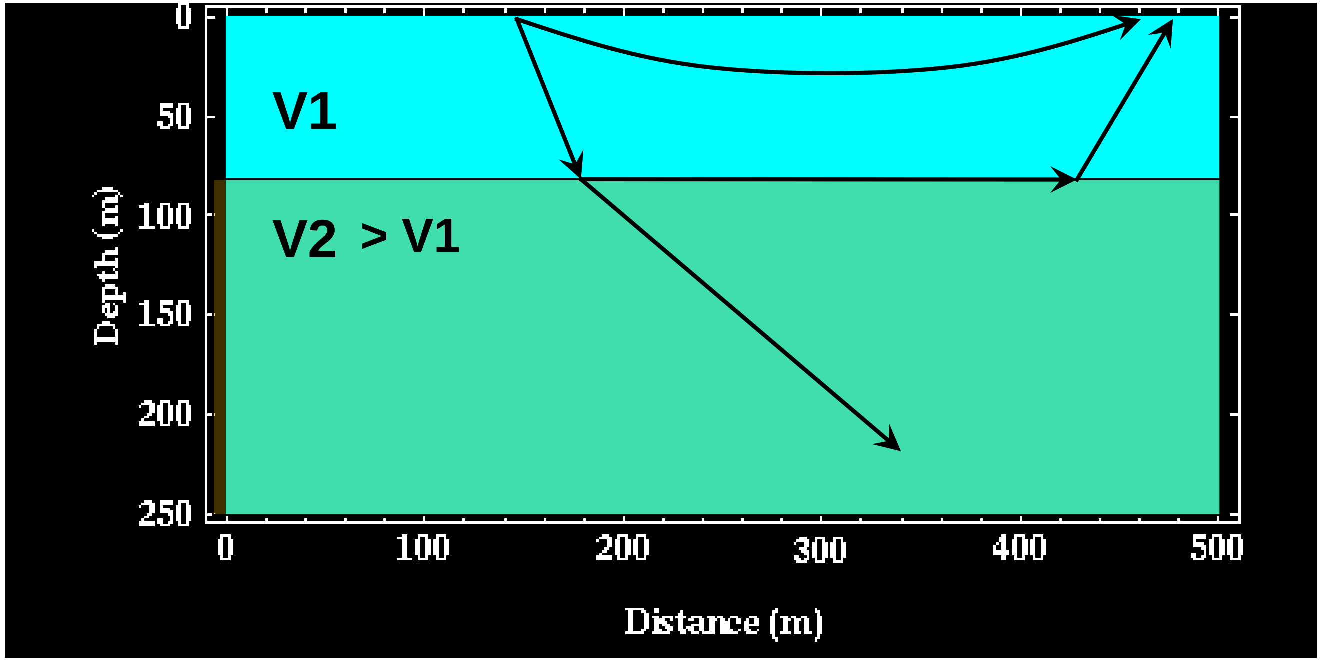
From last time

- **Ray path:** travel path that seismic signals can take to get from one point to another

- **Wave front:** shows the propagation of energy

- Seismic energy decays over time/distance



From last time

- Seismic waves reflect, refract and transmit at interfaces

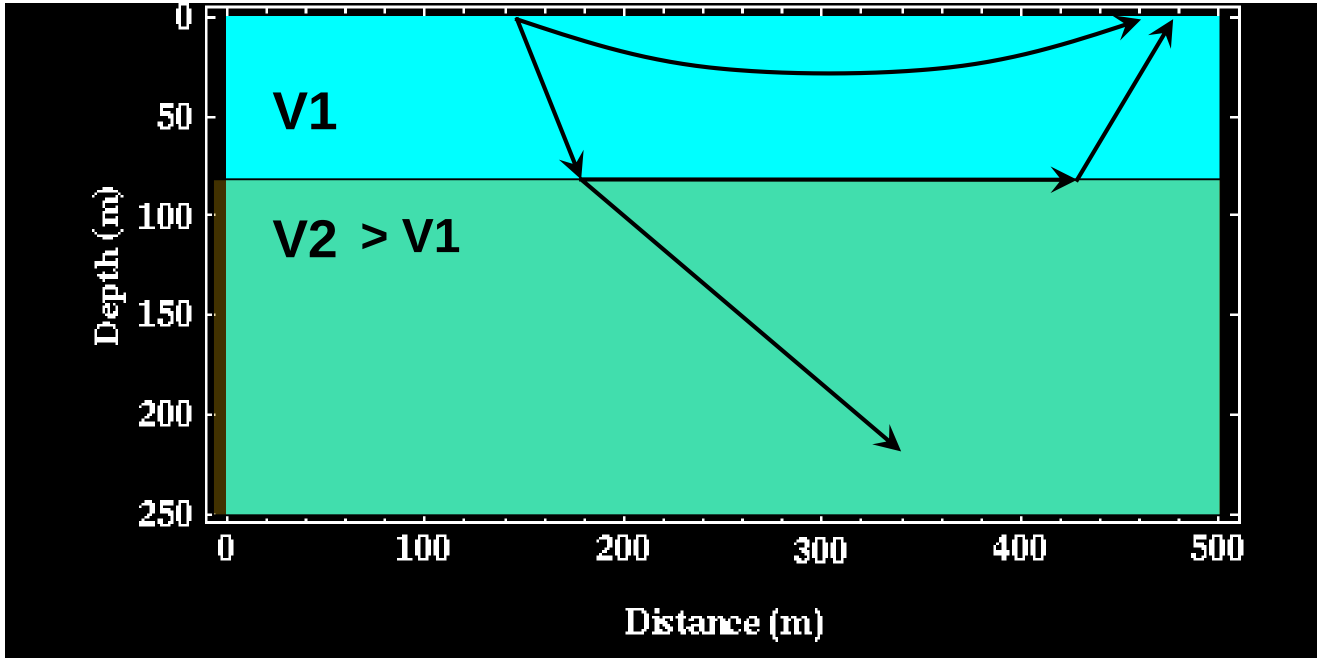
- Acoustic impedance

$$Z = \rho V$$

- Reflection depends on acoustic impedance

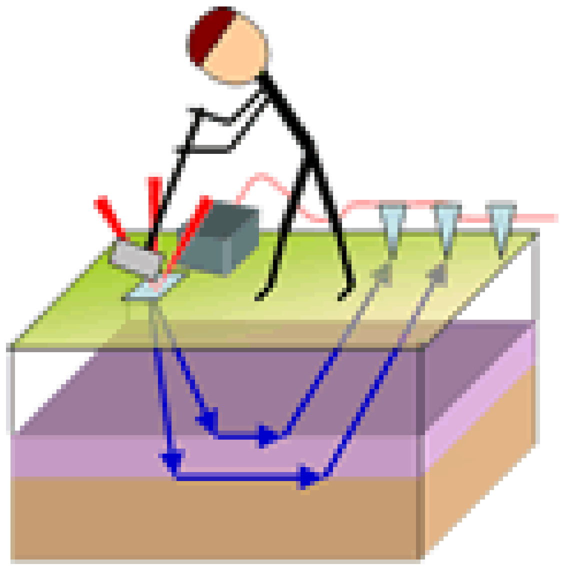
$$R = \frac{A_1}{A_0} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, -1 \le R \le 1$$

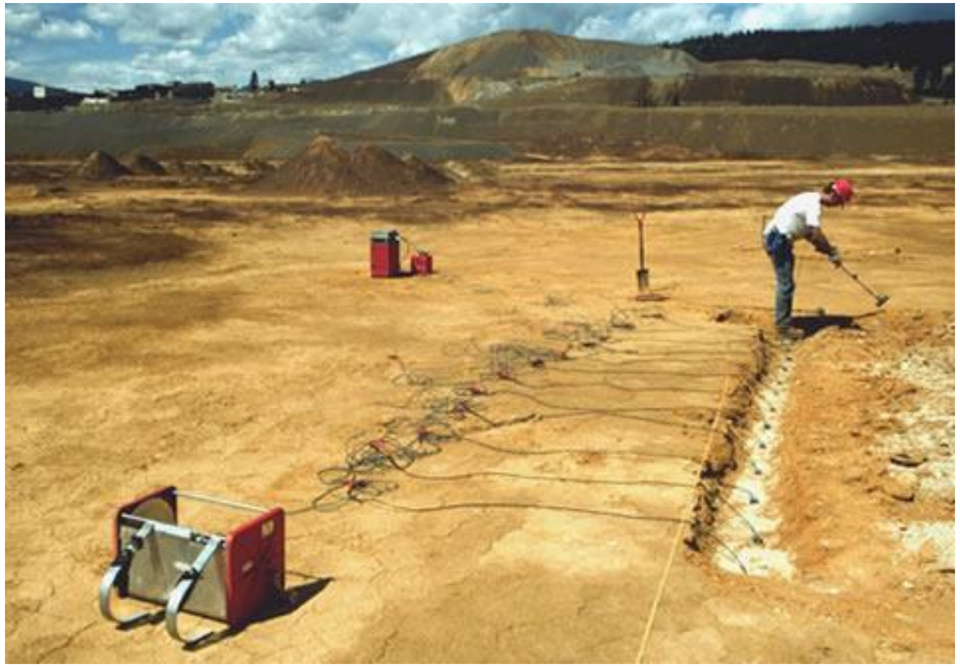
$$T = \frac{A_2}{A_0} = \frac{2Z_1}{Z_2 + Z_1}, 0 \le T \le 2$$



Today’s Topics

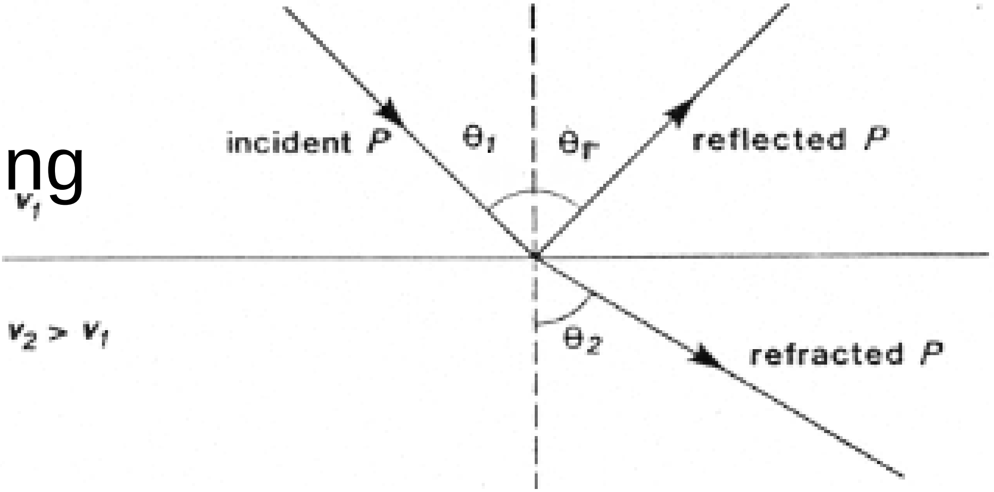
- Basic Principles (continued)
  - Reflection and refraction
  - Travel times: 2 Layer example
  - Extracting layer properties
  - 3 Layer example





Basic Principles:
Reflection and Refraction

Angles of reflection and refraction



- Now consider an plane wave propagating at an angle θ_1

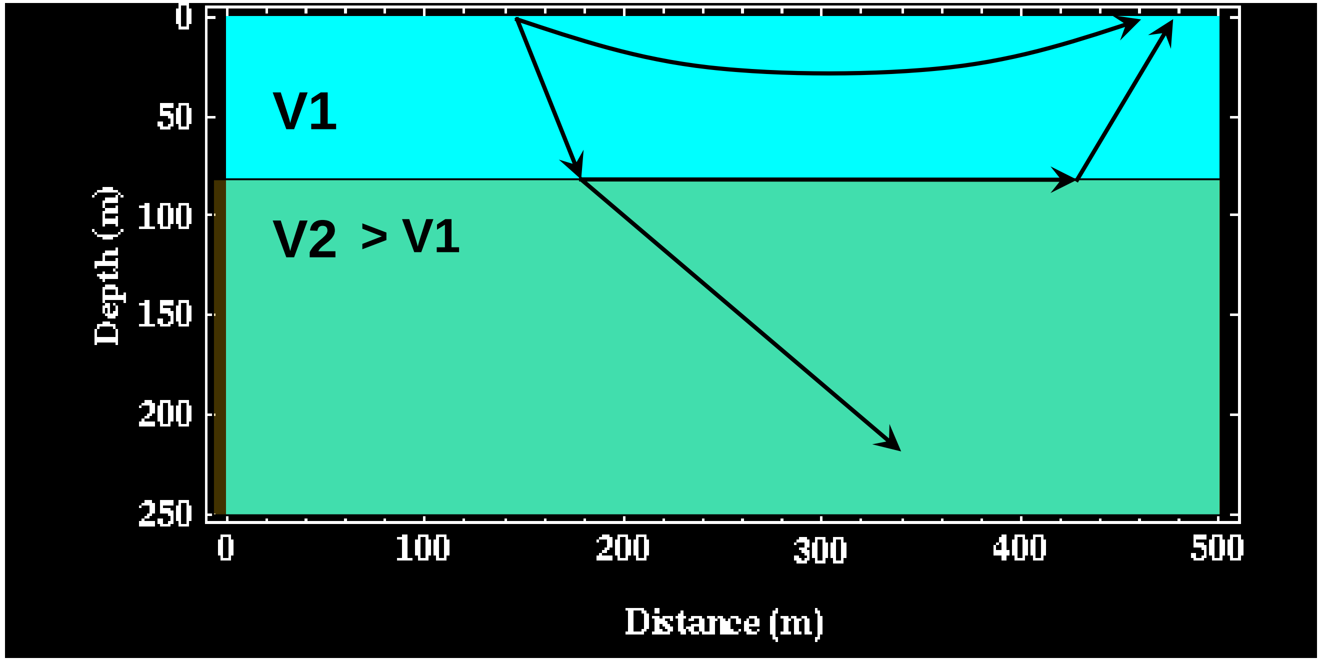
- Law of reflection: reflection angle = incident angle $\boldsymbol{\theta_1 = \theta_r}$

- Law of refraction: refraction angle from **Snell's law**

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

Animation of waves

- Slower over faster (most common): v2 > v1



- Notice the relation between wavefronts and rays (arrows)

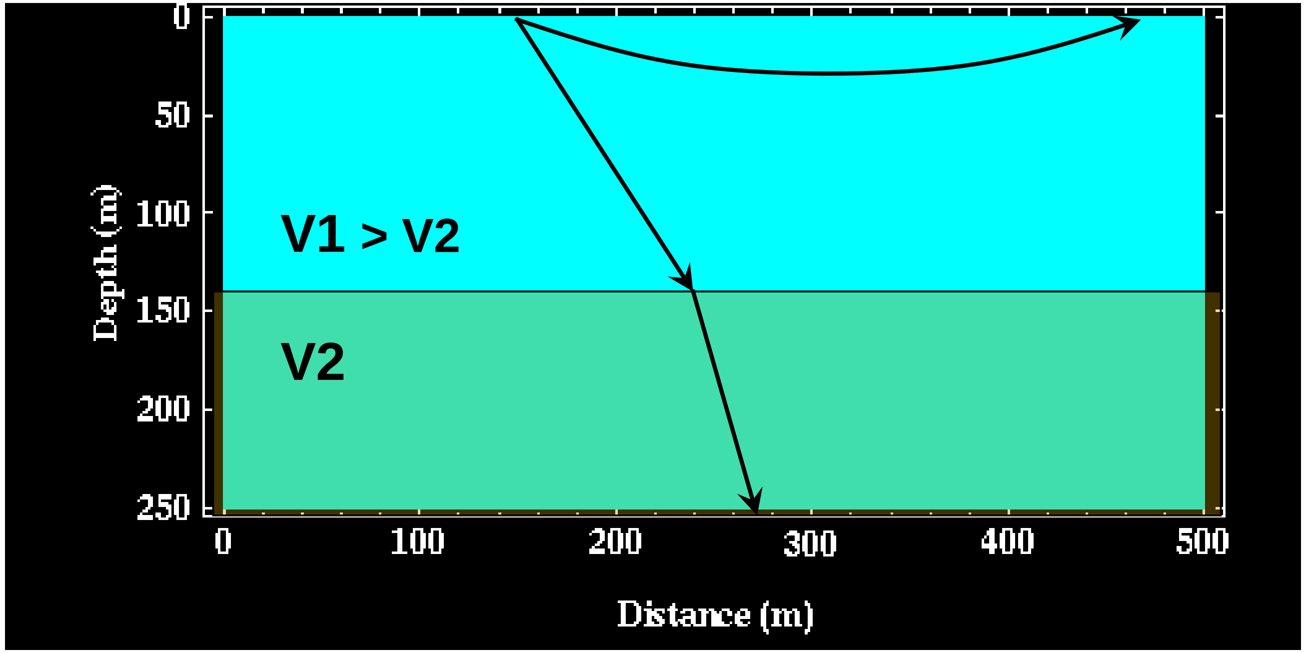
What if…. ?

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

- What if $v_1 > v_2$?
  (ie, faster in top layer)

- Is refraction possible in this situation?

- Implications?



What if…. ?

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

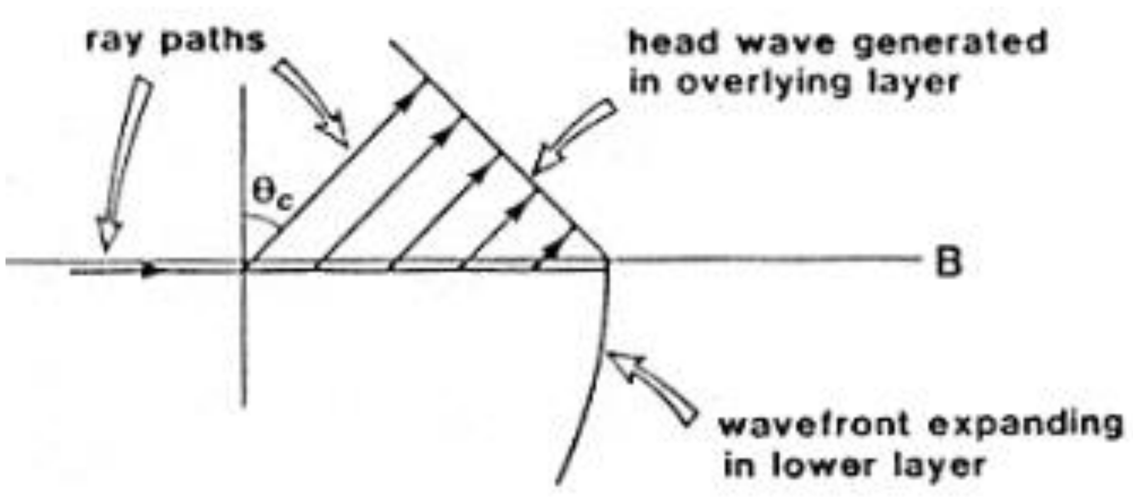
- What if $\theta_2 = 90°$? This is called the critical angle.
- $\sin\theta_1 = v_1/v_2$
- The refracted ray travels horizontally along the interface

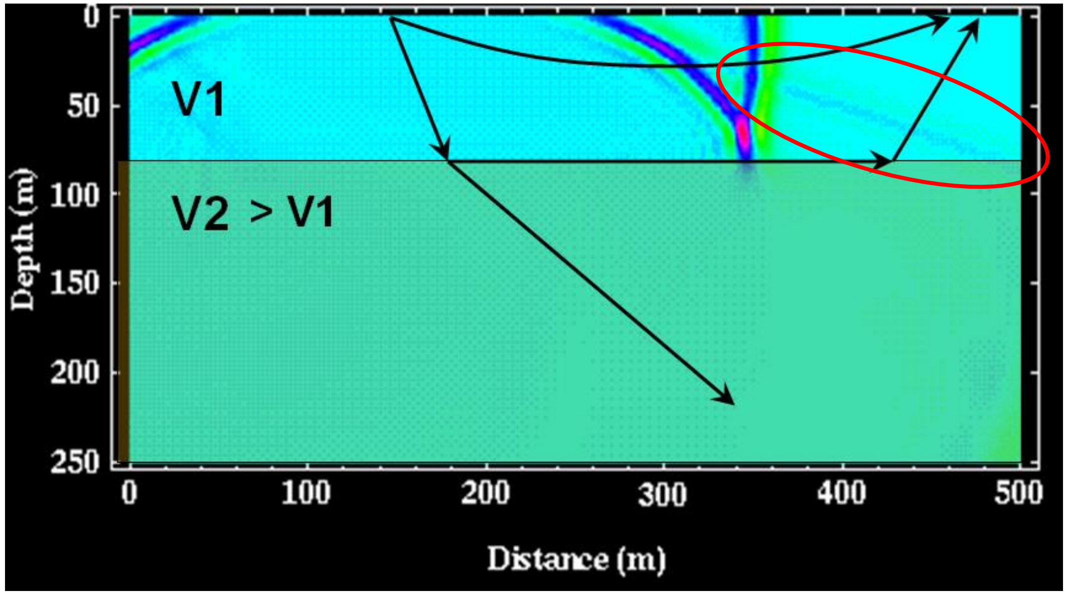
What if…. ?

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

- What if $\theta_2 = 90°$? This is called the critical angle.
- $\sin\theta_1 = v_1/v_2$

- "Head waves" or critically refracted rays send energy back to the surface.





Apps on the web

- http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=16
  - Illustration of reflection and refracted wavefronts using Fresnel-Huygens principles.

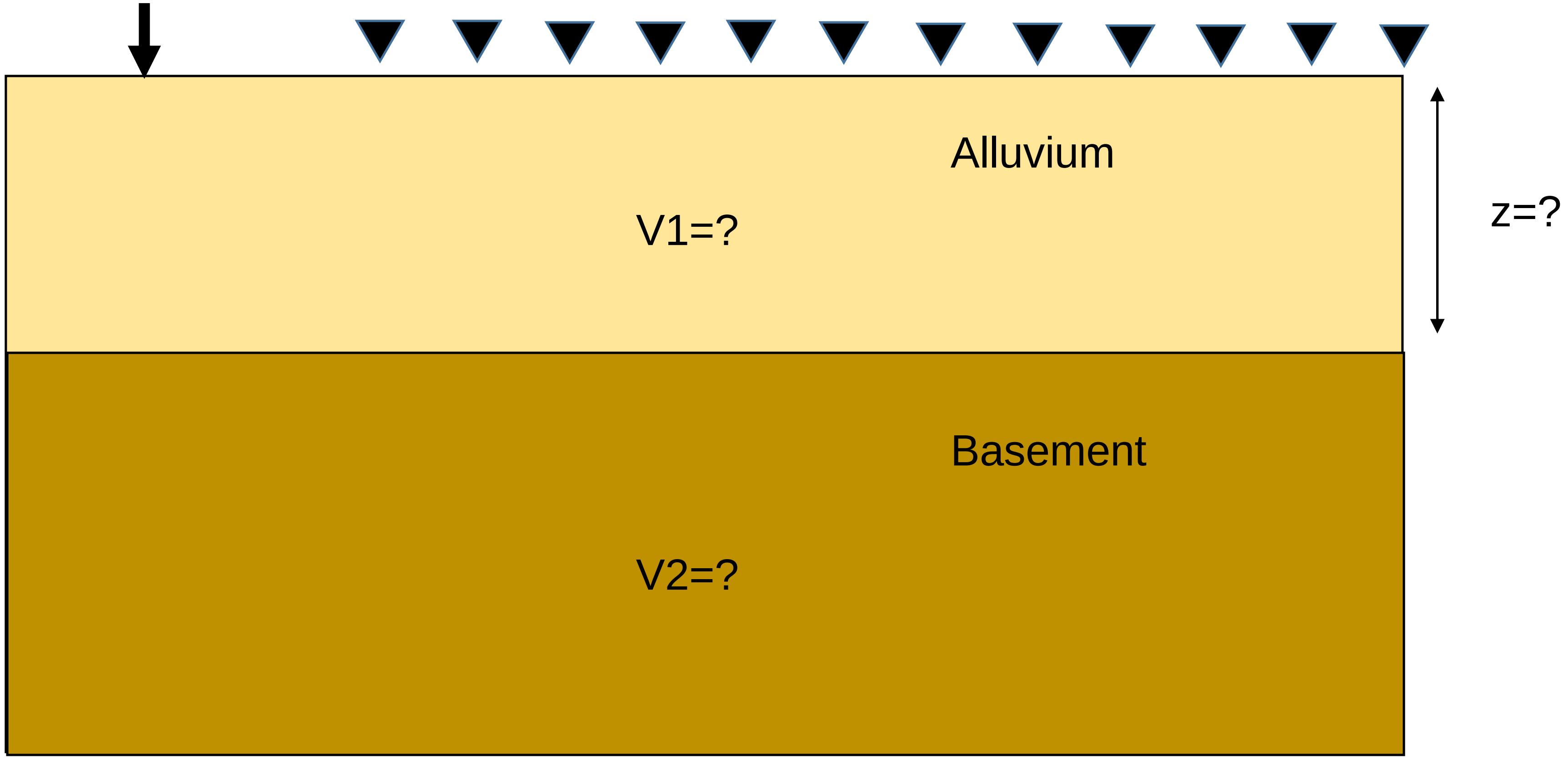
- http://staff.washington.edu/aganse/raydemo/RayDemo2.med.html
  - Ray paths in arbitrary 1D earth. Generate the velocity model and observe first arrivals and curved ray paths. (Visualizing bending rays in linearly increasing velocities)

- http://www.iris.edu/hq/programs/education_and_outreach/visualizations
  - Global Earthquakes recorded by US seismometer arrays. Learn about particle motions, wave .

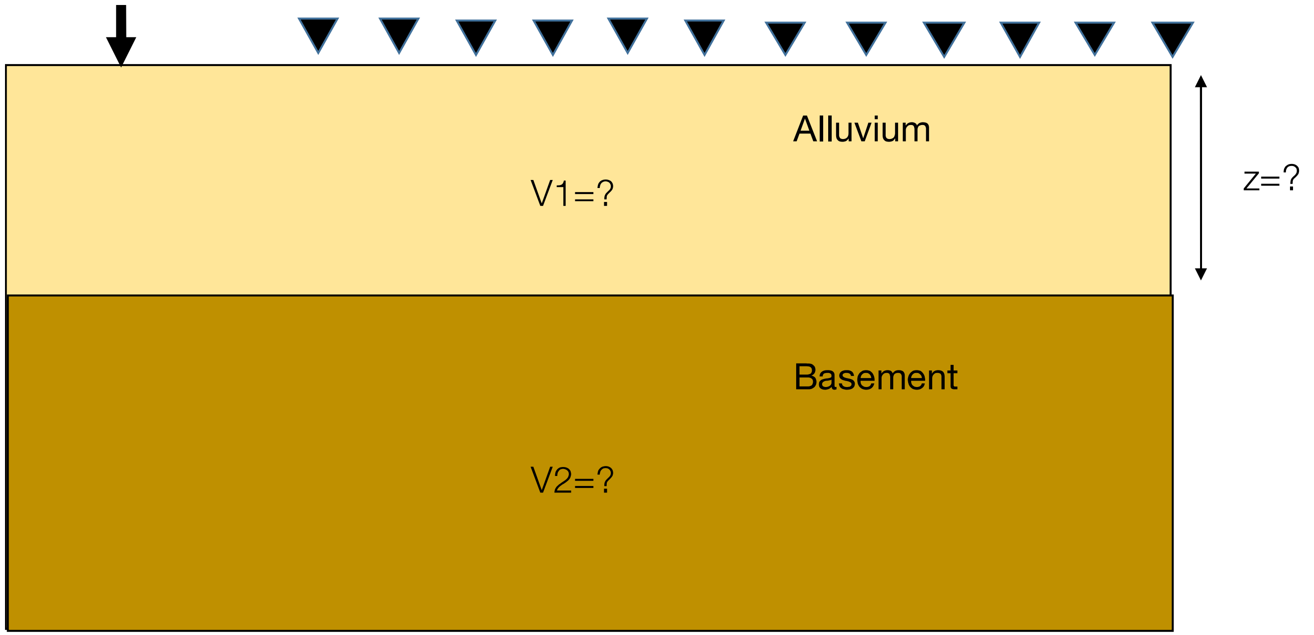
Travel Times

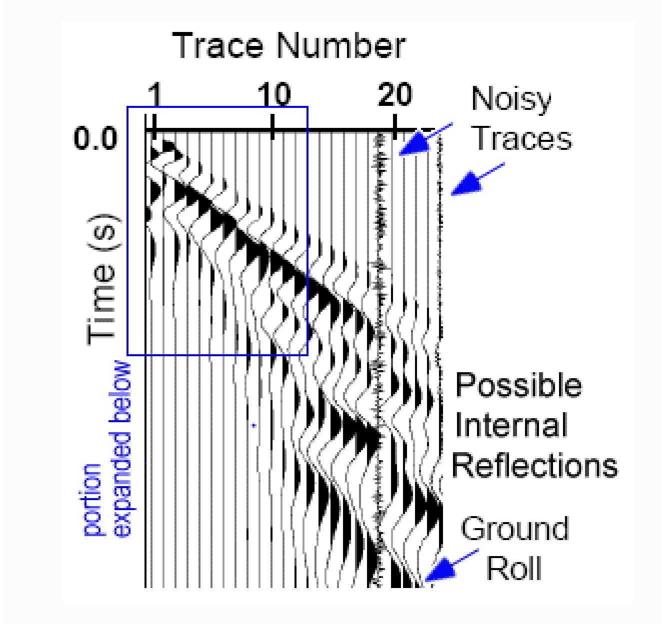
Travel time

- Many ways (ray paths) for signal to get from source to sensor
- How long do these signals take to arrive?
- What can we learn from measuring arrival times?



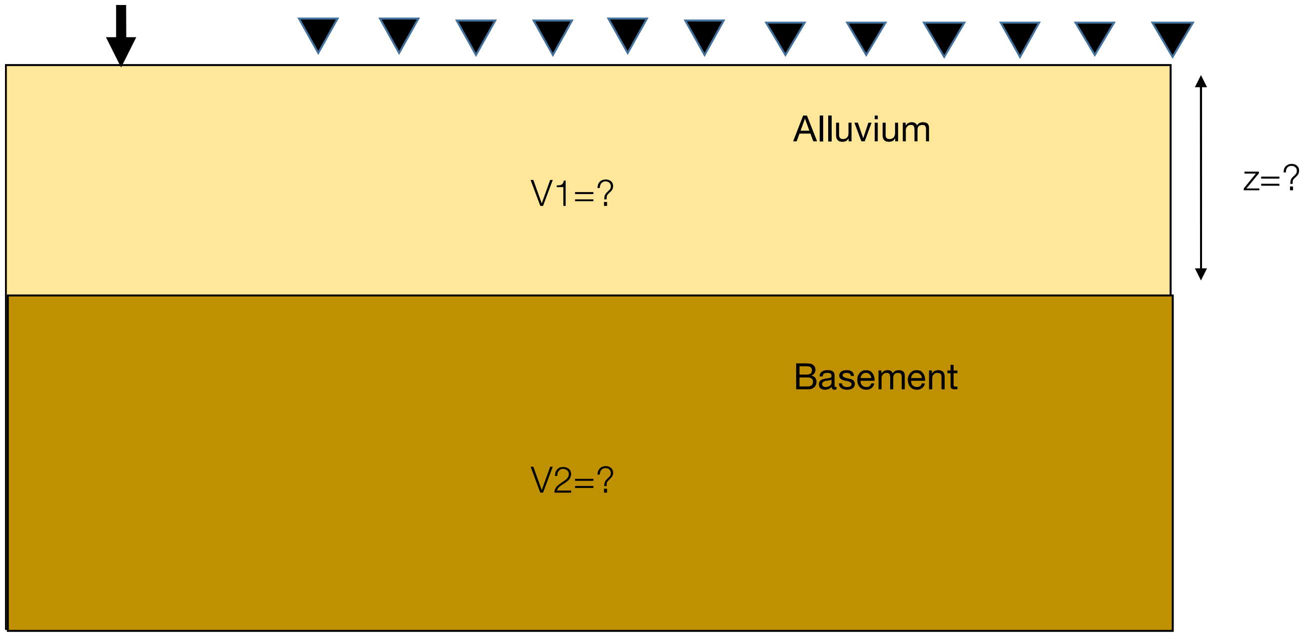
Travel time used for data?



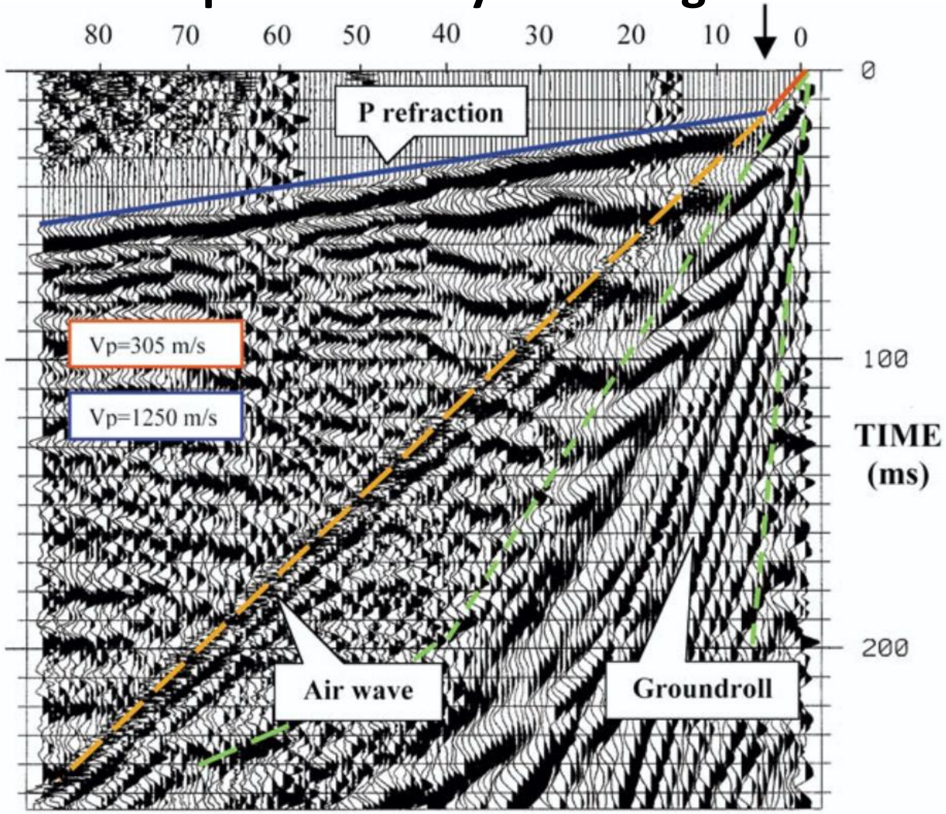


Trace: Time-dependent signal recorded for a single source-receiver pair

Travel time used for data?



Data represented by "seismogram"



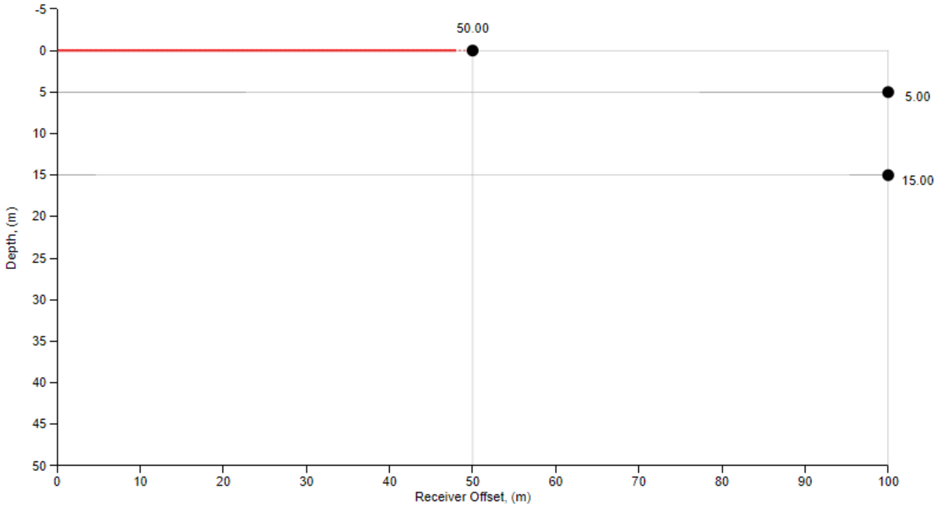
Seismogram: An arrival time vs distance plot representing many traces

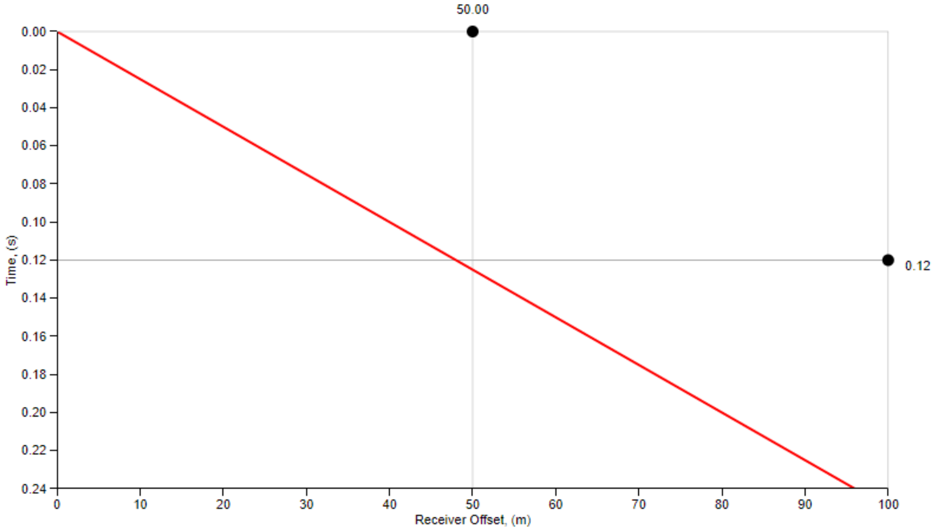
Travel times

- Time required for a seismic wave to travel from source to receiver.
- In a homogeneous medium:

$$t = \frac{d}{v}$$

- Seismic survey: measures signal amplitude as a function of time
- Then estimate subsurface properties





Travel times: 3 principal waves

- Direct waves

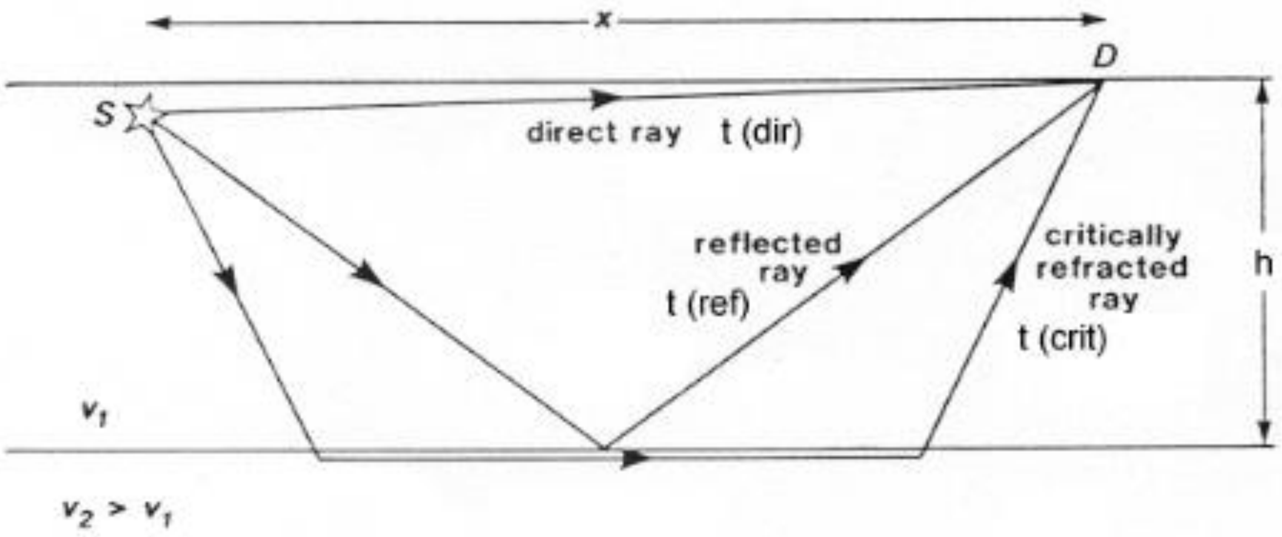
$$t_{dir} = \frac{x}{v_1}$$

- Reflected waves

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

- Critically refracted waves (head waves)

$$t_{refr} = \frac{x}{v_2} + \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$



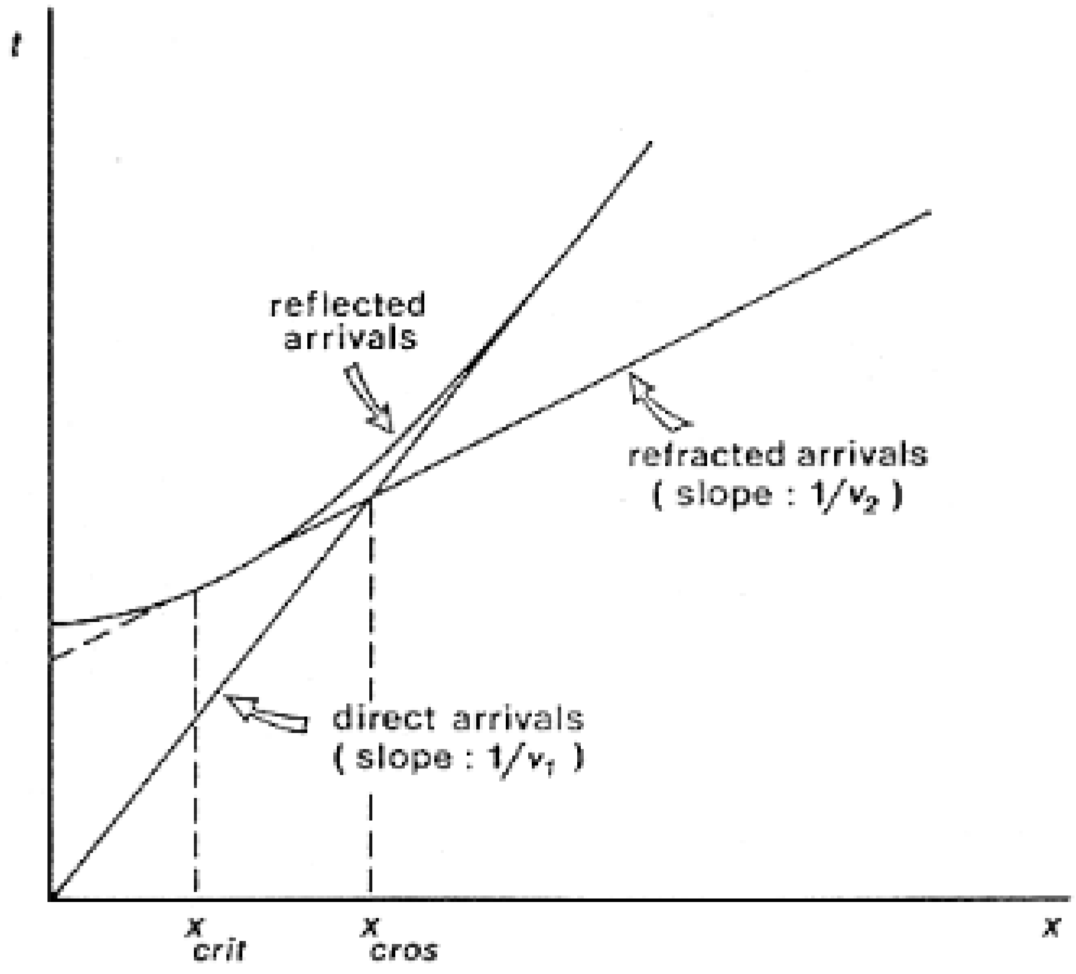
Travel times: Direct wave

- Direct wave travel time

$$t_{dir} = \frac{x}{v_1}$$

- Linear time curve

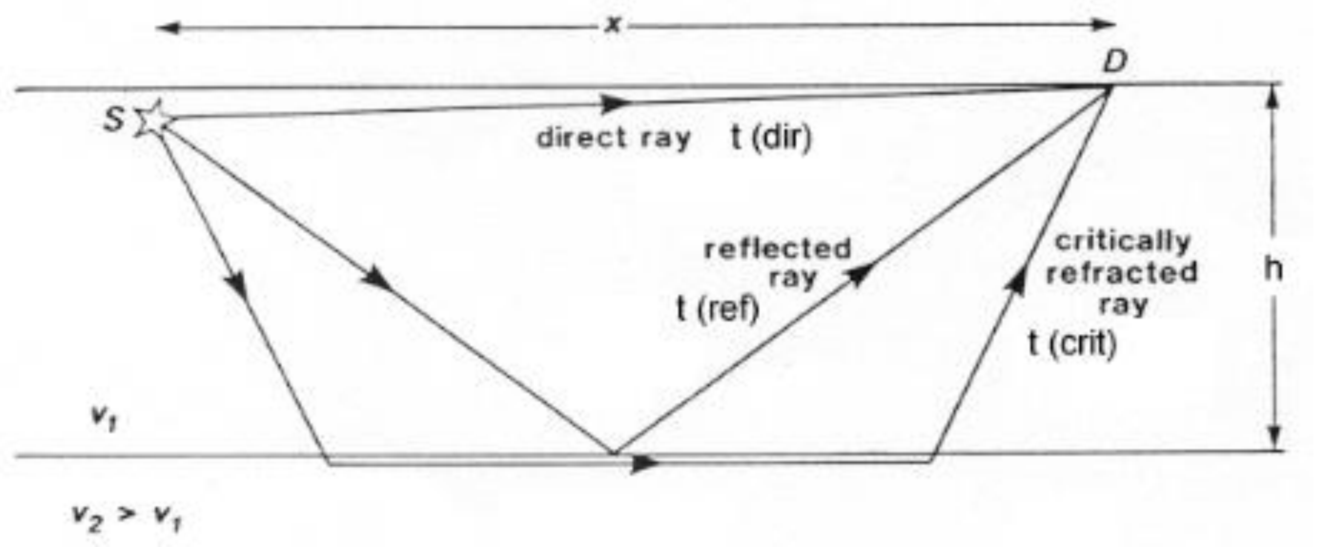
- Slope of $\mathbf{1}/\boldsymbol{v_1}$



Travel times: Reflected wave

- Travel time:

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$



- Linear time curve at sufficient distance

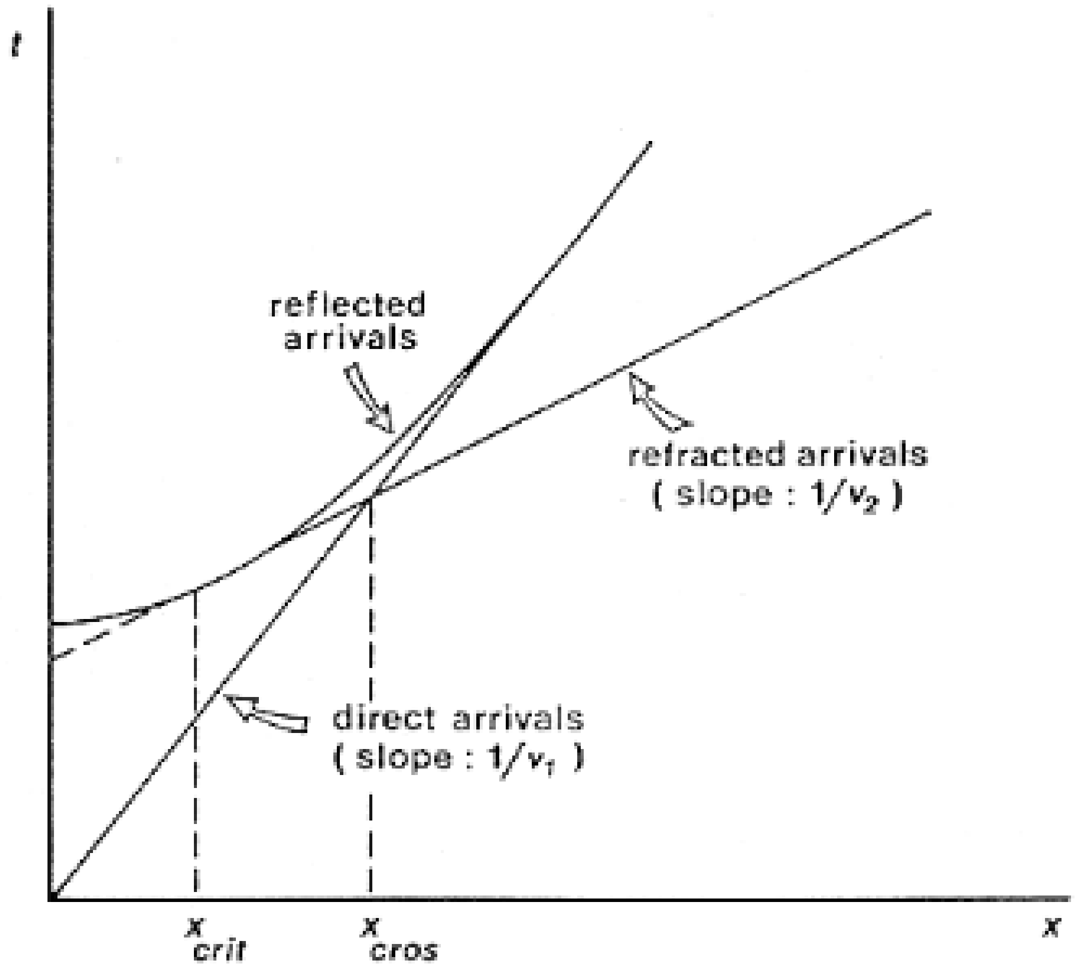
- Slope of $\mathbf{1}/\boldsymbol{v_1}$

Travel times: Reflected wave

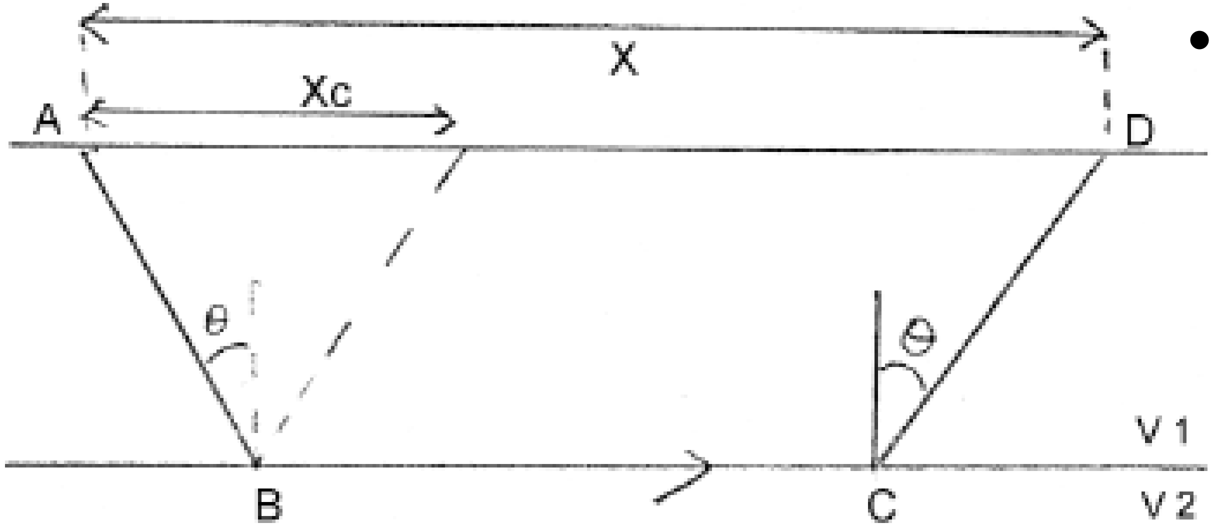
- Travel time:

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

- Linear time curve at sufficient distance with slope of $\mathbf{1}/\boldsymbol{v_1}$



Travel times: Refracted ray in 2-layer earth



- Critical distance

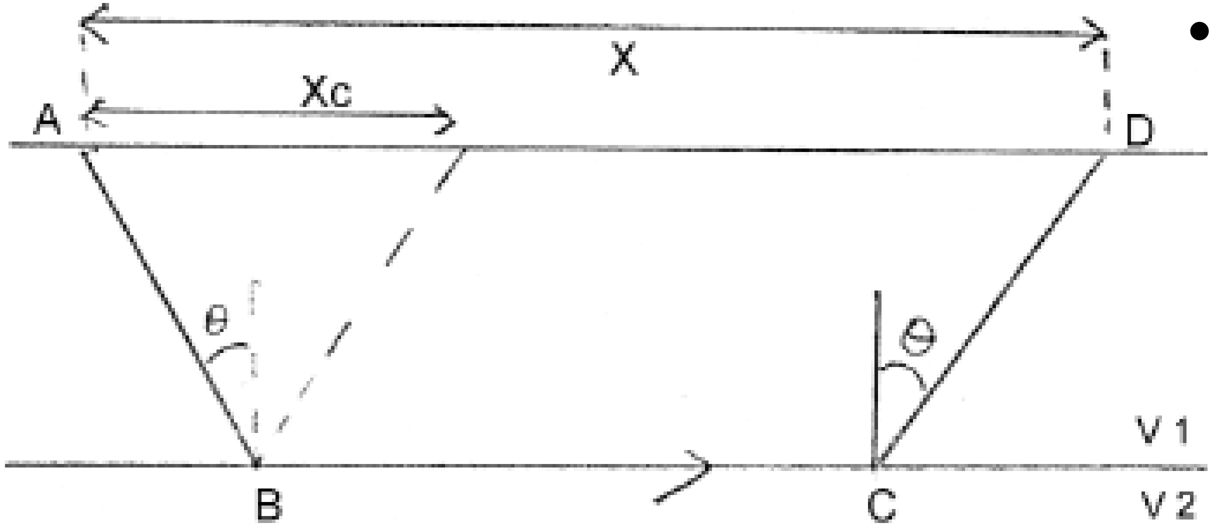
$$x_c = 2h\tan\theta \quad l = \frac{h}{\cos\theta}$$

$$\tan\theta = \frac{x_c}{2h} \quad \cos\theta = \frac{h}{l}$$

- Total travel time is cumulative time for wave to traverse the path ABCD

$$t = t_{AB} + t_{BC} + t_{CD} = \frac{2h}{v_1\cos\theta} + \frac{x - 2h\tan\theta}{v_2}$$

Travel times: Refracted ray in 2-layer earth



- Total travel time
  - Do some trigonometry to get the following useful relations

$$t_{refr} = \frac{x}{v_2} + \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

$$= \frac{x}{v_2} + t_i$$

- This says that the travel time curve is a straight line with a slope of $\frac{1}{v_2}$ and an intercept time of t_i

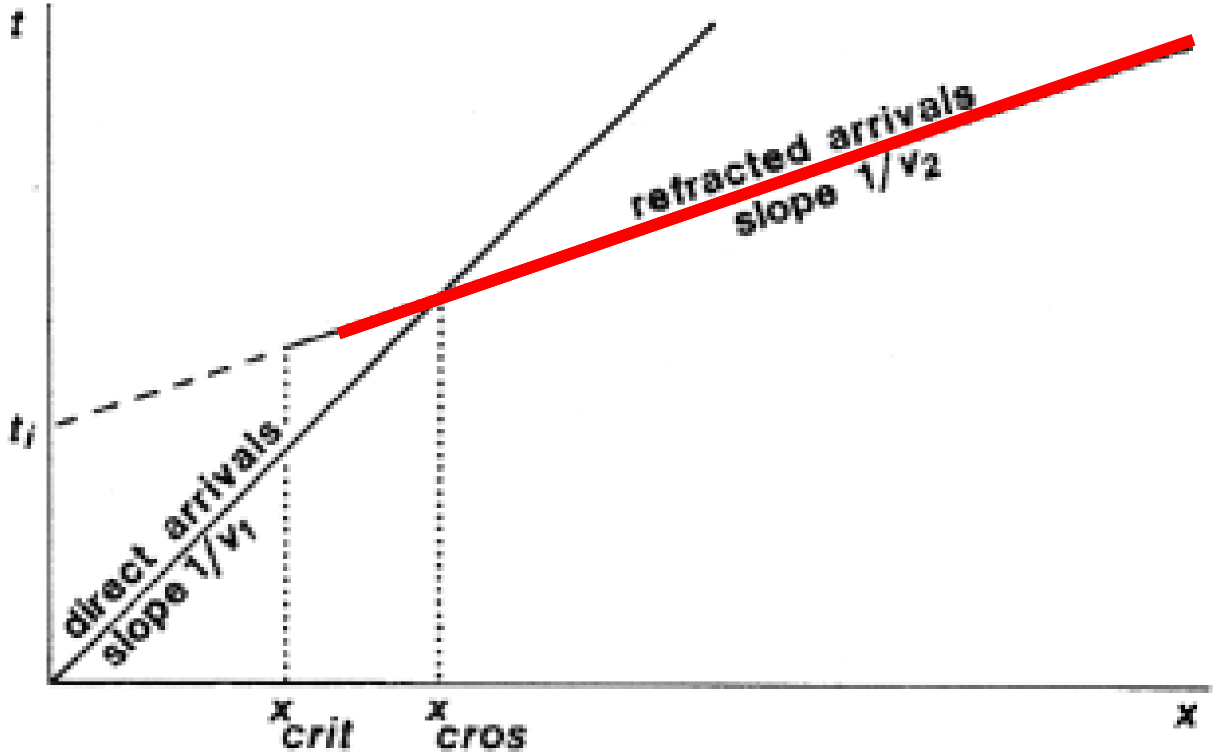
Travel times: Refracted ray in 2-layer earth

- Travel time:

$$t_{refr} = \frac{x}{v_2} + \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

$$= \frac{x}{v_2} + t_i$$

- Linear time curve with slope of $\mathbf{1/v_2}$



All together

- Direct waves

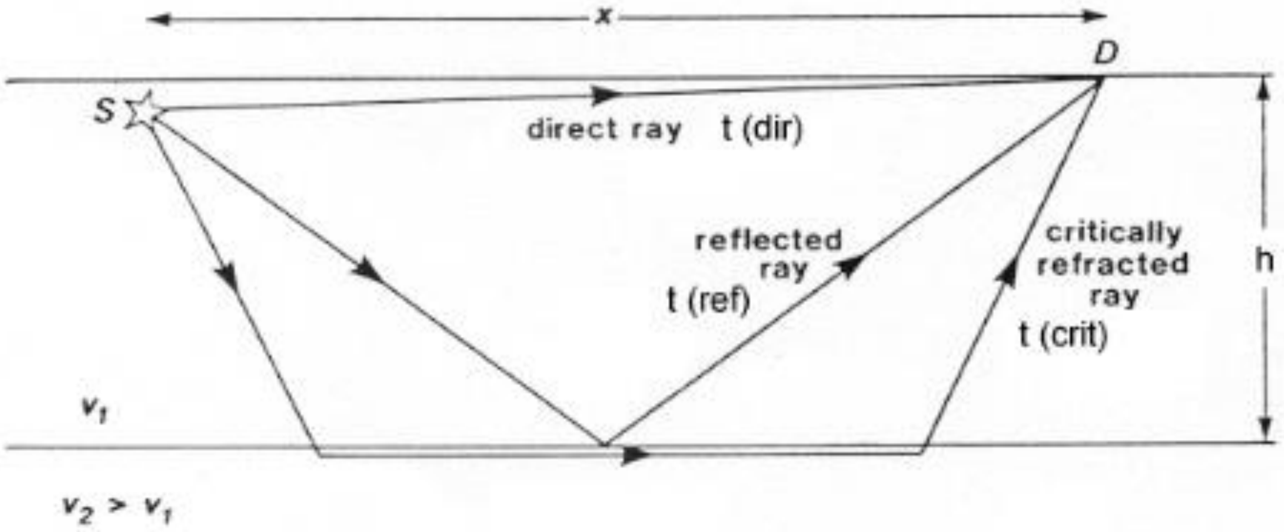
$$t_{dir} = \frac{x}{v_1}$$

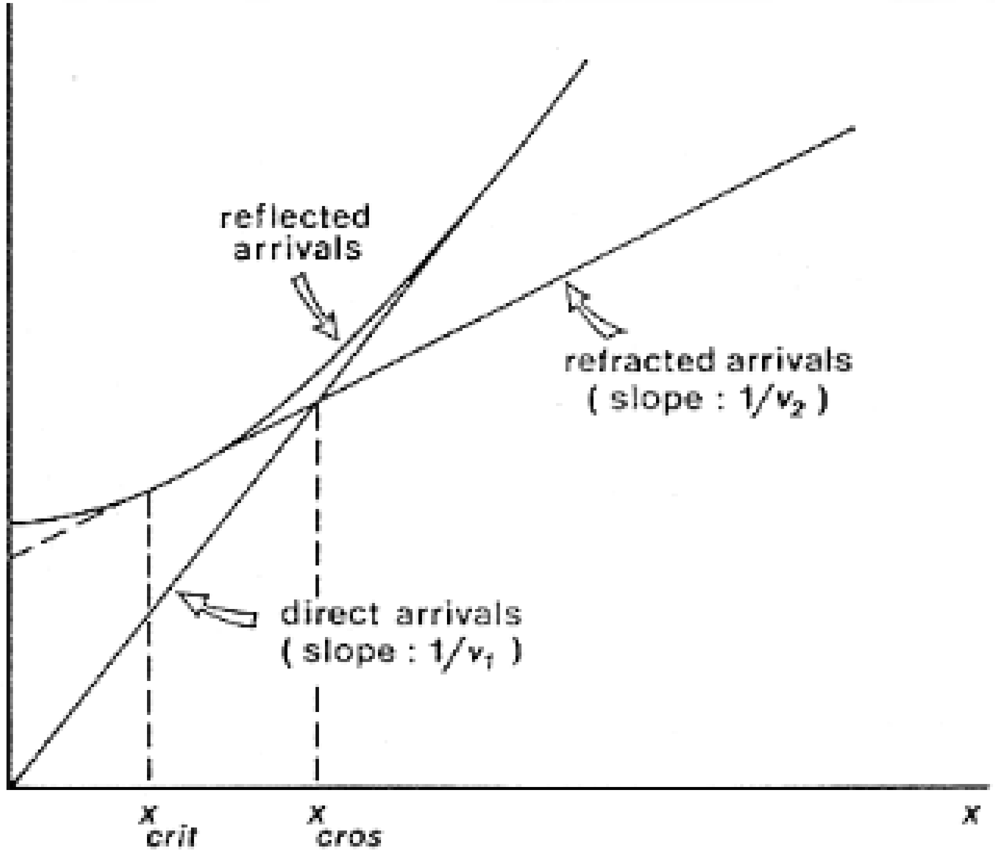
- Reflected waves

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

- Critically refracted waves (head waves)

$$t_{refr} = \frac{x}{v_2} + \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$





Important points

- Critical distance x_{crit}
  Closest surface point to the source at which the refracted ray can be observed

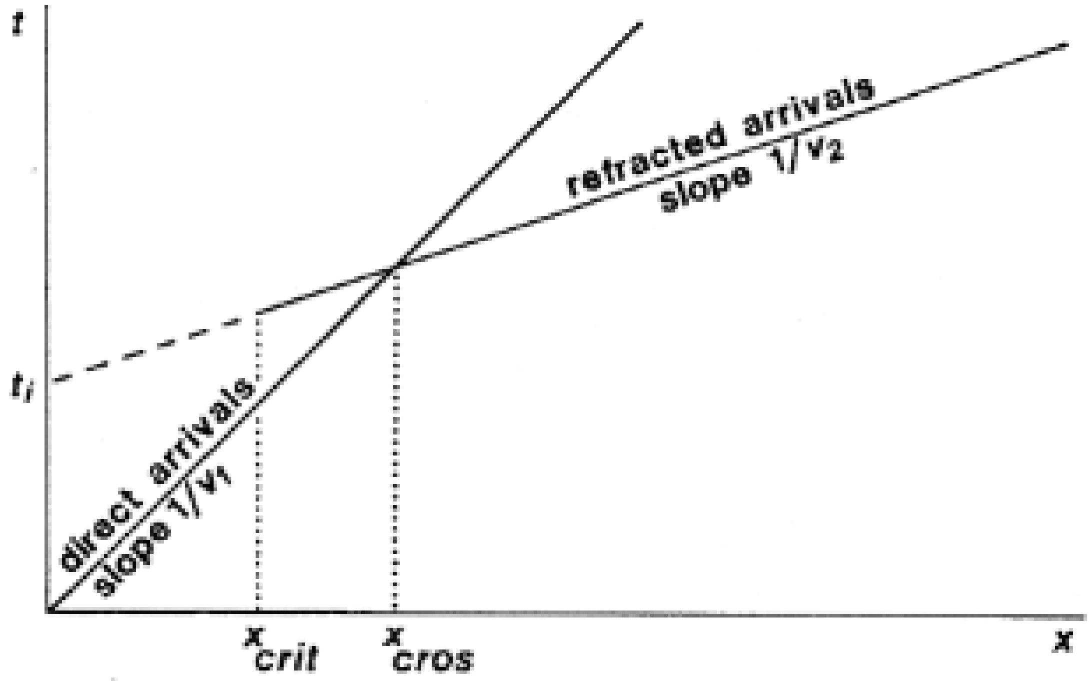
$$x_c = 2h\tan\theta \quad l = \frac{h}{\cos\theta} \qquad \tan\theta = \frac{x_c}{2h} \quad \cos\theta = \frac{h}{l}$$

- Crossover distance x_{cros}
  Surface point at which the direct and refracted rays arrive at the same time

$$x_{\mathrm{cross}} = 2h\sqrt{\frac{v_2+v_1}{v_2-v_1}}$$

- Intercept time t_i

$$t_i = \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$



Basic Principles:
Extracting Layer Properties

Layer 1 velocity

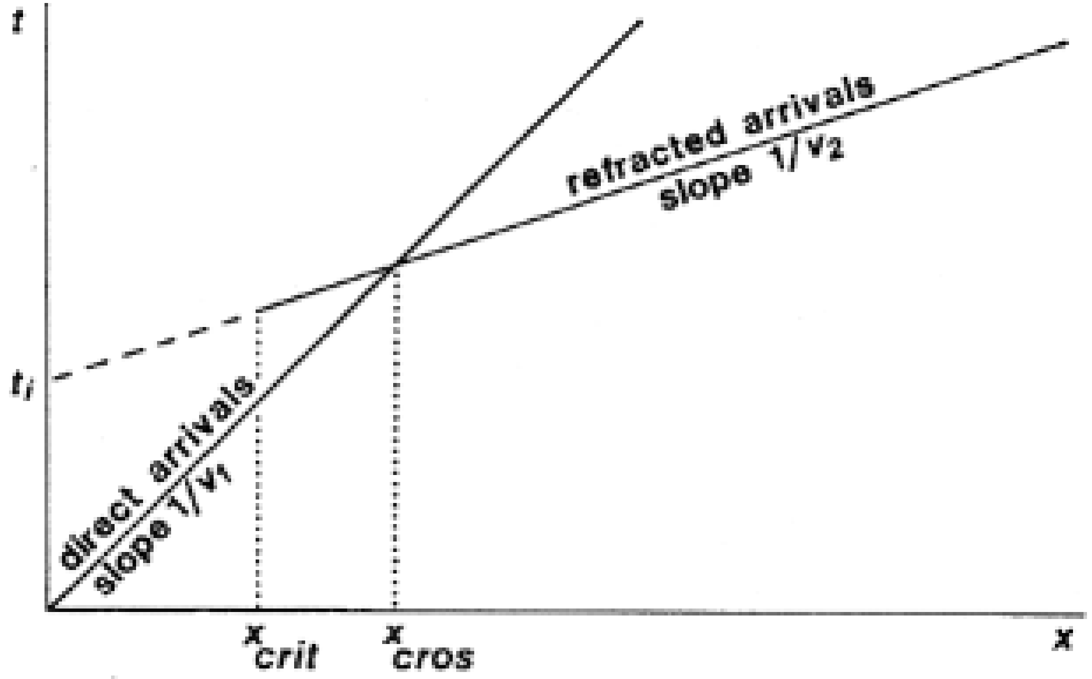
- Direct waves

$$t_{dir} = \frac{x}{v_1}$$

- Reflected waves

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

- Can be estimated from slope of direct and/or reflected wave

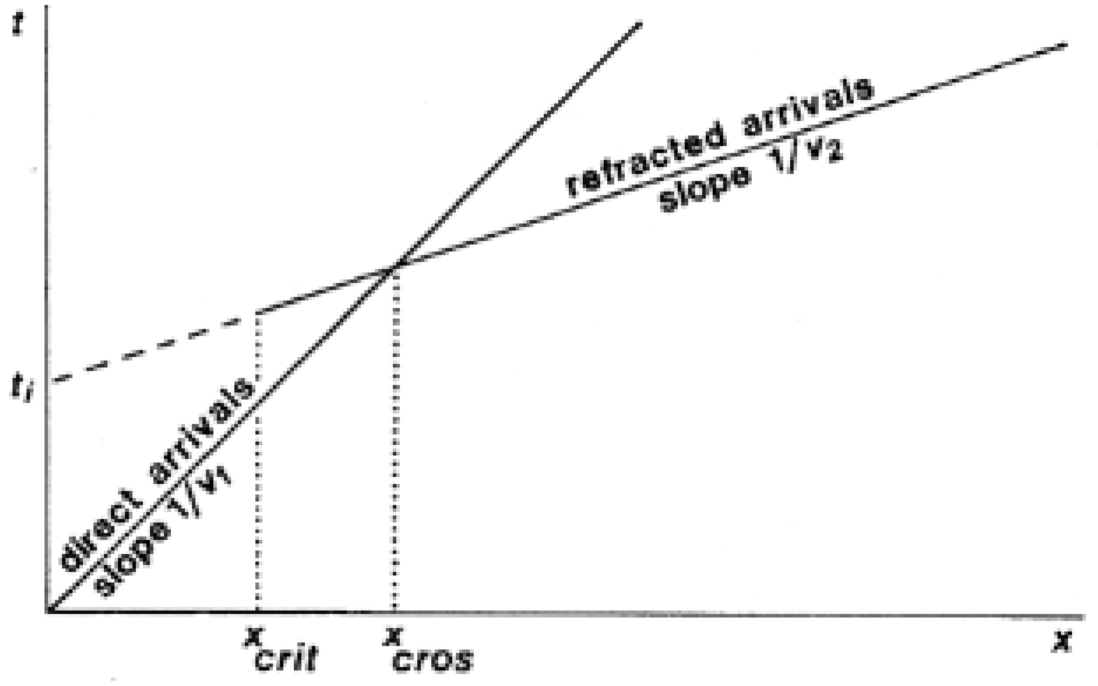


Layer 2 velocity

- Critically refracted waves (head waves)

$$t_{refr} = \frac{x}{v_2} + \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

- Can be estimated from slope of refracted wave



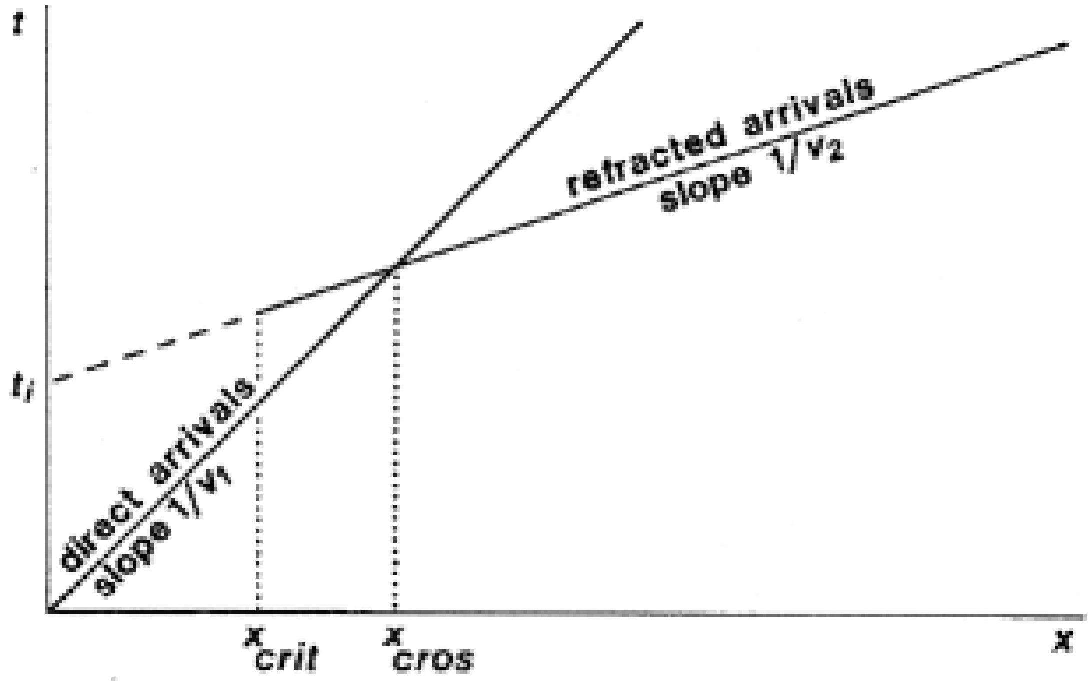
Estimate layer thickness

- Intercept time:

$$t_i = \frac{2h\sqrt{v_2^2 - v_1^2}}{v_1 v_2}$$

- If v1 and v2 are known, solve for h:

$$h = \frac{t_i v_1 v_2}{2\sqrt{v_2^2 - v_1^2}}$$

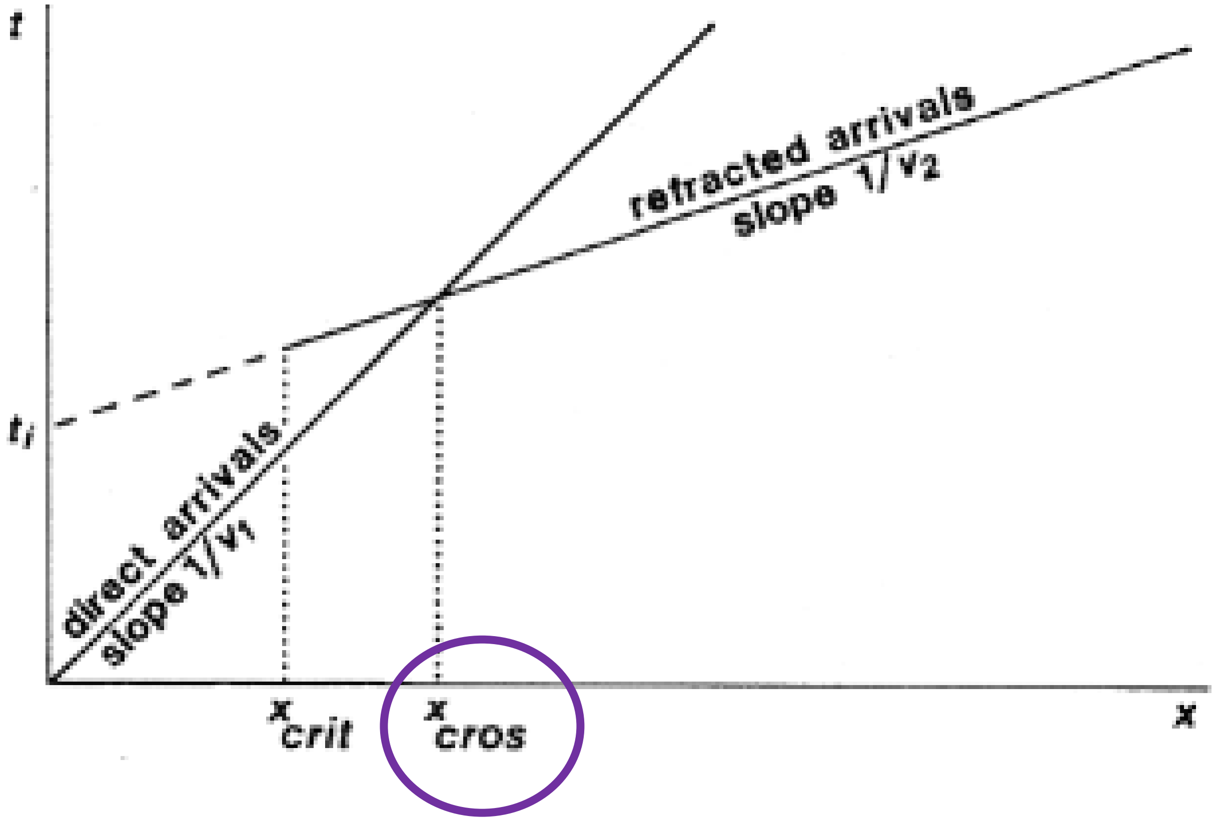


One more thing

- Cross-over distance: where the direct wave and the refracted wave arrive at the same time

$$\boxed{\frac{x_{\text{cross}}}{v_1}} = \boxed{\frac{x_{\text{cross}}}{v_2} + t_i}$$

$$x_{\text{cross}} = \left(\frac{v_1 v_2}{v_2 - v_1}\right) t_i$$
$$= 2h\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}$$



- Another way to calculate one of the variables given values for the others.

What happens if v2<v1?

- Direct waves

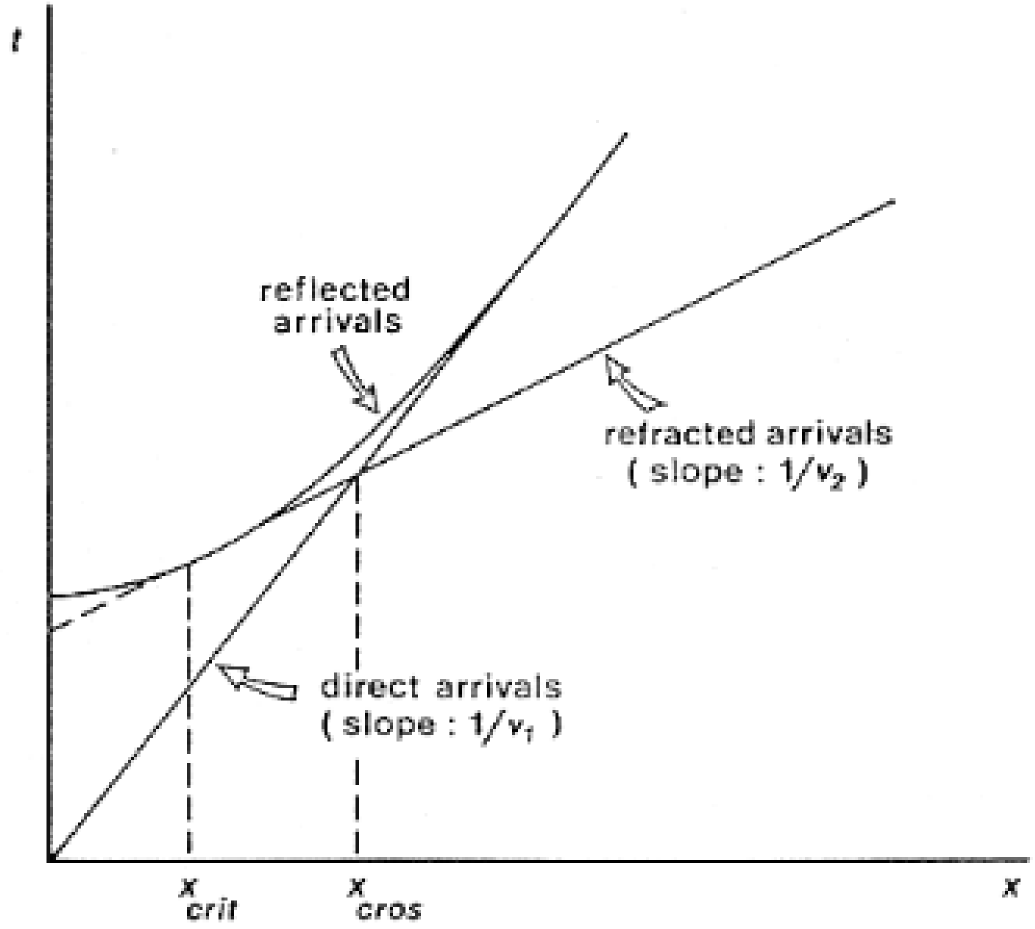
$$t_{dir} = \frac{x}{v_1}$$

- Reflected waves

$$t_{refl} = \frac{\sqrt{x^2 + 4h^2}}{v_1}$$

- Critical refraction

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$



What happens if v2<v1?

- Direct waves

$$t_{dir} = \frac{x}{v_1}$$

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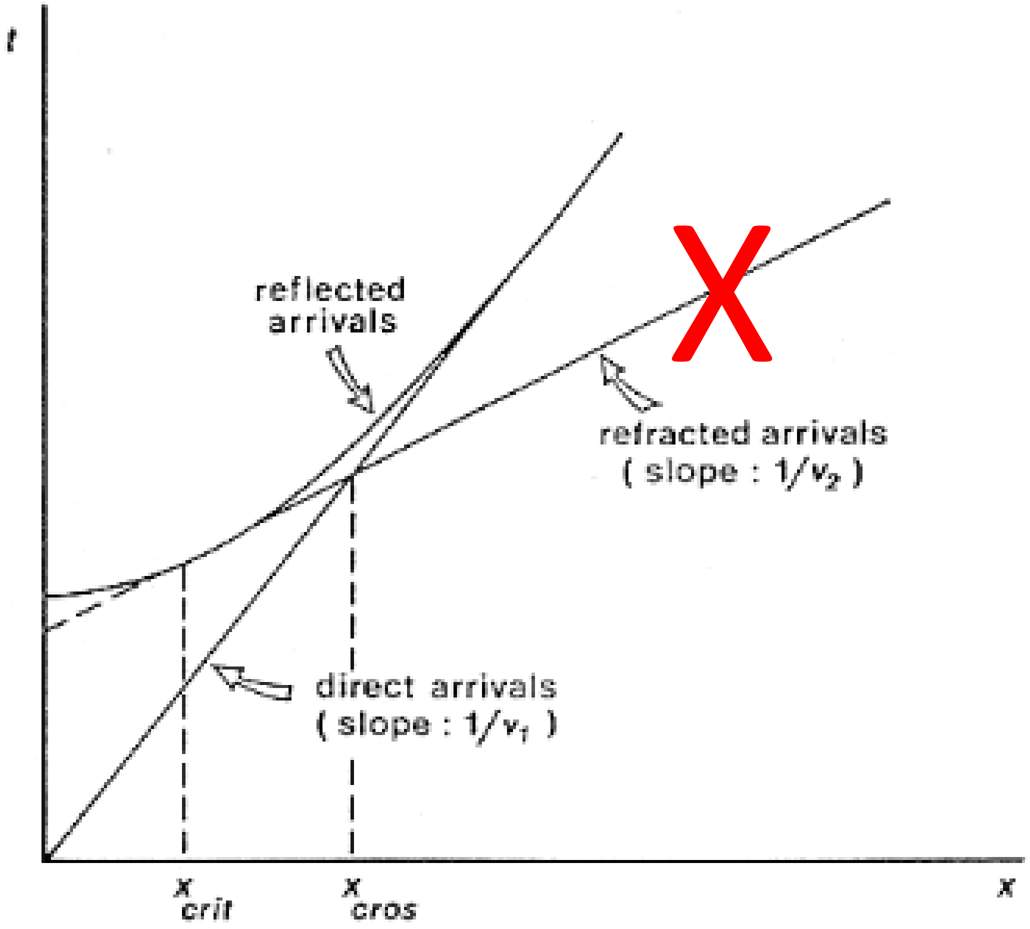
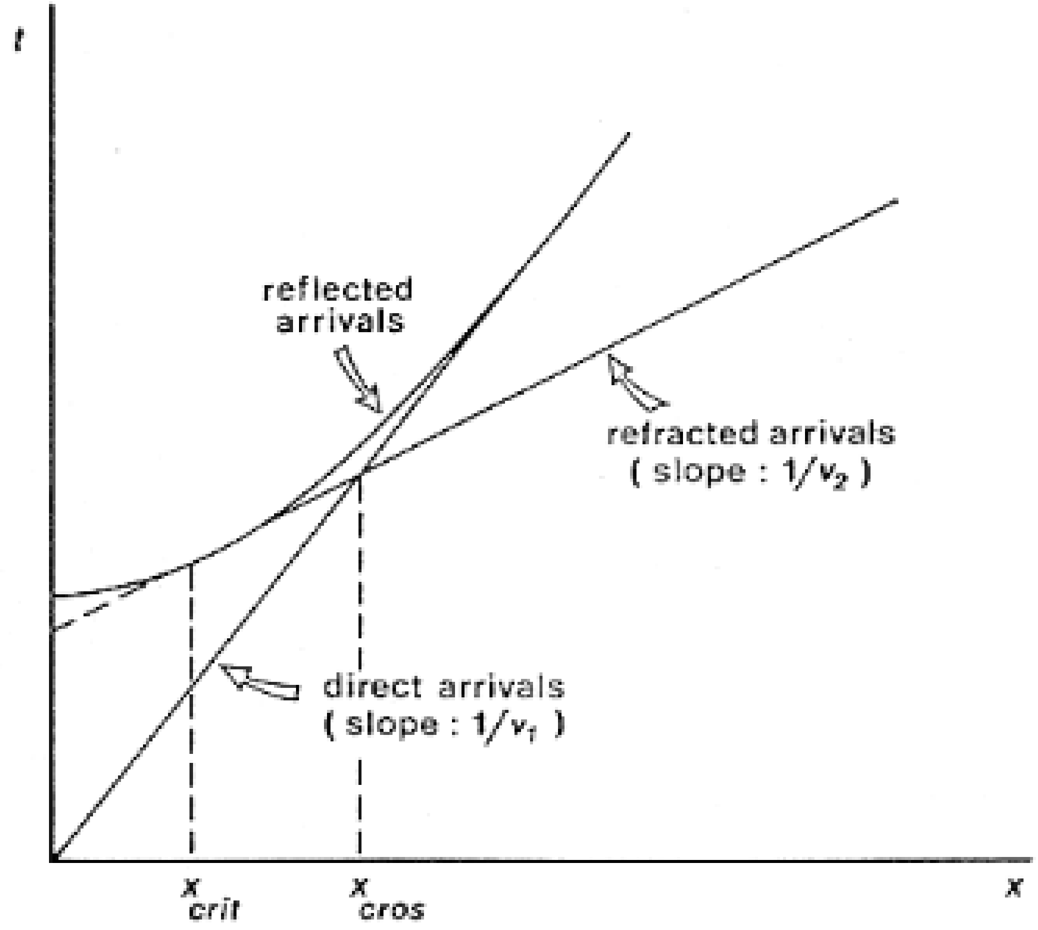
- Critical refraction

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$



Recap

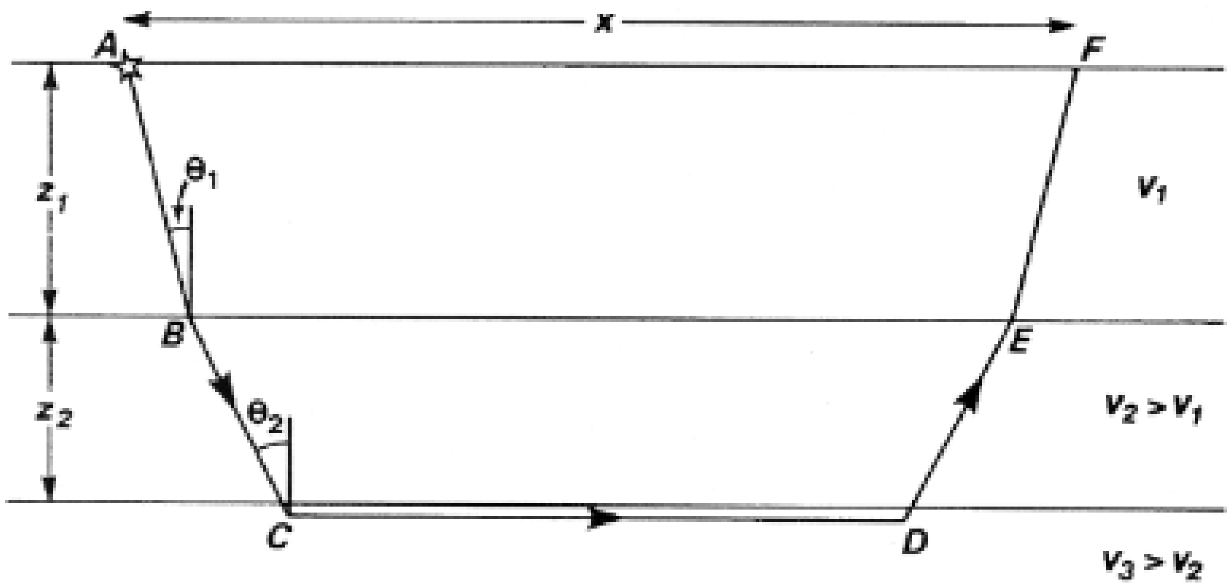
- Plot the arrival times
- Identify each wave
- Use plot and equations to estimate layer velocity and thickness
- Most effective if velocity increases with depth

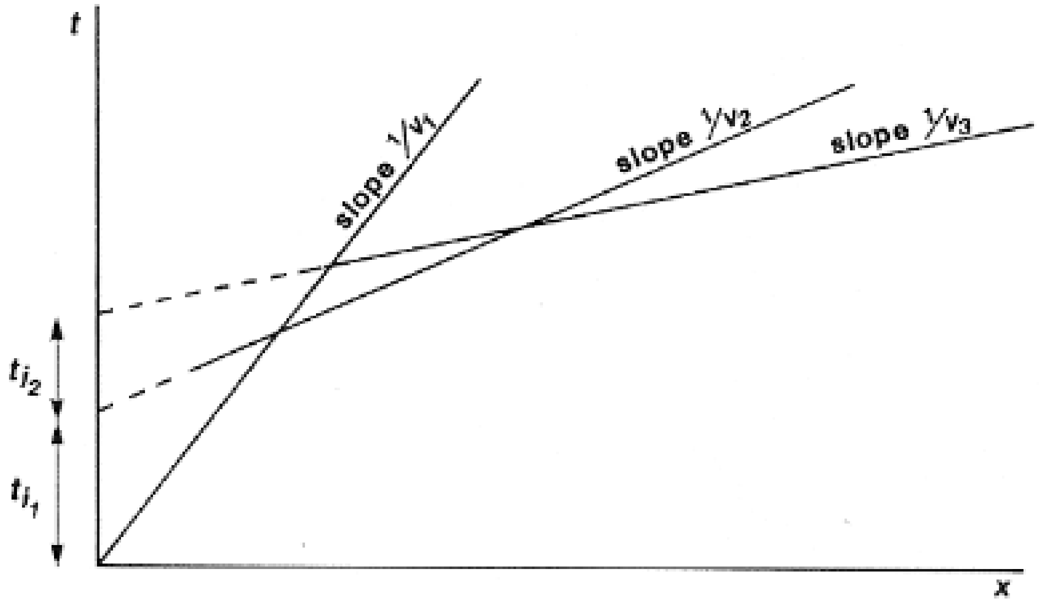


Basic Principles:
3 Layer Example

What about three layers?

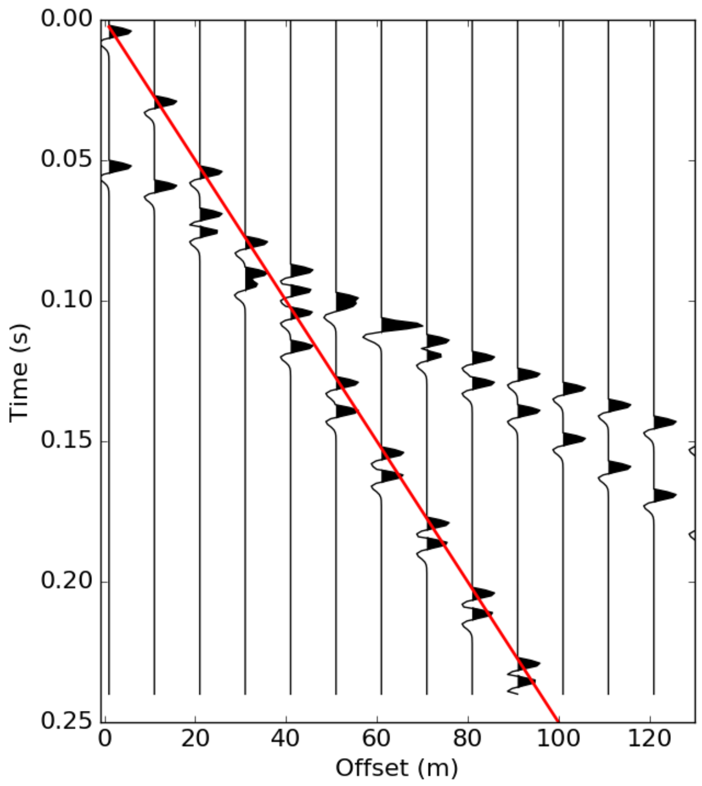
- Snell's law holds: $\dfrac{\sin\theta_1}{v_1} = \dfrac{\sin\theta_2}{v_2} = \dfrac{\sin\theta_3}{v_3} = \ldots$

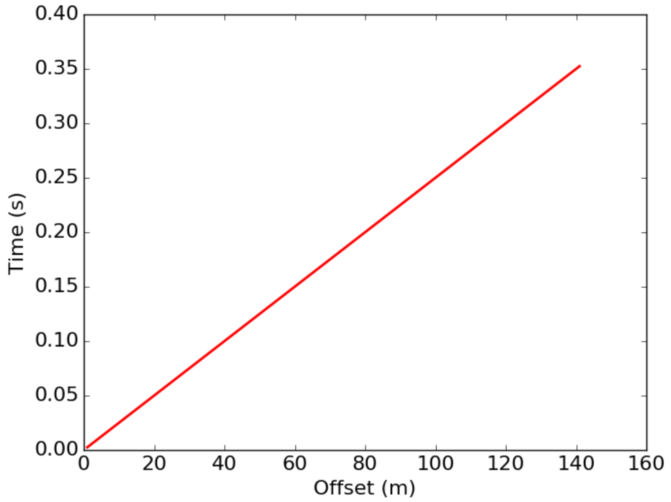


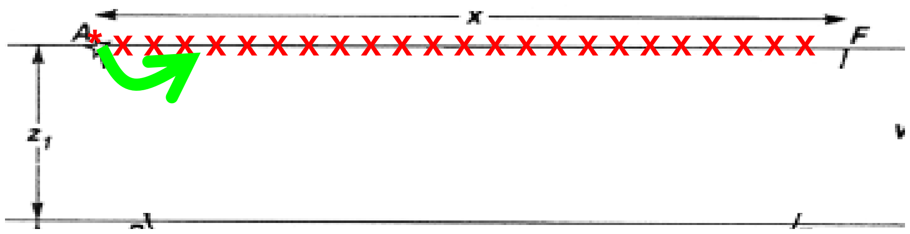


Raw data

- Direct arrivals
- The T-X plot

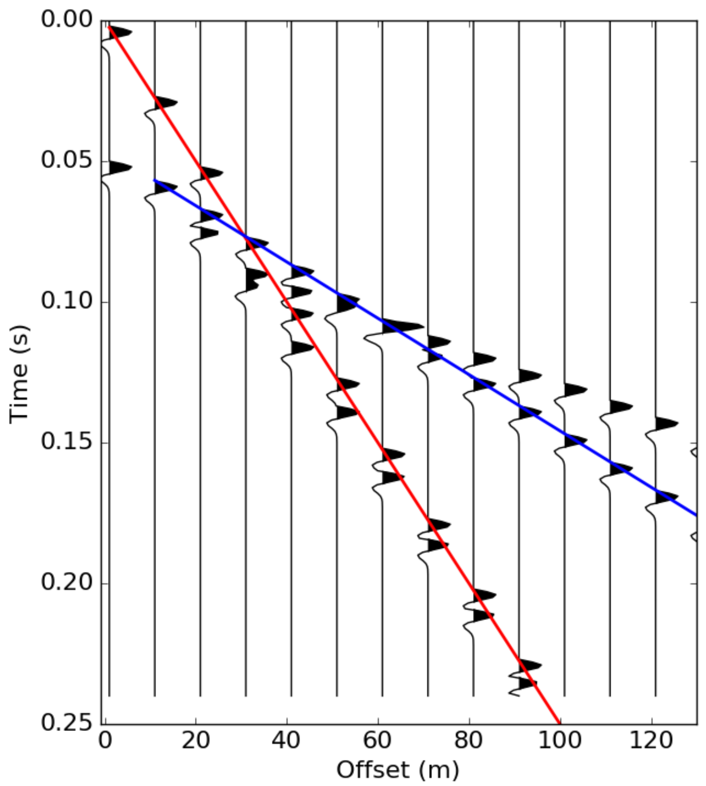


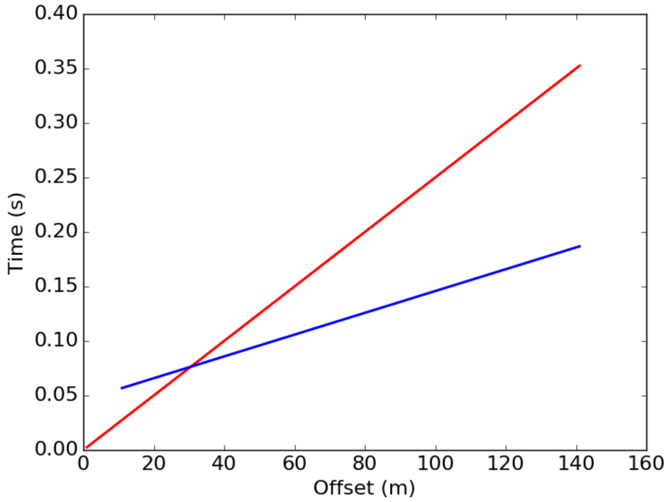


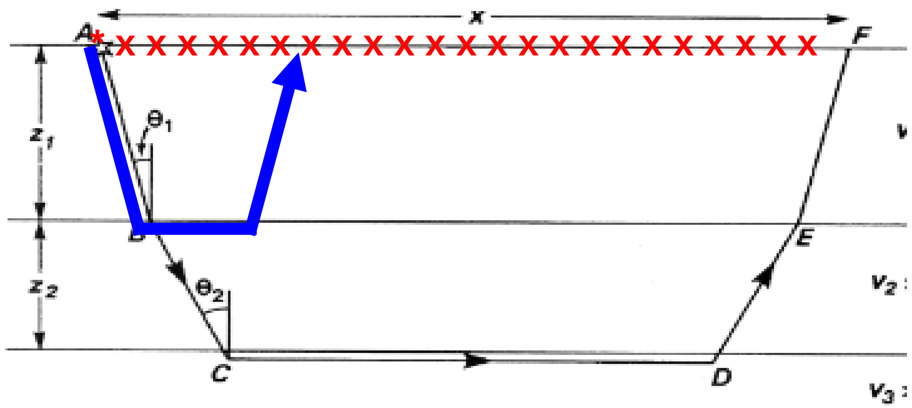


Raw data

- First refractions
- The T-X plot

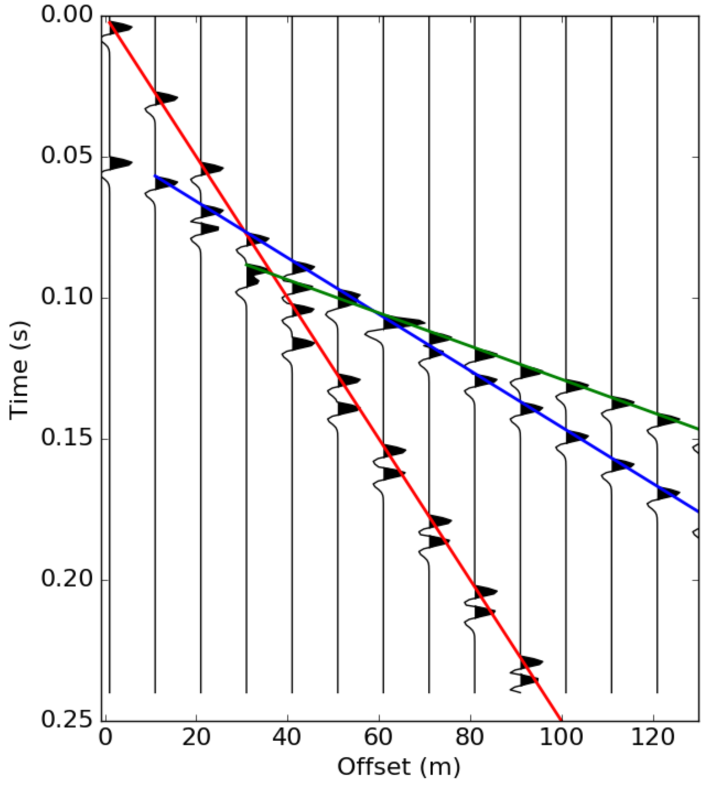


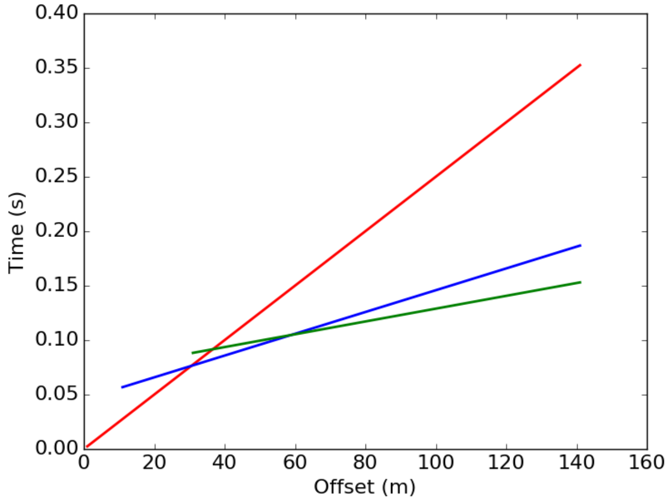


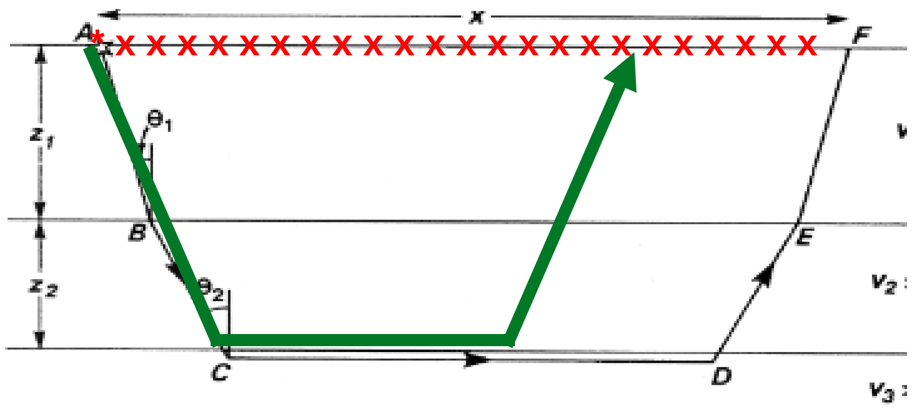


Raw data

- Second refractions
- The T-X plot



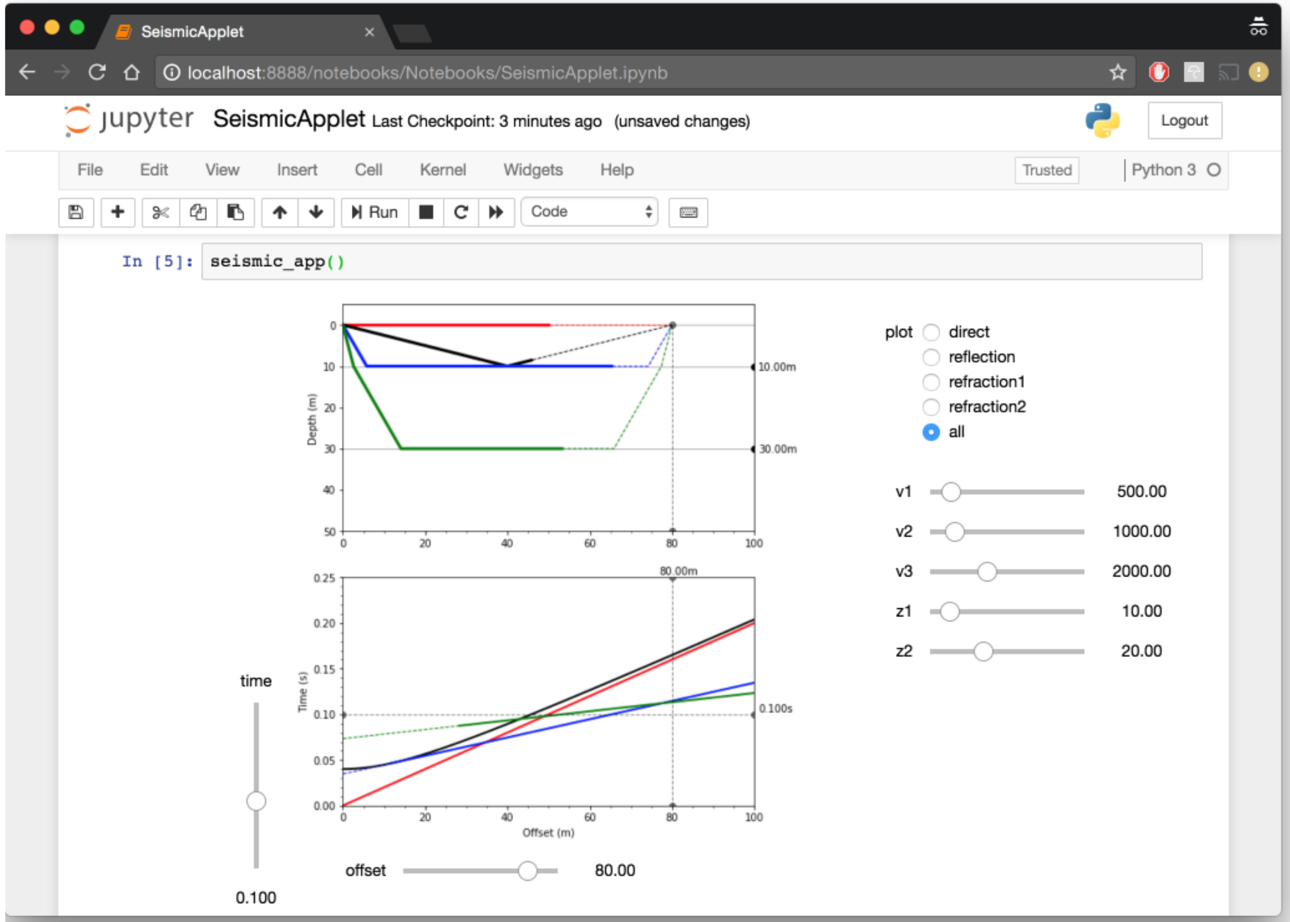




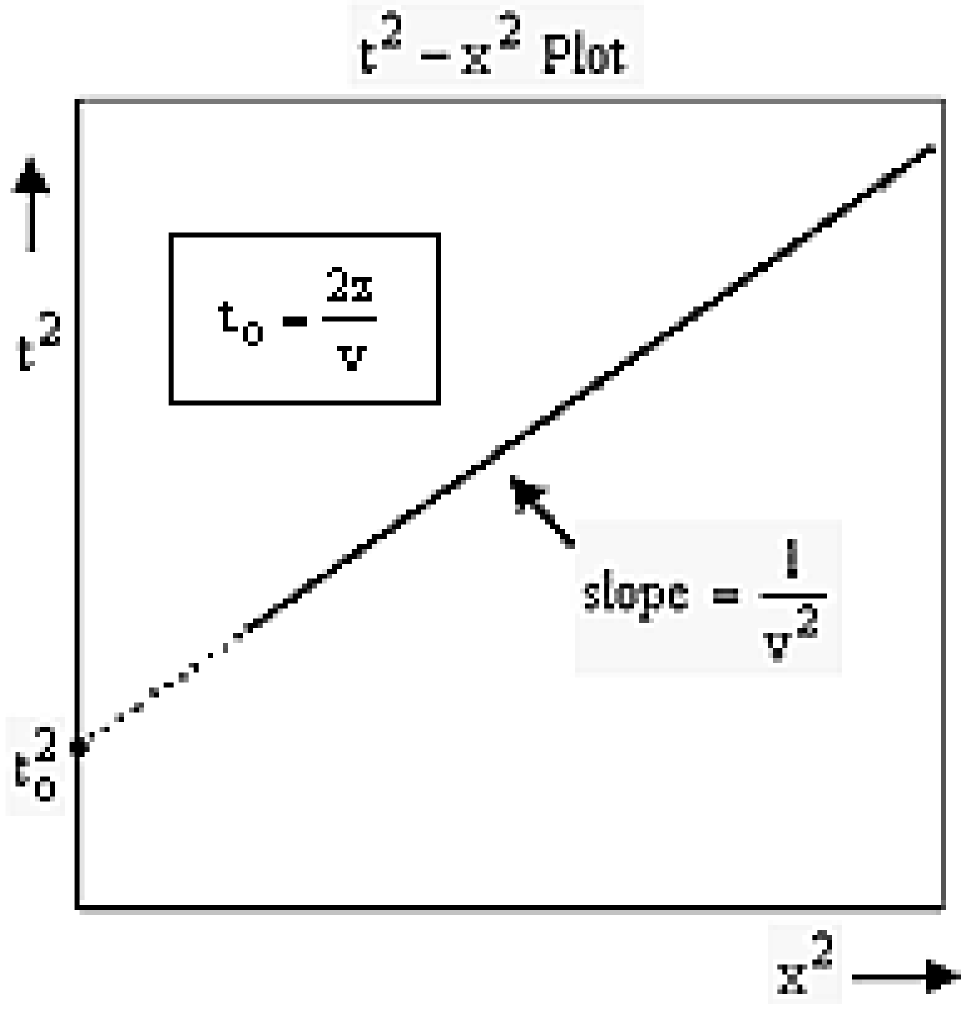
What about three layers?

- Explore using the seismic app

- What happens if v2<v1?



t^2-x^2 plot



$$t = \frac{(x^2 + 4h^2)^{\frac{1}{2}}}{v}$$

$$t^2 = \frac{x^2}{v^2} + \underbrace{\frac{4h^2}{v^2}}_{t_0^2}$$

Can estimate velocity and thickness of the layer

Upcoming activities

- **Reading materials:**
  - GPG: seismic

- **Upcoming activities:**
  - Next Monday/Tuesday – Seismic Lab I

Unit Activities

- **Labs: (Seismic I)**
  - Monday, September 30th
  - Tuesday, October 1st
- **Labs: (Seismic II)**
  - Monday, October 7th
  - Tuesday, October 8th
- **TBL:**
  - Monday, October 7th
- **Quiz:**
  - Monday, October 7th