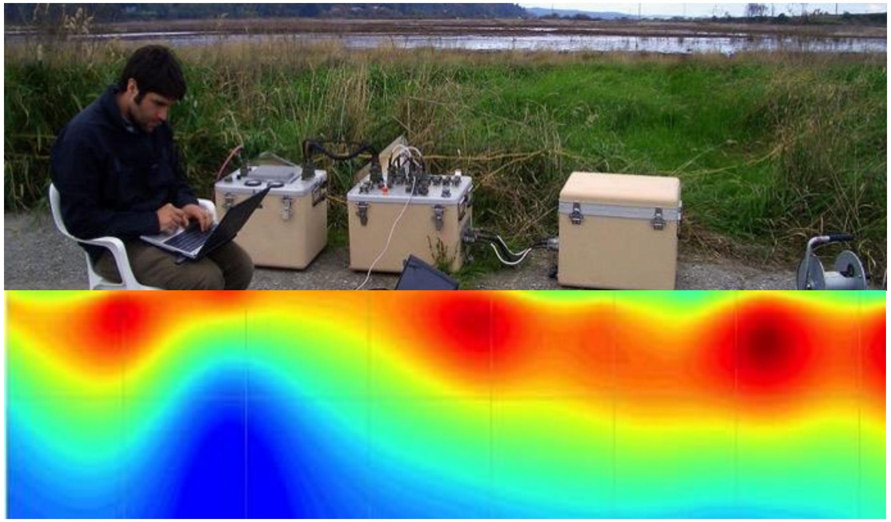
EOSC 350 : Environmental, Geotechnical and Exploration Geophysics I

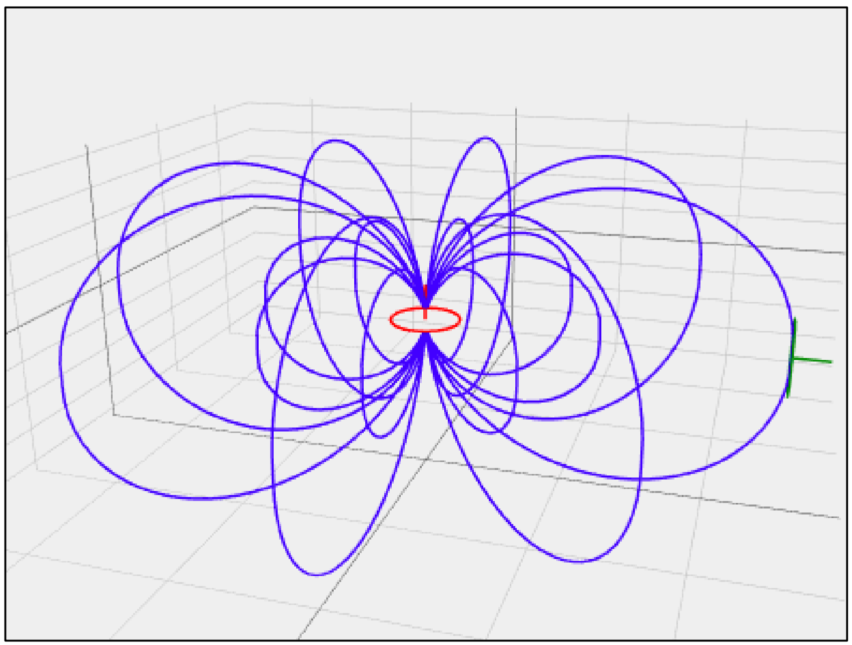
# EM 3-loop Model



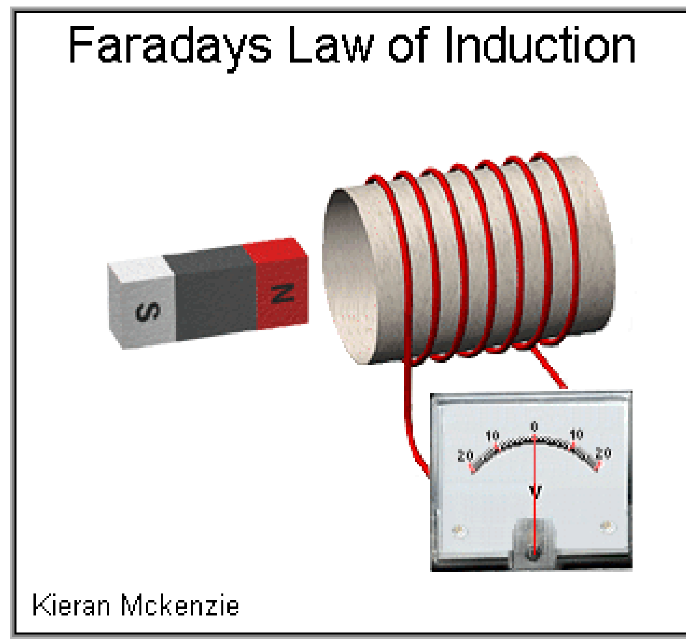
September – December, 2017

# Maxwell's Equations

## Ampere's law
electric -> magnetic

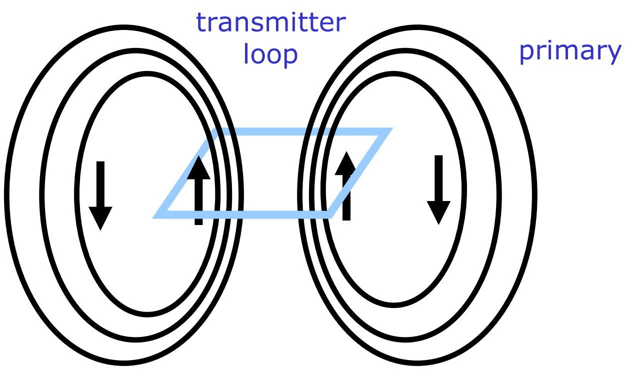


## Faraday's law
magnetic -> electric



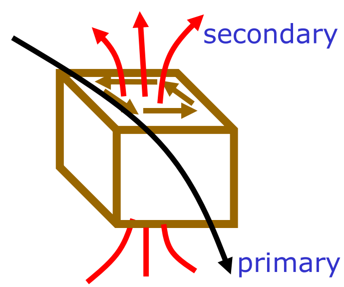
# Communicate with the earth

## Transmitter loop



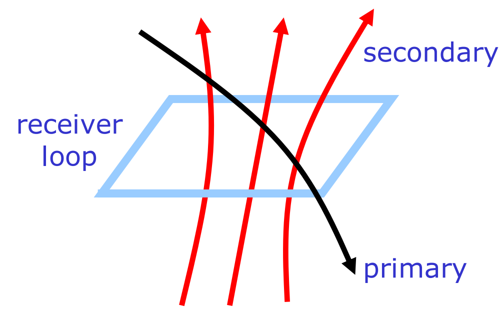

**Ampere**: time-varying current and changing primary magnetic field

## Target




**Faraday**: current induced by the changing primary field; **Ampere**: induced current generates a secondary magnetic field

## Receiver loop

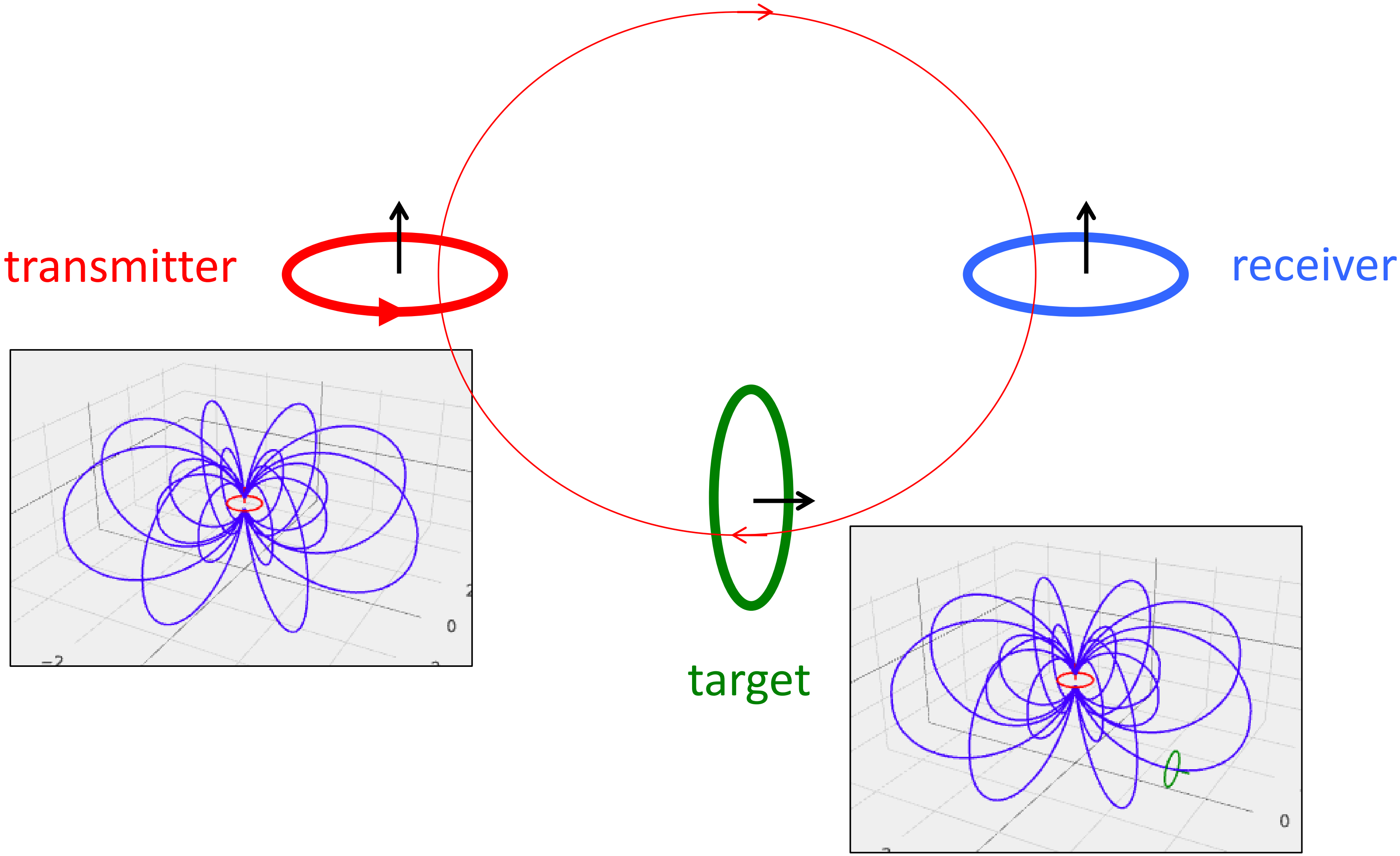



**Faraday**: measurable current induced in the loop by the changing secondary field

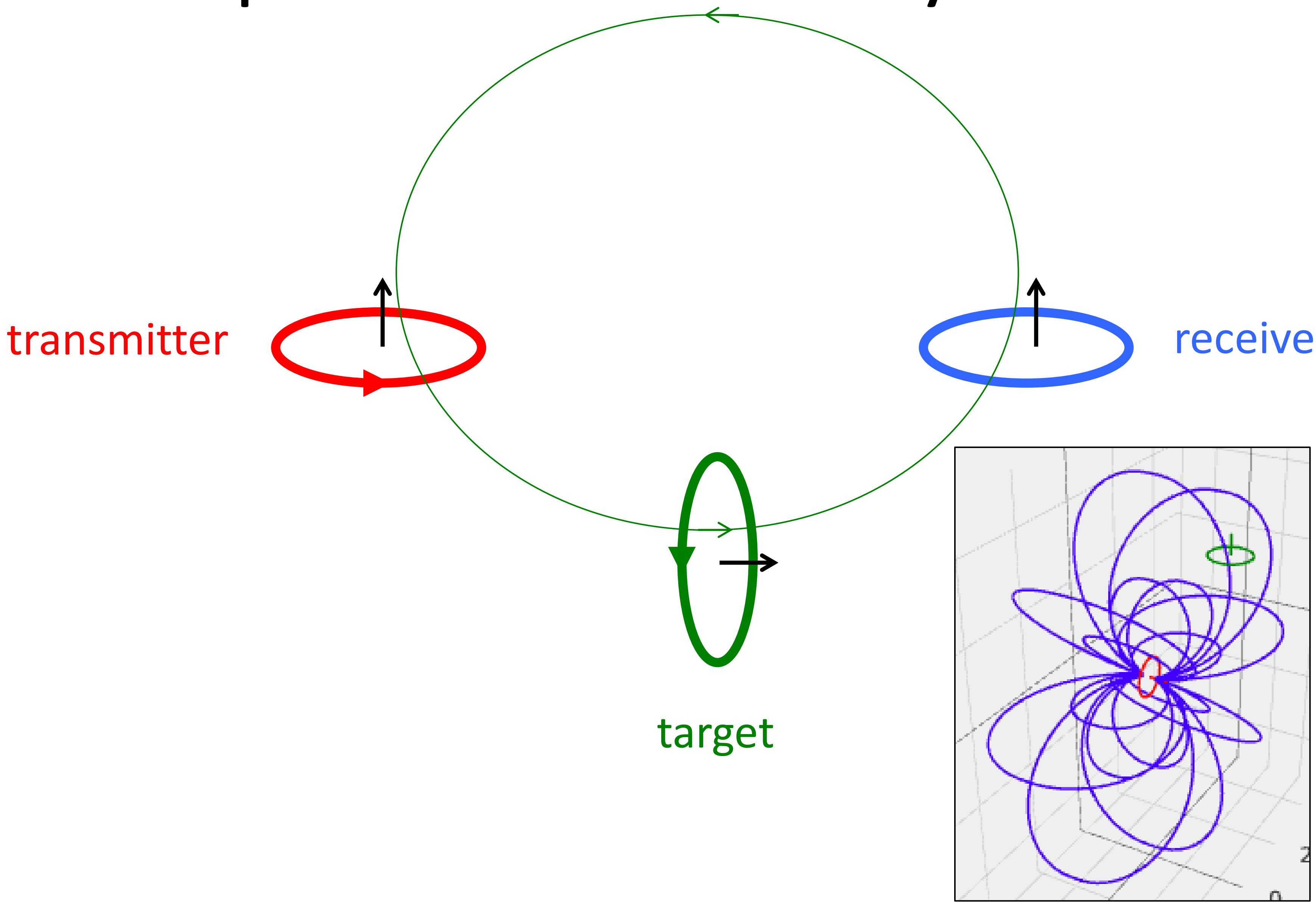
# 3-loop Model


Interactive 3D visualization in “MagDipoleLoops3D.ipynb”
clone from https://github.com/yangdikun/magLab.git

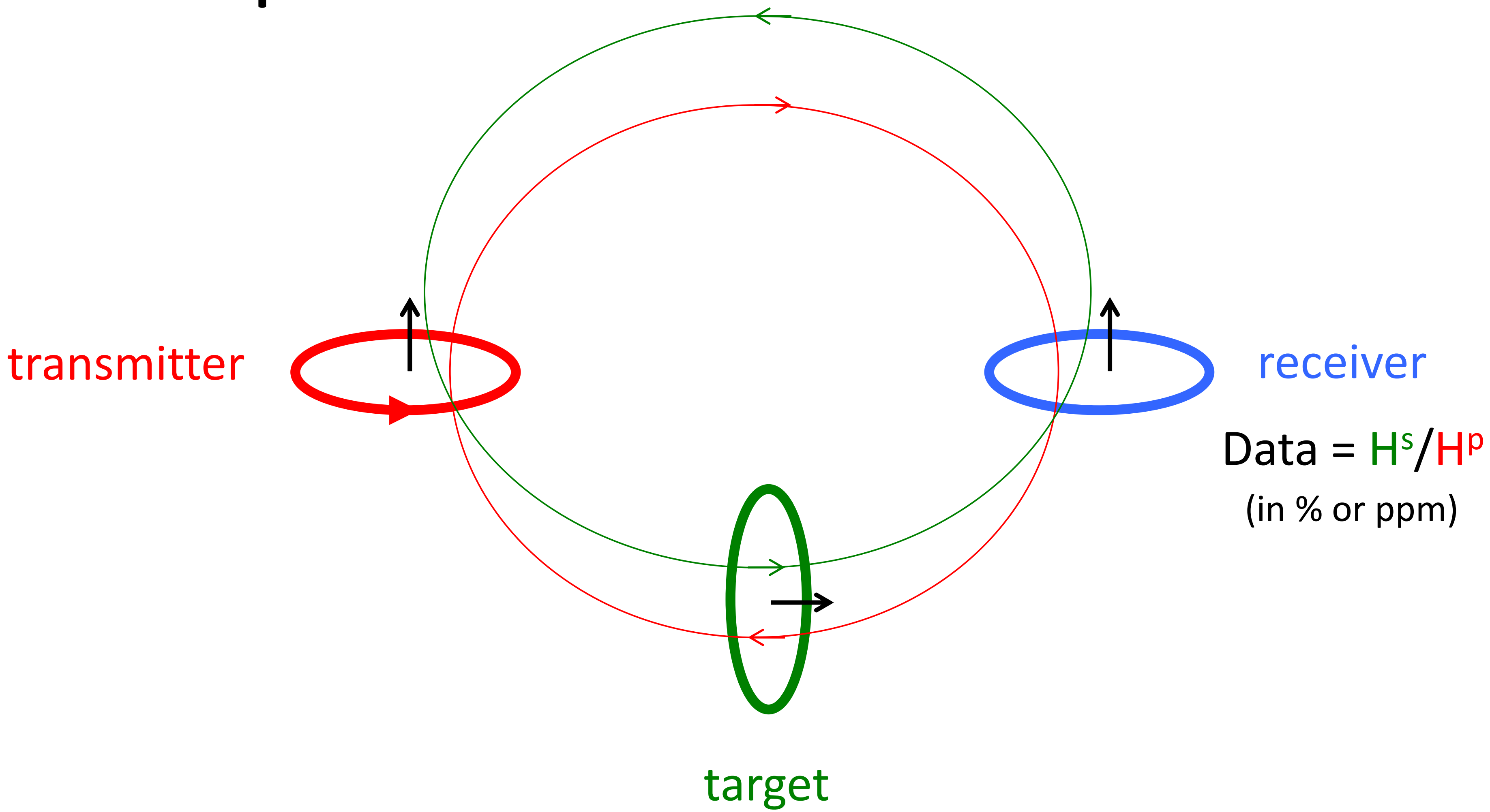
# 3-loop Model: Primary $H^p$



# 3-loop Model: Secondary $H^s$



# 3-loop Model




**Question**: Is the data positive or negative for the scenario on this page?

Hint: Think about the positive and negative anomalies in total field magnetics.

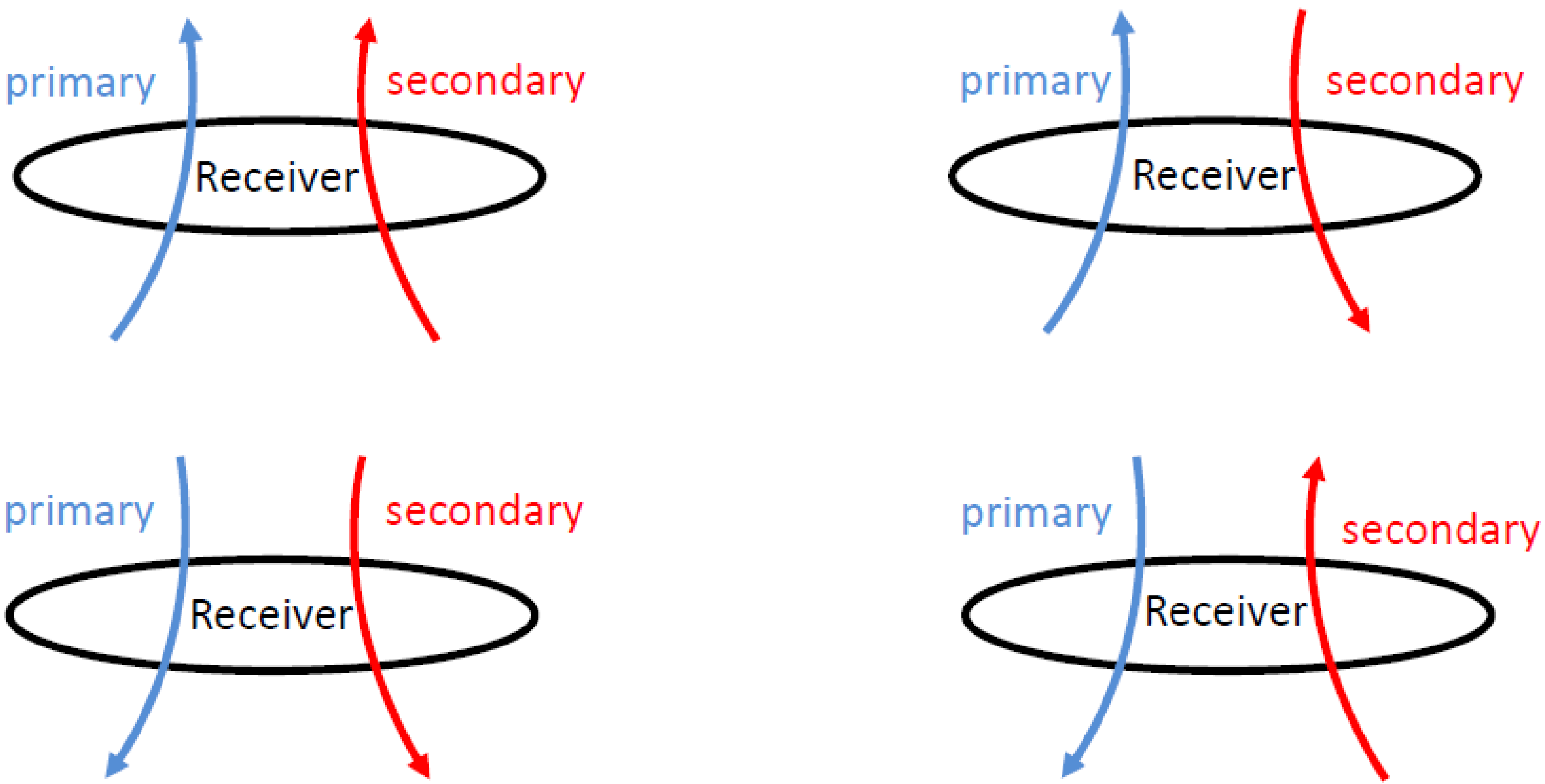
# Data (Hs/Hp) Sign Convention

Positive

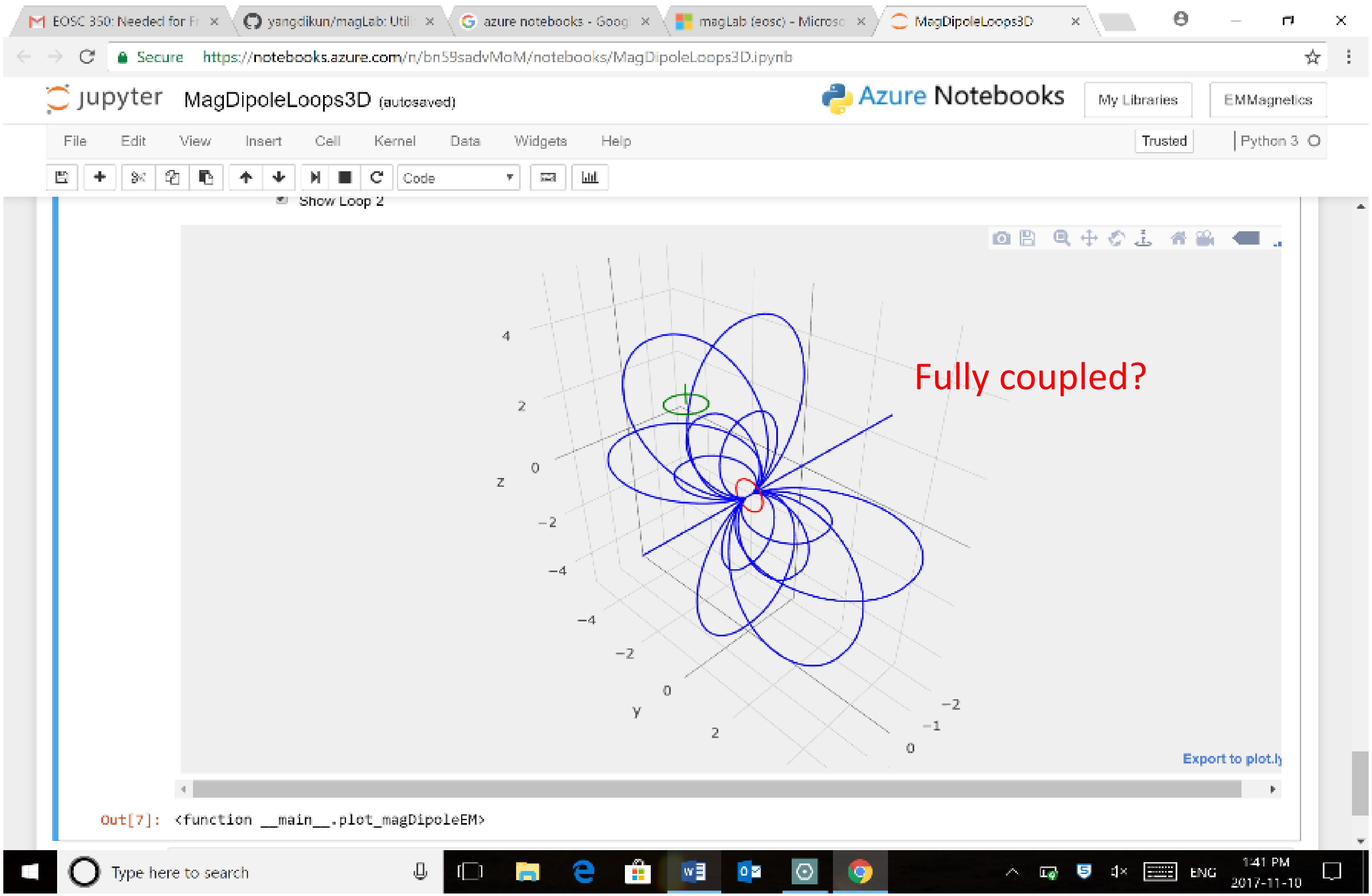
primary and secondary in same direction

Negative

primary and secondary in opposite directions



# Coupling between Two Loops Through Magnetic Flux Linkage



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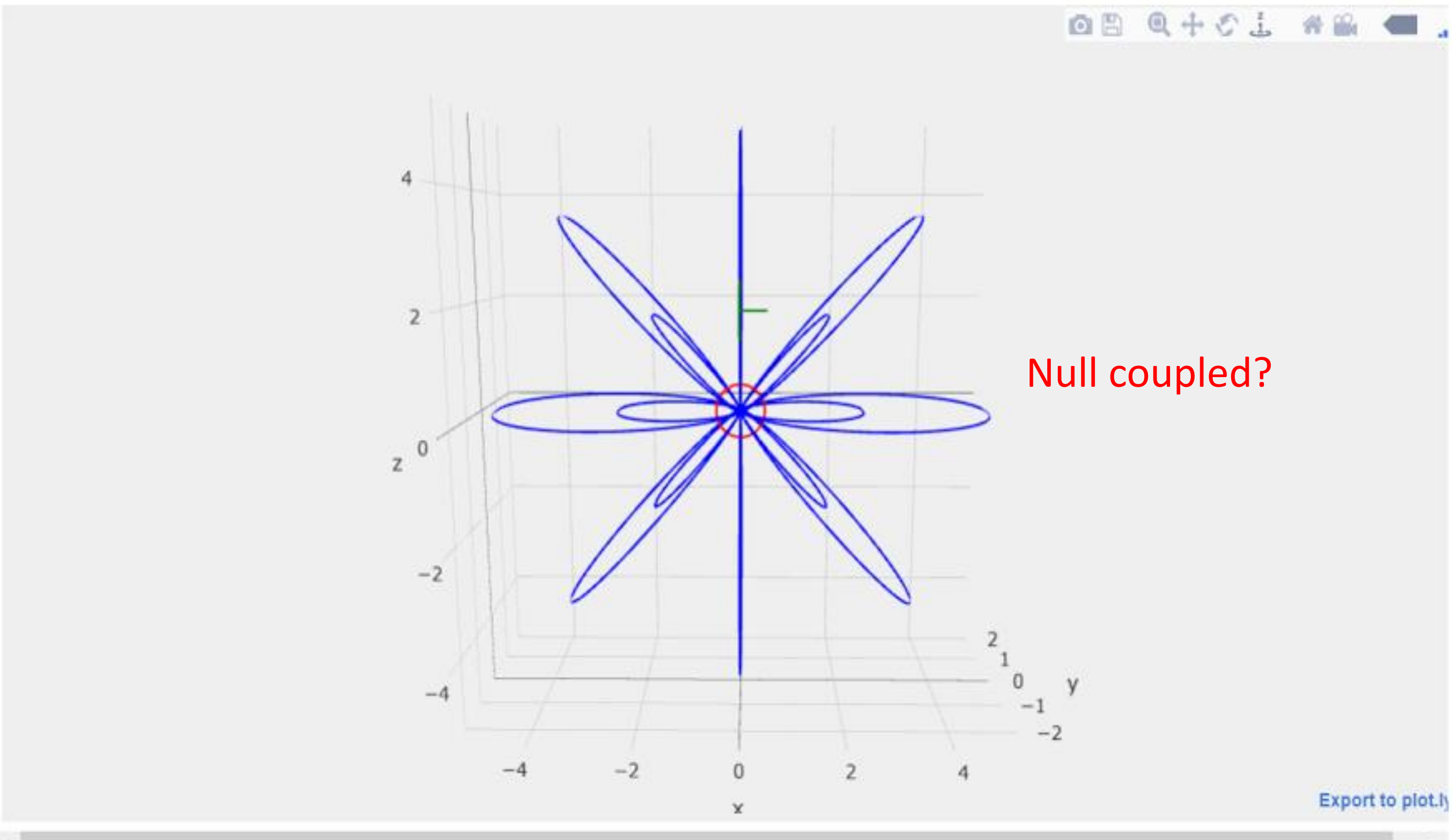
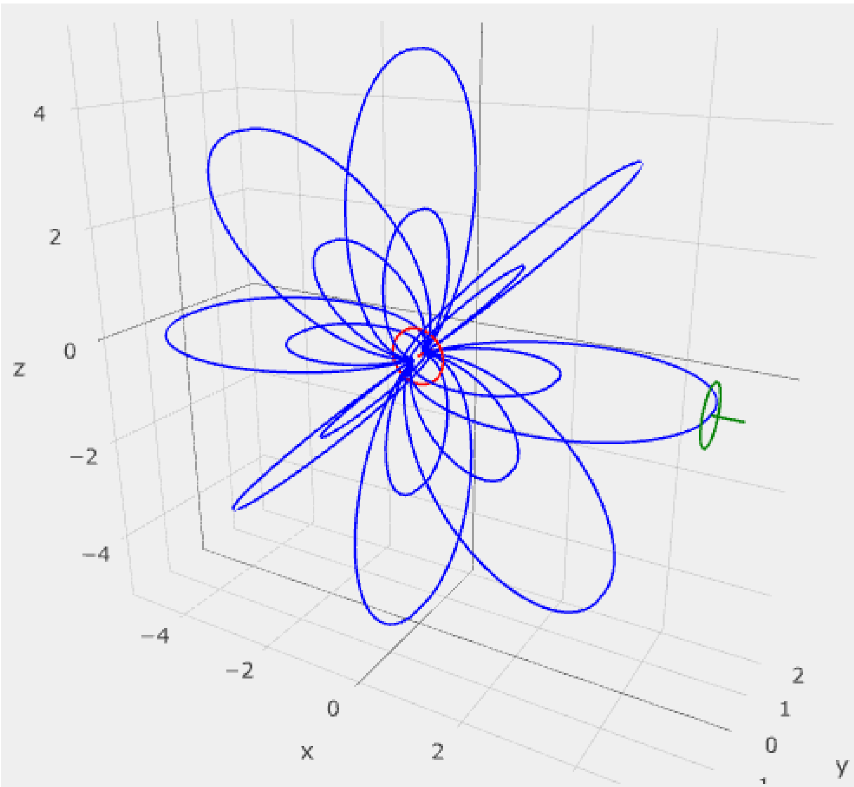
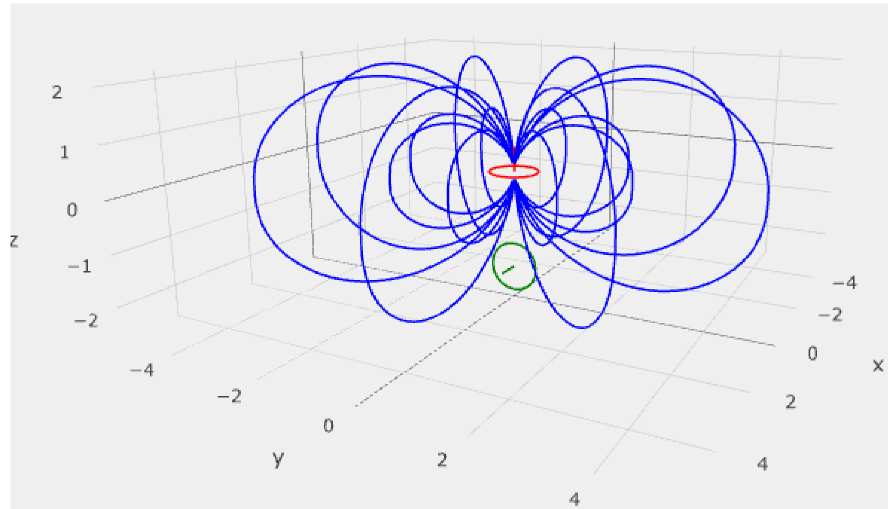
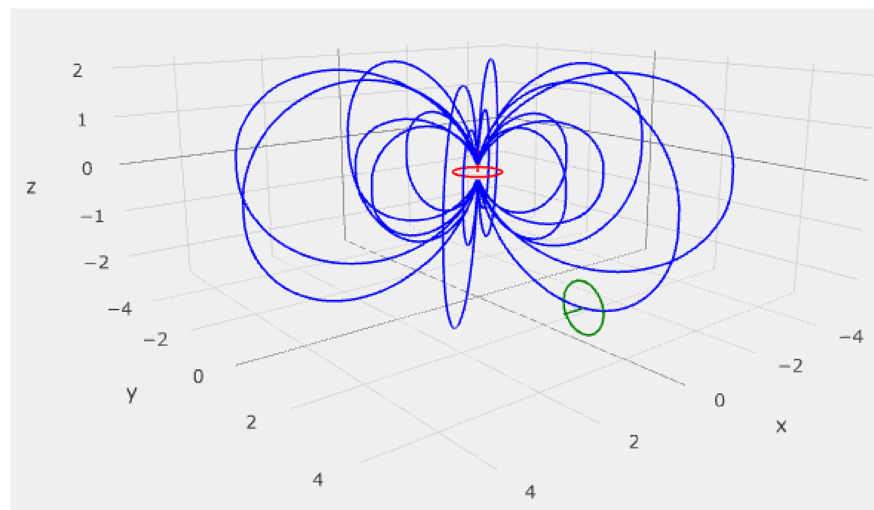



Null coupled?

# Coupling between Two Loops Through Magnetic Flux Linkage

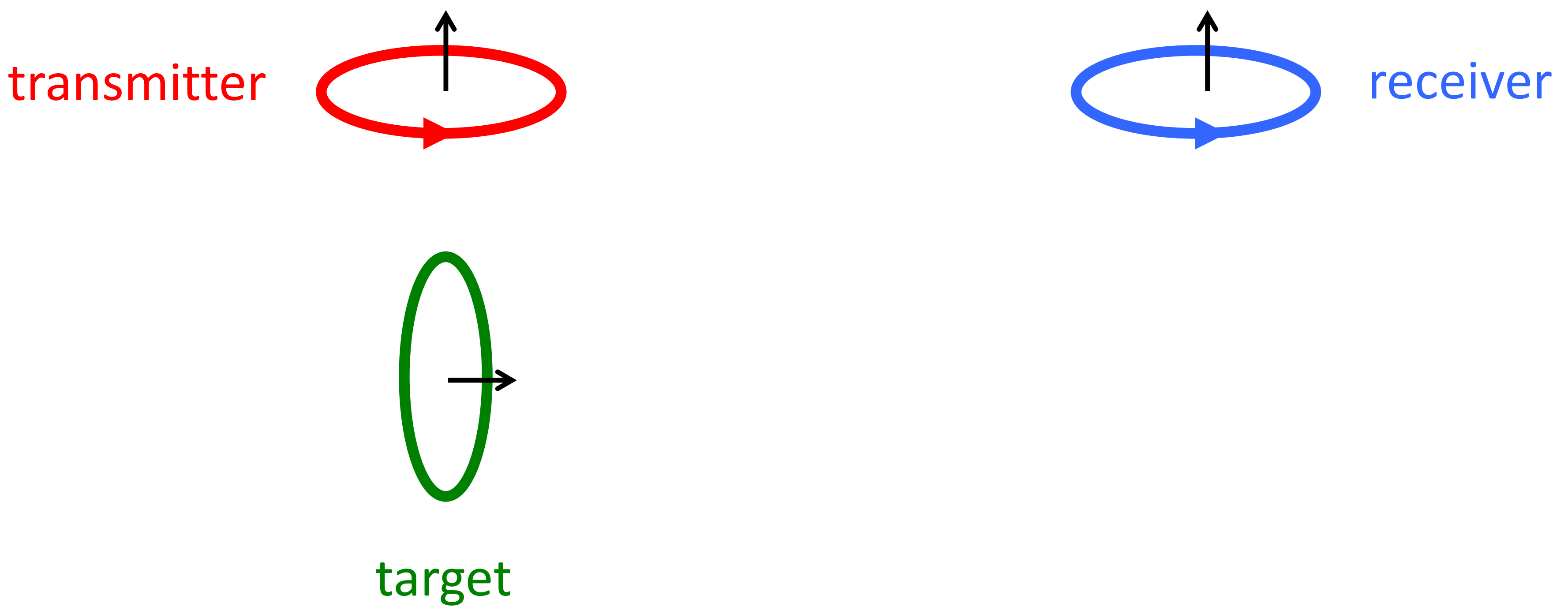


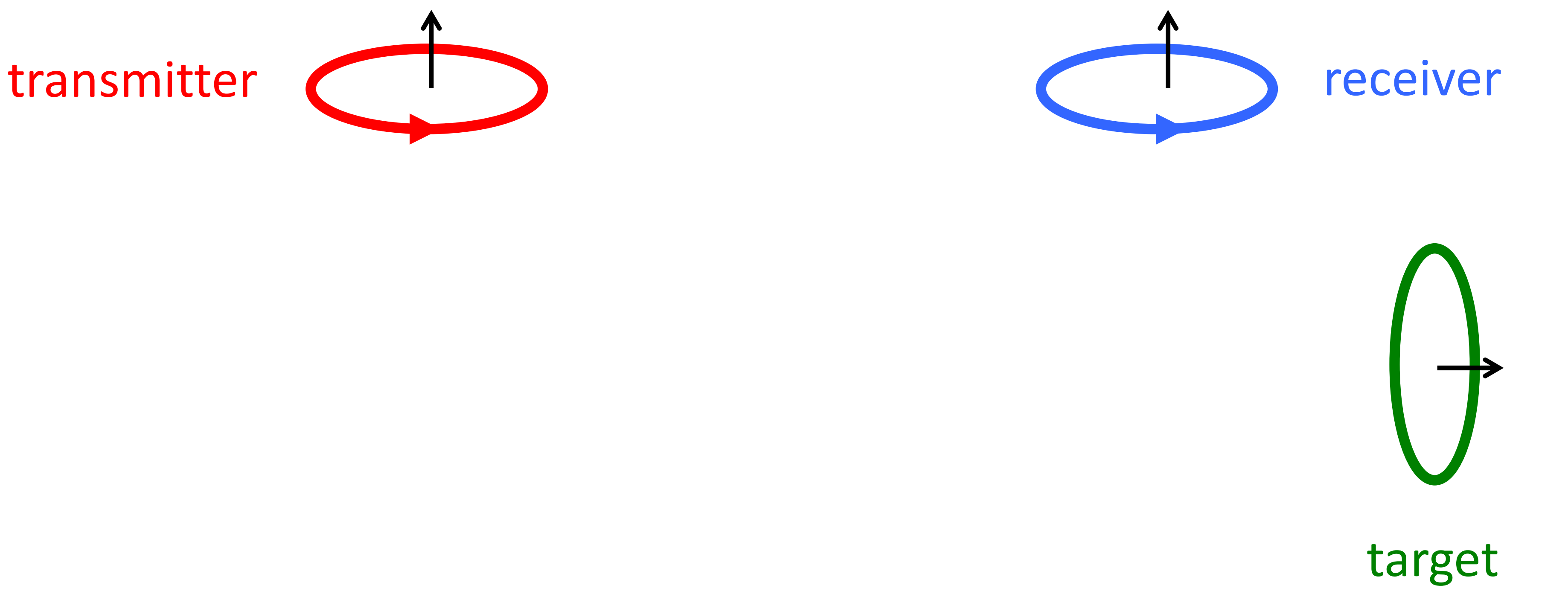




Null coupled

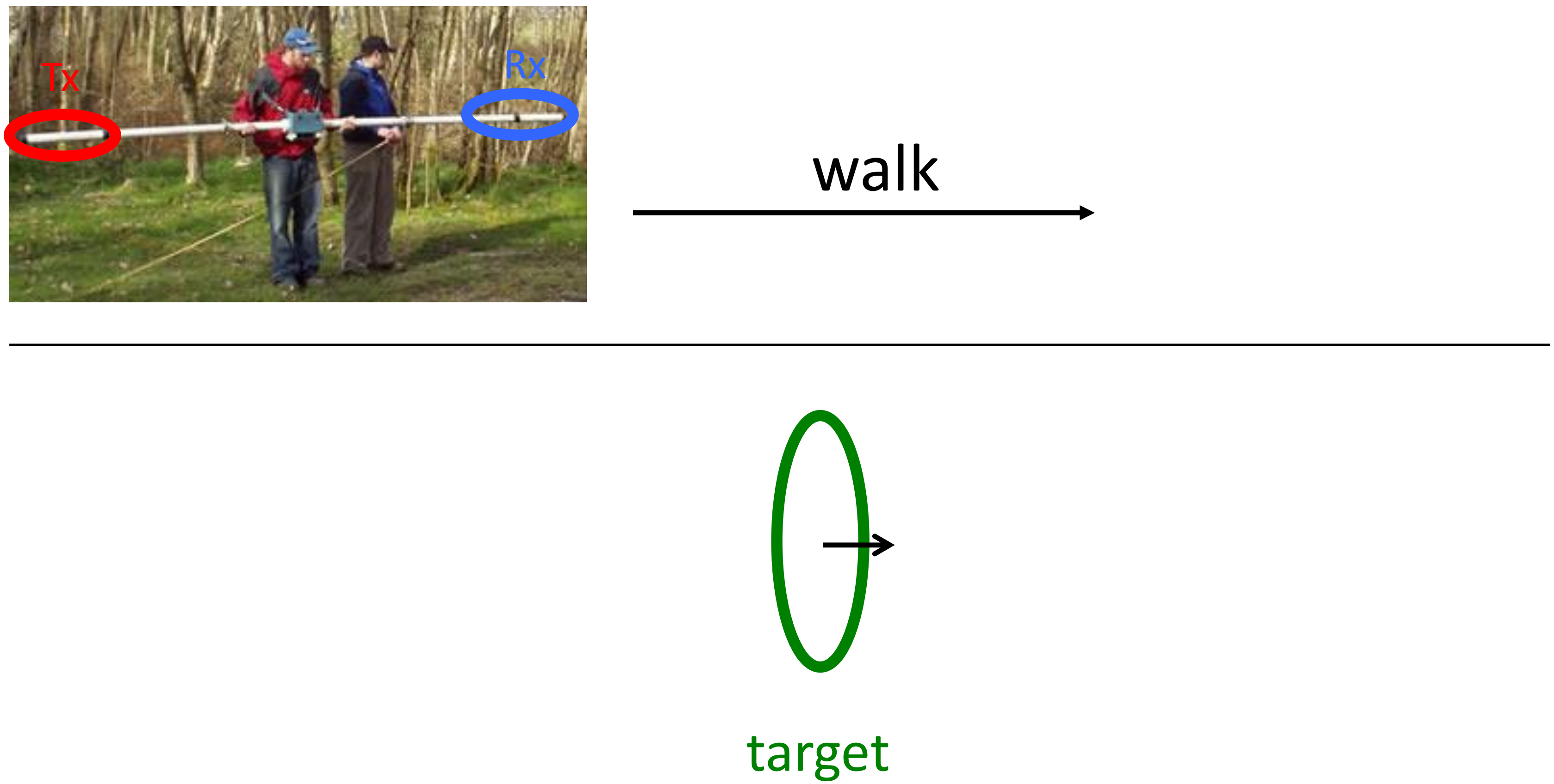
# $H^s/H^p$: Positive or Negative?



# $H^s/H^p$: Positive or Negative?



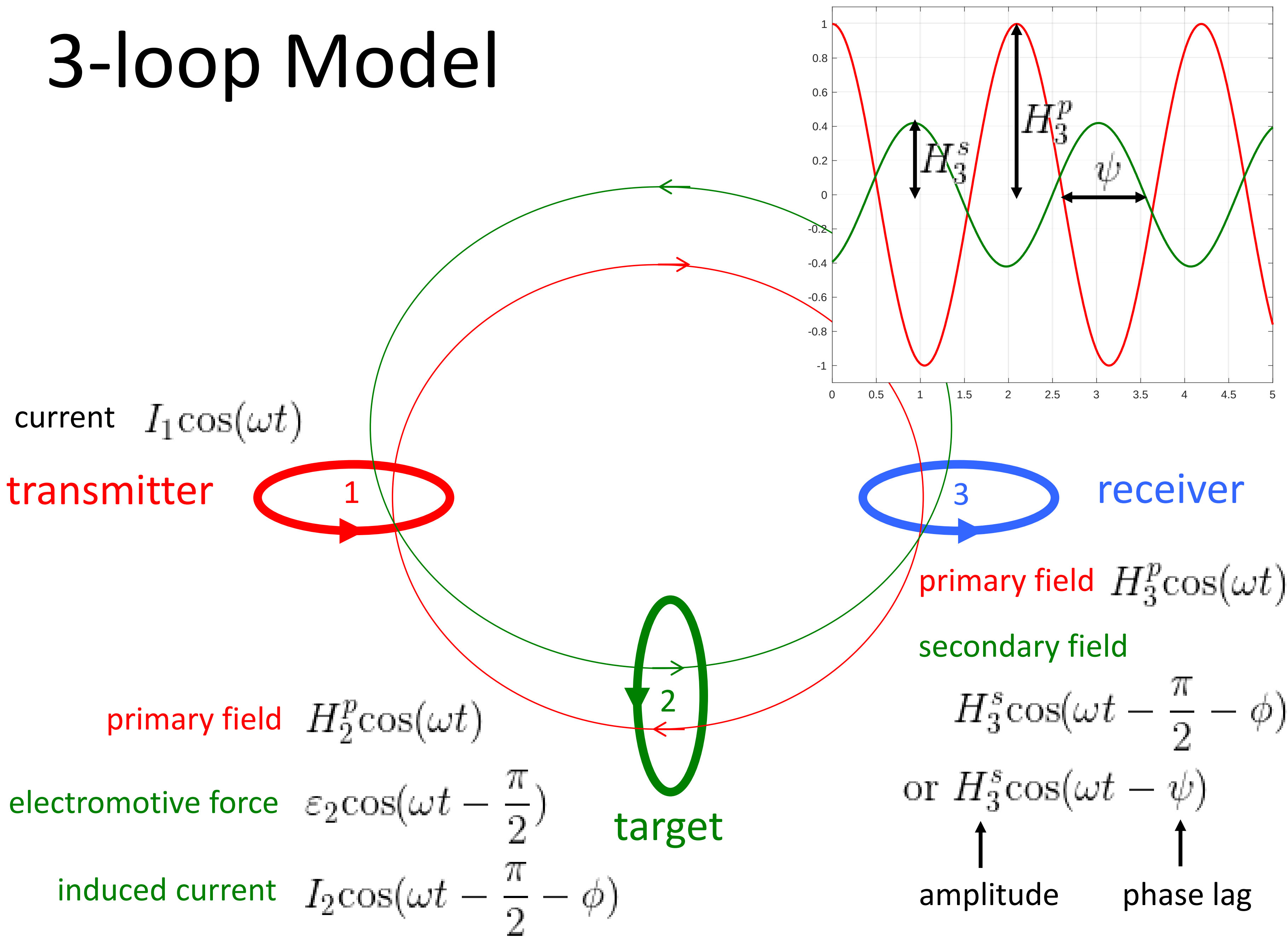
# Hs/Hp Profile



Drawing lines only helps qualitative understanding.

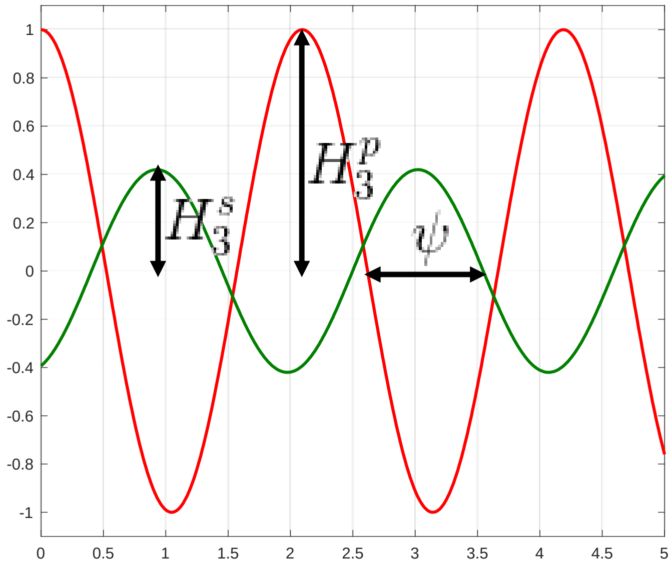
We need more math to do a quantitative interpretation.

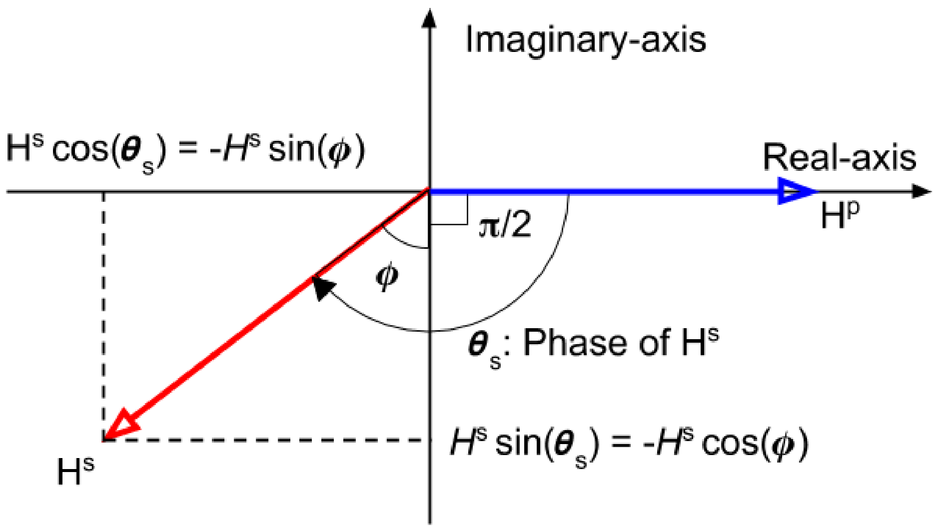
# 3-loop Model



current $I_1 \cos(\omega t)$
transmitter
1
receiver
3
primary field $H_3^p \cos(\omega t)$
secondary field
$H_3^s \cos(\omega t - \frac{\pi}{2} - \phi)$
or $H_3^s \cos(\omega t - \psi)$
amplitude
phase lag
2
target
primary field $H_2^p \cos(\omega t)$
electromotive force $\varepsilon_2 \cos(\omega t - \frac{\pi}{2})$
induced current $I_2 \cos(\omega t - \frac{\pi}{2} - \phi)$
$H_3^s$
$H_3^p$
$\psi$
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

# Decompose Secondary Field




$$\phi = tan^{-1}(\frac{\omega L}{R}) = tan^{-1}(\alpha)$$

primary field $H_3^p \cos(\omega t)$

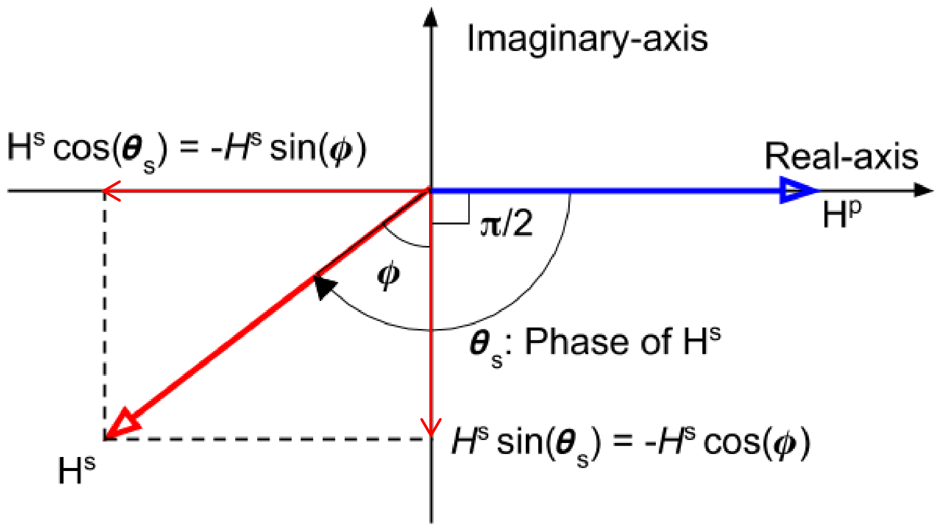
secondary field

$$H_3^s \cos(\omega t - \frac{\pi}{2} - \phi)$$

or $H_3^s \cos(\omega t - \psi)$

- $H^s$ swings in the third quadrant: $0 < \phi < 90°$
- $\phi$ depends on the induction number **$\alpha$**
- **$\alpha$** is a function of frequency **$\omega$**, self inductance **L** and resistance **R** of Loop 2

# Decompose Secondary Field




$$\phi = tan^{-1}(\frac{\omega L}{R}) = tan^{-1}(\alpha)$$

**Question**: What happens to the $H^s$ (red arrow) for a very conductive or very resistive target?

Decompose $H^s$ to two orthogonal components then normalize by $H^p$:

90° phase lag: called "out-of-phase", "quadrature", "imaginary"

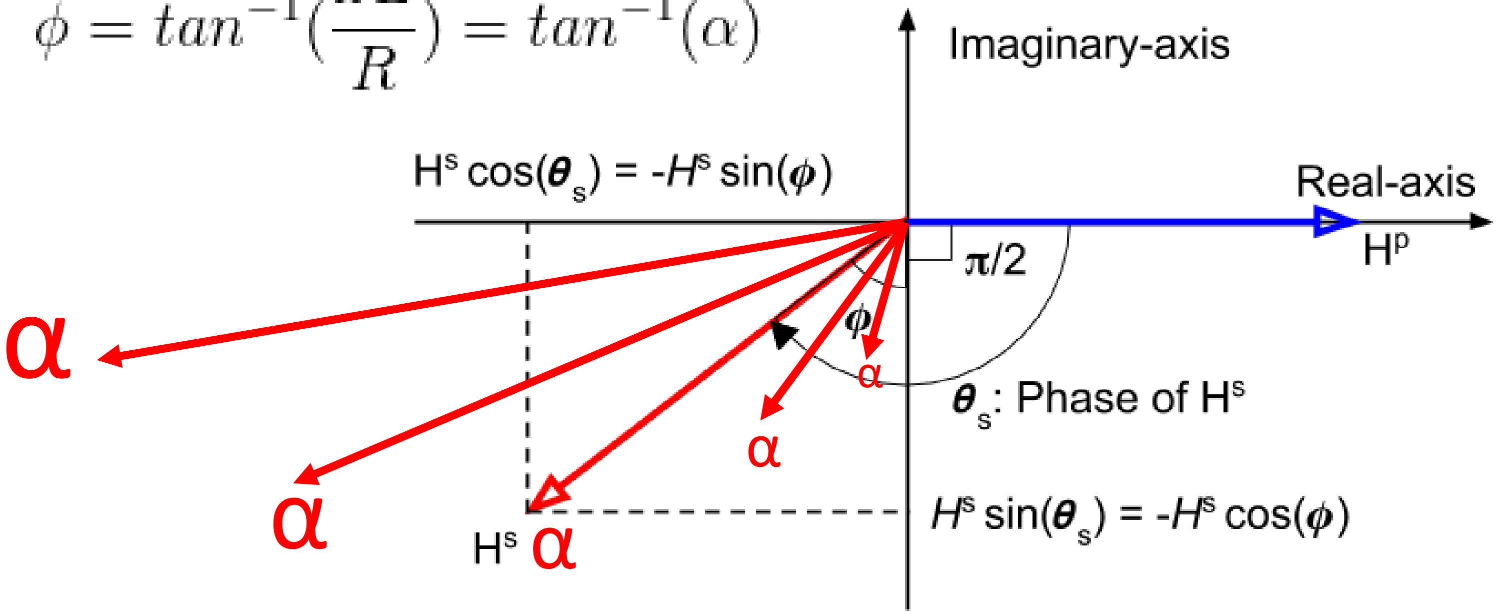
$$\frac{H^s \cos(\phi)}{H^p}$$

180° phase lag: called "in-phase", "real"

$$\frac{H^s \sin(\phi)}{H^p}$$

# Response Function

$$\phi = tan^{-1}(\frac{\omega L}{R}) = tan^{-1}(\alpha)$$



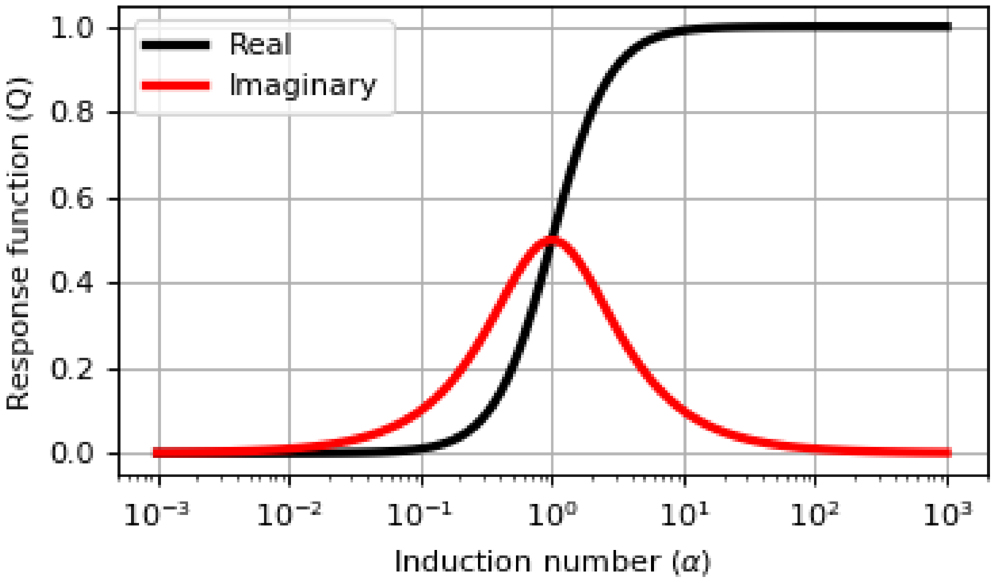

**Question**: How would the real and imaginary data change with the induction number α?

# Response Function

$$Q(\alpha) = \frac{i\alpha}{1+i\alpha} = \frac{\alpha^2 + i\alpha}{1+\alpha^2} \qquad \alpha = \frac{\omega L}{R}$$

Resistive limit:

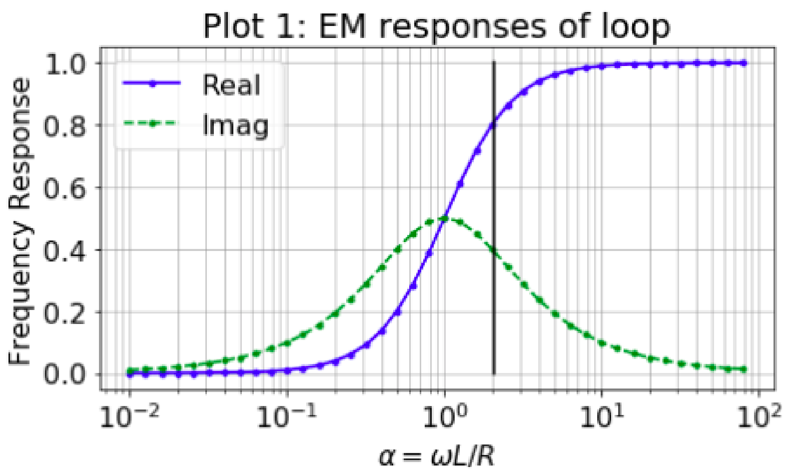
- low frequency
- low conductivity

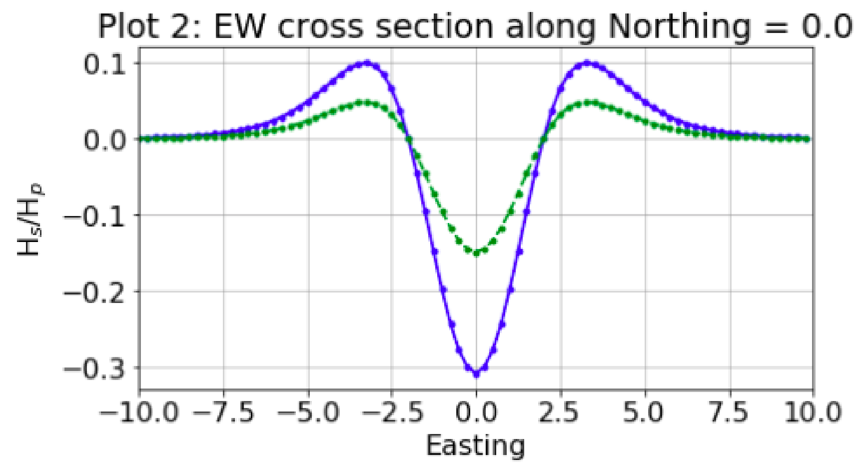



Inductive limit:

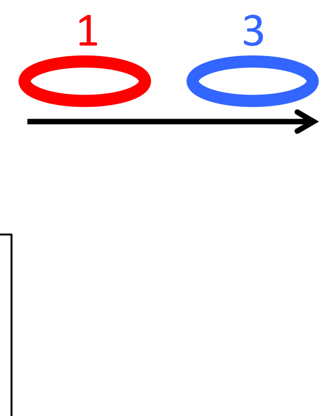
- high frequency
- high conductivity

# Expected Data From a Loop Target




$$\frac{H_3^s}{H_3^p} = -\frac{M_{12}M_{23}}{M_{13}L}\left[\frac{\alpha^2 + i\alpha}{1+\alpha^2}\right]$$




Coupling
- location, orientation
- overall magnitude

Induction
- properties of loop 2
- how much in Re & Im

Interactive demo in “EM_ThreeLoopModel.ipynb”, clone from https://github.com/geoscixyz/gpgLabs.git

# Summary

- 3-loop circuit model for EM-31 over compact conductive objects
  - EM energy transmission via magnetic flux linkage
  - Sign and overall magnitude of data: draw field lines and using mutual inductance
  - Real and imaginary portion of data: response function as a function of the induction number
- Be able to sketch data on a profile
- Be able to infer conductivity using the response function plot