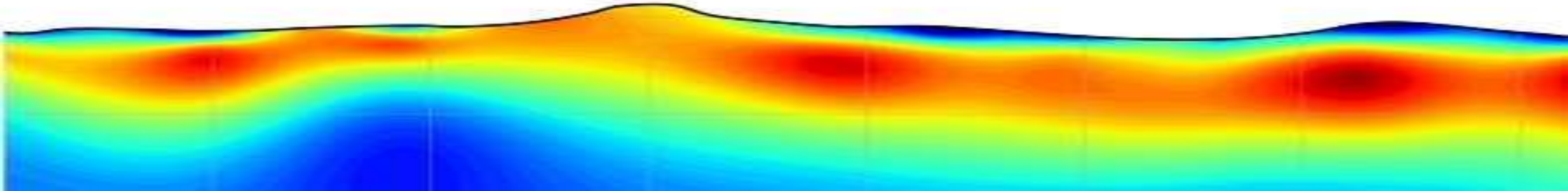
SUSTech Southern University of Science and Technology
南方科技大学 SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
地球与空间科学系 DEPARTMENT OF EARTH AND SPACE SCIENCES

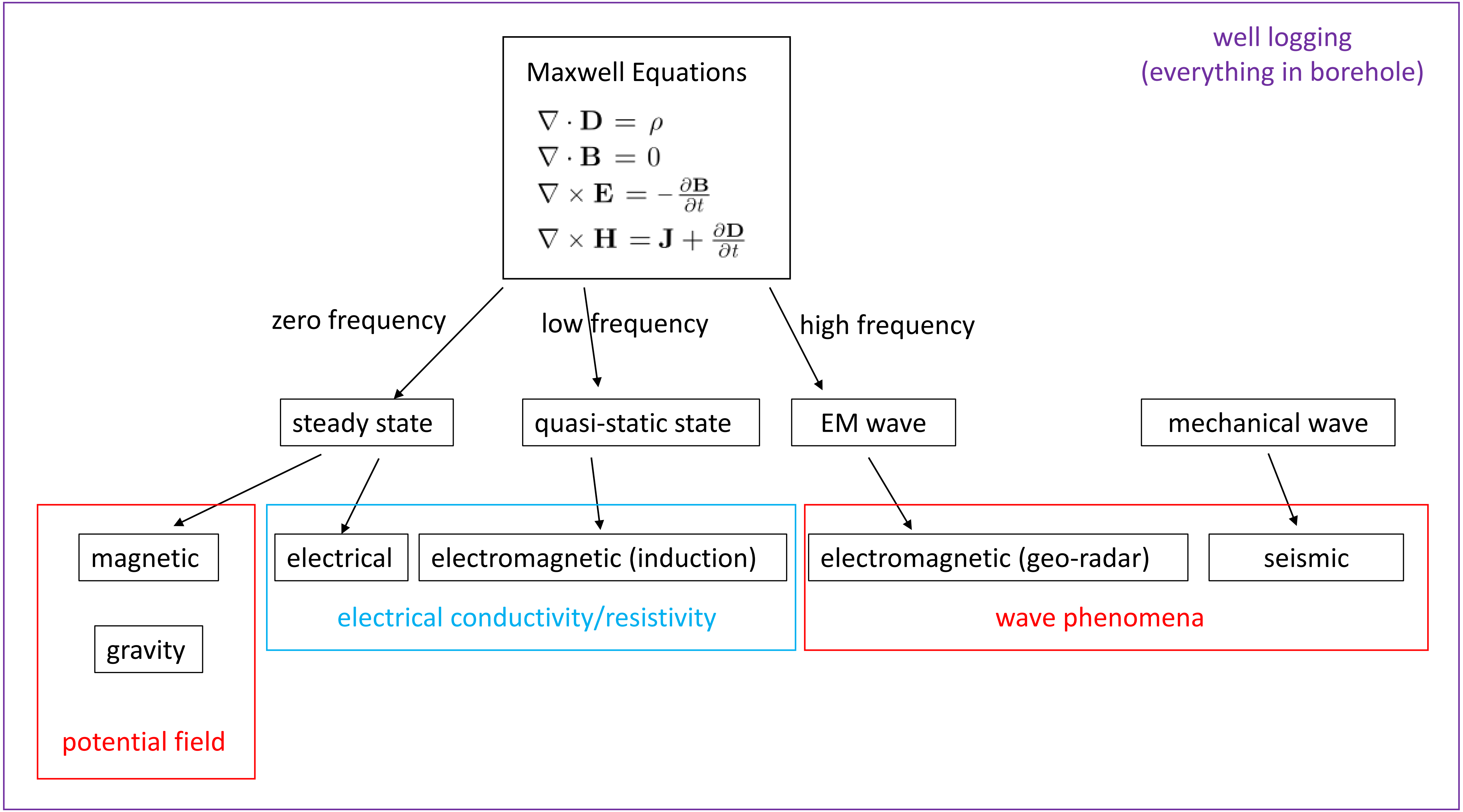
ESS302 Applied Geophysics II

Gravity, Magnetic, Electrical, Electromagnetic and Well Logging

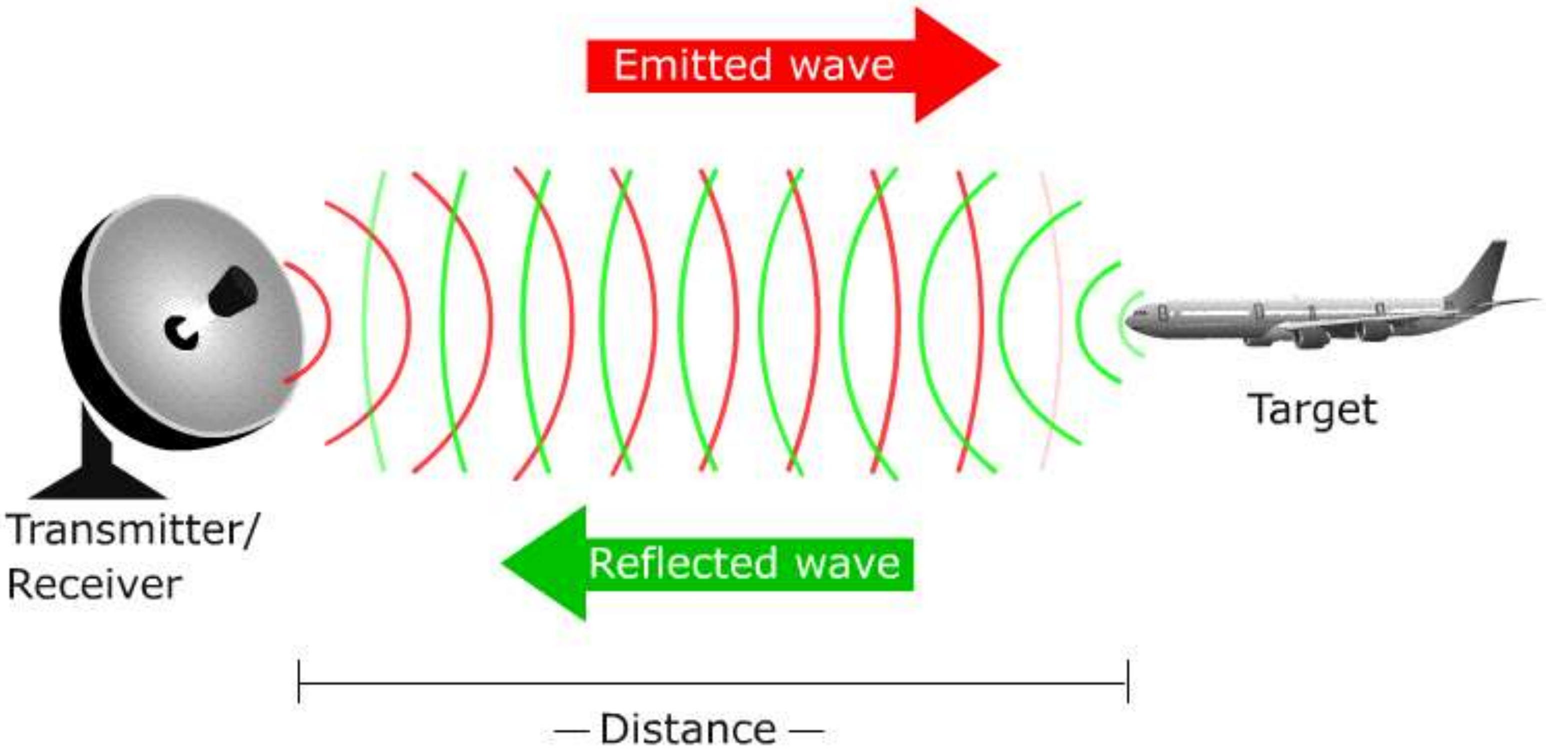
Electromagnetic 1: GPR Theory

Instructor: Dikun Yang   Feb – May, 2019

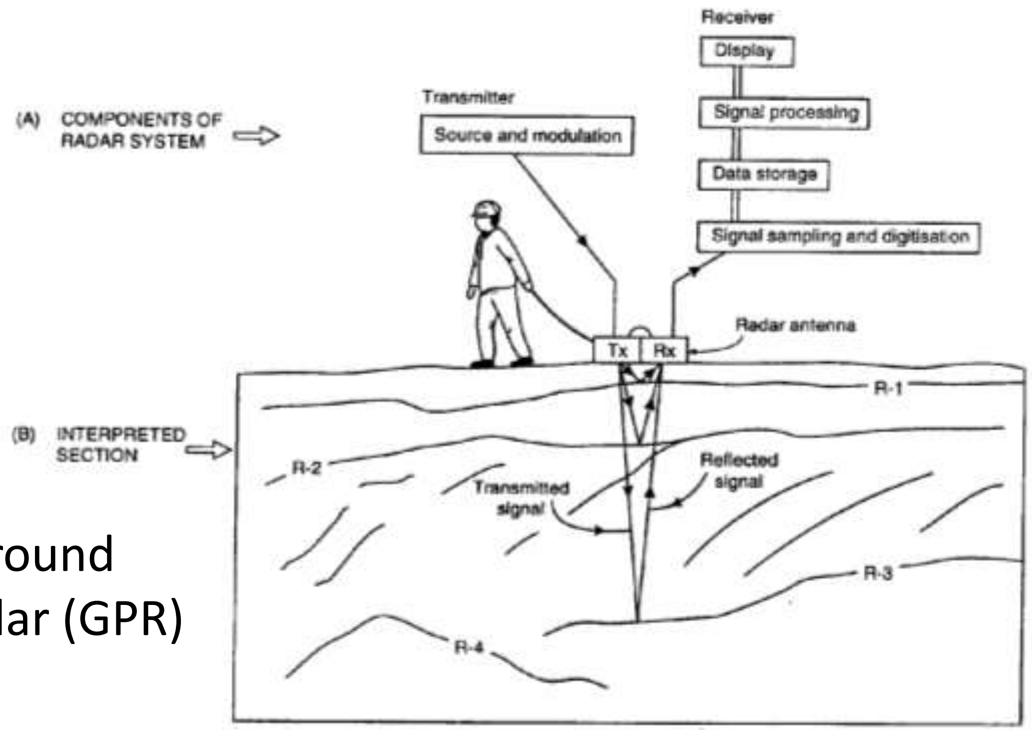



well logging
(everything in borehole)
Maxwell Equations
$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
zero frequency
low frequency
high frequency
steady state
quasi-static state
EM wave
mechanical wave
magnetic
gravity
potential field
electrical
electromagnetic (induction)
electrical conductivity/resistivity
electromagnetic (geo-radar)
seismic
wave phenomena

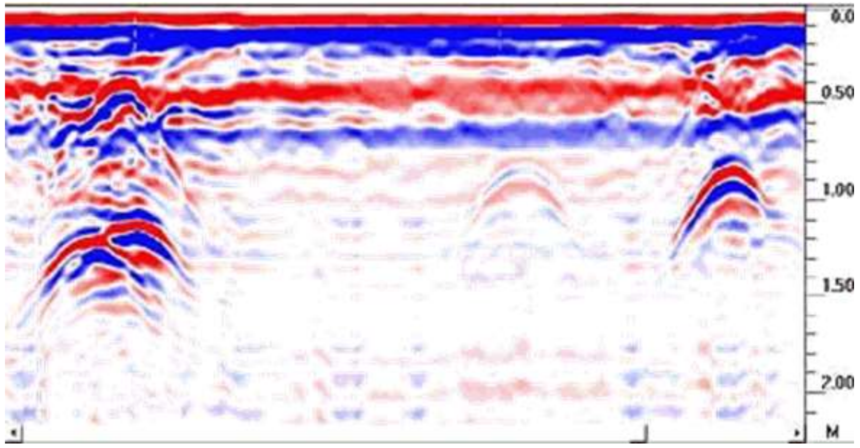
Radar



- Can we do the same thing to the subsurface?
- What are the differences between finding an object in the air and underground using EM waves?

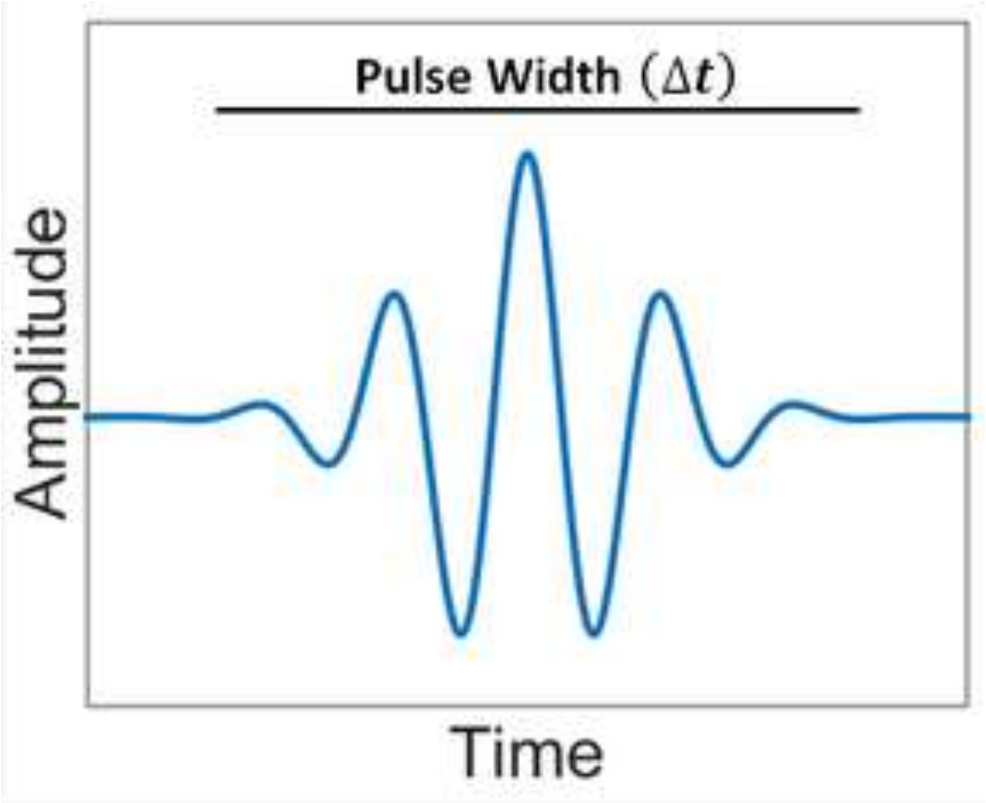


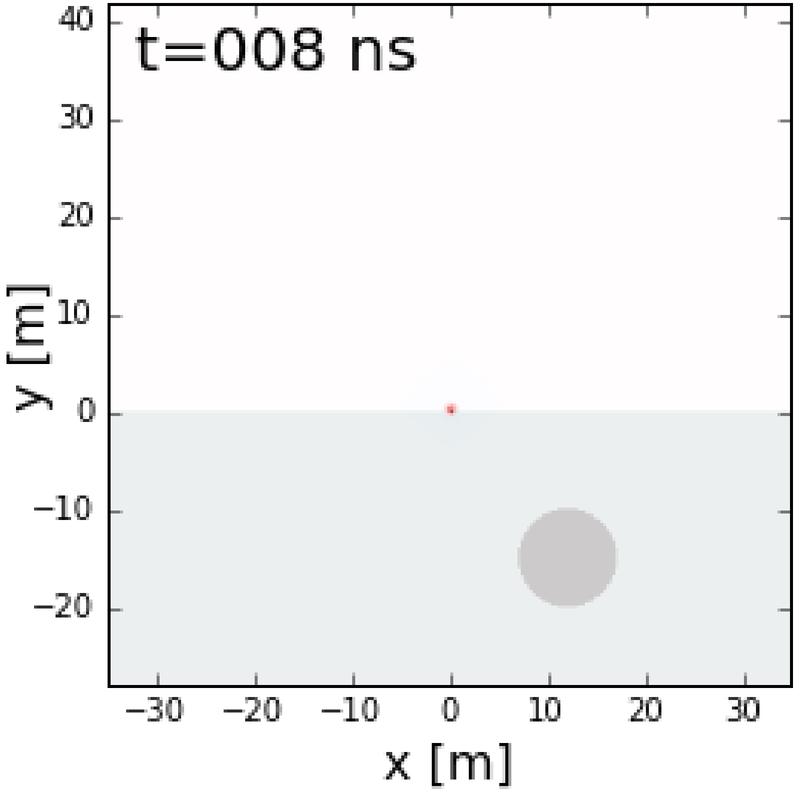
Geo-radar or ground penetrating radar (GPR)



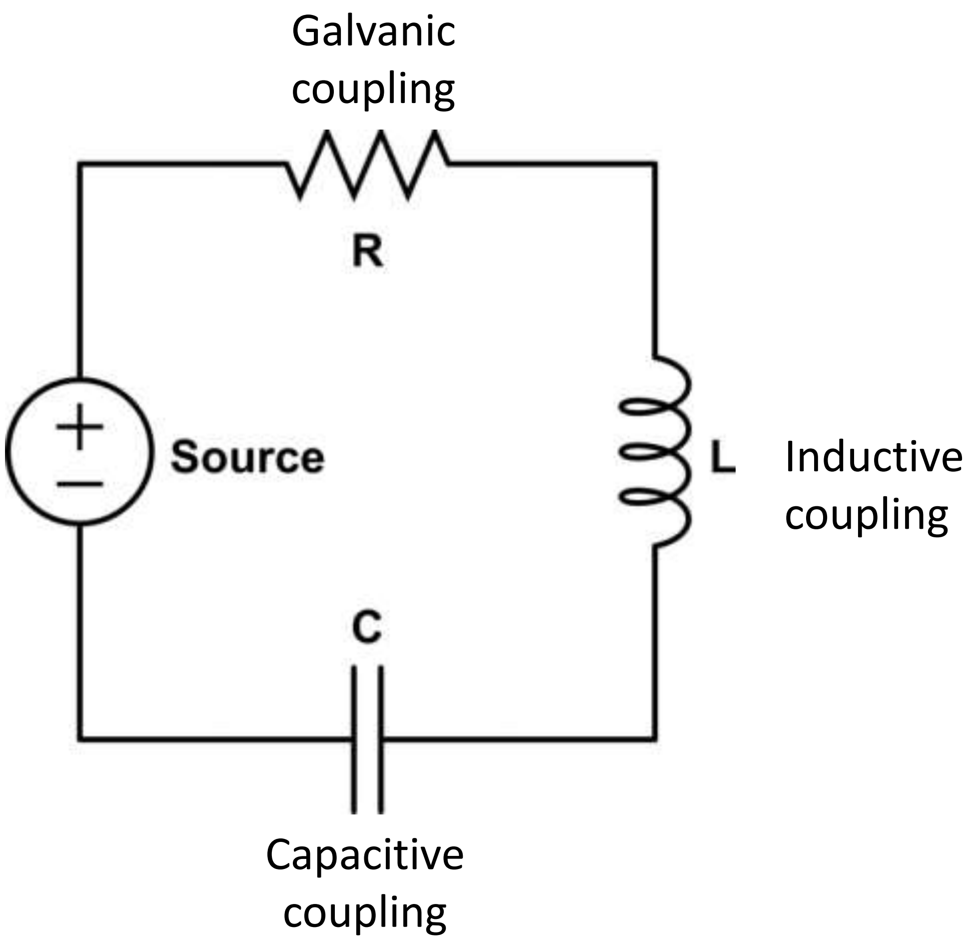
Radargram

EM Field at High Frequencies – Wave

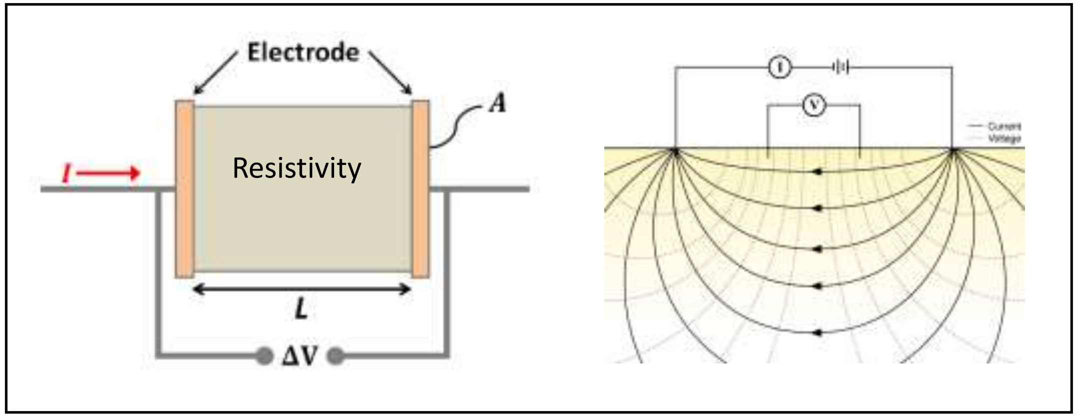


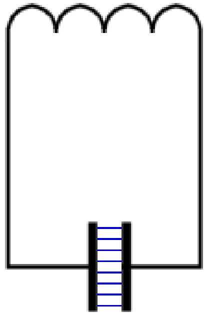


Ground Penetrating Radar (GPR)



Recall…





Capacitive coupling

- High frequency EM field
- Dielectric constant (ε_r)
- Wave phenomenon

Wave Propagation

Medium characterized by three physical properties:
σ (electrical conductivity), ε (electrical permittivity),
μ (magnetic permeability)

In general:

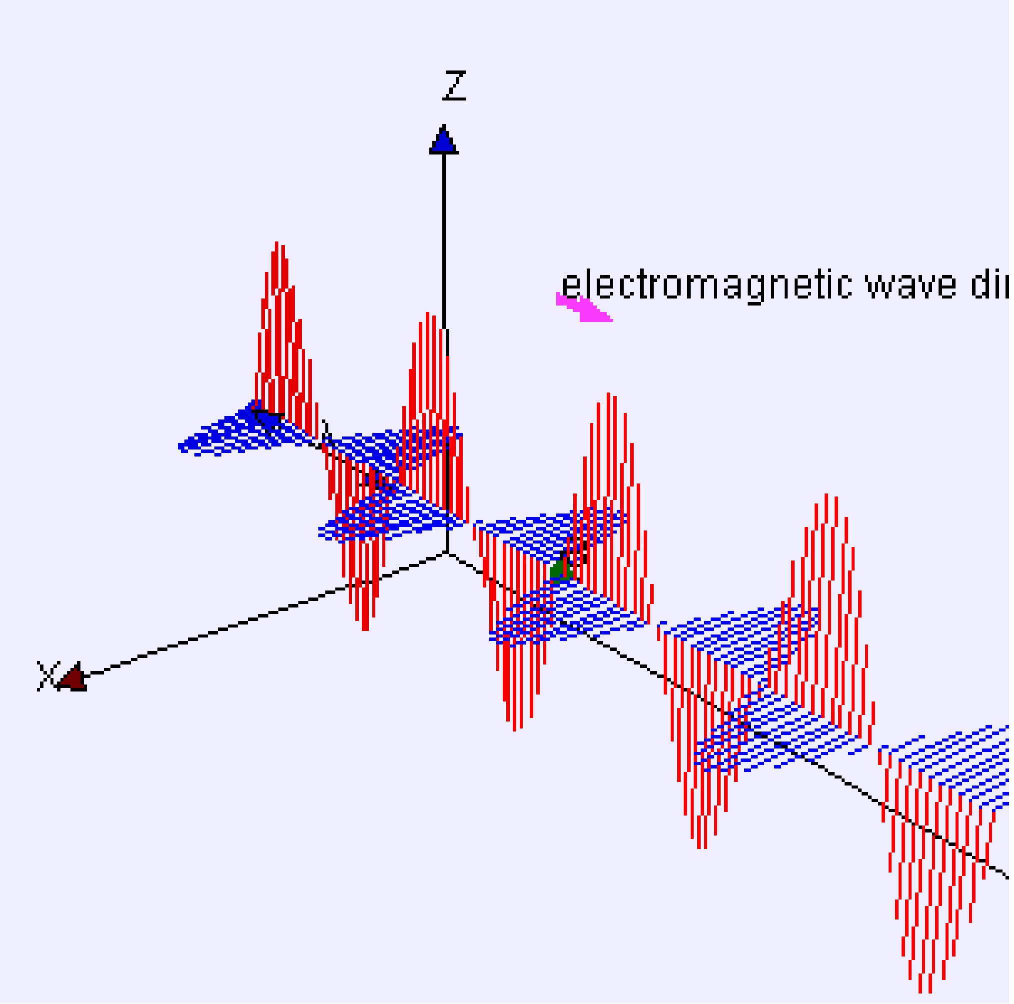
$$V = \sqrt{\frac{2}{\mu\varepsilon}}\left[\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2}+1\right]^{-1/2}$$

Wave regime ($\sigma \ll \omega\varepsilon$):

$$V = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

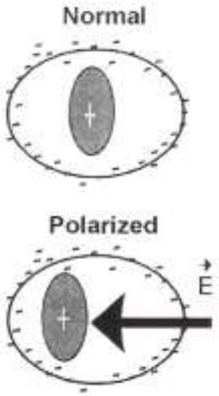
Non-magnetic approximation ($\mu_r = 1$):

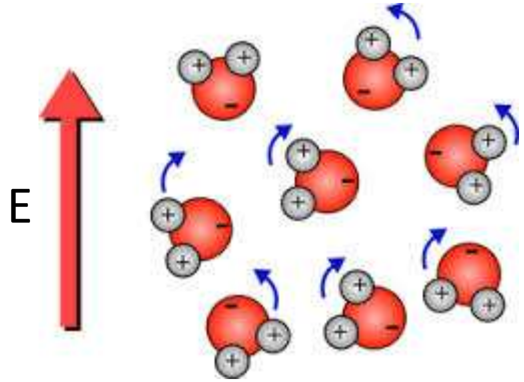
$$\boxed{V = \frac{c}{\sqrt{\varepsilon_r}}}$$

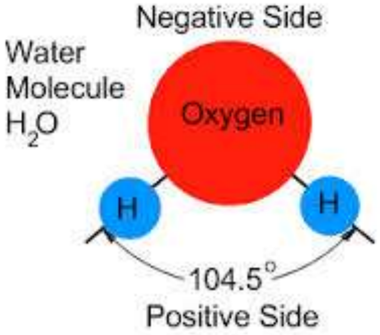


Question: How does EM wave propagate in perfect conductors?

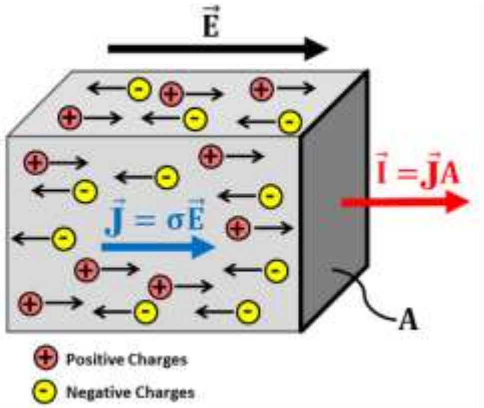
Dielectric Permittivity (ε):
How easily a material is electrically polarized



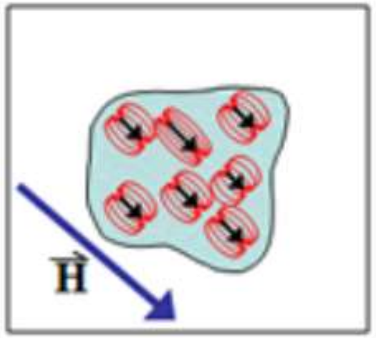




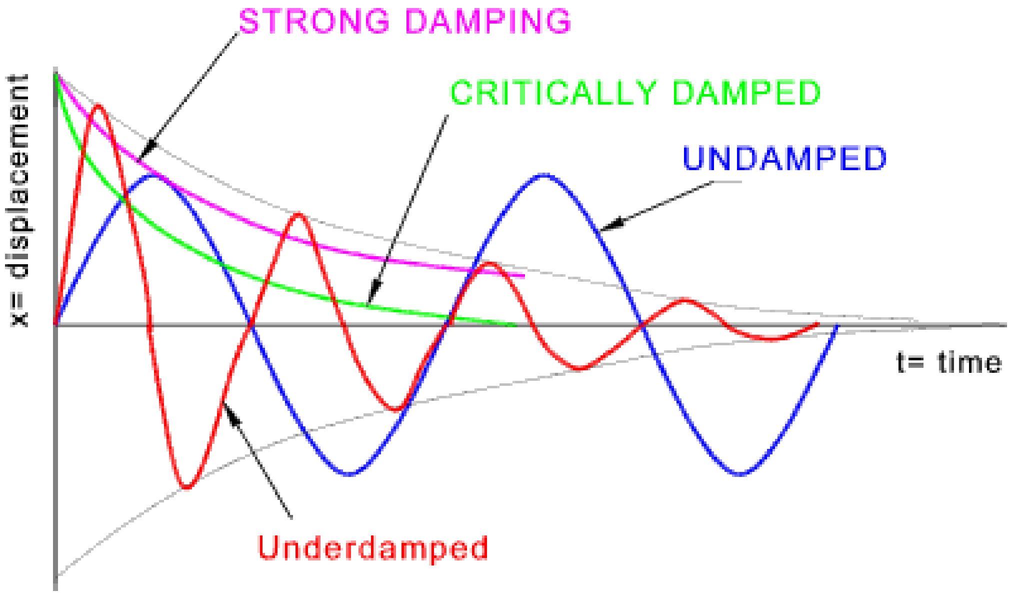
Electrical Conductivity (σ):
How easily electrical charges flow through a material

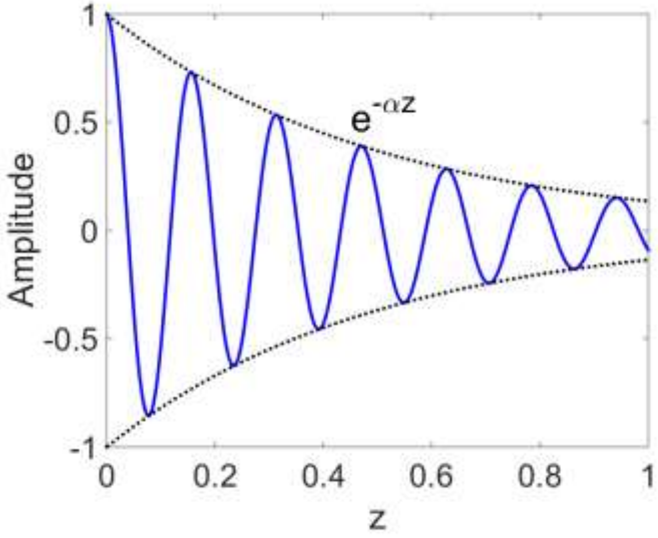


Magnetic Permeability (μ):
How strongly a material supports magnetism



Wave Attenuation





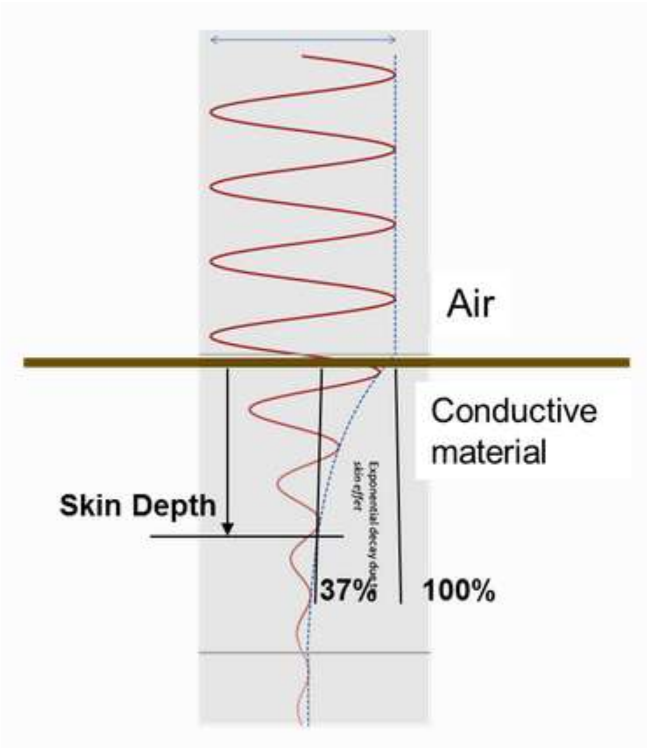
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/2} - 1\right]^{1/2} \approx \begin{cases} \sqrt{\dfrac{\omega\mu\sigma}{2}} & \text{for } \omega\varepsilon \ll \sigma \\ \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}} & \text{for } \sigma \ll \omega\varepsilon \end{cases}$$

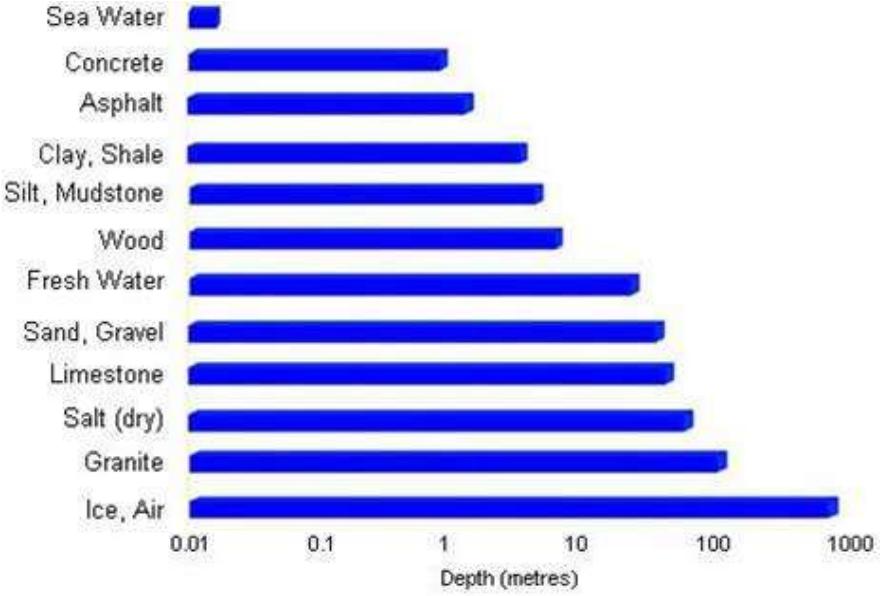
- Quasi-Static ($\omega\varepsilon << \sigma$): Conductive/Low-frequency
- Wave Regime ($\sigma << \omega\varepsilon$): Resistive/High-frequency

Skin Depth and Probing Distance

- Skin Depth: Distance at which a wave is reduced to 37% (1/e) of its original amplitude

- The probing distance is approximated 3 skin depths.





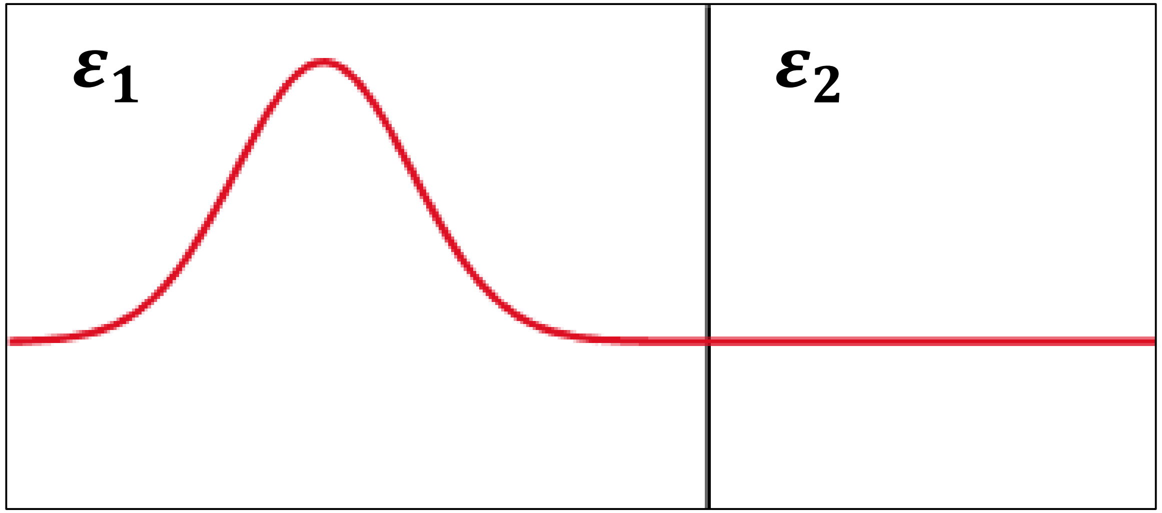
$$\delta \approx \begin{cases} 503\sqrt{\dfrac{1}{\sigma f}} & \text{for } \omega\varepsilon \ll \sigma \\ 0.0053\dfrac{\sqrt{\varepsilon_r}}{\sigma} & \text{for } \sigma \ll \omega\varepsilon \end{cases}$$

Table of relative dielectric permittivity (e_R), electrical conductivity (σ), and velocity.			
Material	**e_R**	**σ (mSeimens/m)**	**V avg (m/ns)**
Air	1	0	.3
Distilled water	**80**	**0.01**	0.033
Fresh water	80	0.5	0.033
Sea water	80	**3000**	**0.01**
Dry sand	**3**- 5	0.01	0.15
Saturated sand	20-30	0.1-1.0	0.06
Limestone	4-8	0.5-2.0	0.12
Shales	5-15	1-100	0.09
Silts	5-30	1-100	0.07
Clays	5-40	2- 1000	0.06
Granite	4-6	0.01-1.0	0.13
Dry salt	5-6	0.01-1.0	0.13
Ice	3-4	0.01	**0.16**

Reflection and Transmission

$$R = \frac{\text{Reflected Amplitude}}{\text{Incident Amplitude}} = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

$$T = \frac{\text{Transmitted Amplitude}}{\text{Incident Amplitude}} = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$



- If $\varepsilon_1 \approx \varepsilon_2$, most of the wave is transmitted
- If $\varepsilon_1 \ll \varepsilon_2$ or $\varepsilon_1 \gg \varepsilon_2$, most of the wave is reflected

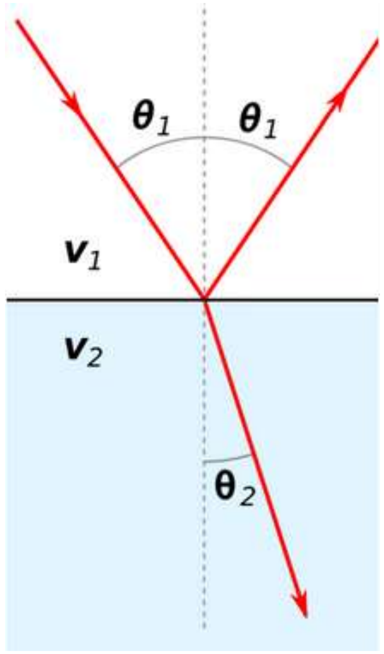
Refraction

- Snell's Law:

$$\frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2}$$

$$V = c/\sqrt{\varepsilon_r}$$

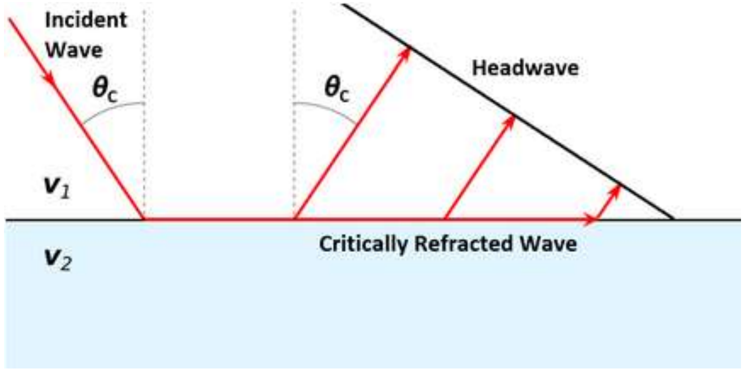
$$\sqrt{\varepsilon_1}\,\sin\theta_1 = \sqrt{\varepsilon_2}\,\sin\theta_2$$



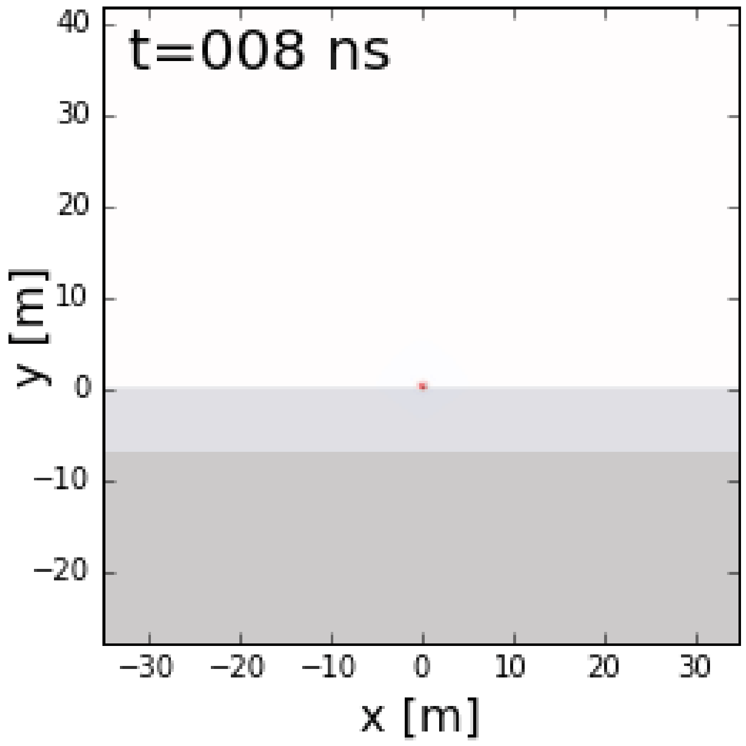
- Critical refraction

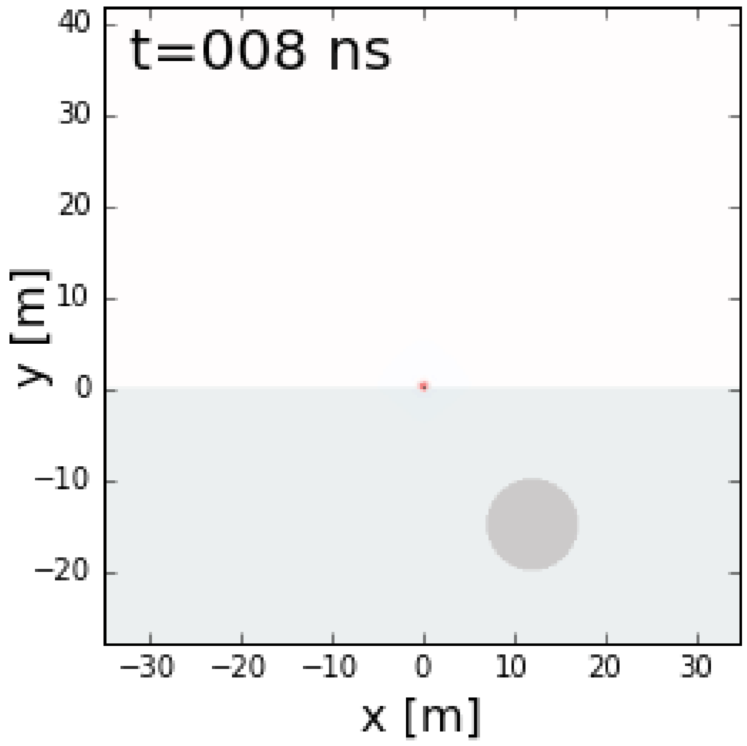
$$\sin\theta_c = \frac{V_1}{V_2}$$

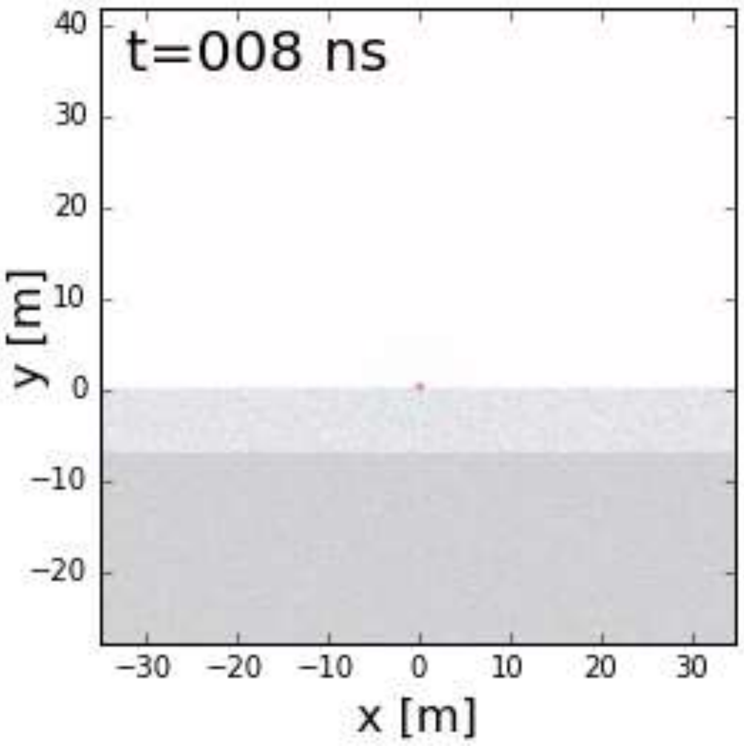
Requires $V_1 < V_2$



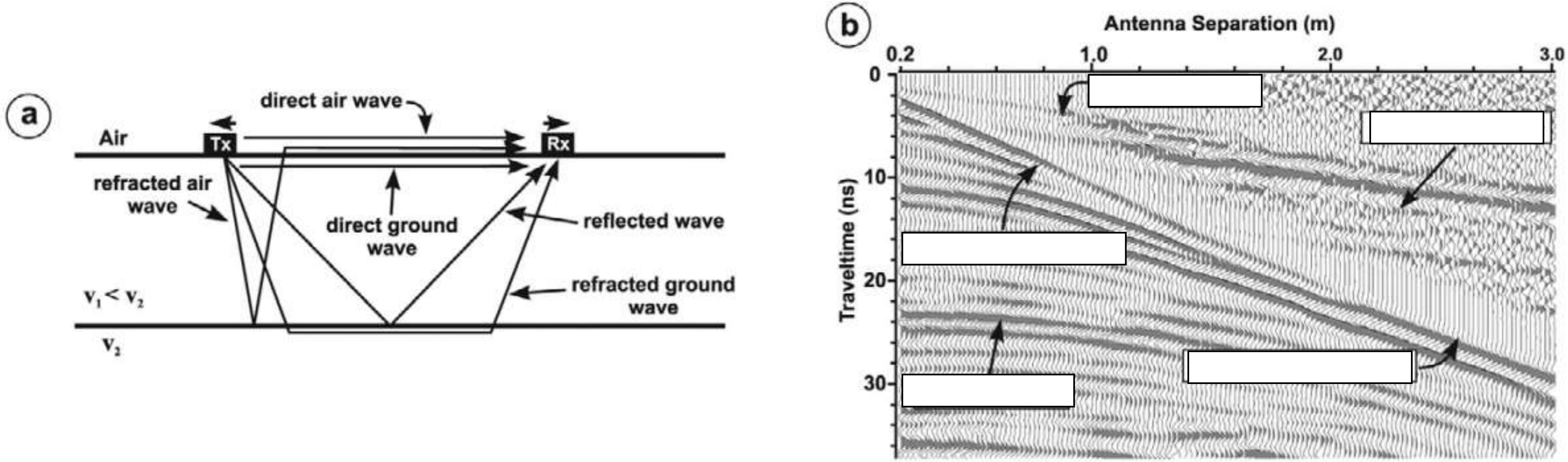
Reflection, Transmission, Refraction, Scattering





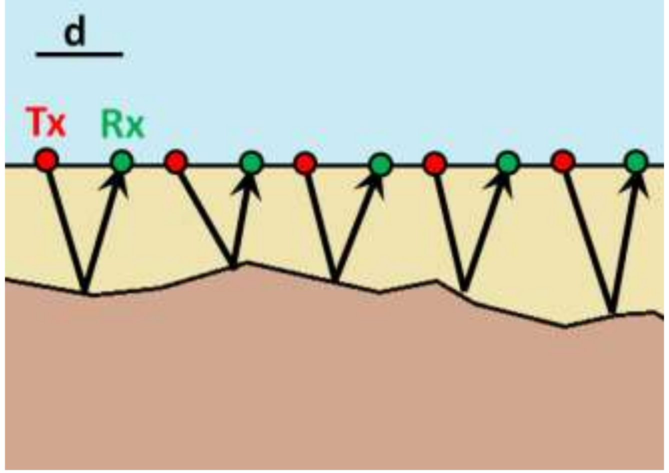


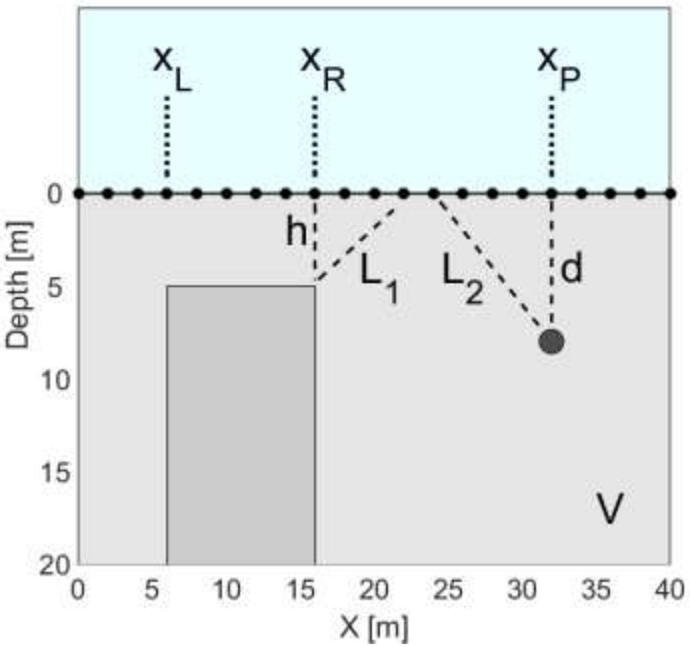
EM Wave Propagation in a Two-layer Earth

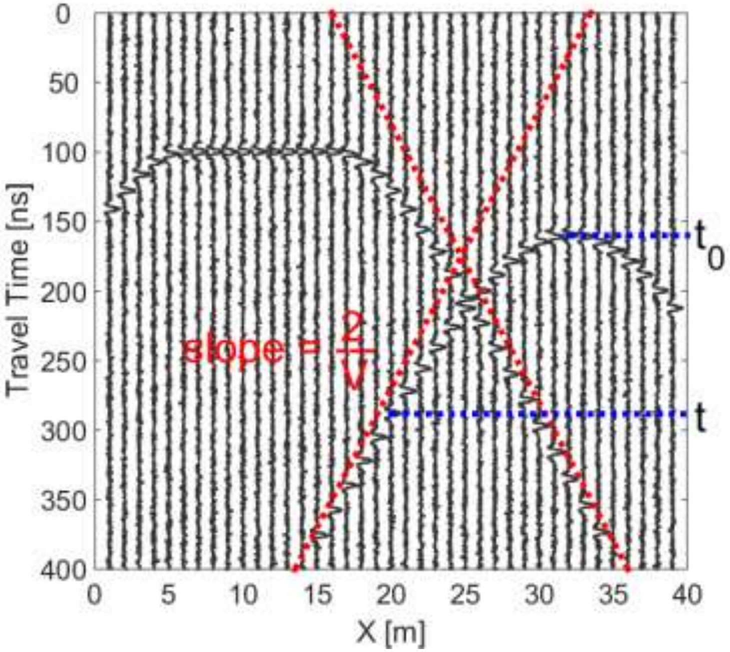


GPR Anomaly on Radargram

$$t_p = \frac{2L_2}{V} = \frac{2\sqrt{(x-x_p)^2+d^2}}{V}$$







- Determine background medium velocity
- Determine the depth of burial
- Determine the size of extended objects

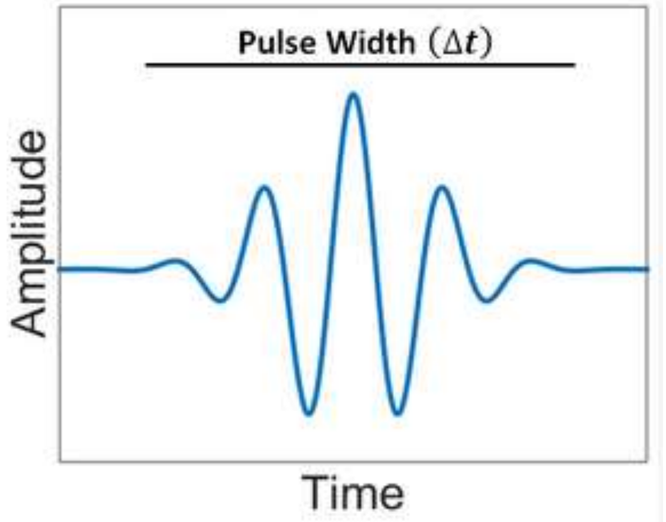
GPR Source Signal

- Wavelet: A wave-like oscillation of short duration
- Bandwidth: Range of frequencies in the wavelet
- Pulse Width: Time-duration of wavelet
- Spatial Length: Wavelength of the wavelet
- Central Frequency: Operating frequency of GPR survey

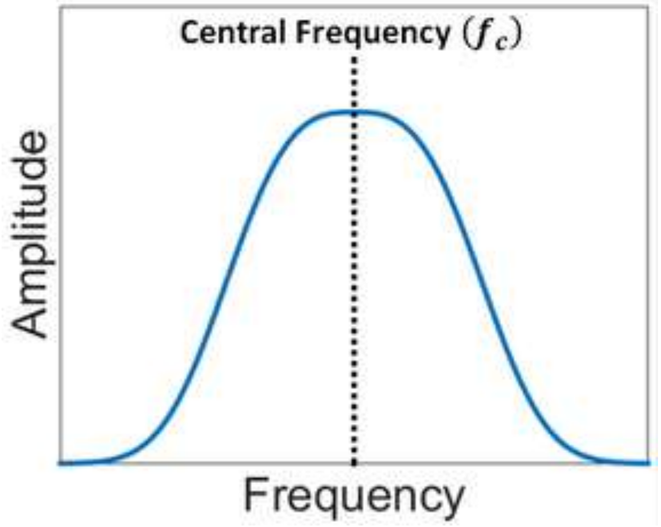
$$f_c = \frac{1}{\Delta t}$$

Typically 50 MHz to 1 GHz

Wavelet



Frequencies in Wavelet



GPR Source Signal: Spatial Length

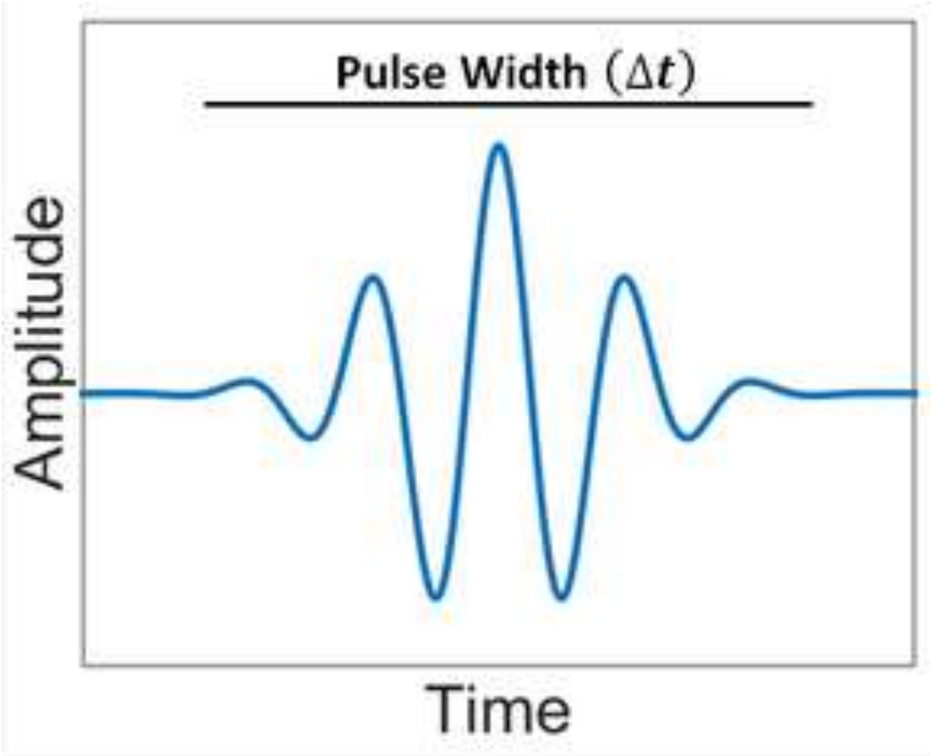
- The spatial length (wavelength) of the GPR pulse is dependent on the central frequency and velocity

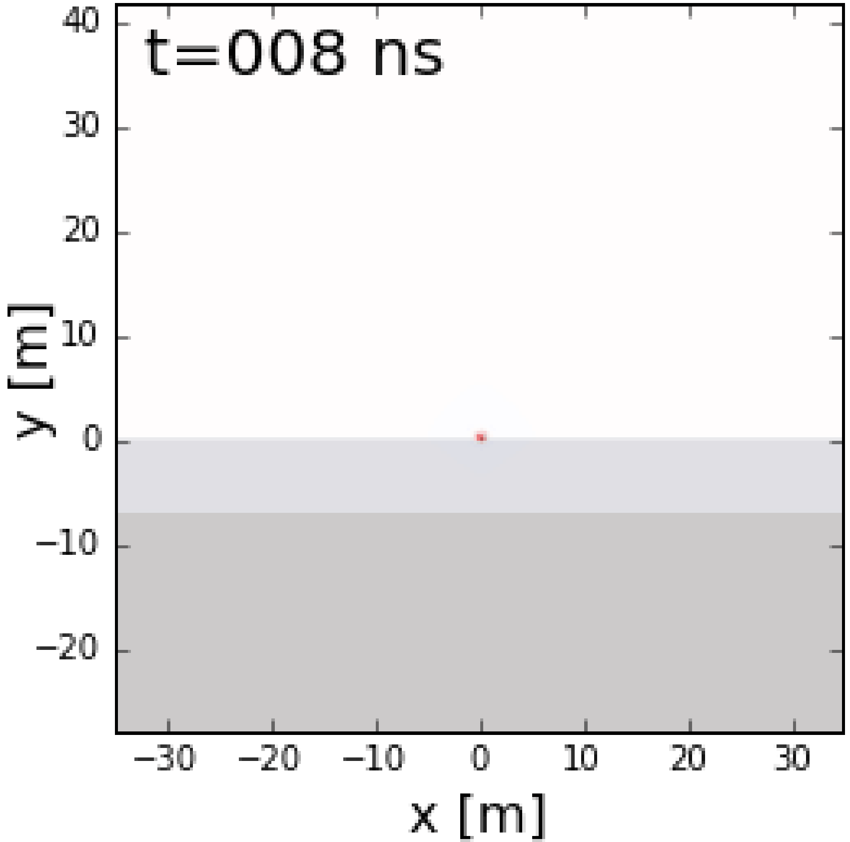
$$\lambda = \frac{V}{f_c} = \frac{c}{f_c\sqrt{\varepsilon_r}} = \frac{c\,\Delta t}{\sqrt{\varepsilon_r}}$$

- When the GPR signal at some frequency is transmitted across an interface, it can be stretched or contracted

- Lower velocity → Shorter spatial length
- Lower frequency → Larger spatial length

Signal Stretched or Contracted?





Resolution of GPR

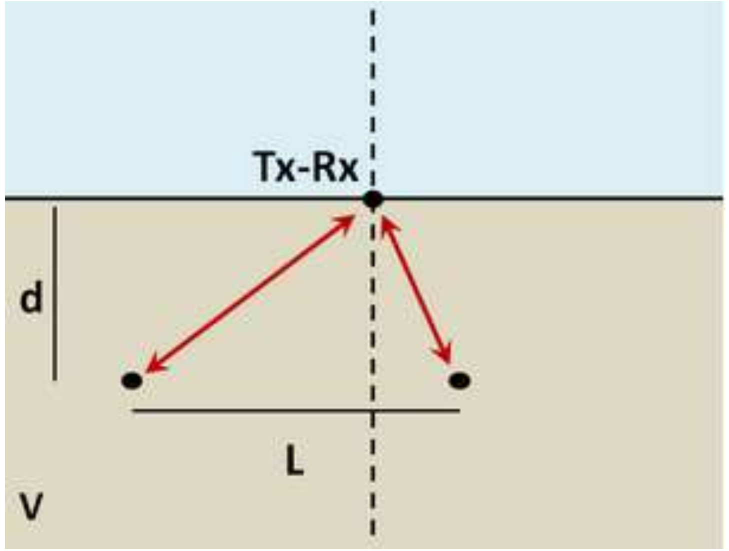
- ¼ wavelength rule:

  The thickness of a layer must be at least ¼ the wavelength of the GPR signal.

$$L > \frac{c}{4f_c\sqrt{\varepsilon_r}} = \frac{c\Delta t}{4\sqrt{\varepsilon_r}}$$

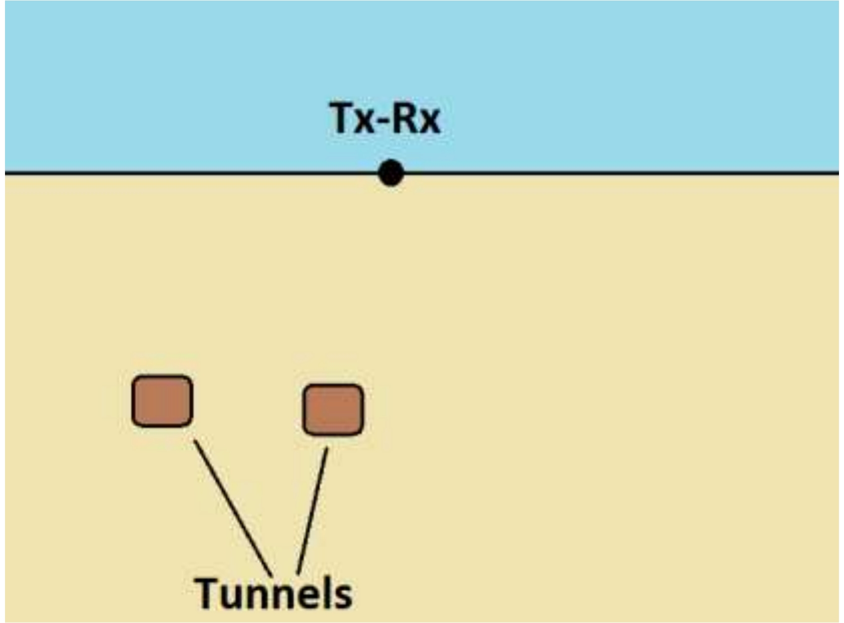
- For zero offset survey

$$L > \sqrt{\frac{V\,d}{2f_c}}$$



Probing Distance vs. Resolution

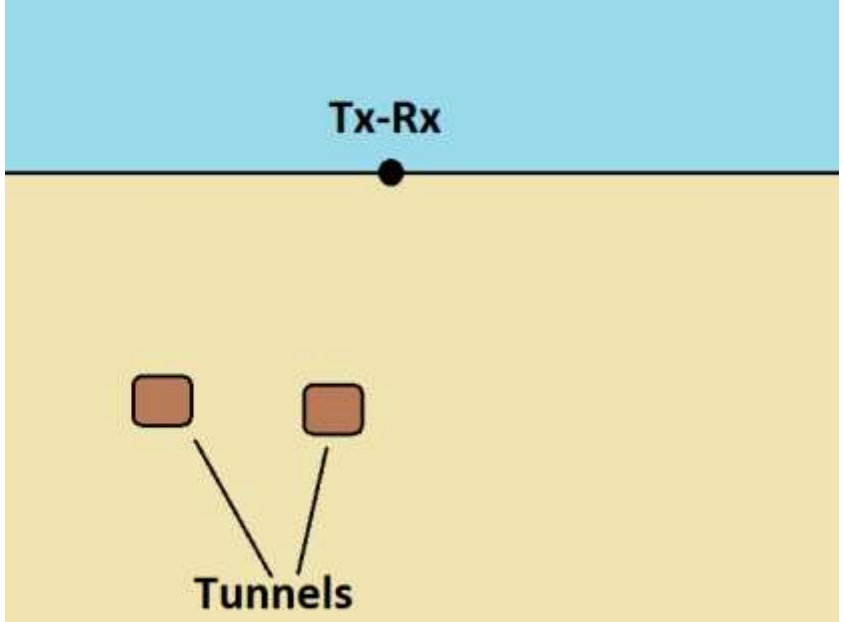
- Want to find two buried tunnels.
- Using a zero offset survey configuration.
- Higher frequencies give better resolution
- Lower frequencies give larger probing distance

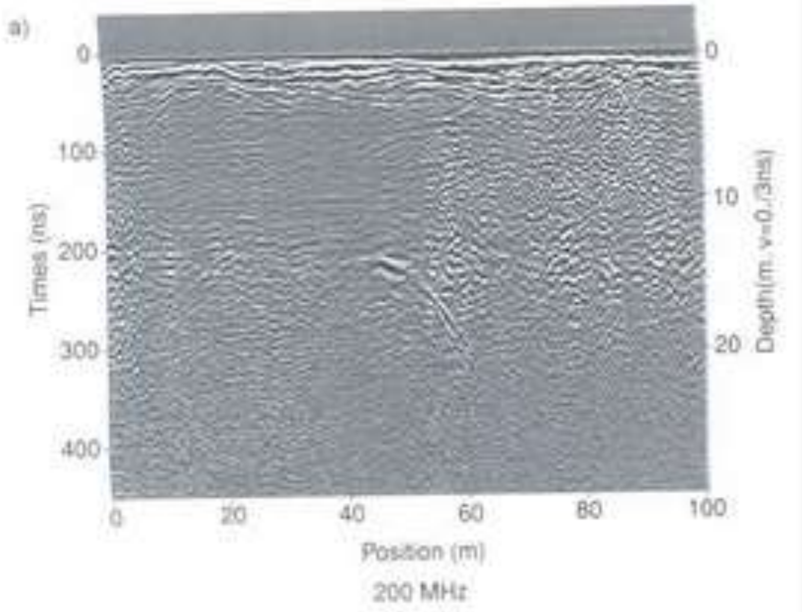


Radargram 200 MHz

- Little to no useful signal after 200 ns
- Can't see features from the tunnels

- Too much attenuation of signal
- Probing distance insufficient

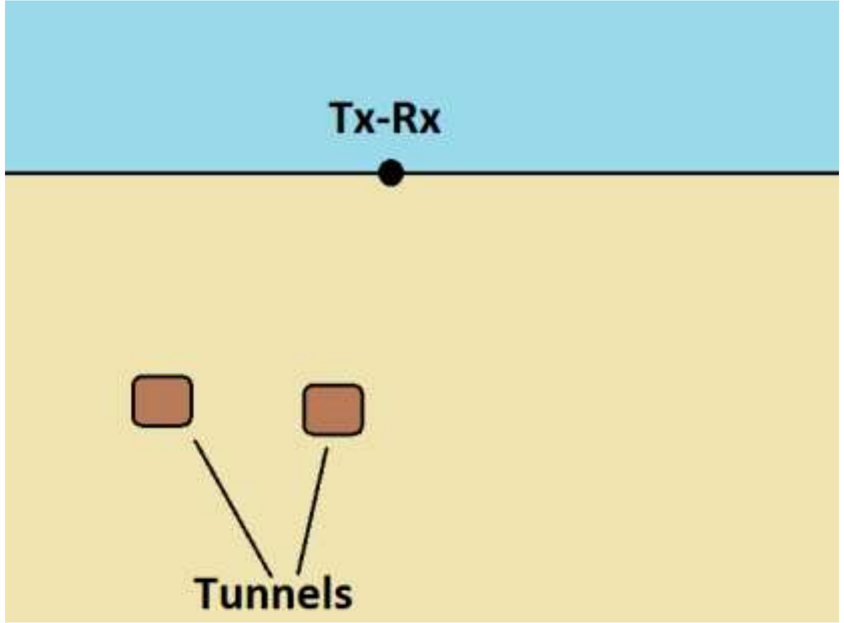


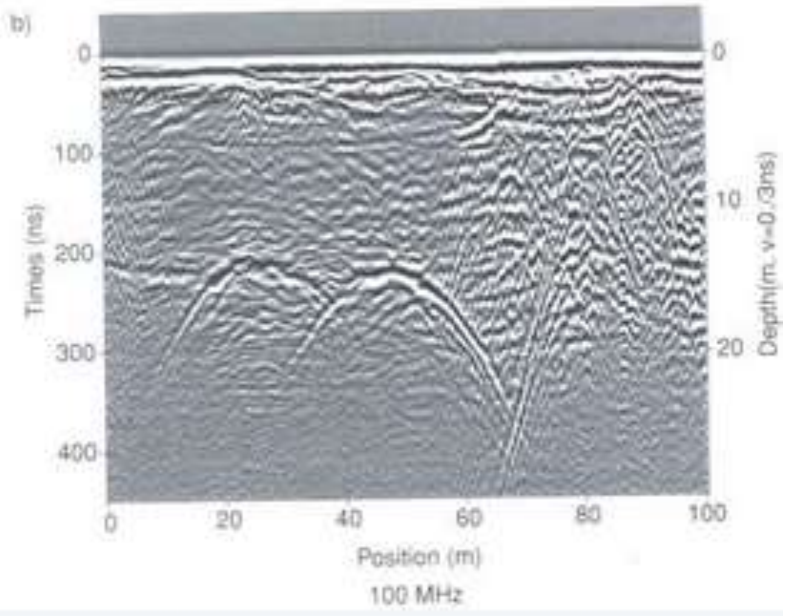


Radargram 100 MHz

- Useful signals up to 300 ns
- See top of hyperbolas from tunnels

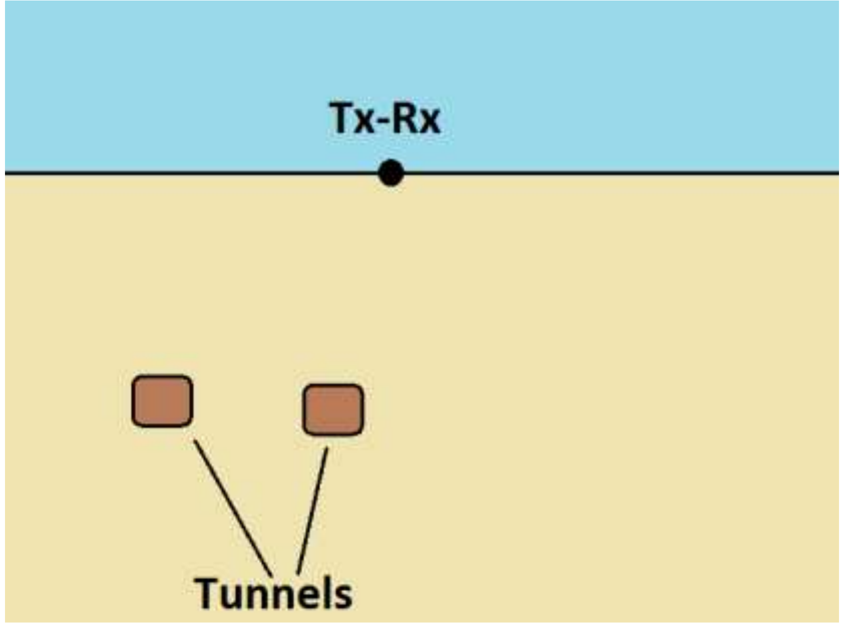
- Lower resolution
- Can see tunnels

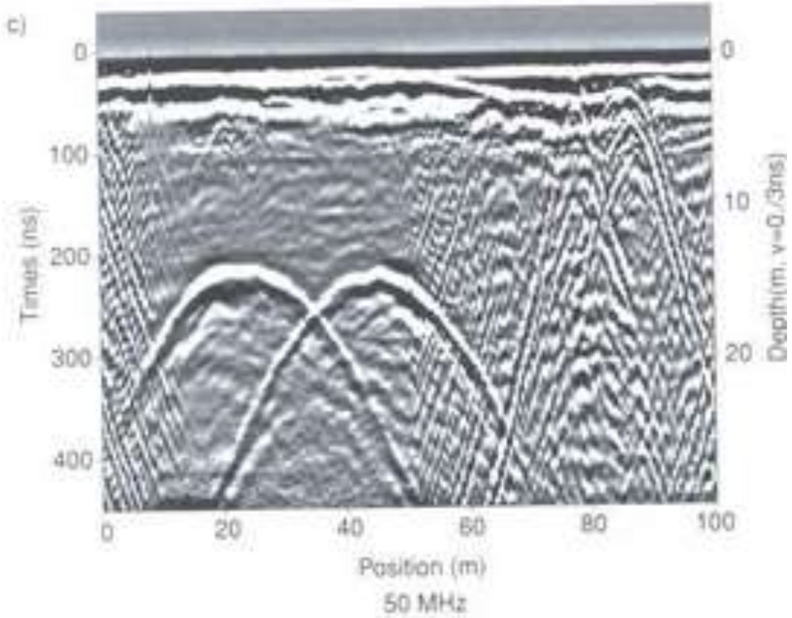




Radargram 50 MHz

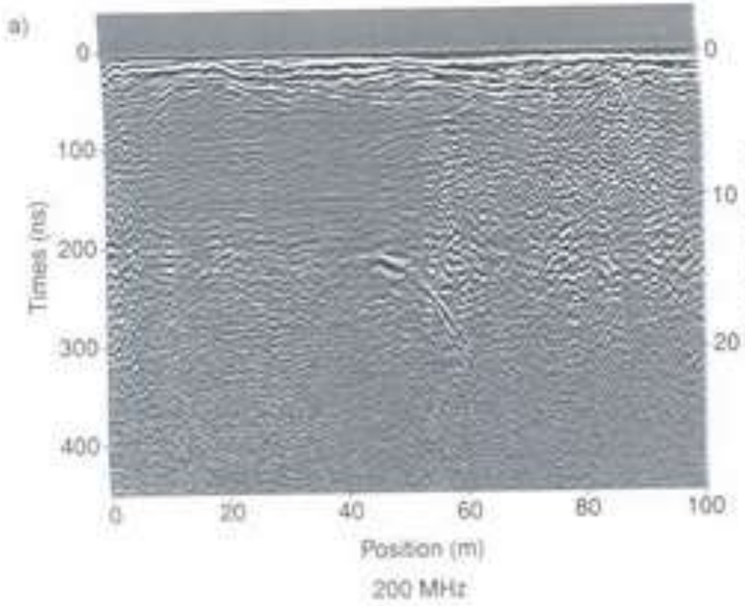
- Useful signals through 400 ns
- Well-defined hyperbolas from tunnels
- Lower resolution image
- Best frequency for what we want to observe



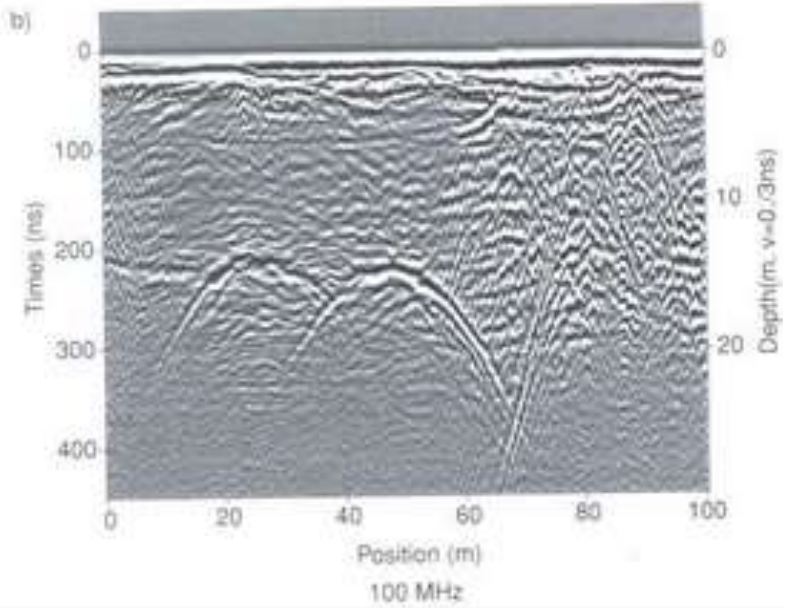


Depth vs. Resolution

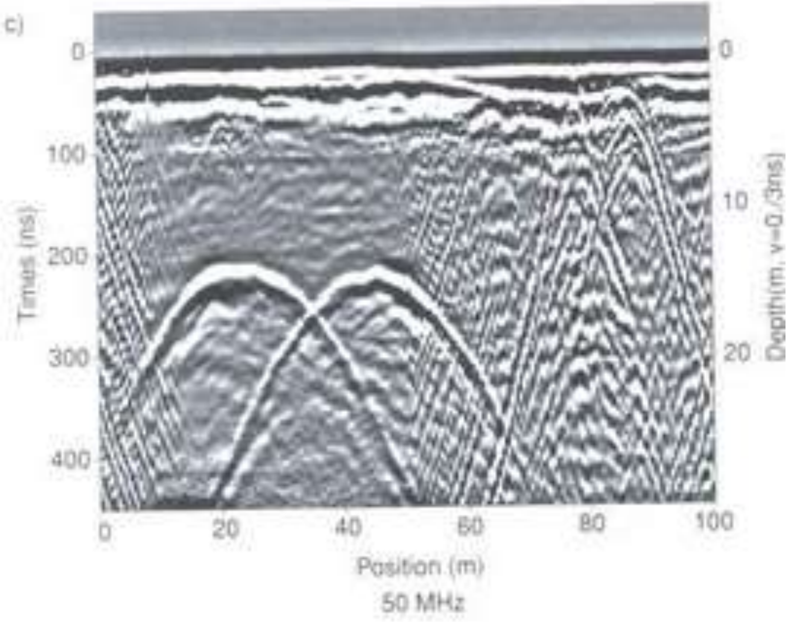
200 MHz



100 MHz



50 MHz



Summary

- EM at high frequency: Wave regime
- Physical properties utilized by EM/GPR
- Reflection, transmission, refraction and scattering
- Signal length scale and resolution
- Depth vs. spatial resolution