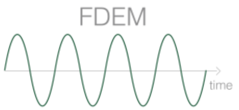
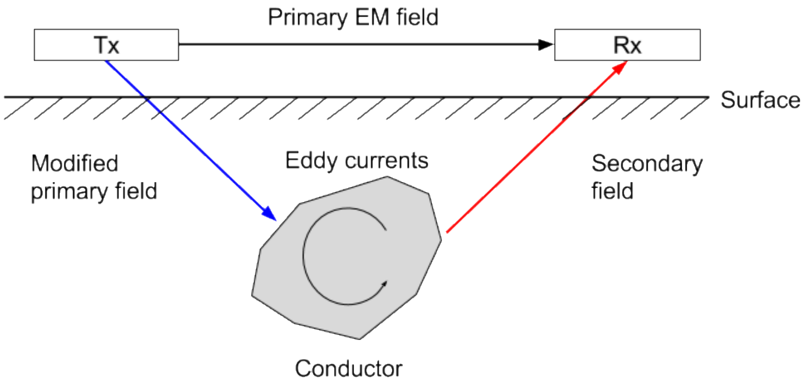
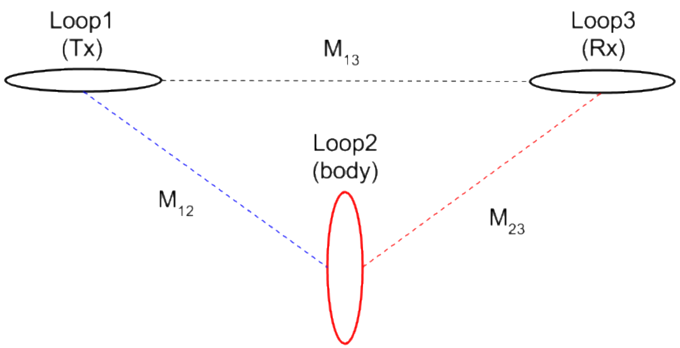
From last time



Circuit model of EM induction





Coupling coefficient:

- Depends on loop geometry

$$M_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|\mathbf{r} - \mathbf{r}'|^2}.$$

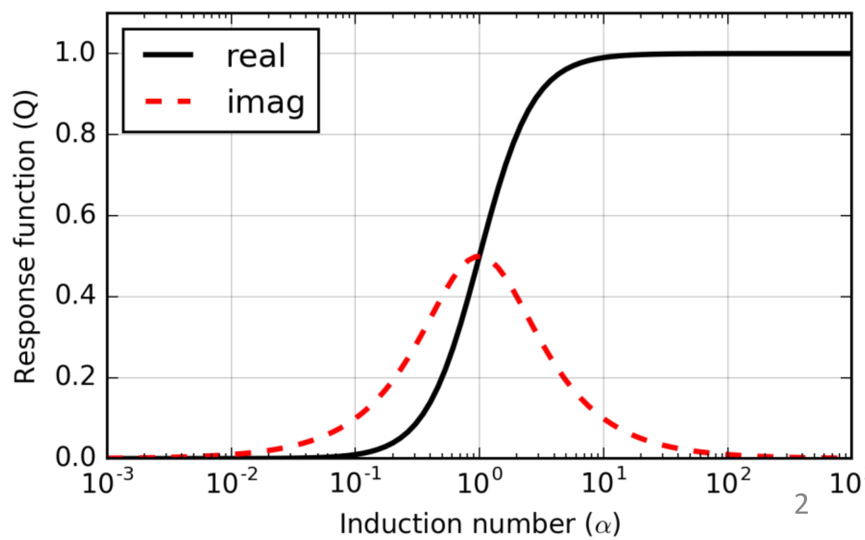
Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L} \underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}$$

Induction Number

- Depends on properties of target

$$\alpha = \frac{\omega L}{R}$$



Circuit model of EM induction

- If coupling between transmitter and target is bad ($M_{12} \sim 0$)
  → $H_s \sim 0$

- If coupling between target and receiver is bad ($M_{23} \sim 0$)
  → $H_s \sim 0$

- If transmitter frequency is low ($\omega \sim 0$)
  → $\alpha \sim 0$
  → $H_s \sim 0$

- If L/R is smaller, higher frequencies required for large response

Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L}\underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}$$

Induction Number

- Depends on properties of target

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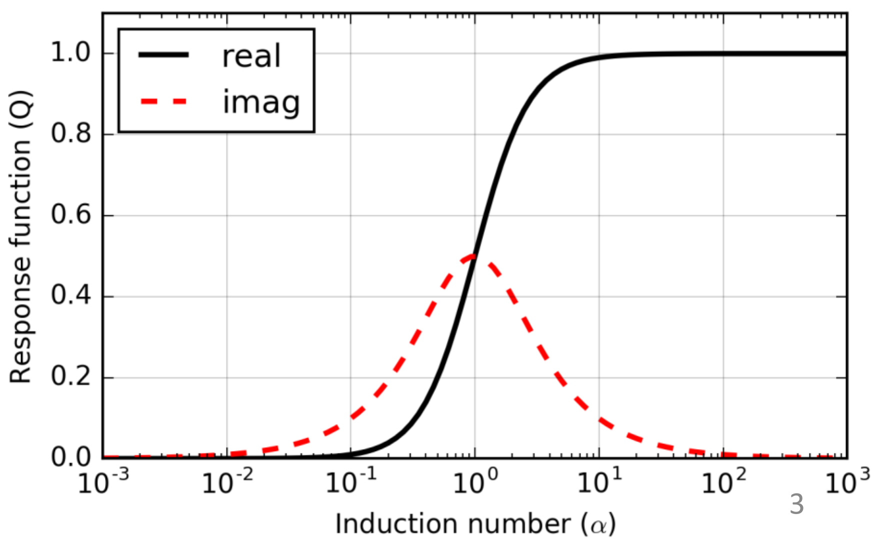
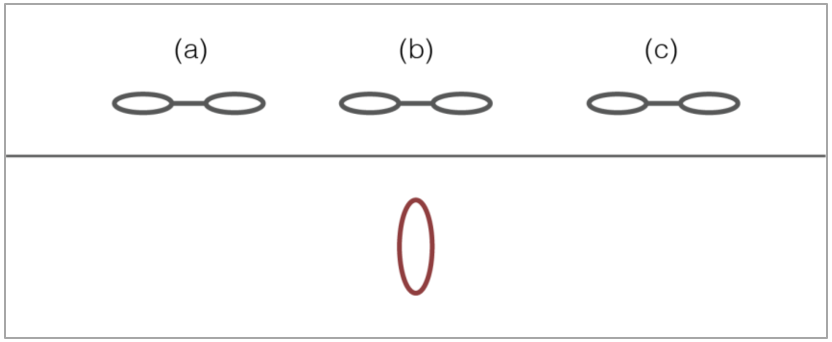
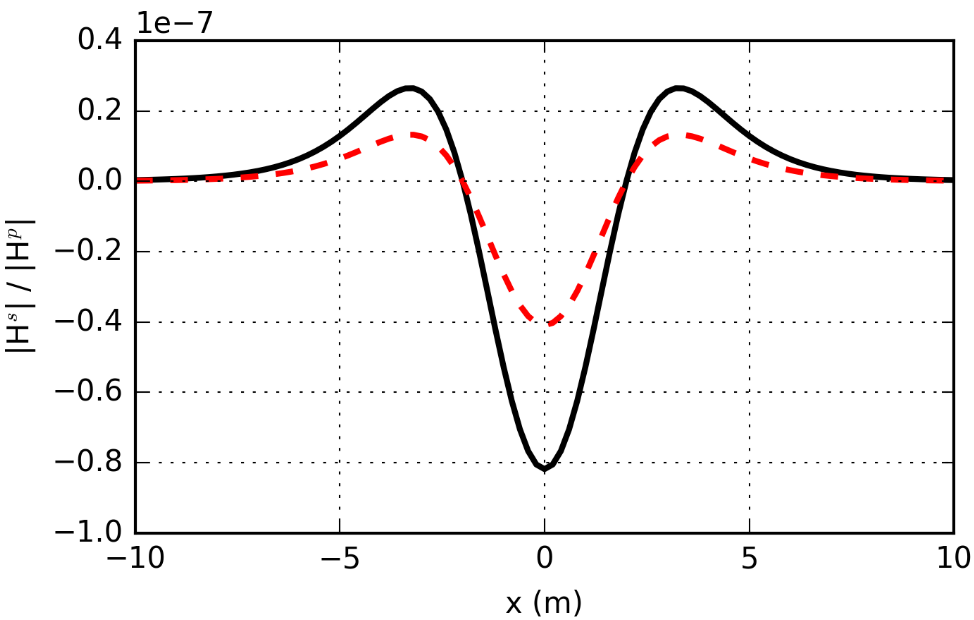


Response from conductor in resistive Earth

Profile over the loop



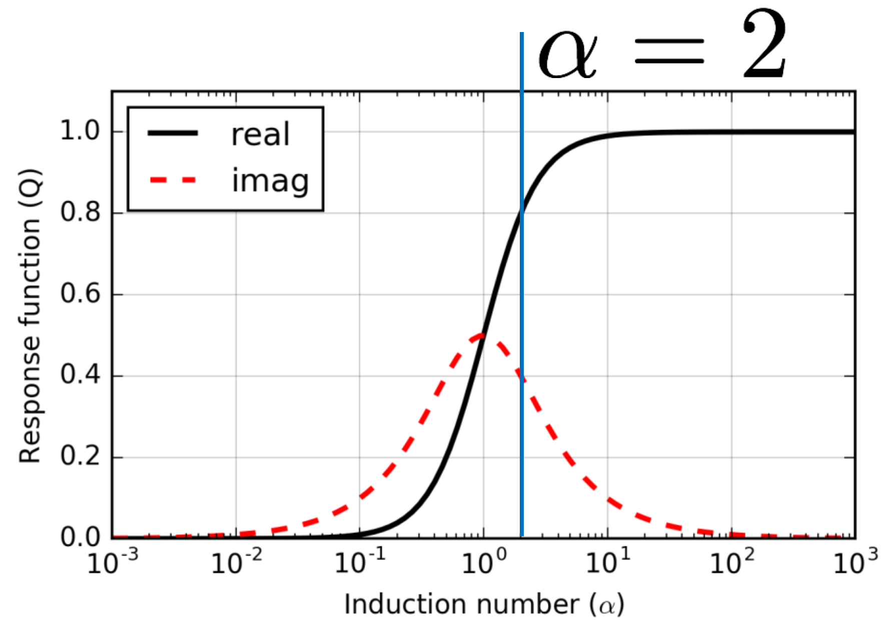


- Induction number

$$\alpha = \frac{\omega L}{R}$$

$$\alpha > 1$$

- When Real > Imag

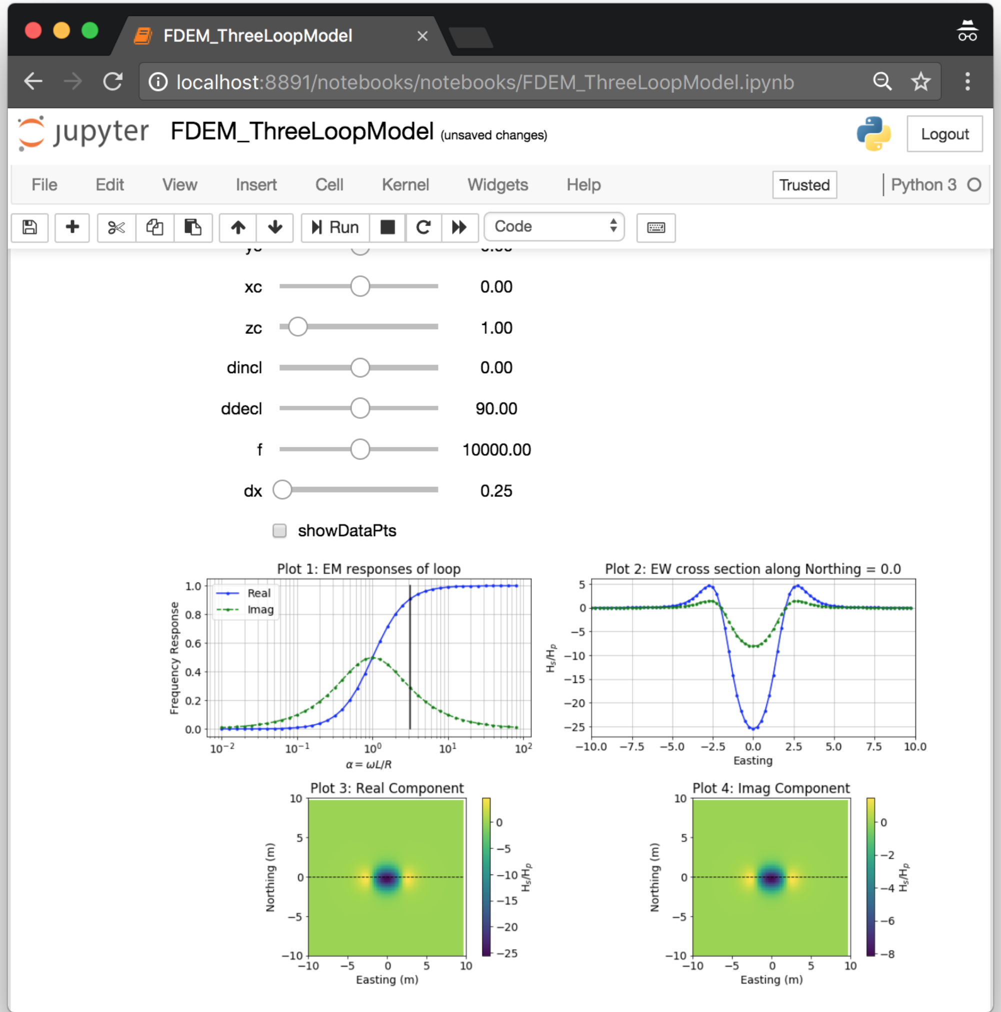


App: Three Loop Model

- FDEM_ThreeLoopModel

- Parameters:
  - Location, separation of transmitter and receiver
  - Number of sounding locations
  - Orientation of target loop
  - Resistance, inductance of target loop

- View:
  - Response function
  - Real and imaginary components (plan view and a profile line)



http://em.geosci.xyz/apps.html

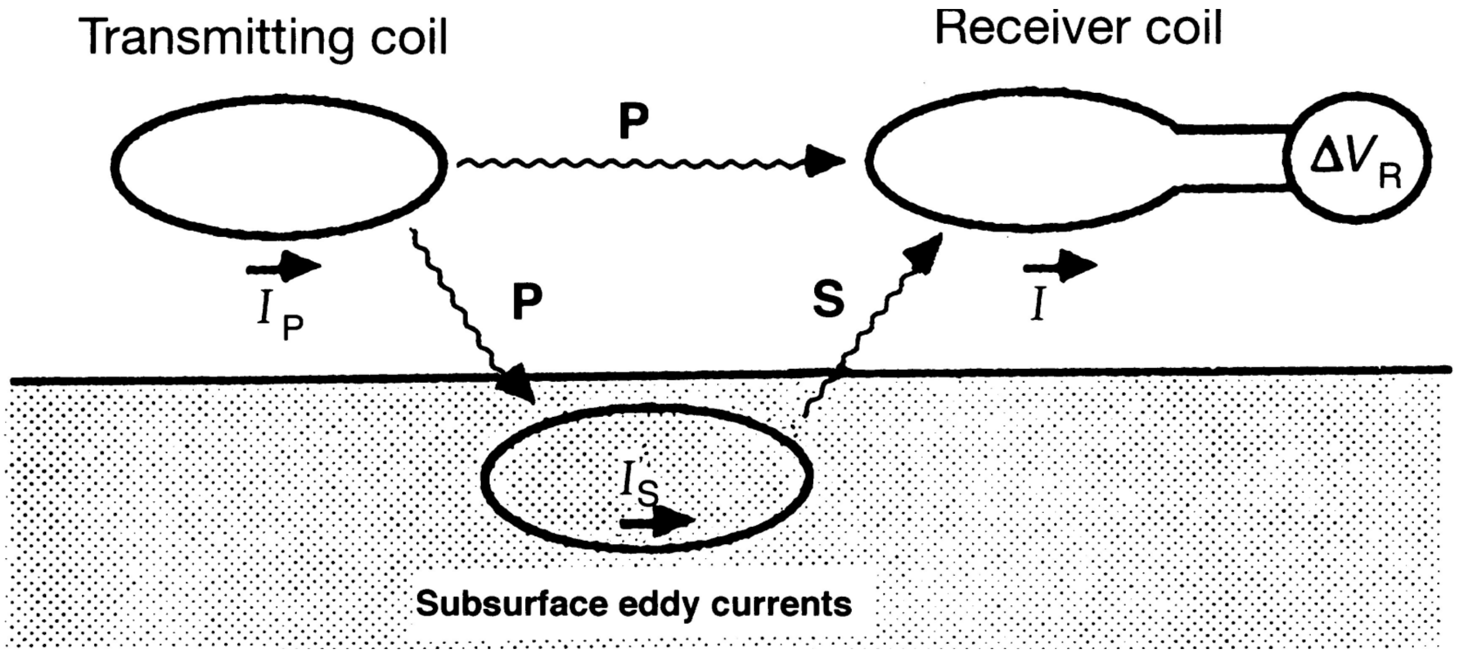
Today's Topics

- Basic Physics:
  - Attenuation and skin depth
  - Conductor in a non-resistive background
  - Basic physics recap

- Survey
  - Sources
  - Receivers
  - Sensitivity

Basic Physics:
Attenuation and Skin Depth

Earth has non-zero conductivity



This results in 2 things:

1) EM signal attenuates on the way down and on the way up from the conductor

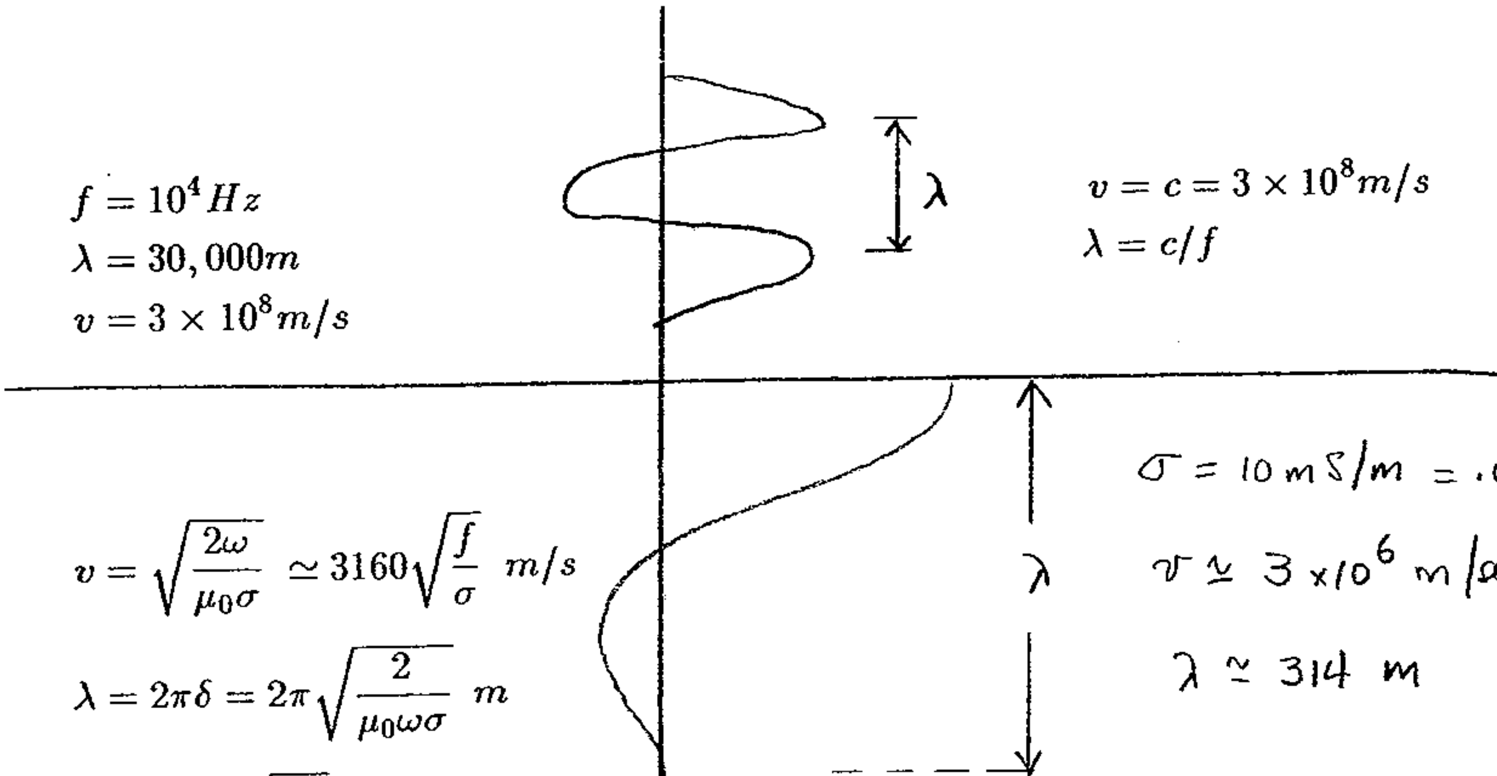
2) Currents induced in Earth resulting in secondary fields (we will ignore this effect for now)

EM waves inside the earth

$f = 10^4 Hz$

$\lambda = 30,000m$

$v = 3 \times 10^8 m/s$



$v = c = 3 \times 10^8 m/s$

$\lambda = c/f$

$\sigma = 10\, mS/m = .01 S/m$

$v \simeq 3 \times 10^6\, m/sec$

$\lambda \simeq 314\ m$

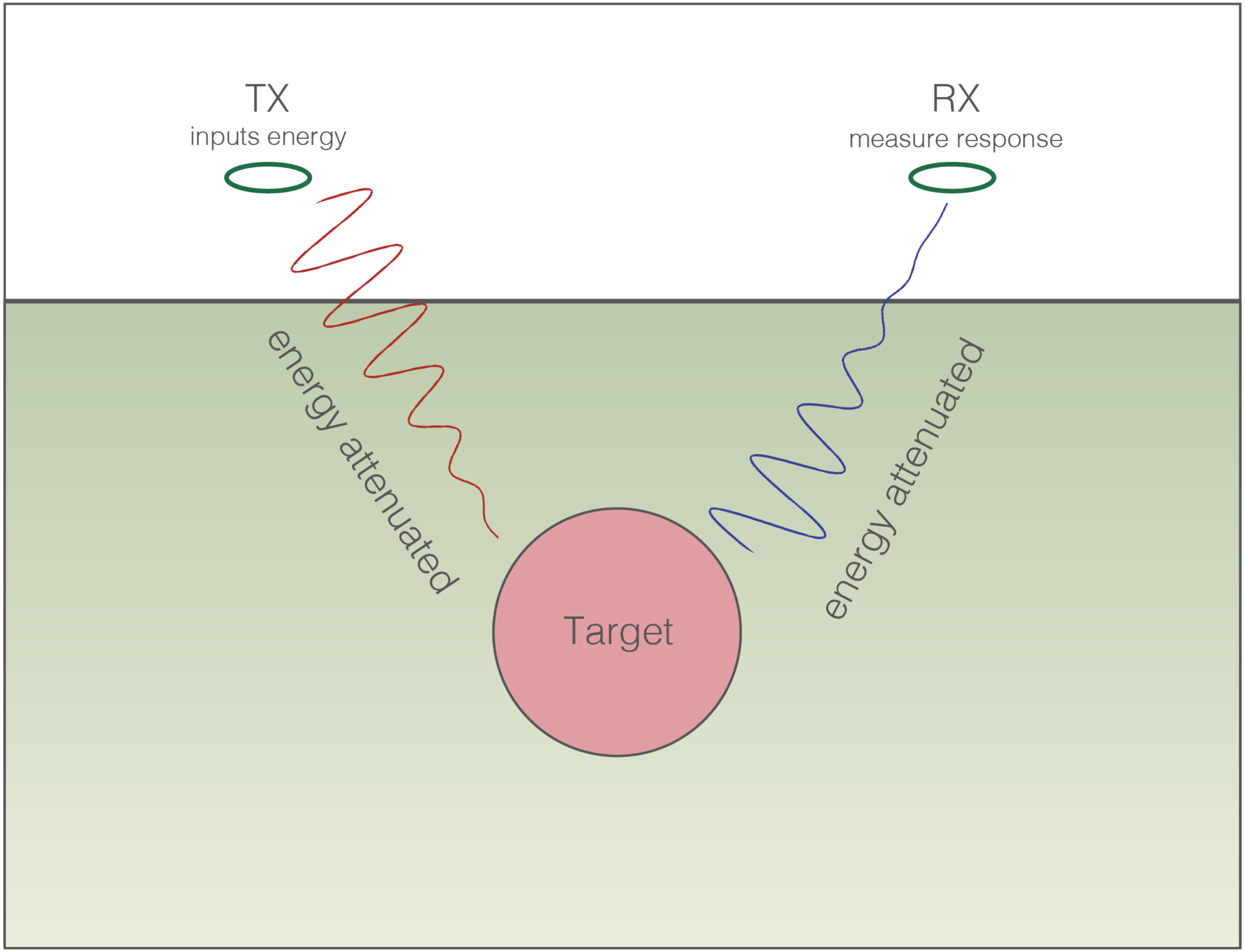
$$v = \sqrt{\frac{2\omega}{\mu_0 \sigma}} \simeq 3160 \sqrt{\frac{f}{\sigma}}\ m/s$$

$$\lambda = 2\pi\delta = 2\pi \sqrt{\frac{2}{\mu_0 \omega \sigma}}\ m$$

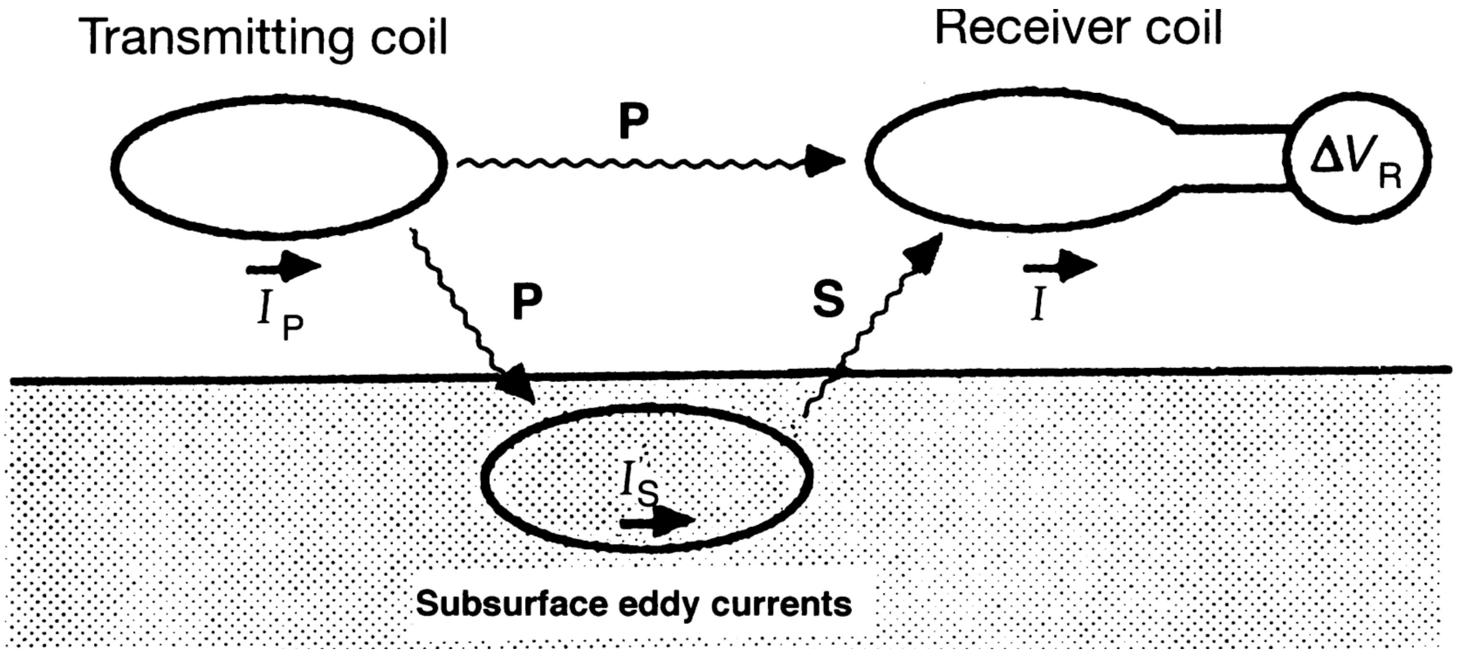
$$\lambda = 3140 \sqrt{\frac{1}{f\sigma}}\ m$$

Vertical dimension is not to scale

Attenuation of EM Signal



Depth of Investigation



Depth of investigation depends upon

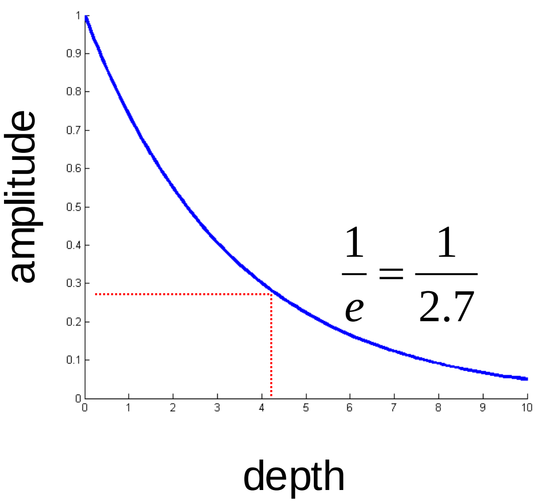
- skin depth
- source receiver geometry

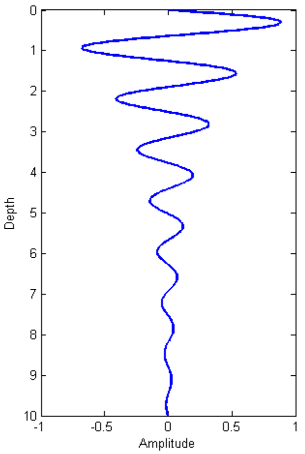
Skin Depth

- EM waves decay when propagating in a conducting earth

- Skin depth

$$\delta \approx 500\sqrt{\frac{\rho}{f}} \quad \text{meter}$$





where ρ is resistivity in Ωm
and *f* is frequency in Hz

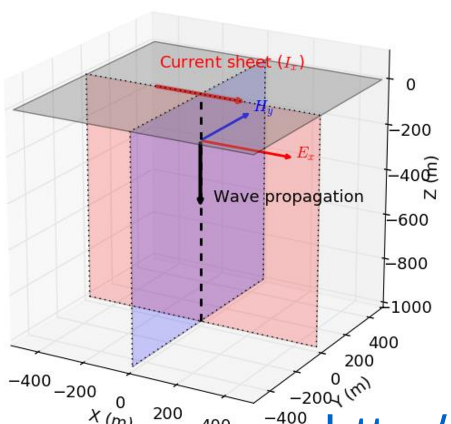
Plane Wave apps

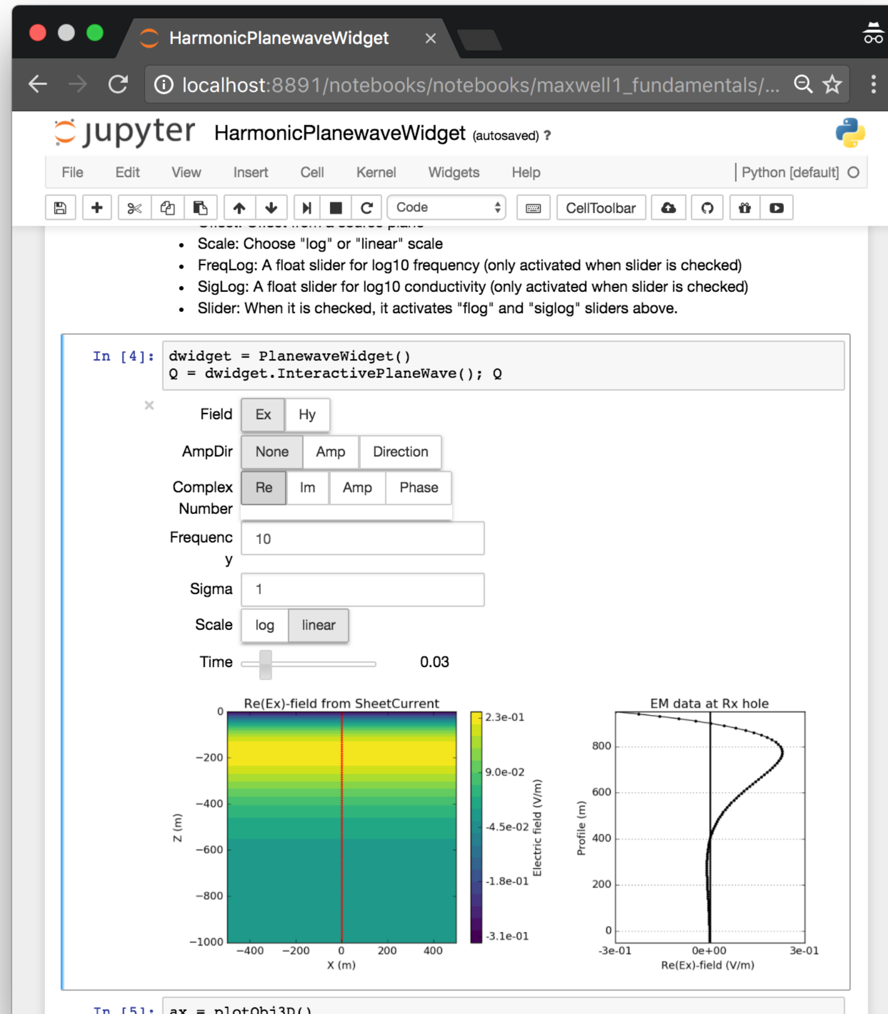
- 2 apps:
  - Transient

$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2} z}{2\pi^{1/2} t^{3/2}} e^{-\mu\sigma z^2/(4t)}$$

  - Harmonic

$$\mathbf{H} = \mathbf{H_0} \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$





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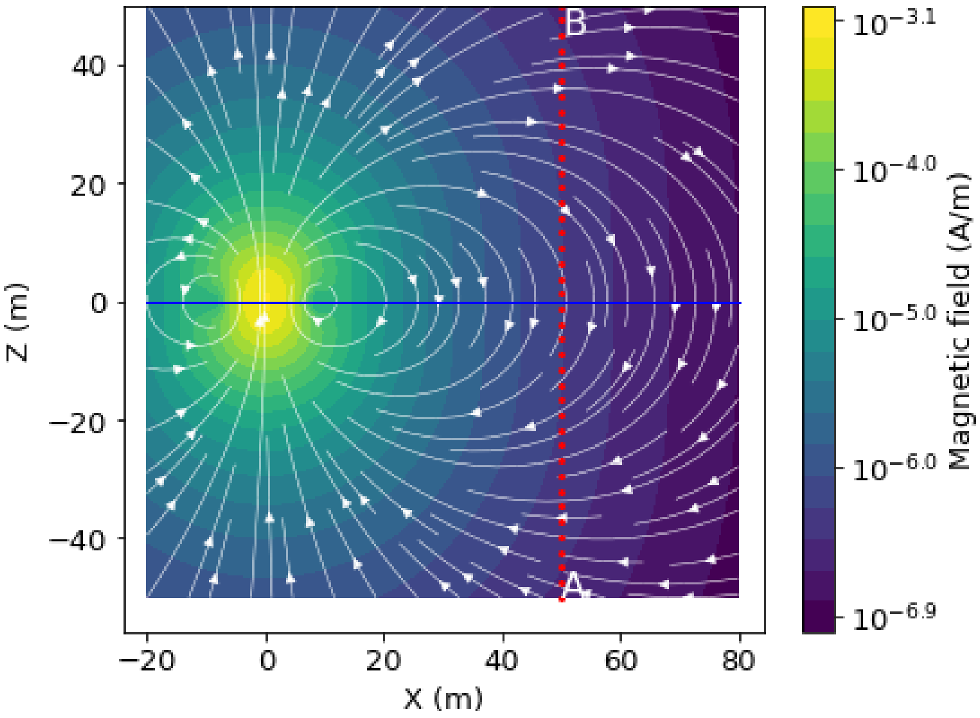
Geometric Decay: dipole sources

- Primary field has a geometric decay away from the transmitter
  - very different from a plane wave source
- Two principal sources (for small transmitters characteristic of airborne surveys):
  - VMD: vertical magnetic dipole
  - HMD: horizontal magnetic dipole

Magnetic field from a vertical magnetic dipole in a wholespace

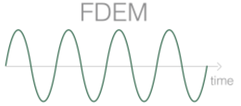


Basic Physics:
Conductor in a Non-Resistive Background

Effects of background conductivity (frequency)

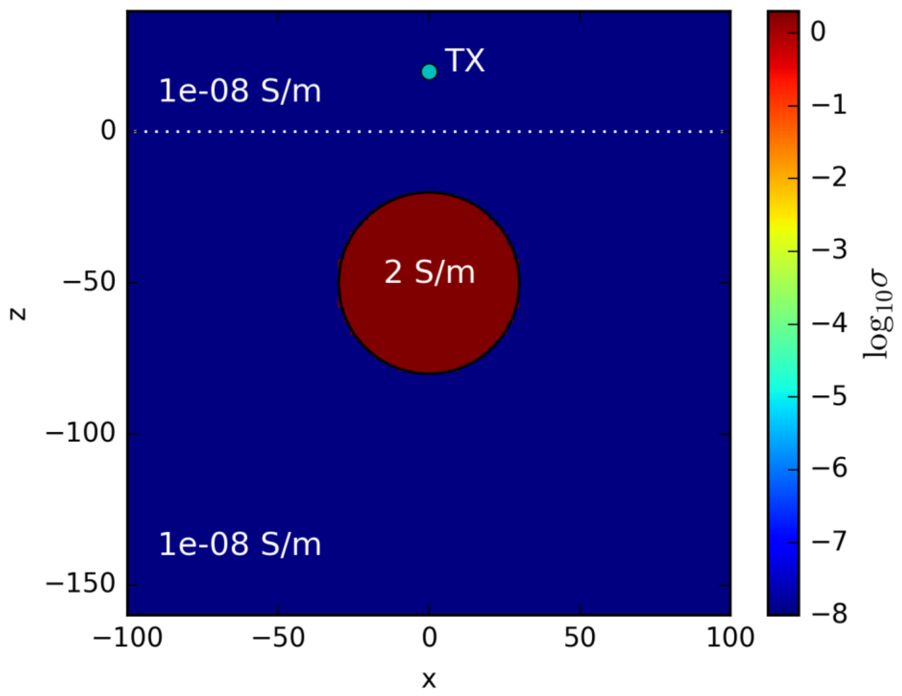
- Buried, conductive sphere
- Vary background conductivity

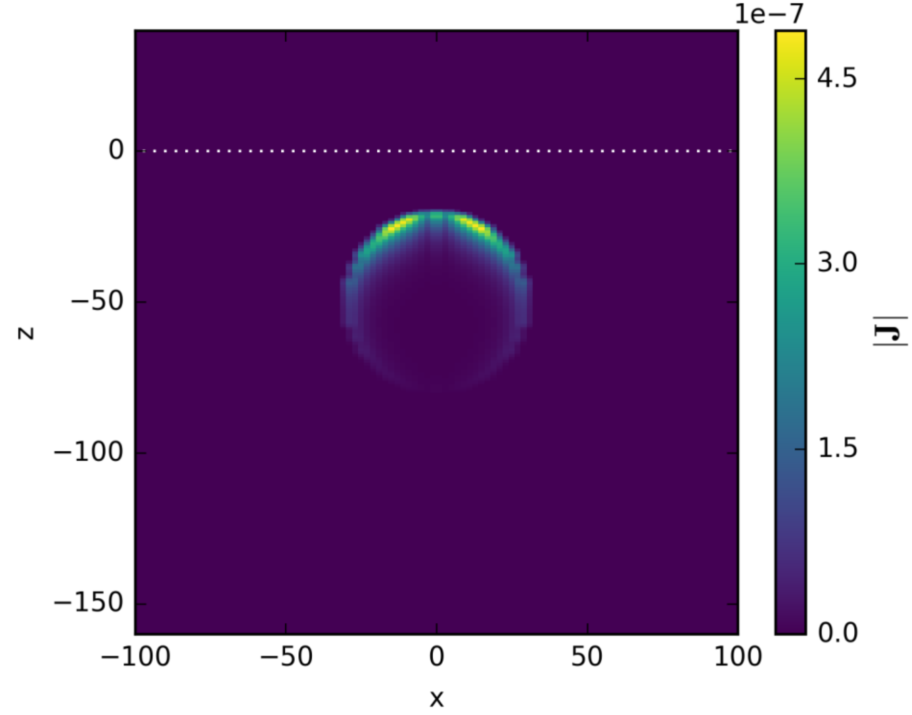
- Frequency: 10^4 Hz



10^{-8} S/m background

Current Density





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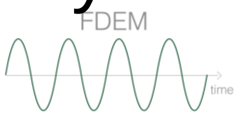
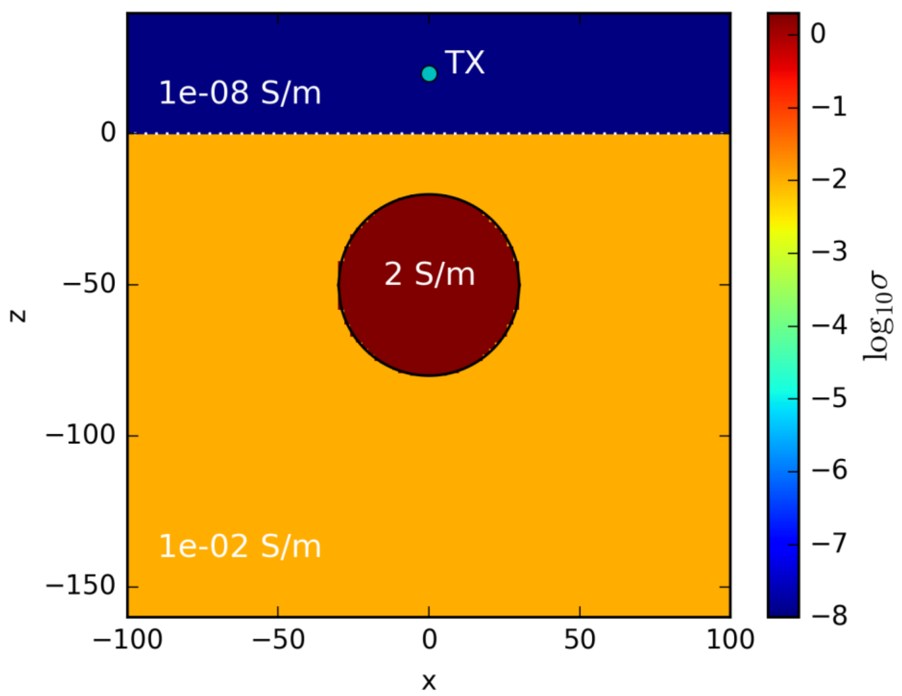
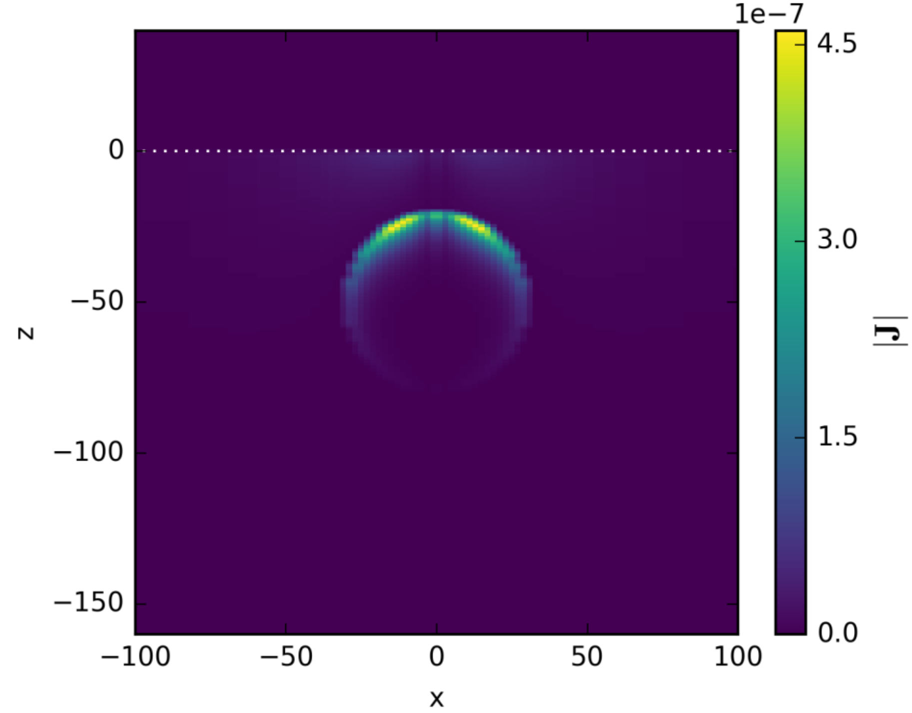


10^{-2} S/m background

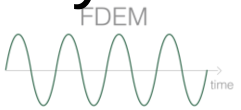


Current Density

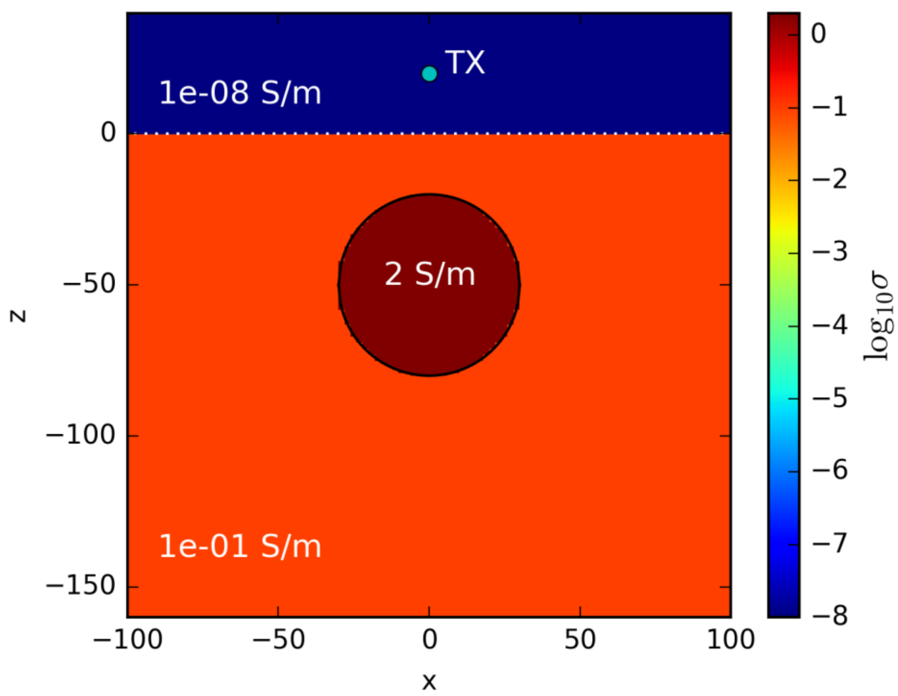


Effects of background conductivity (frequency)

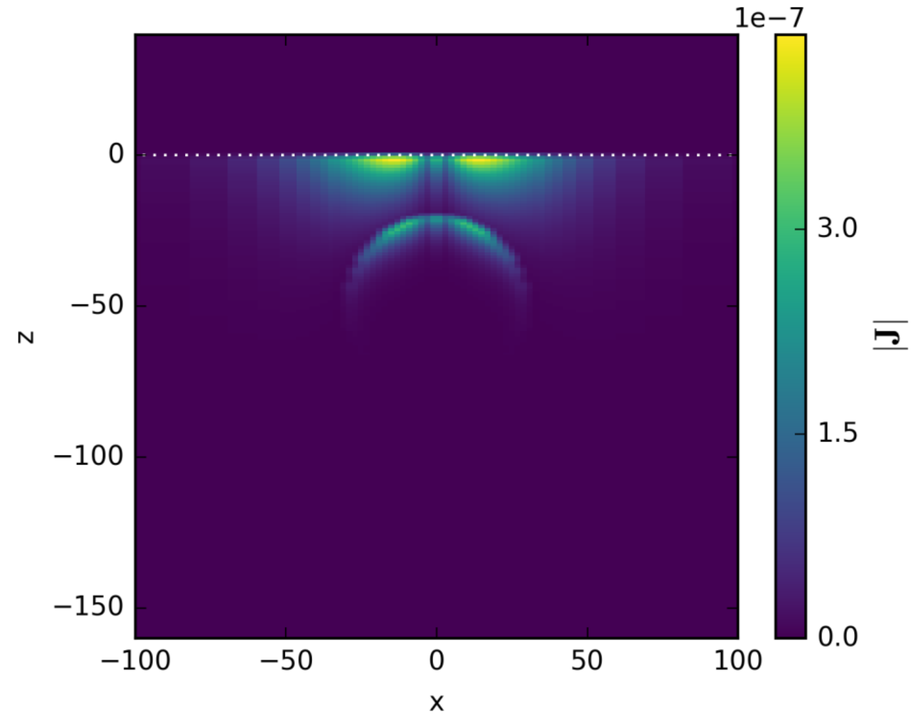
- Buried, conductive sphere
- Vary background conductivity
- Frequency: 10^4 Hz



10^{-1} S/m background



Current Density

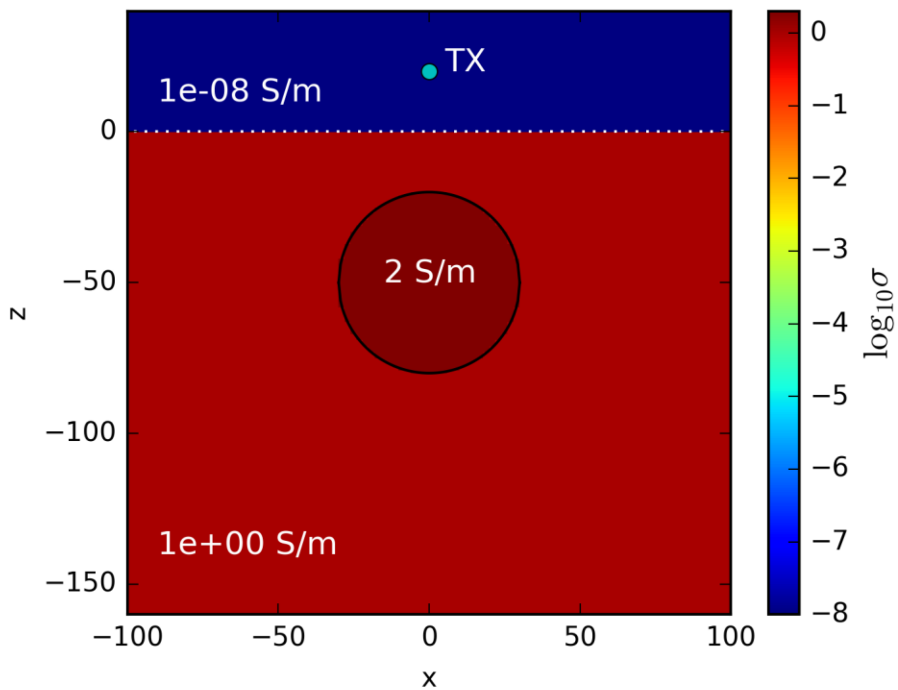


Effects of background conductivity (frequency)

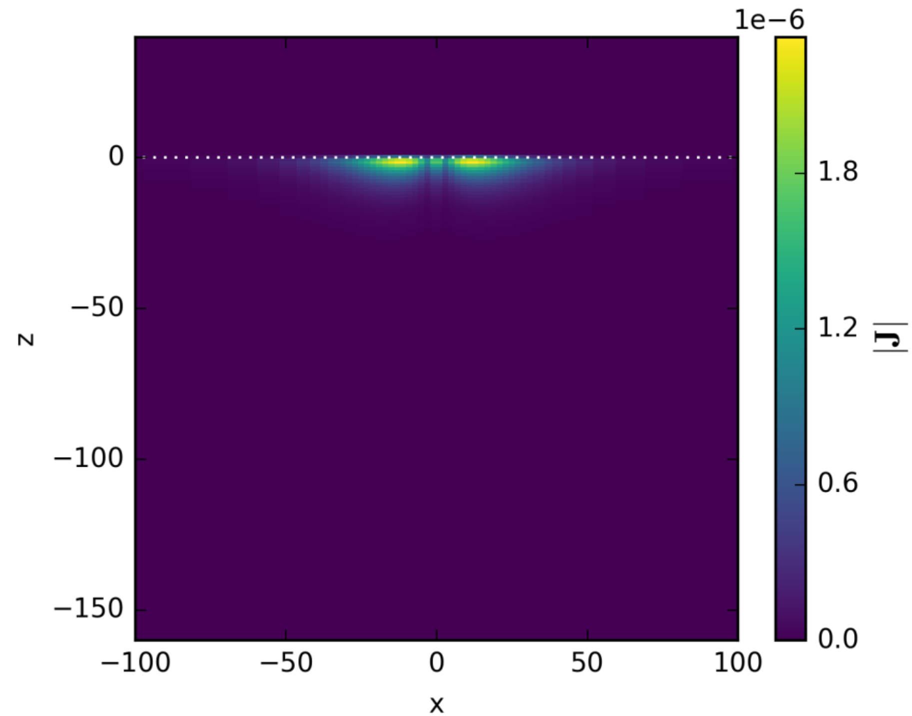
- Buried, conductive sphere
- Vary background conductivity

- Frequency: 10^4 Hz

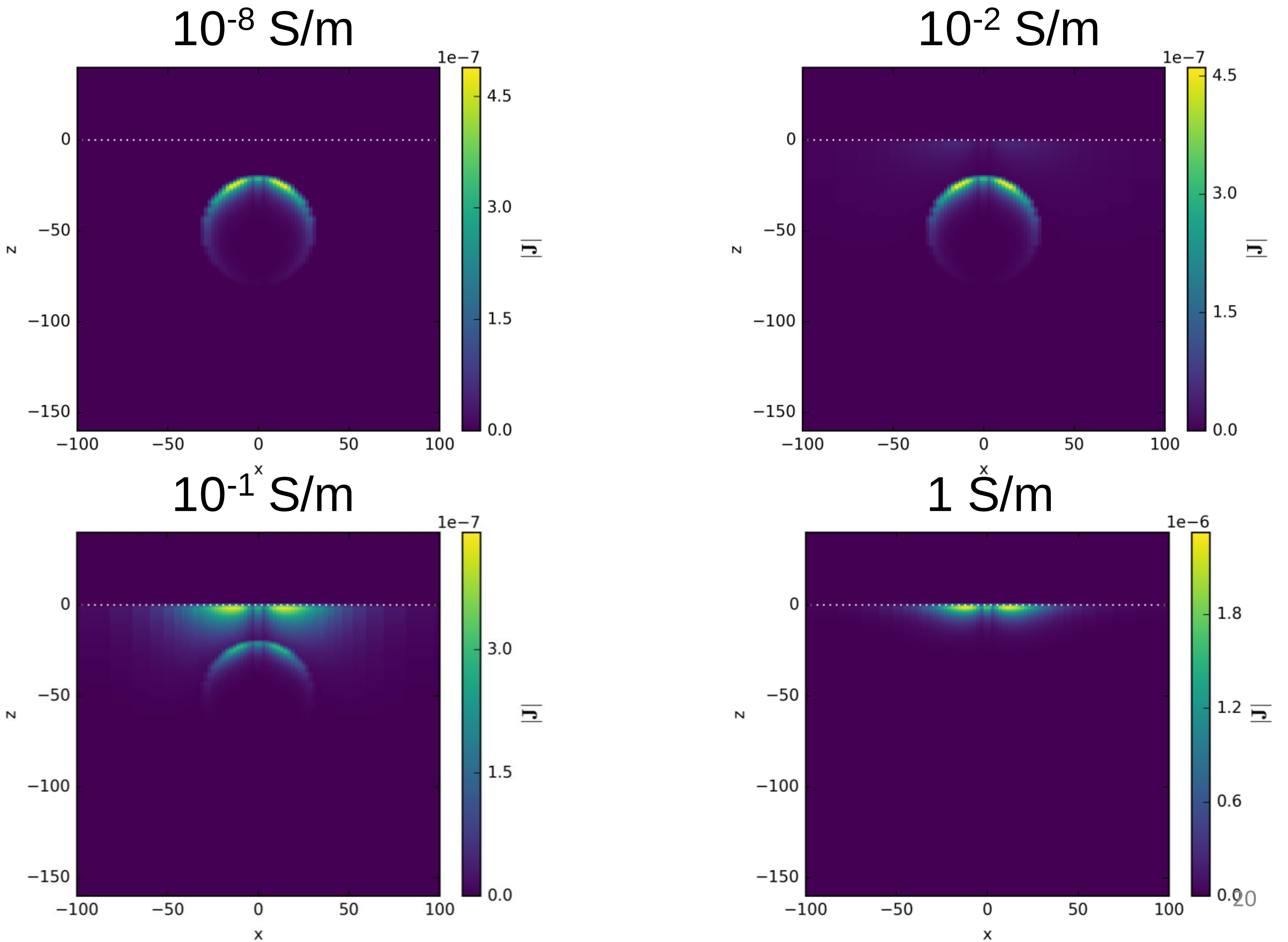
1 S/m background



Current Density



Effects of background conductivity (frequency)



Basic Physics:
Recap

Basic Experiment

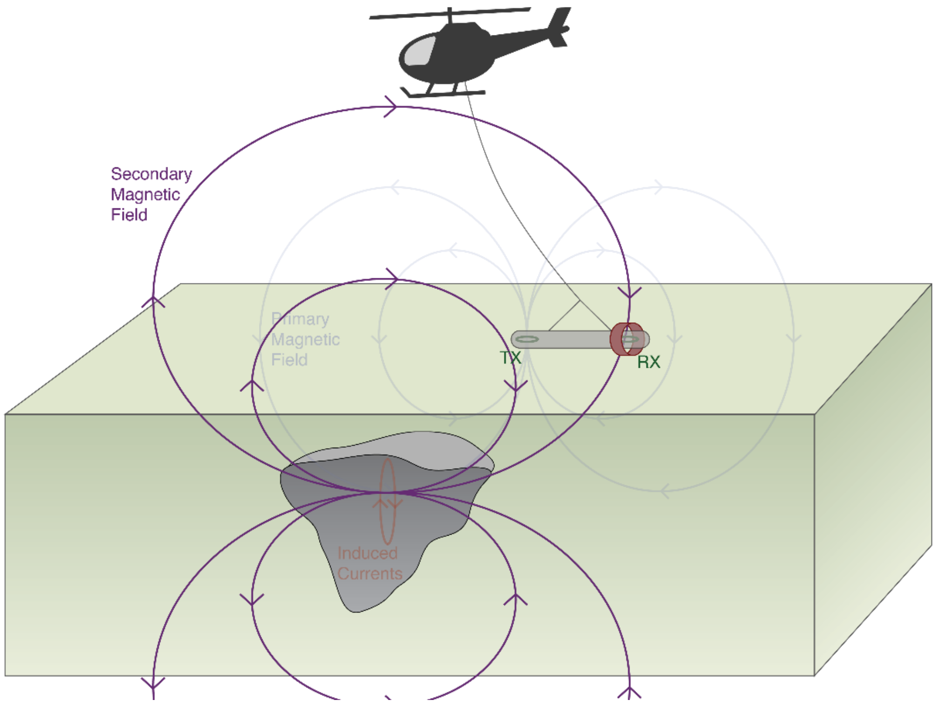
- **Source (Tx):**
  Current loop makes primary magnetic field

- **Induction:**
  Time-varying magnetic fields induce electric fields everywhere

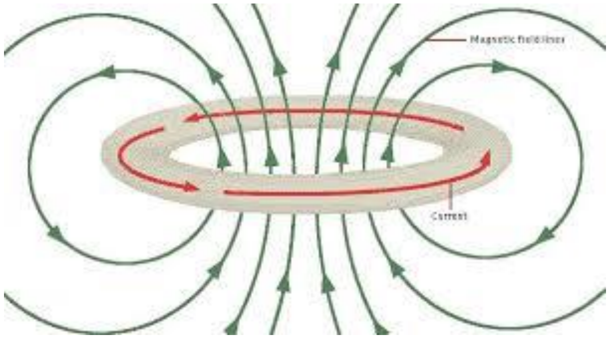
  → Large induced currents in conductors

- **Secondary Fields:**
  Induced currents in conductors produce secondary magnetic fields

- **Receiver (Rx):**
  Measures magnetic fields



Directions

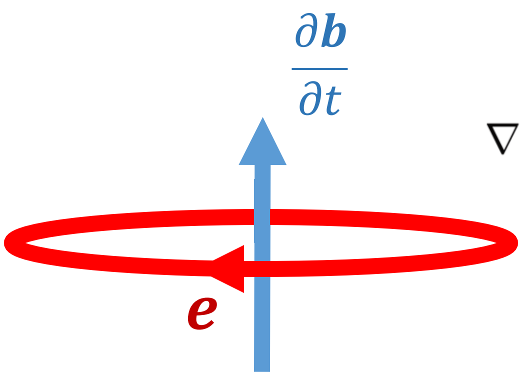


Ampere's Law: $\nabla \times \mathbf{H} = \mathbf{J}$

- Currents produce magnetic fields
- Right-hand rule

Faraday's Law

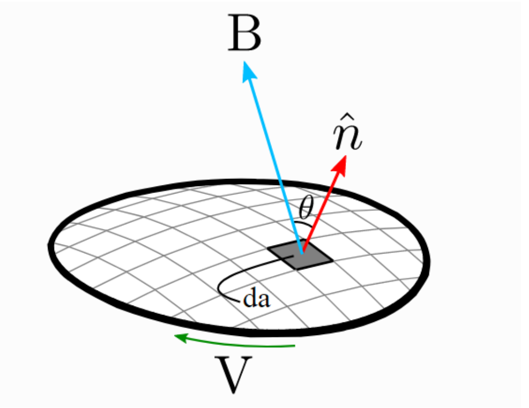
- Time/frequency varying magnetic fields produce electric fields
- Time/frequency varying magnetic flux generates voltage in wire loops
- Left-hand rule



Lenz' Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt}$$

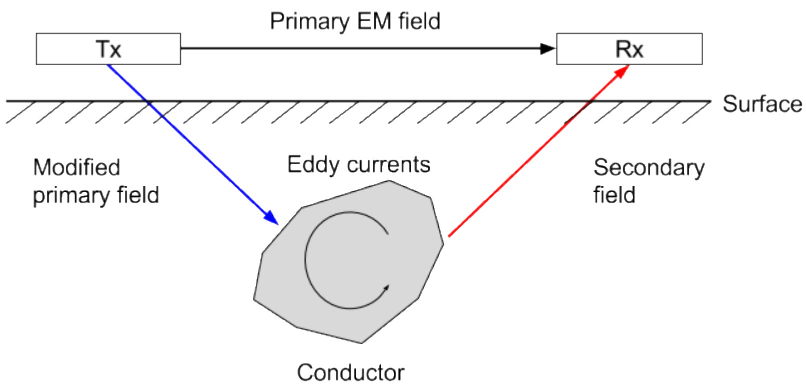


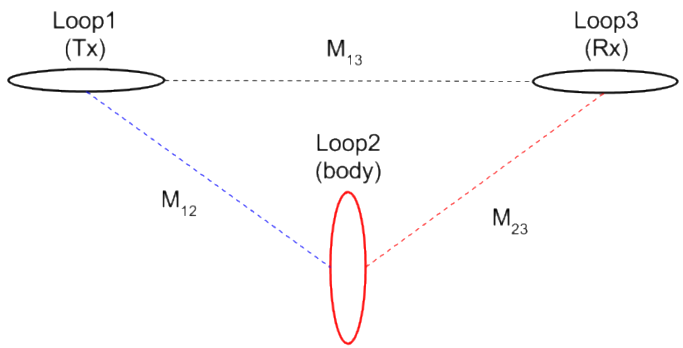
Ohm's Law

- Current resulting from electric fields depends on conductivity/resistivity

$$\vec{J} = \sigma \vec{E}$$

Circuit model of EM induction





Coupling coefficient:

- Depends on loop geometry

$$M_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|\mathbf{r} - \mathbf{r}'|^2}.$$

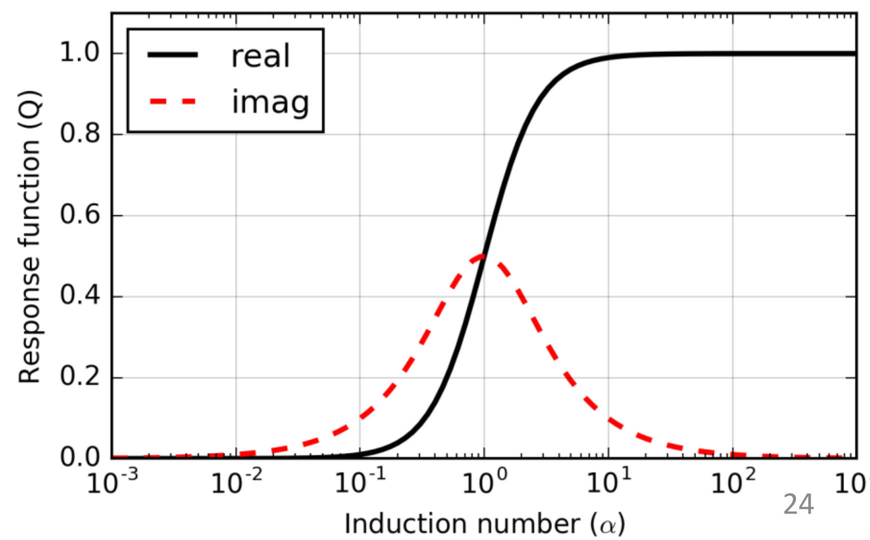
Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L} \underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}$$

Induction Number

- Depends on properties of target

$$\alpha = \frac{\omega L}{R}$$



Coupling

- If coupling between transmitter and target is bad ($M_{12}\sim 0$)
  → $H_s \sim 0$

- If coupling between target and receiver is bad ($M_{23}\sim 0$)
  → $H_s \sim 0$

- If transmitter frequency is low ($\omega\sim 0$)
  → $\alpha \sim 0$
  → $H_s \sim 0$

- If L/R is smaller, higher frequencies required for large response

- Coupling and EMF

$$
\begin{aligned}
EMF &= -\frac{\partial \phi_{\mathbf{B}}}{\partial t} \\
&= -\frac{\partial}{\partial t}\left(\mathbf{B}_p \cdot \hat{\mathbf{n}}\right) A
\end{aligned}
$$

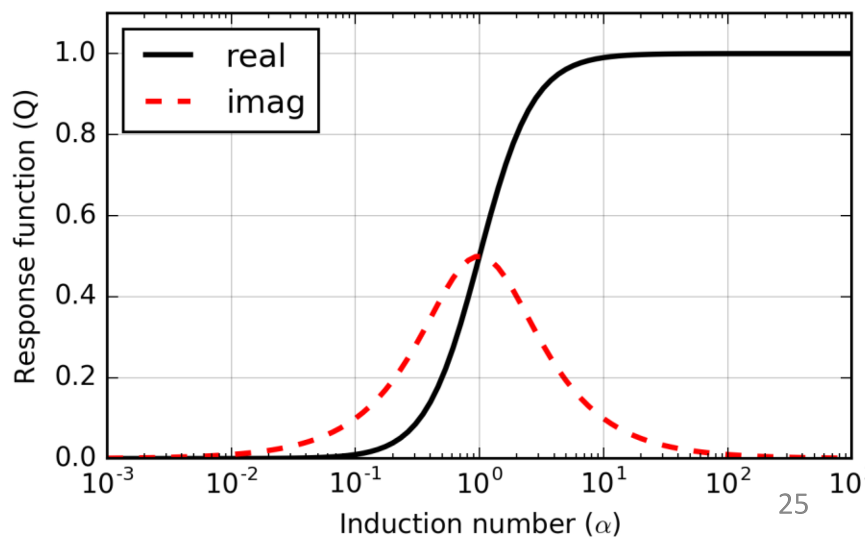
Magnetic field at the receiver

$$
\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L}\underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}
$$

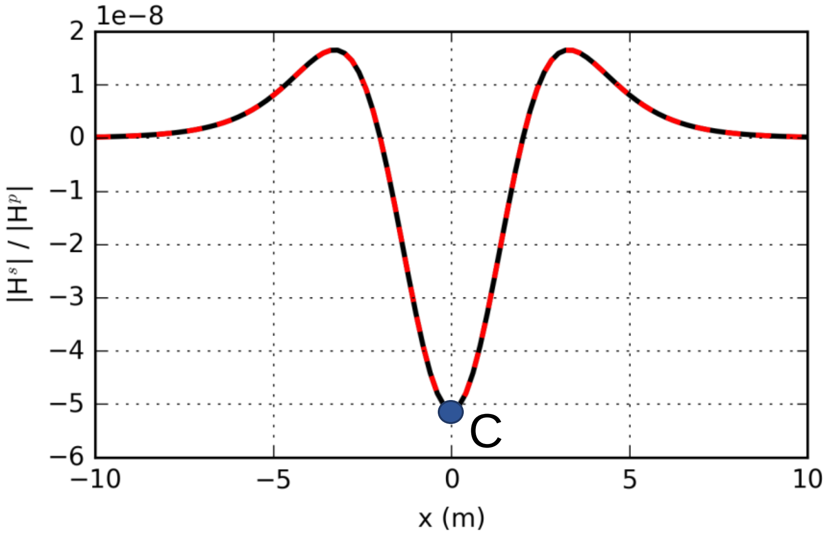
Induction Number

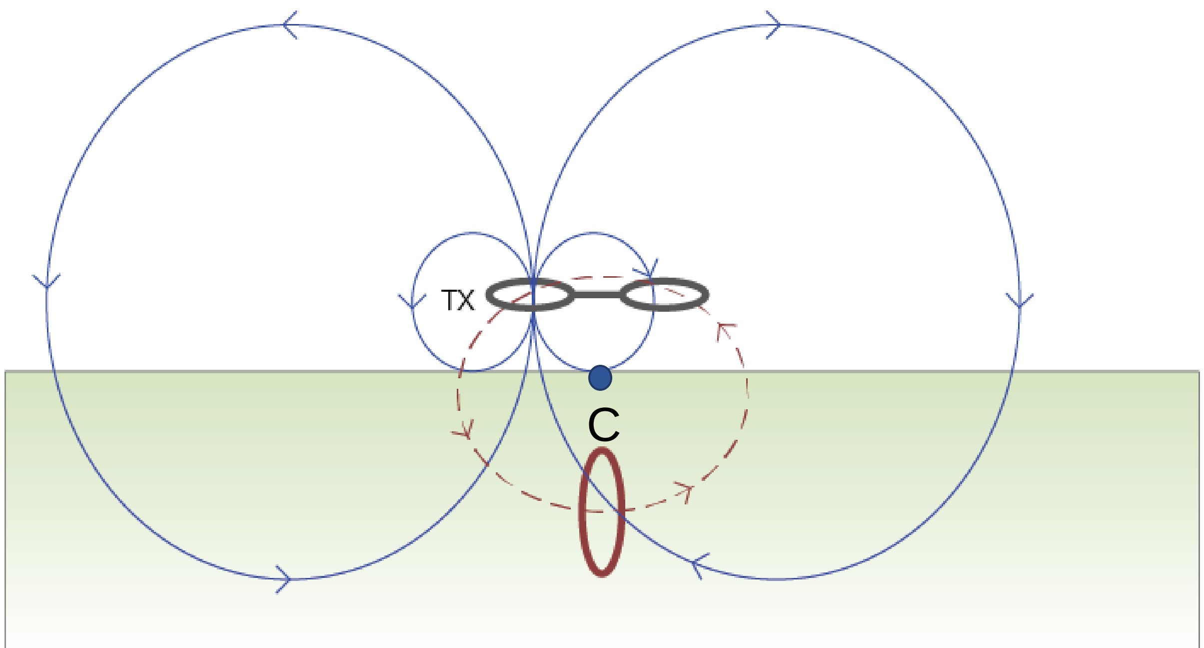
- Depends on properties of target

$$
\alpha = \frac{\omega L}{R}
$$

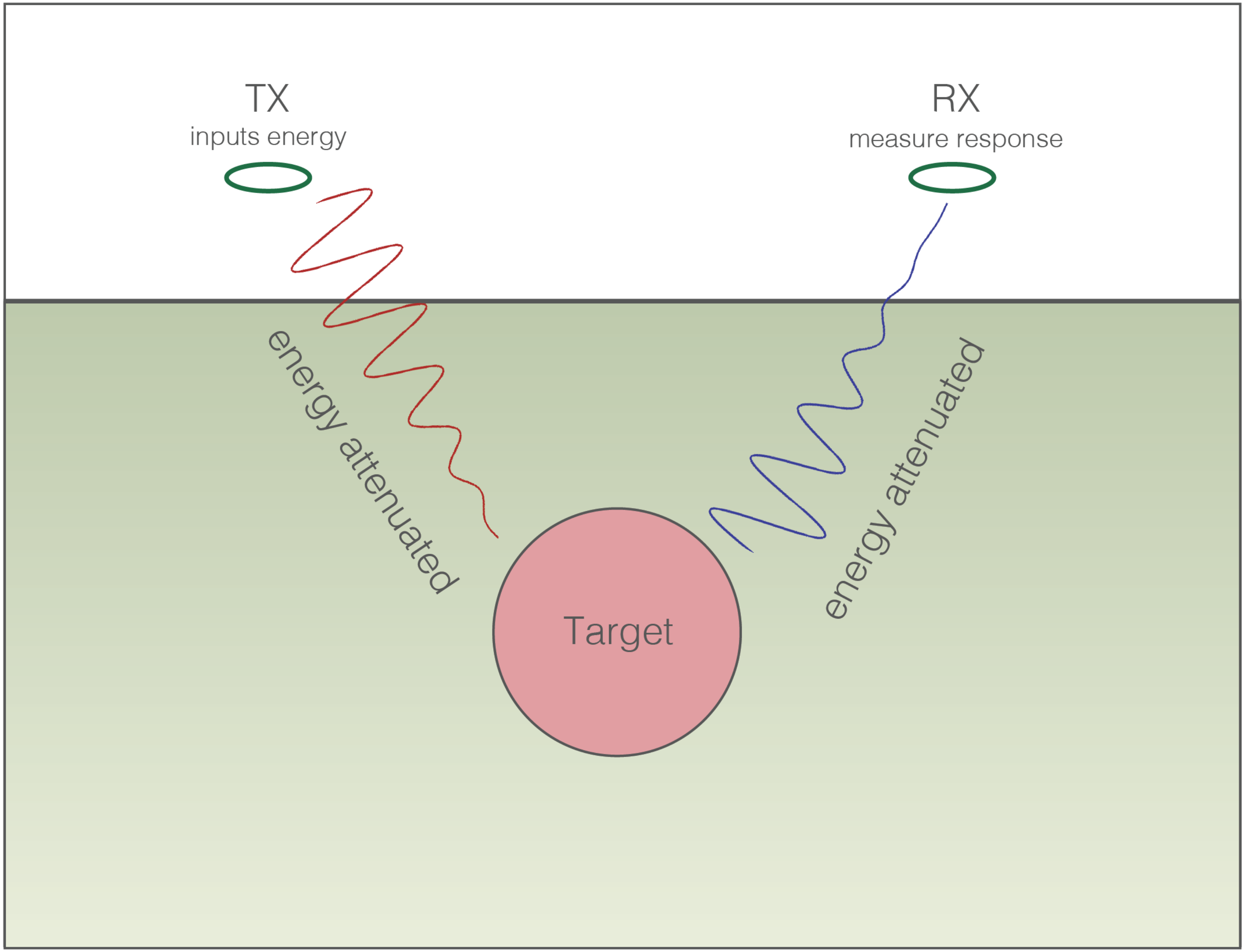


Response over target



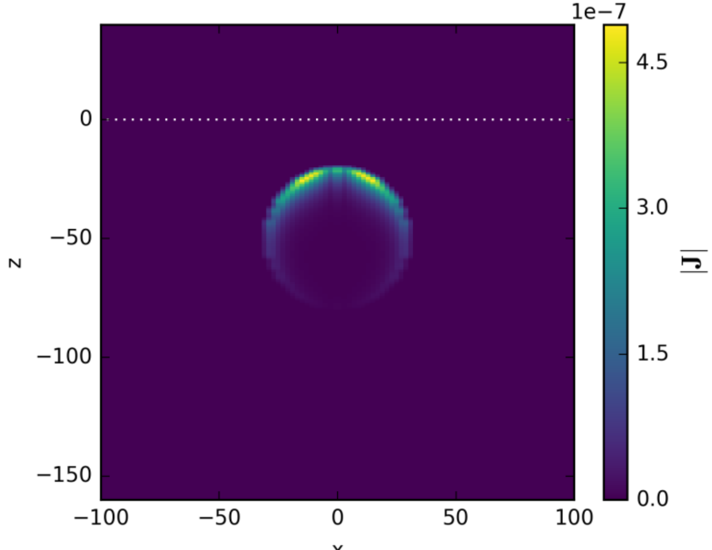


Attenuation of EM Signal

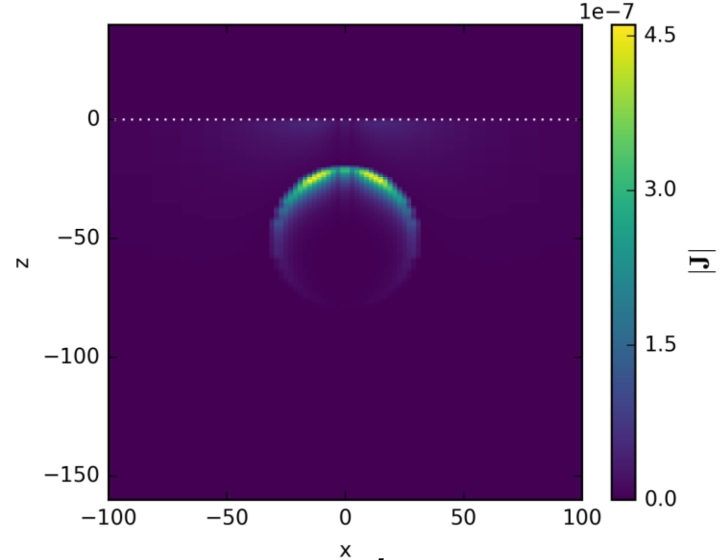


Effects of background conductivity (frequency)

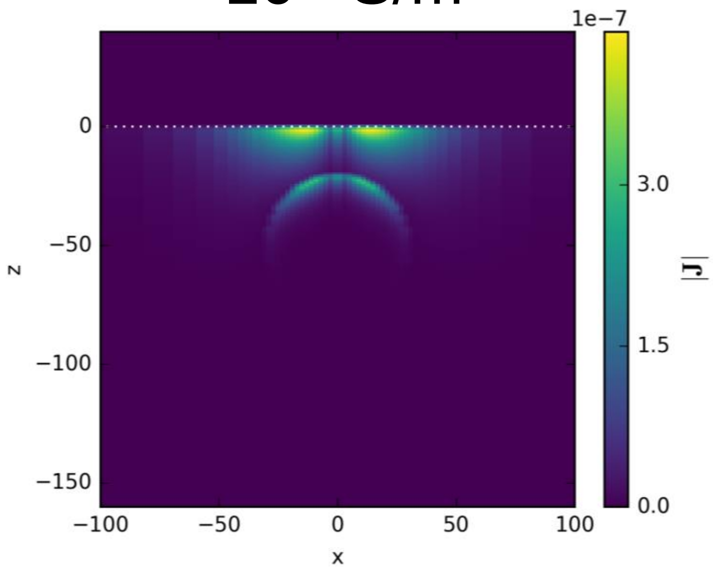
10^{-8} S/m



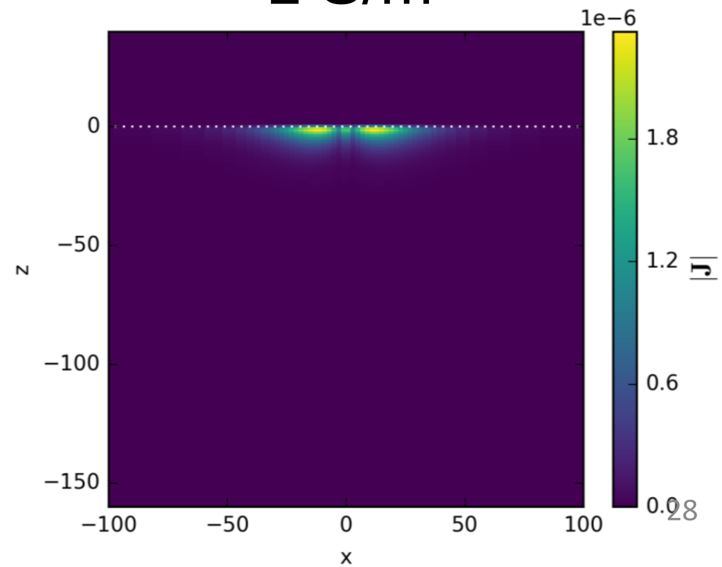
10^{-2} S/m



10^{-1} S/m



1 S/m



Survey

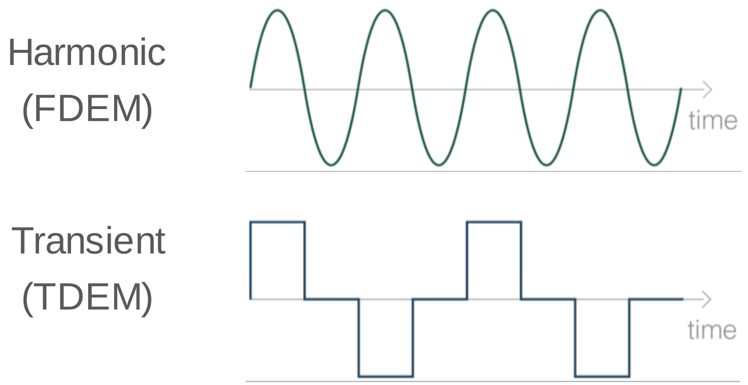
Reading on the GPG:

https://gpg.geosci.xyz/content/DC_resistivity/DC_surveys.html

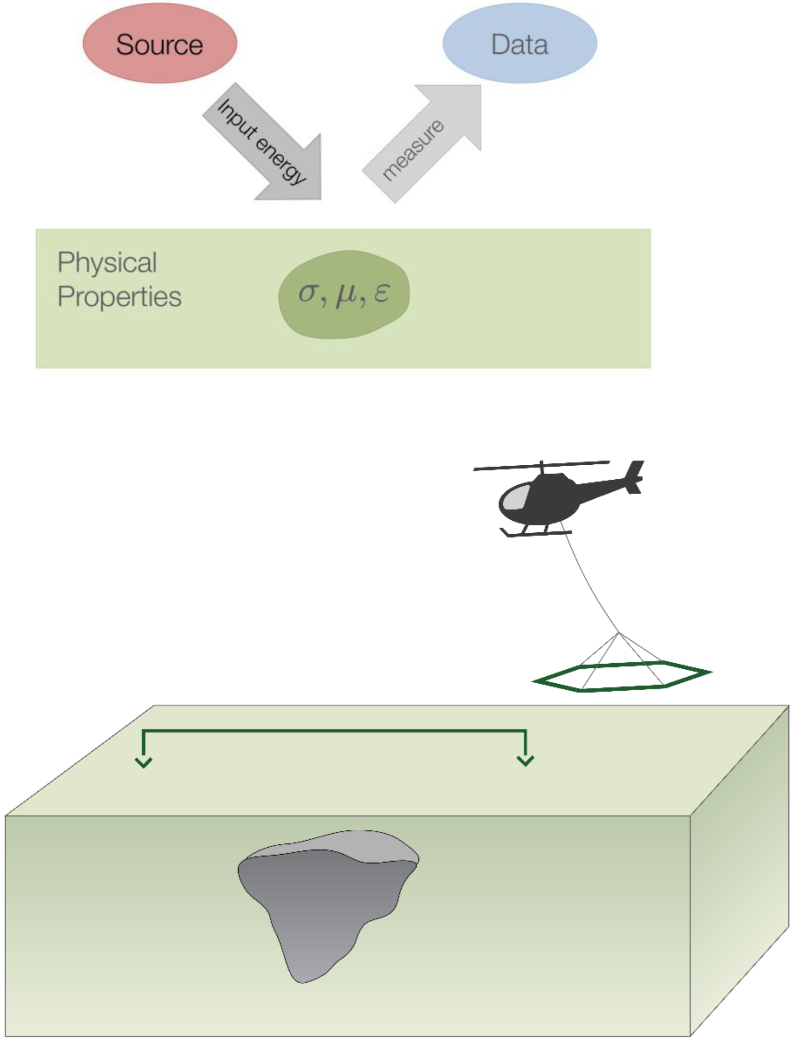
Sources

- Type
  - Inductive
  - Grounded

- Waveform

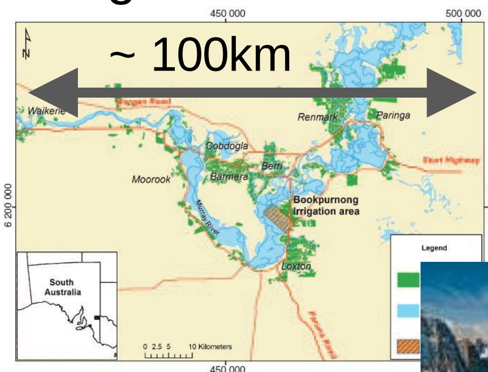


- Location
  - Airborne
  - Ground
  - Borehole



Applications for different sources

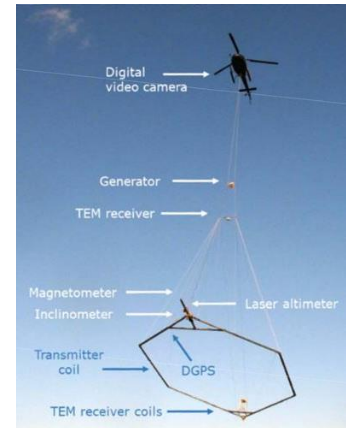
Large areas



Rugged terrain

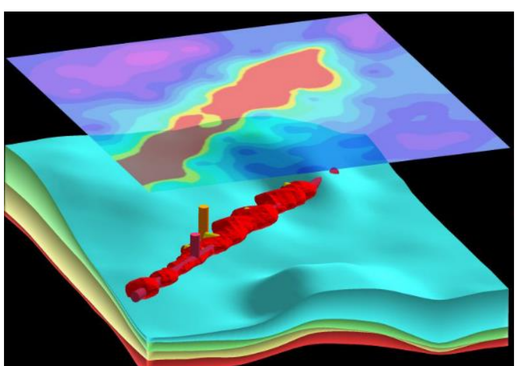
Airborne Survey

Resolve

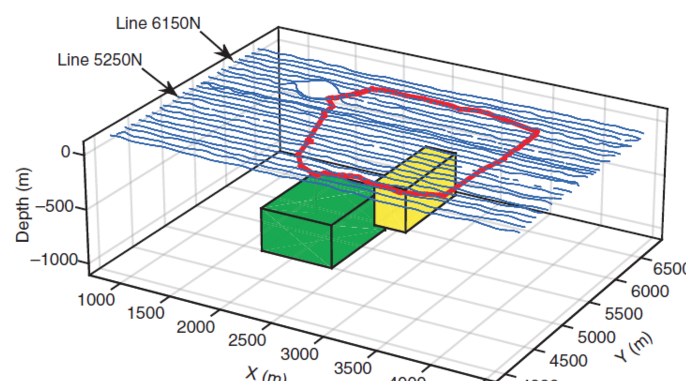


SkyTEM

Deep Targets

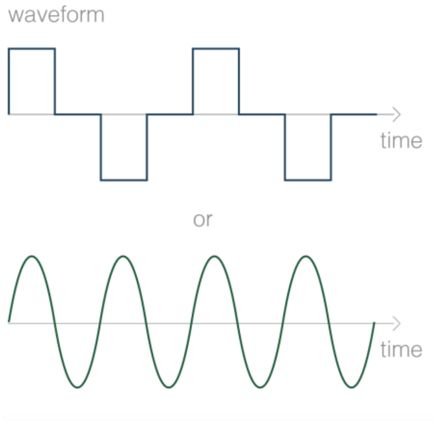


Large Loop



Transmitter considerations

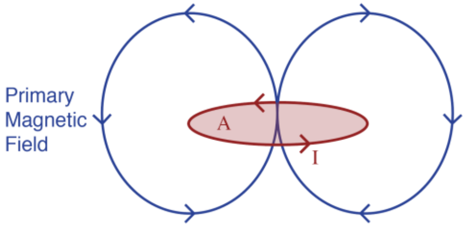
- Time or frequency?



- Key factor is moment

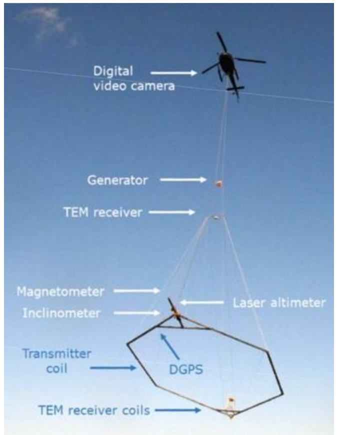
$$m = I \text{ (current) } A \text{ (area) } N \text{ (\# of turns)}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{r}(\mathbf{m}\cdot\mathbf{r})}{|\mathbf{r}|^5} - \frac{\mathbf{m}}{|\mathbf{r}|^3}\right)$$



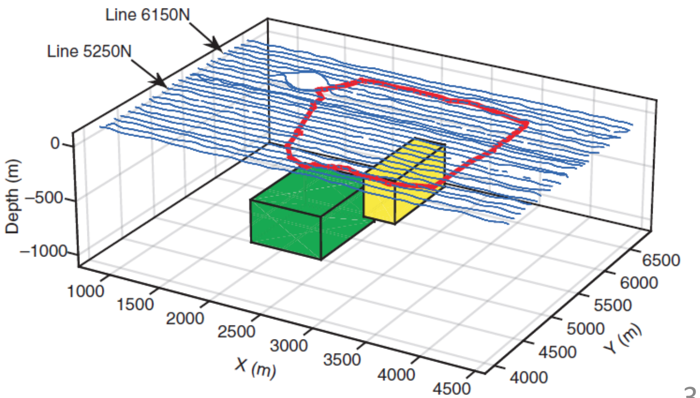
Airborne Survey

Resolve



SkyTEM

Large Loop

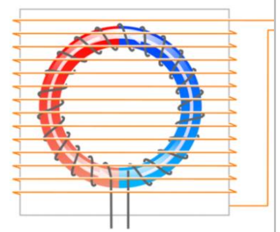


Receivers: Time Domain

Magnetometer

$$\mathbf{b}(t)$$

- Measures:
  - Magnetic fields
  - 3 components
- eg. 3-component fluxgate

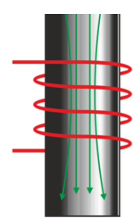


Fluxgate

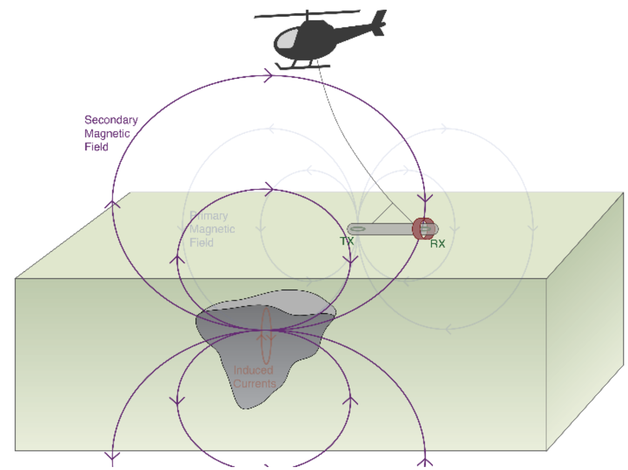
Coil

$$\frac{\partial b}{\partial t}$$

- Measures:
  - Voltage
  - Single component that depends on coil orientation
  - Coupling matters
- eg. airborne frequency domain
- ratio of Hs/Hp is the same as Vs/Vp



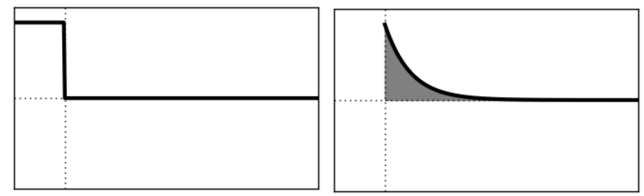
Coil



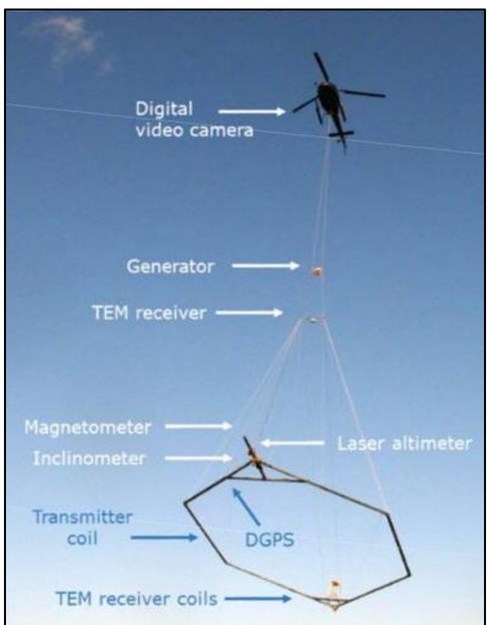
Receivers: Time Domain Data

- Primary field has off-time
- Measure secondary fields
- Receivers can be mounted on transmitter loop or above

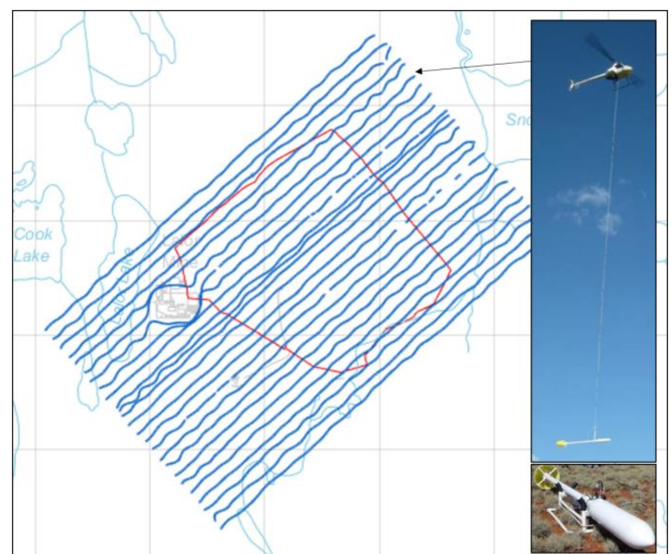
Current Response



SkyTEM

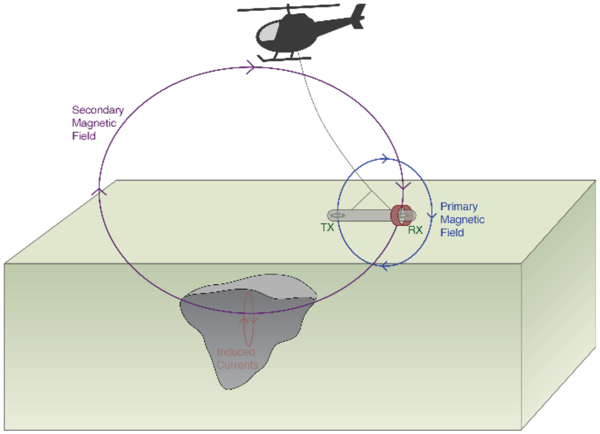


HeliSAM

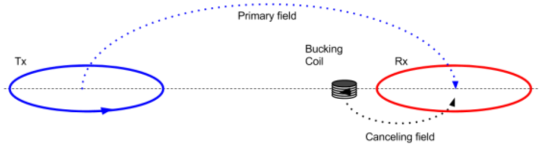


Receivers: Frequency Domain

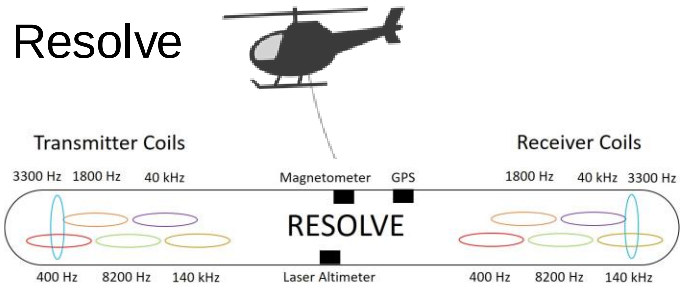
- Primary field
  - always "on"
  - large compared to secondary fields



- Primary removal
  - Compute and subtract
  - Bucking coil

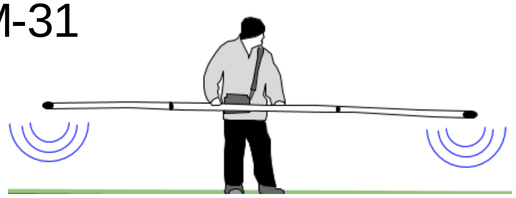


Resolve

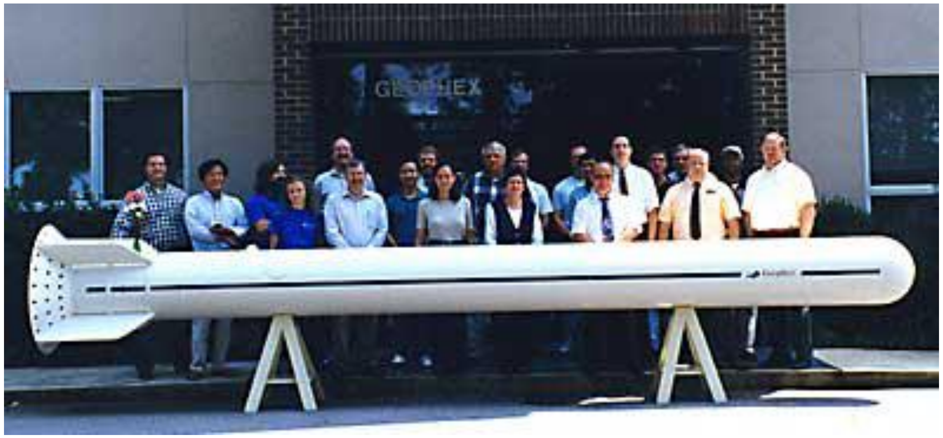


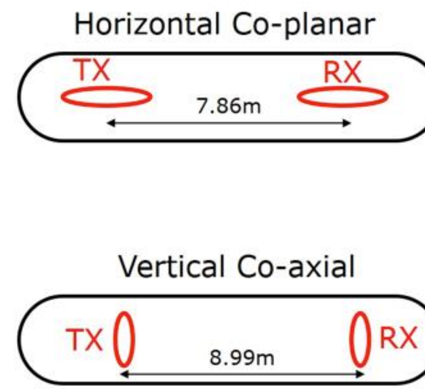
- Main requirement:
  - Know positions of Tx and Rx
  - Keep them in one unit

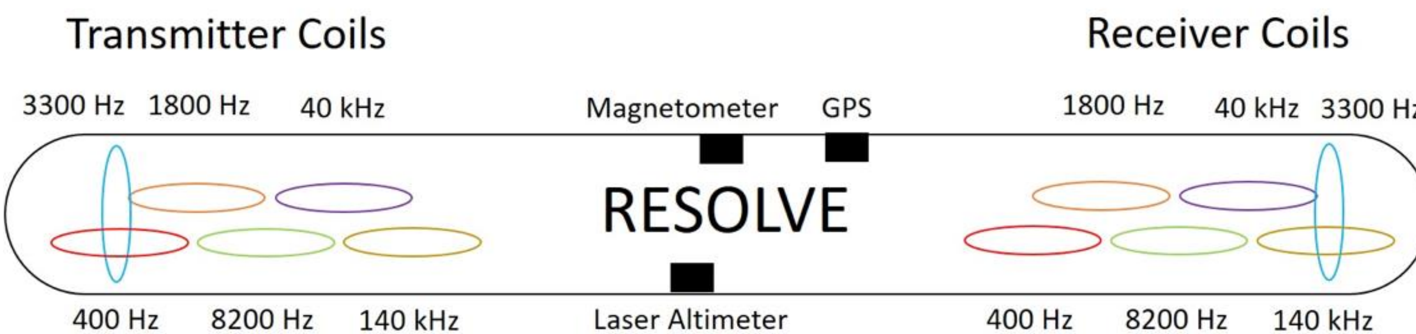
EM-31



Survey: Airborne EM Resolve System







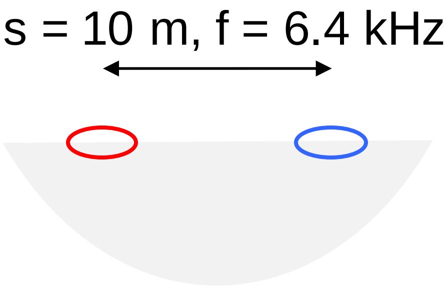
Skin depth: High frequency for shallow; low frequency for deep

Survey: GEM3

- Concentric Tx-Rx
- Frequency 60 Hz to 24 kHz
- Identify an object based on its spectral fingerprints

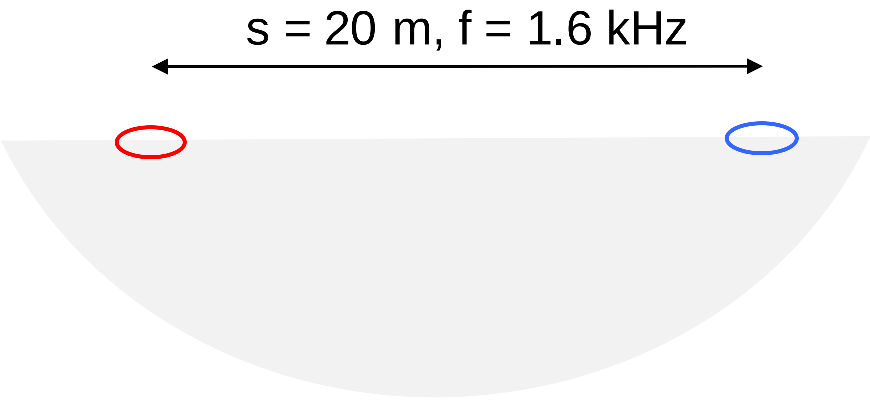
Survey: EM-41

Penetration depends upon frequency and Tx-Rx separation



Survey: EM-41

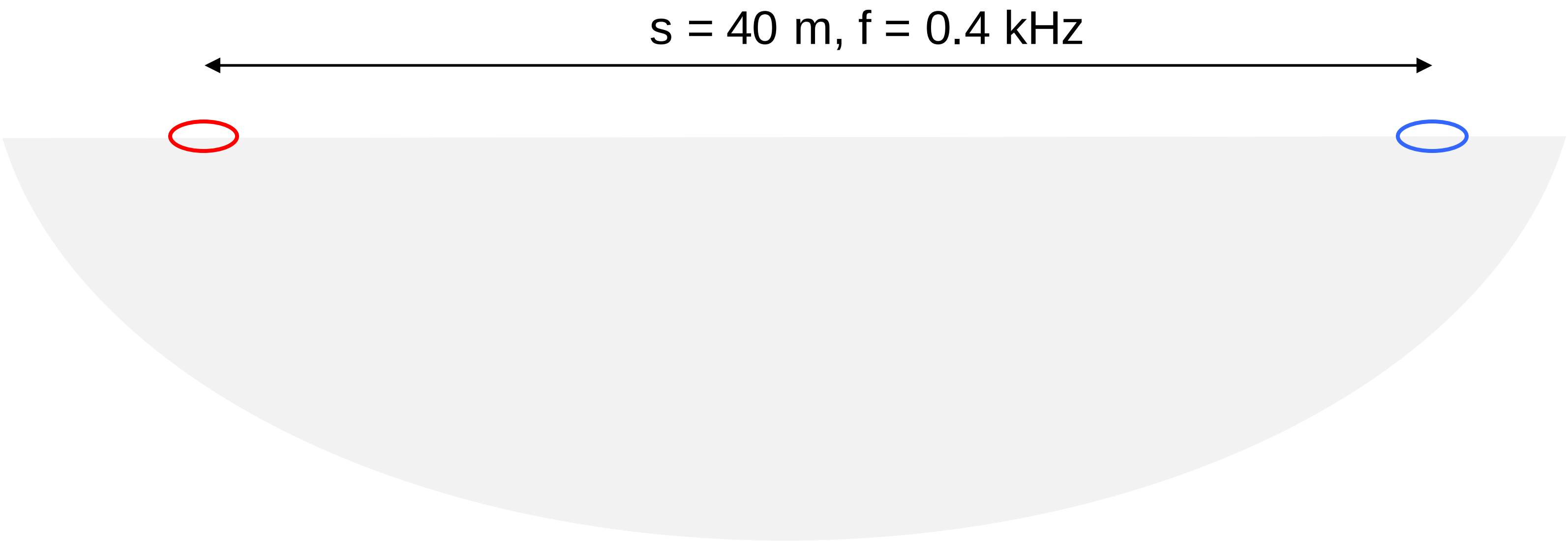
Penetration depends upon frequency and Tx-Rx separation



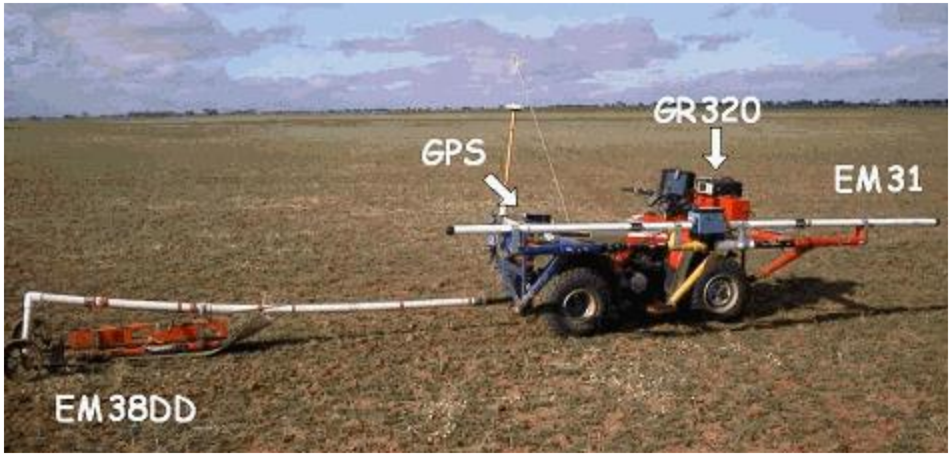
Survey: EM-41

Penetration depends upon frequency
and Tx-Rx separation

- Variable depth of exploration down to 60 m
- HCP or VCP coil configuration
- Groundwater exploration in fractured and faulted bedrock



Survey: EM-41



- Frequency = 9.8 kHz
- Tx-Rx spacing = 3.66 m
- Horizontal or vertical coplanar
- “Ground conductivity meter”

Survey: Recap

- Many FEM and TEM systems for different applications
- Challenging terrain or large scale:
  → Consider airborne survey
- Deep targets and/or conductive overburden:
  → Low frequency and large transmitter dipole moment
- High resolution needed:
  → Ground-based survey and possibly higher frequencies
- Penetration/domain of investigation of ground FEM systems depends on frequency and separation

Unit Activities

- **Labs: (EM I)**
  - Monday, November 4th
  - Tuesday, November 5th
- **Labs: (EM II)**
  - Monday, November 18th
  - Tuesday, November 19th
- **TBL:**
  - Friday, November 15th
- **Quiz:**
  - Wednesday, November 20th