac{\partial}{\partial t} \left(\mathbf{B}_p \cdot \hat{\mathbf{n}}\right) A$$

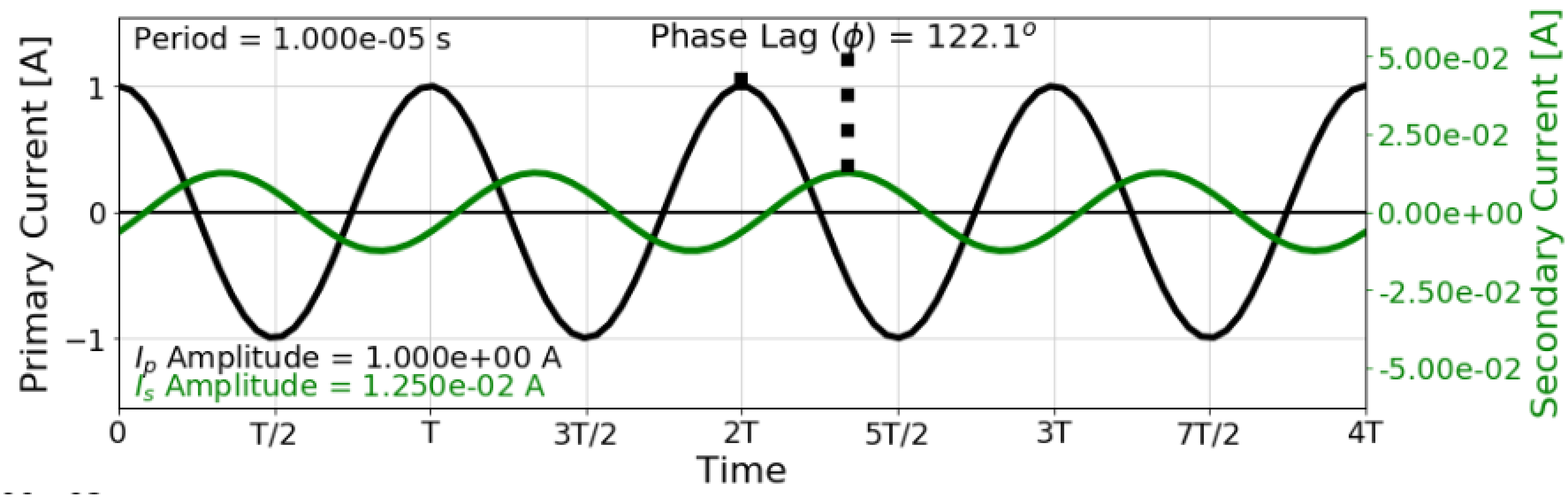
- Stronger EMF if:
  - Larger Bp
  - Larger A
  - Better coupling



Good Coupling

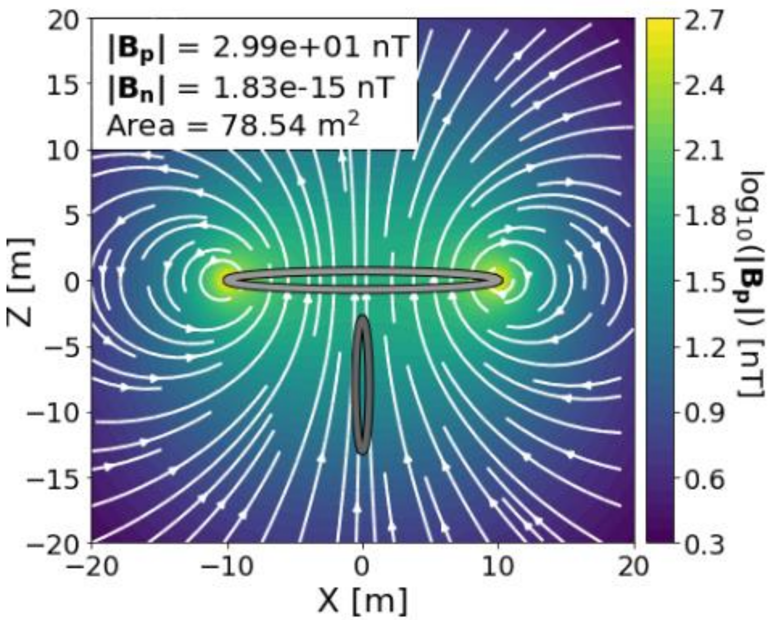
- Loop 2 is normal to primary field from loop 1
- $\boldsymbol{B}_p \cdot \widehat{\boldsymbol{n}} \neq 0$

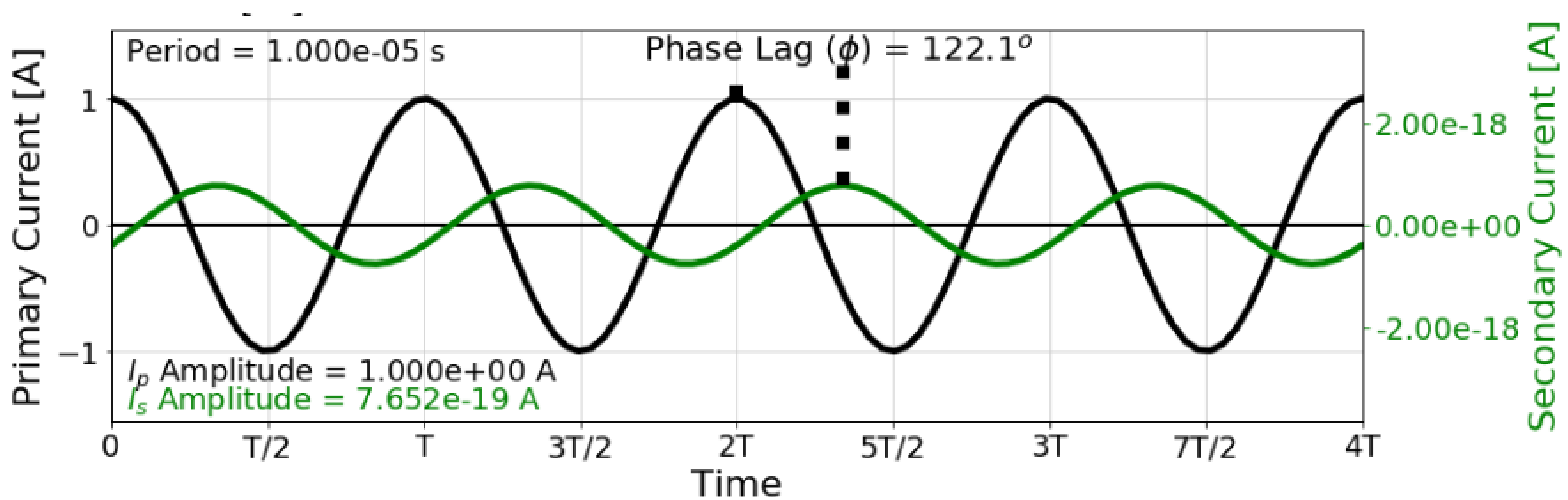




Bad Coupling

- Loop 2 is parallel to primary field from loop 1
- $\boldsymbol{B}_p \cdot \widehat{\boldsymbol{n}} = 0$





Bad Coupling

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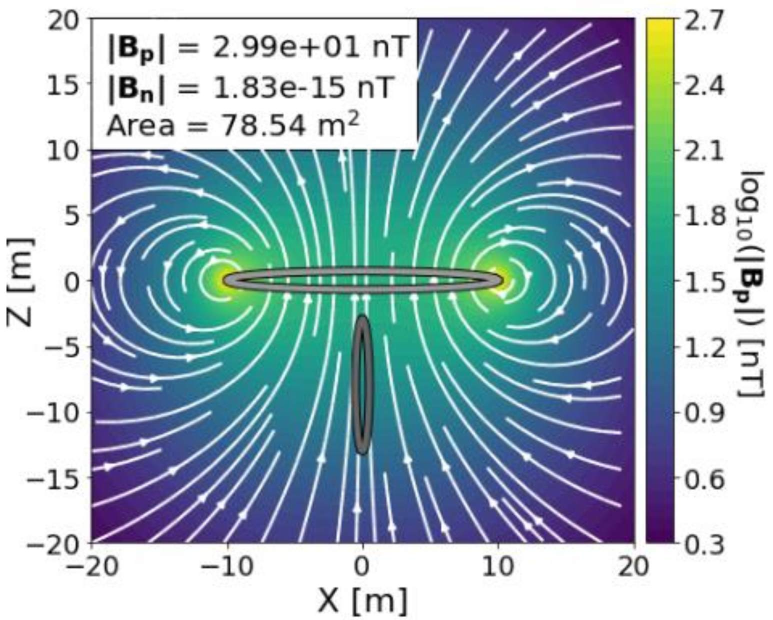
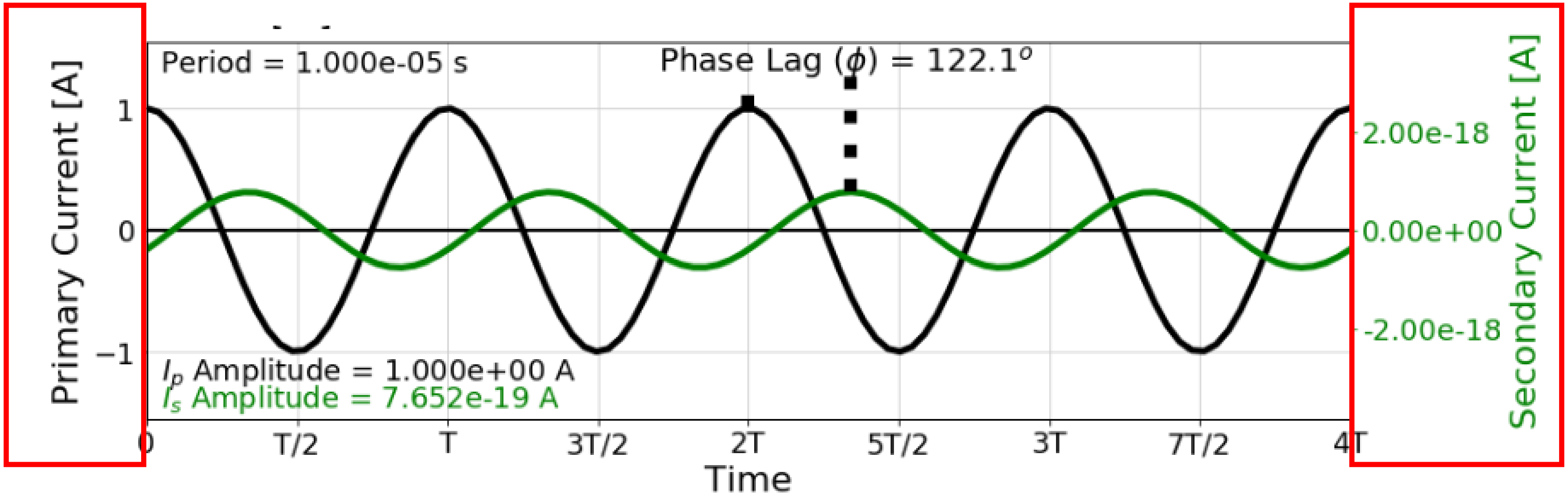
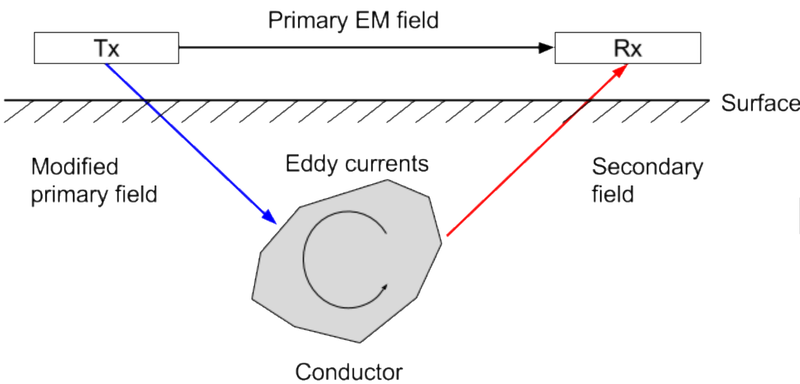
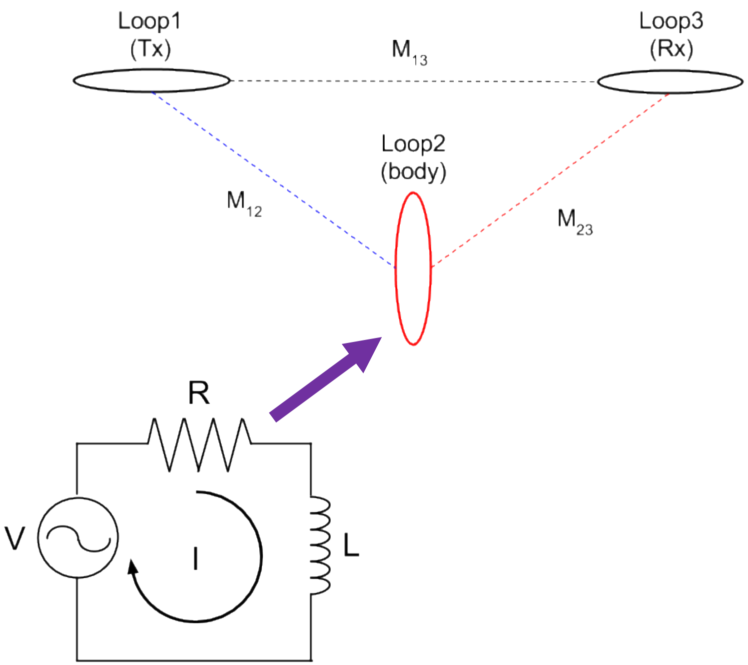




A Circuit Model for EM:
EM Response at Receiver

Induction Recap





EMF in loop 2 is:

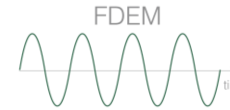
$$EMF = -\frac{\partial \phi_{\mathbf{B}}}{\partial t}$$
$$= -\frac{\partial}{\partial t}\left(\mathbf{B}_p \cdot \hat{\mathbf{n}}\right) A$$

Current in loop 2 is:

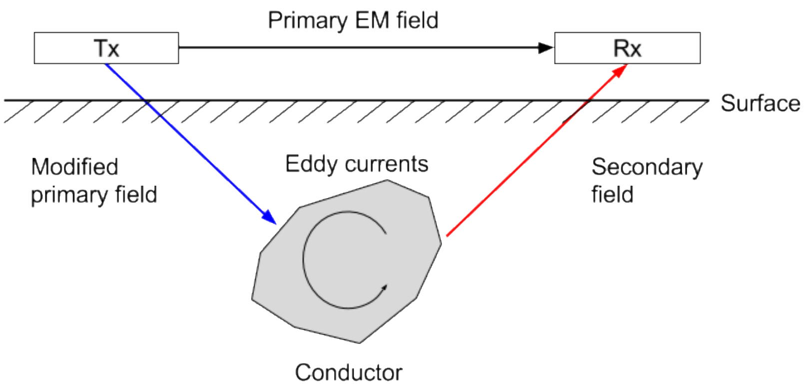
$$I_s(t) = I_s e^{-t/\tau}$$

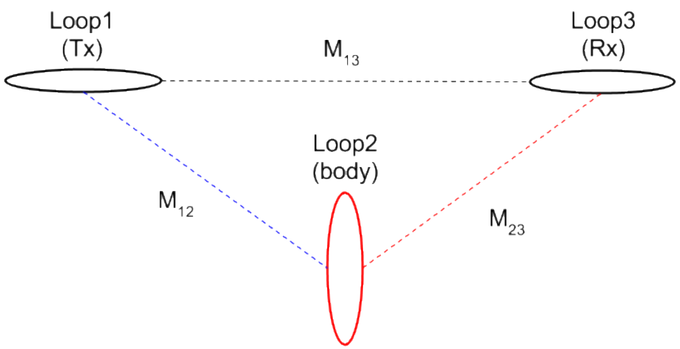
$$I_s(t) = I_s \cos\psi \cos\omega t + I_s \sin\psi \sin\omega t$$

What is Hs at receiver relative to Hp?



Circuit model of EM induction





Coupling coefficient:

- Depends on loop geometry

$$M_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|\mathbf{r} - \mathbf{r}'|^2}.$$

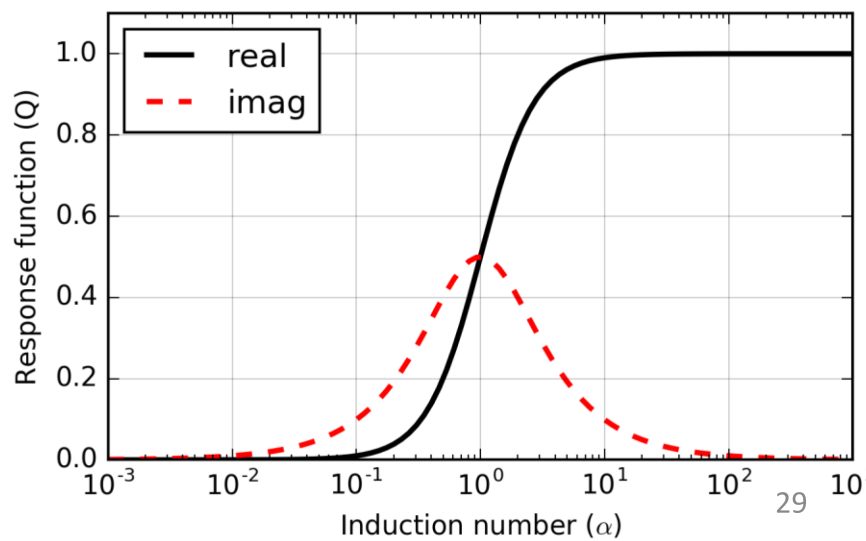
Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L} \underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}$$

Induction Number

- Depends on properties of target

$$\alpha = \frac{\omega L}{R}$$



Circuit model of EM induction

- If coupling between transmitter and target is bad ($M_{12} \sim 0$)
  → $H_s \sim 0$

- If coupling between target and receiver is bad ($M_{23} \sim 0$)
  → $H_s \sim 0$

- If transmitter frequency is low ($\omega \sim 0$)
  → $\alpha \sim 0$
  → $H_s \sim 0$

- If L/R is smaller, higher frequencies required for large response

Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L}\underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}$$

Induction Number

- Depends on properties of target

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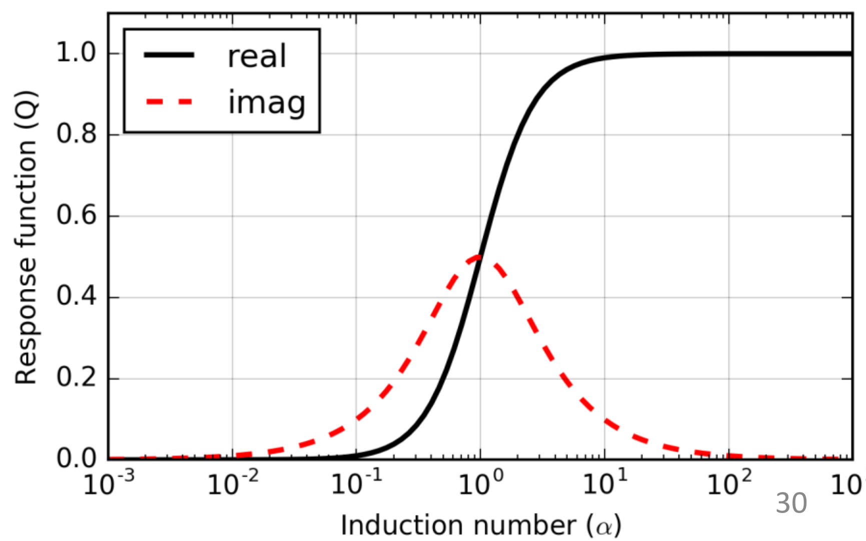
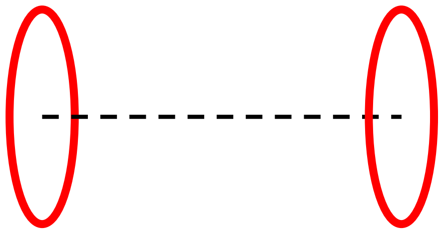


A Circuit Model for EM: Airborne FEM Example

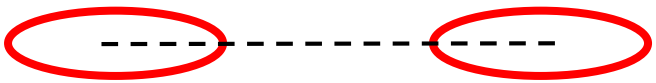
Survey

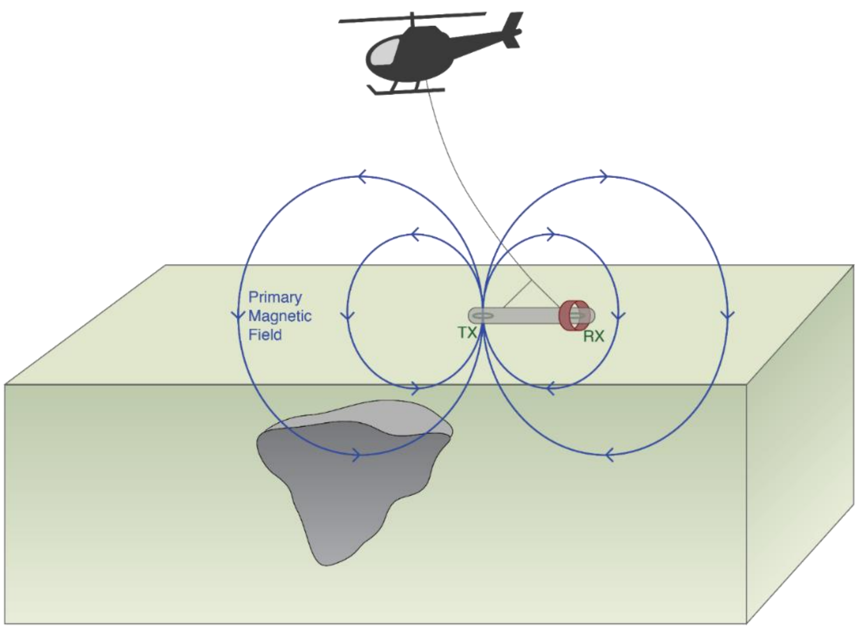
- Airborne or land-based
- Transmitter and receiver loop
- Fly survey lines
- Collect response at 1 or more frequencies

Co-axial



Co-planar



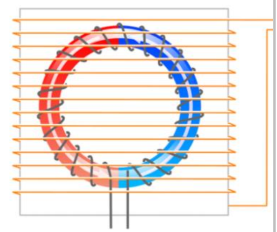


Receiver and Data

Magnetometer

$$\mathbf{b}(t)$$

- Measures:
  - Magnetic fields
  - 3 components
- eg. 3-component fluxgate

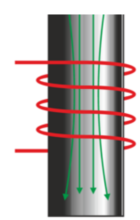


Fluxgate

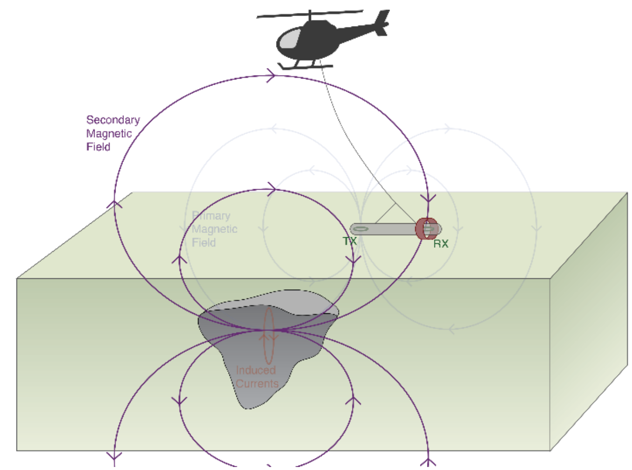
Coil

$$\frac{\partial b}{\partial t}$$

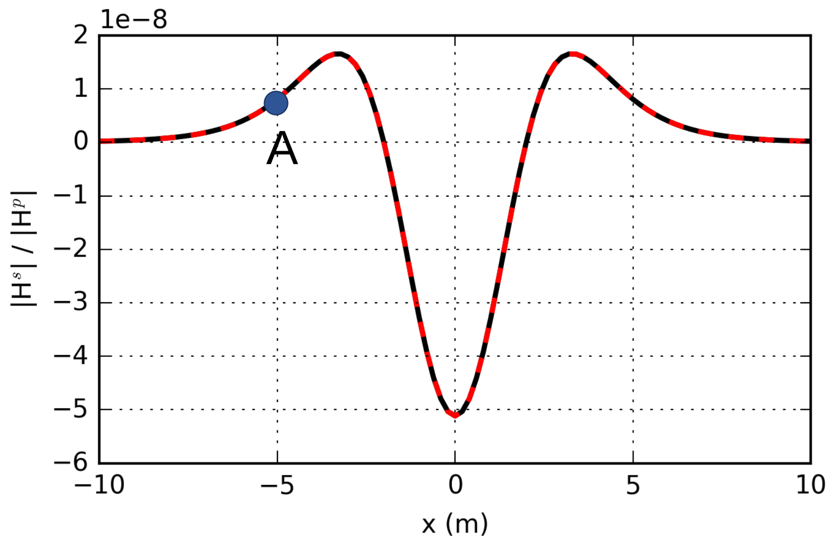
- Measures:
  - Voltage
  - Single component that depends on coil orientation
  - Coupling matters
- eg. airborne frequency domain
- ratio of Hs/Hp is the same as Vs/Vp

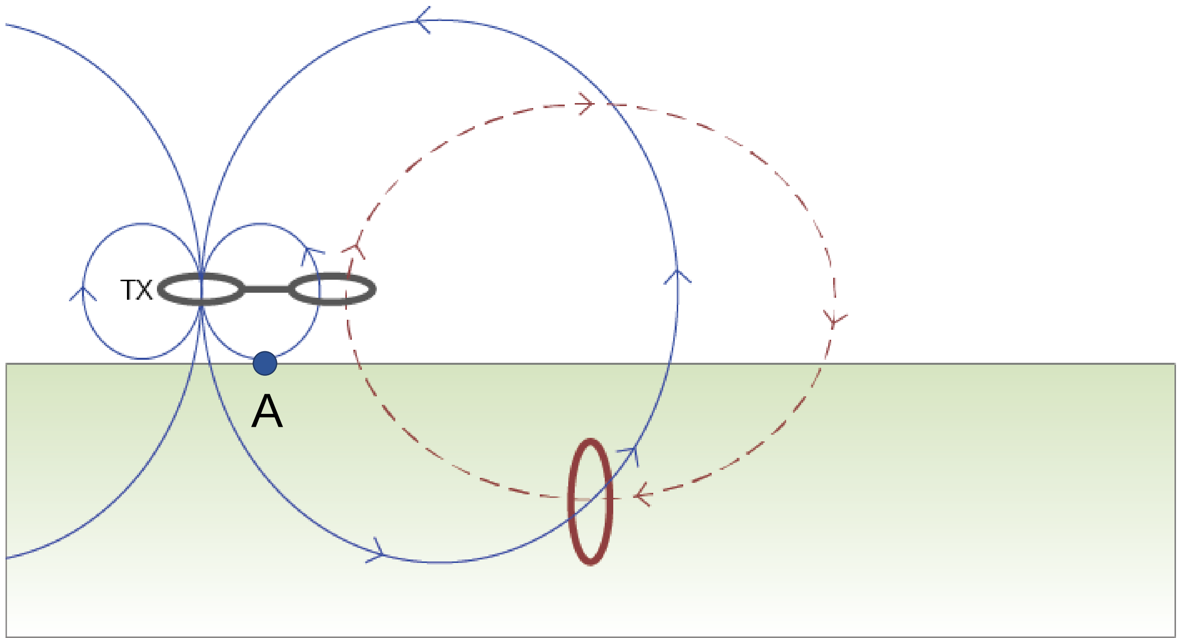


Coil

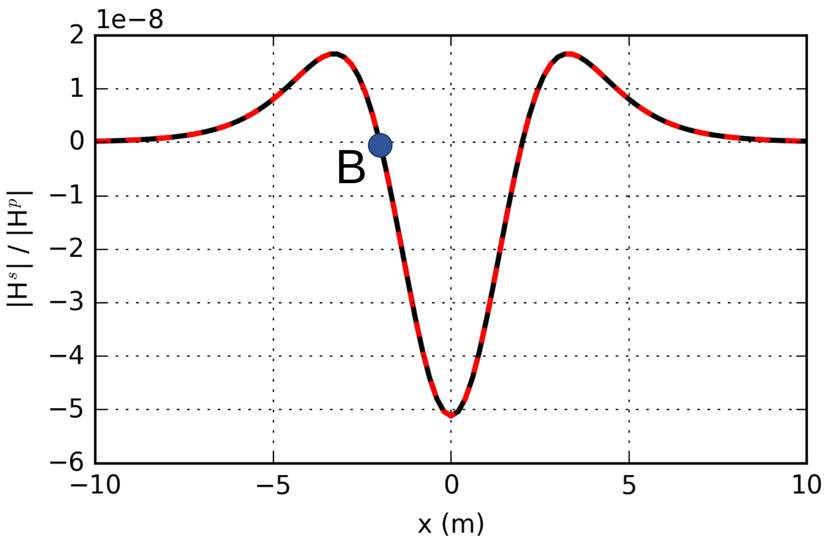


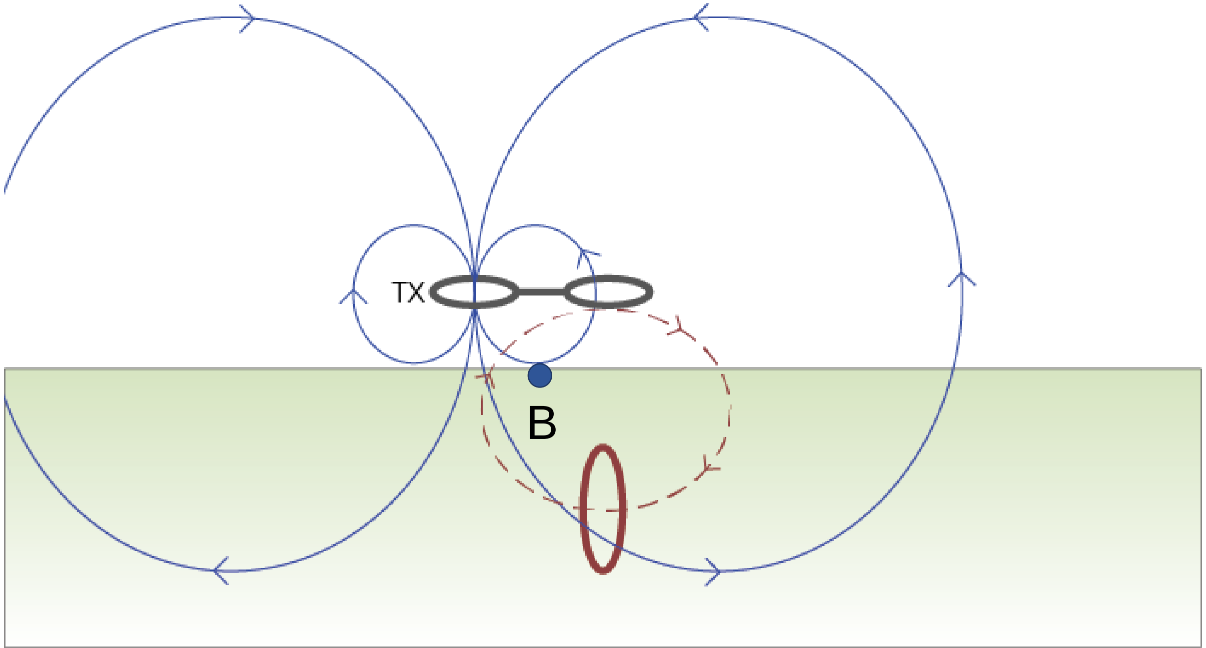
Response away from target



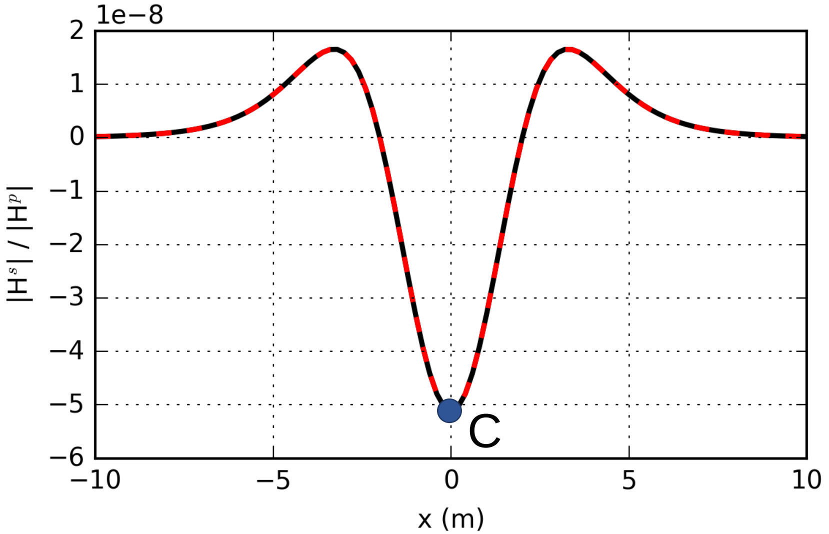


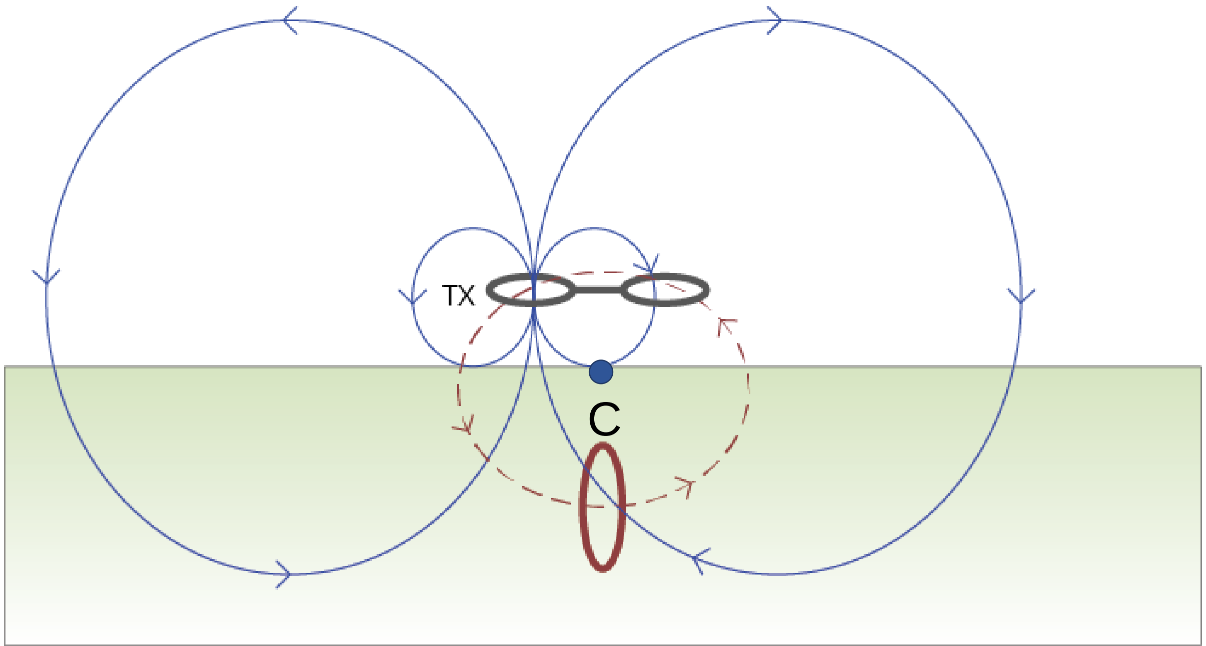
Receiver over target



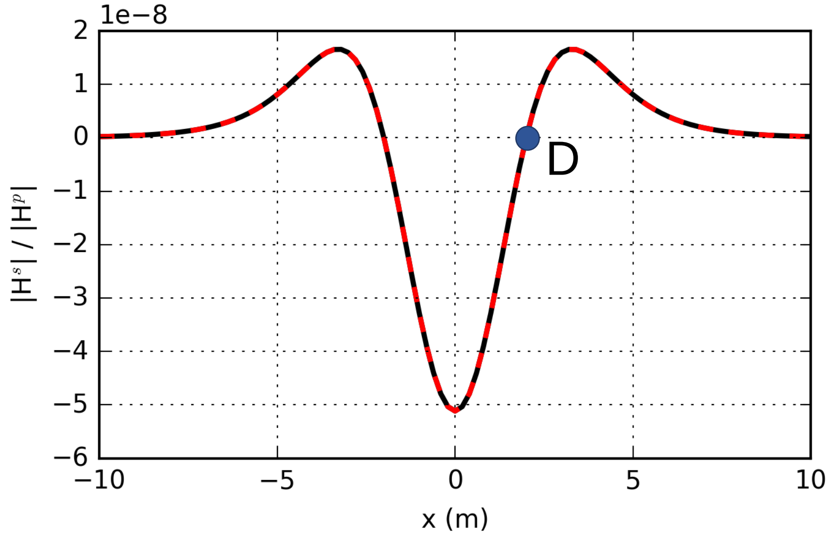


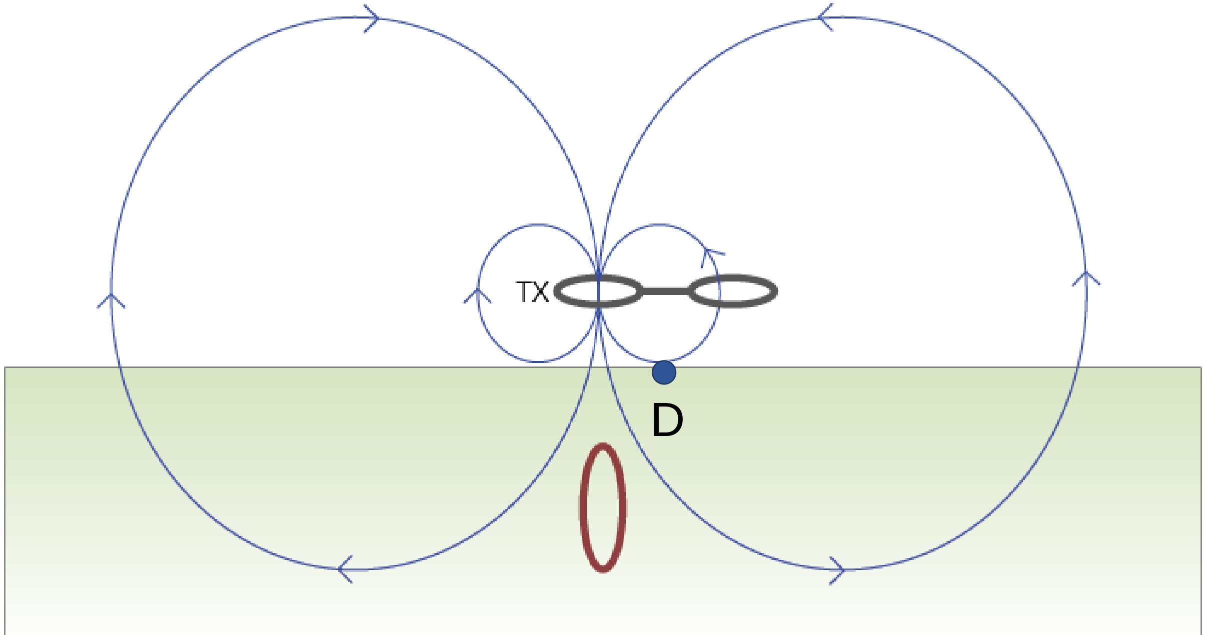
Response over target



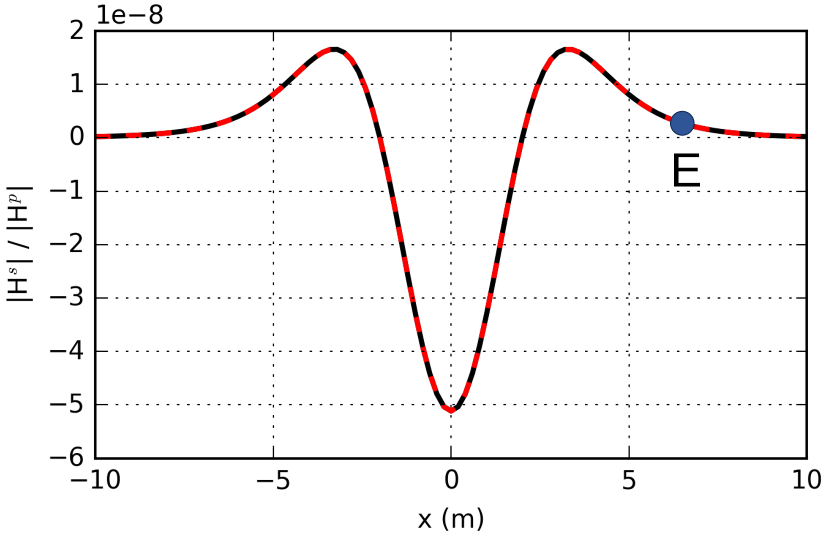


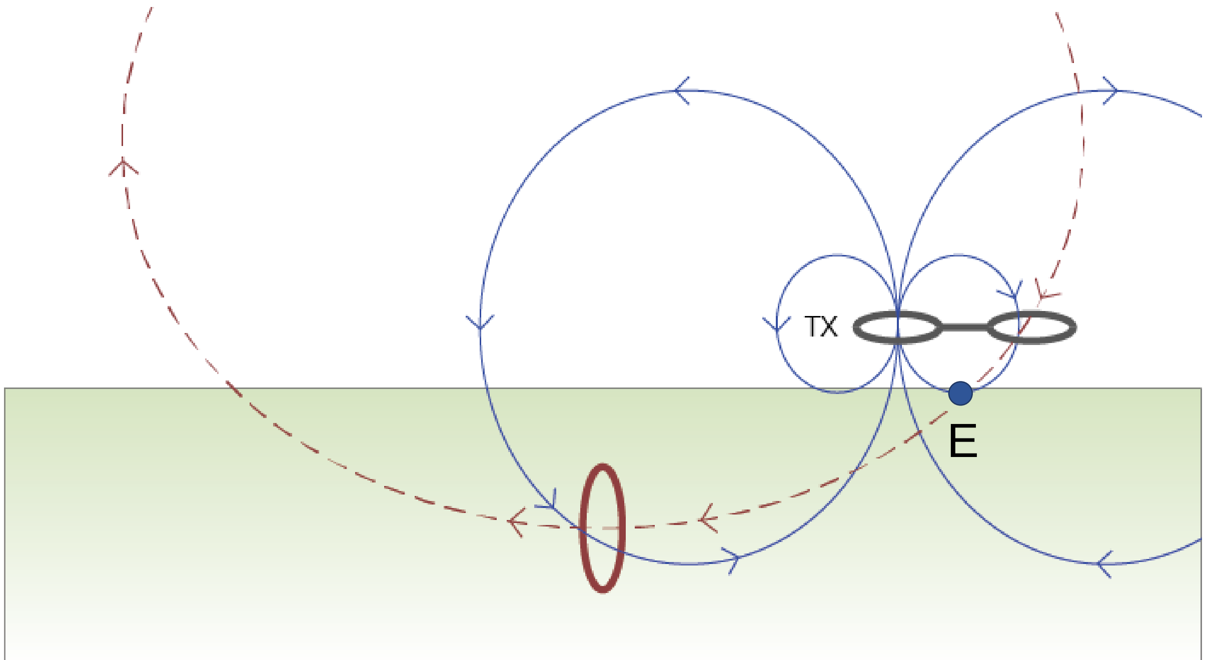
Transmitter over target





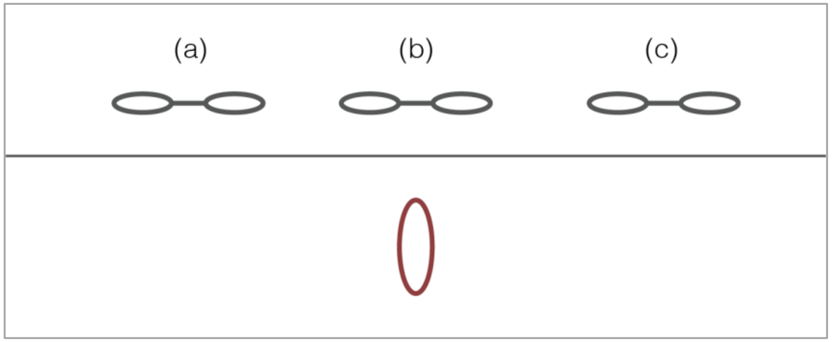
Response away from target

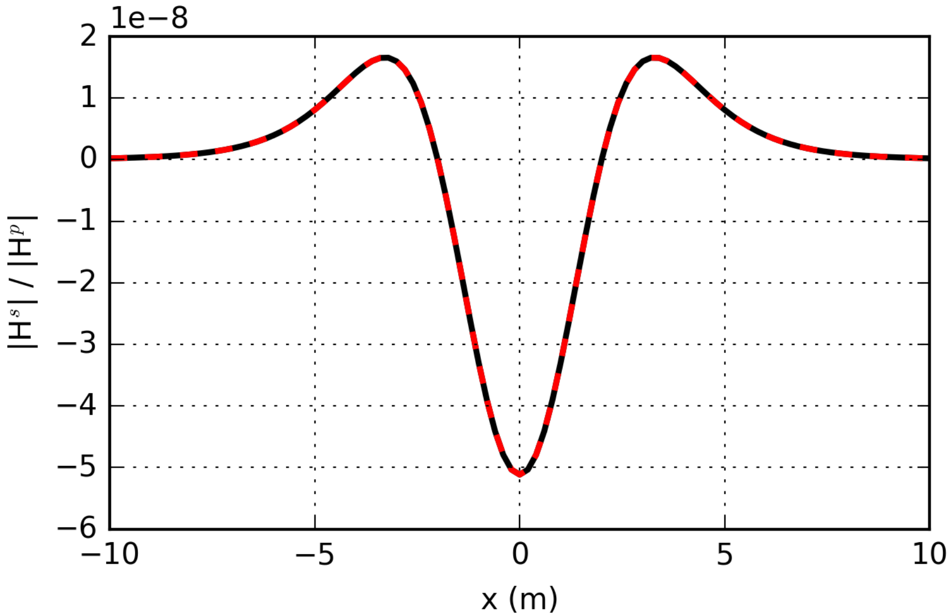




Response from conductor in resistive Earth

Profile over the loop



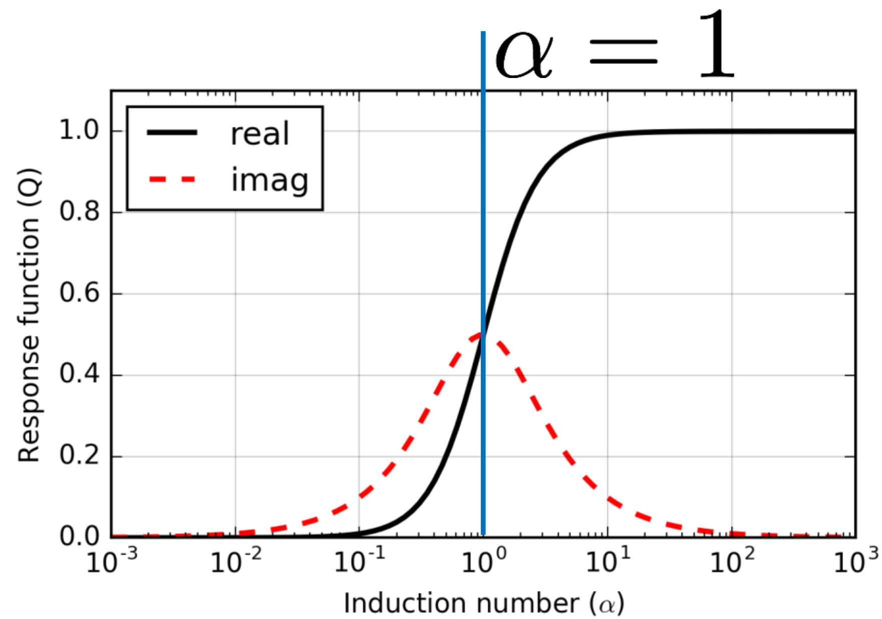


- Induction number

$$\alpha = \frac{\omega L}{R}$$

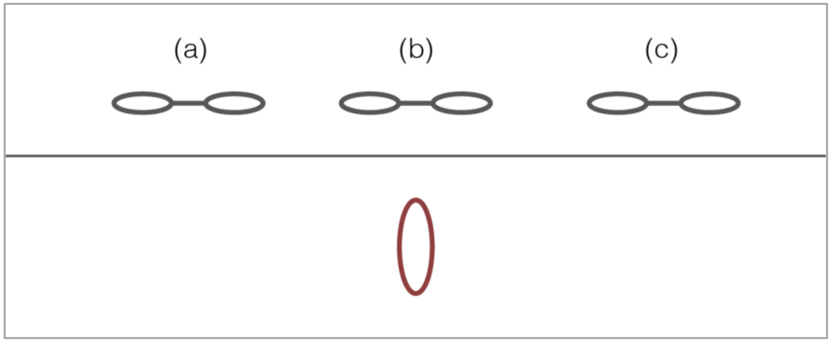
$$\alpha = 1$$

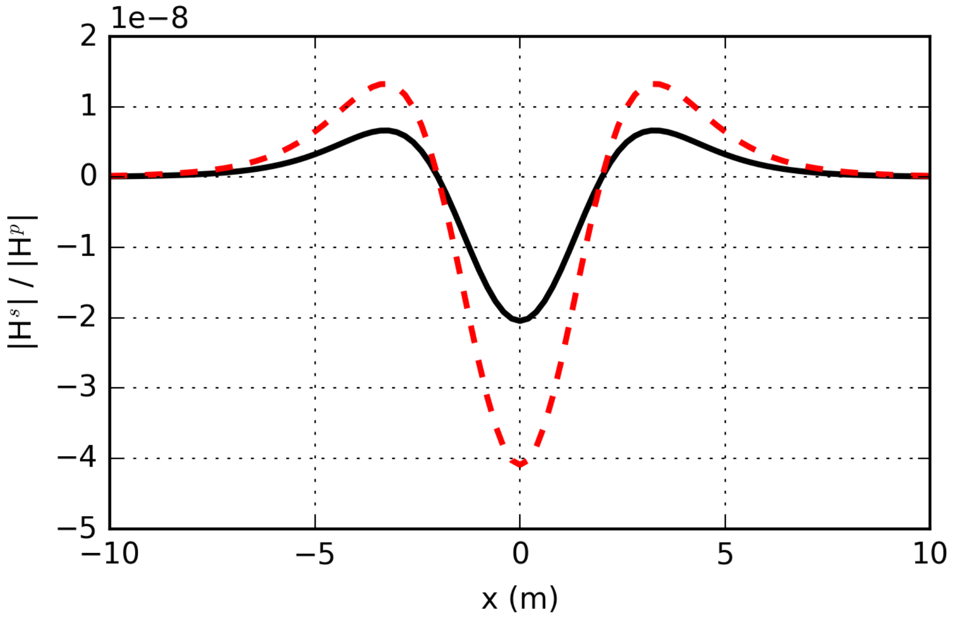
- When Real = Imag



Response from conductor in resistive Earth

Profile over the loop





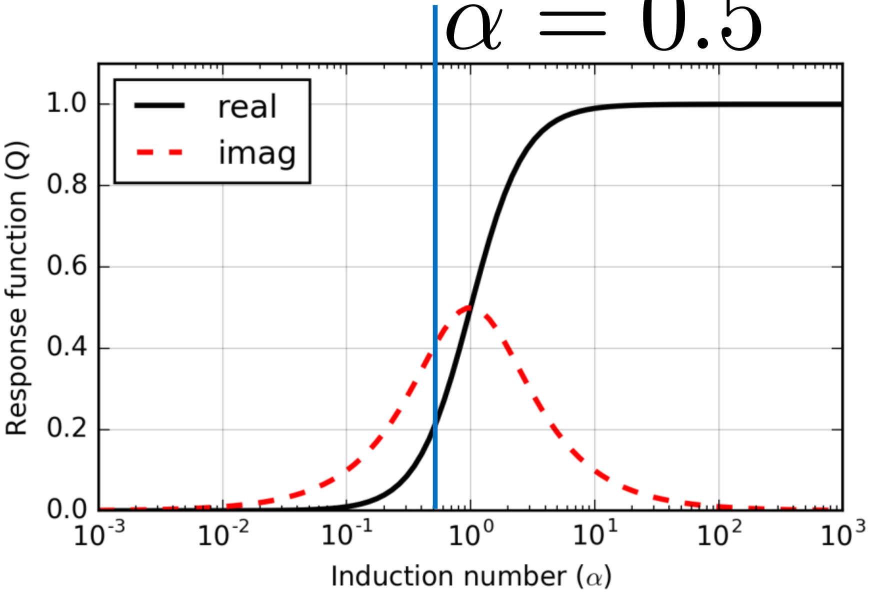
- Induction number

$$\alpha = \frac{\omega L}{R}$$

$$\alpha < 1$$

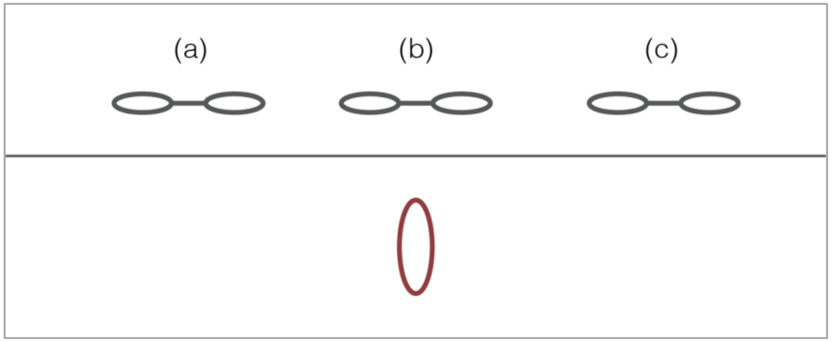
- When Real < Imag

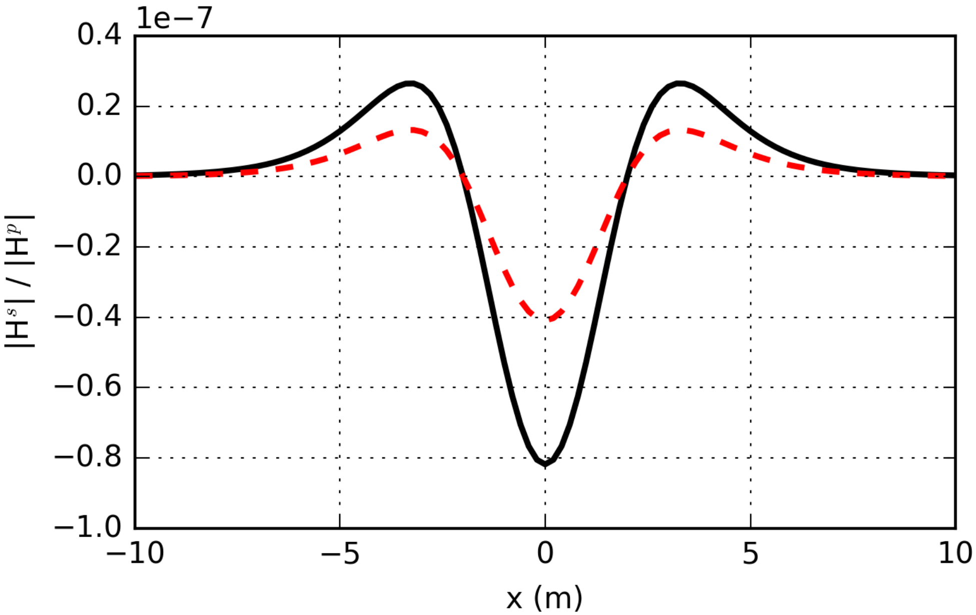
$$\alpha = 0.5$$



Response from conductor in resistive Earth

Profile over the loop



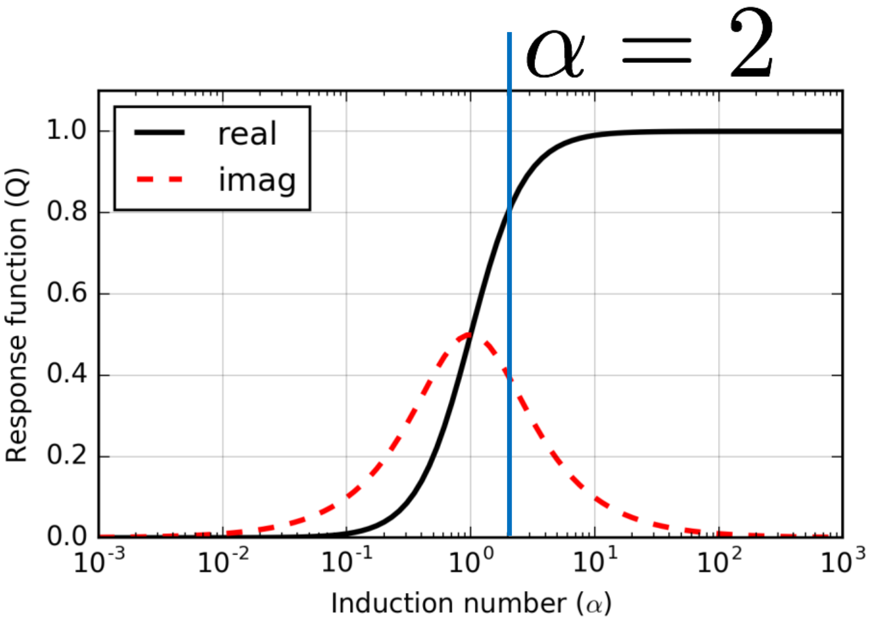


- Induction number

$$\alpha = \frac{\omega L}{R}$$

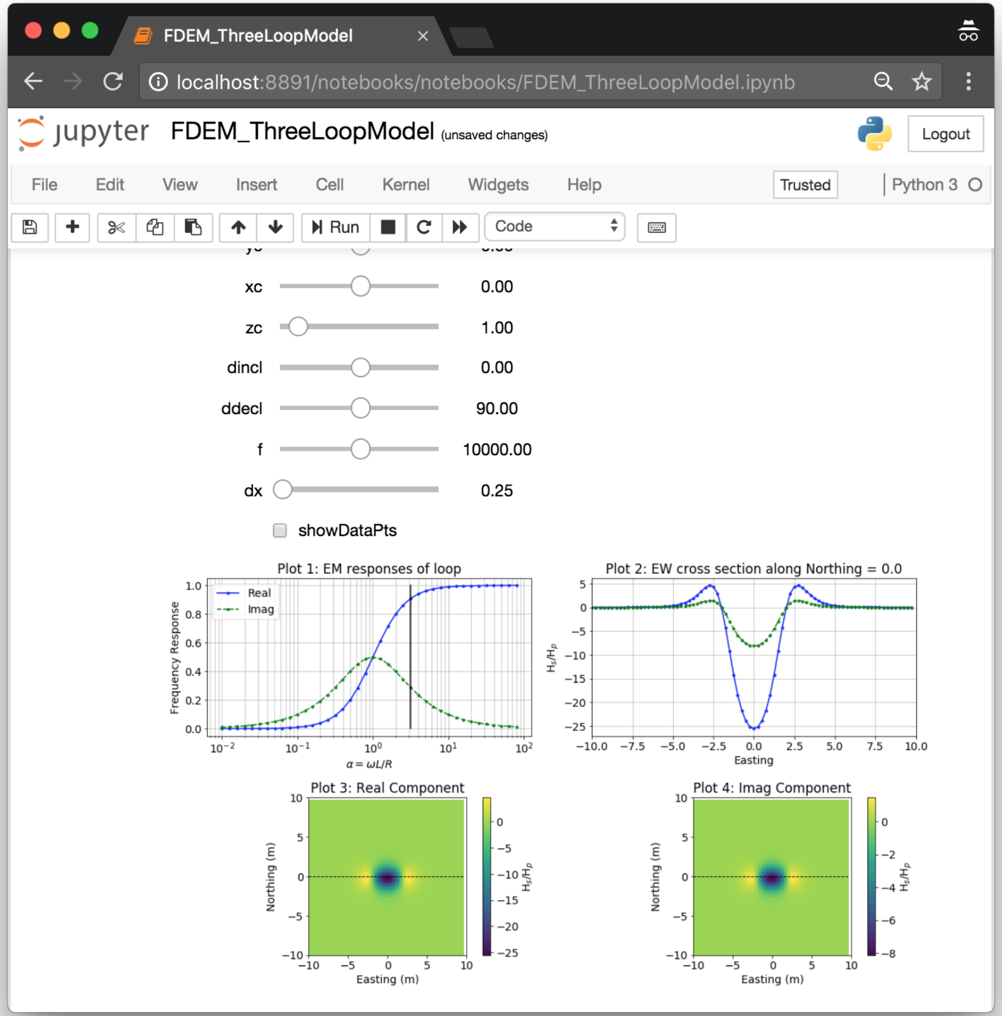
$$\alpha > 1$$

- When Real > Imag



App: Three Loop Model

- FDEM_ThreeLoopModel
- Parameters:
  - Location, separation of transmitter and receiver
  - Number of sounding locations
  - Orientation of target loop
  - Resistance, inductance of target loop
- View:
  - Response function
  - Real and imaginary components (plan view and a profile line)



http://em.geosci.xyz/apps.html

Unit Activities

- **Labs: (EM I)**
  - Monday, November 4th
  - Tuesday, November 5th
- **Labs: (EM II)**
  - Monday, November 18th
  - Tuesday, November 19th
- **TBL:**
  - Wednesday, November 15th
- **Quiz:**
  - Wednesday, November 15th