SUSTech Southern University of Science and Technology

南方科技大学 SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

地球与空间科学系 DEPARTMENT OF EARTH AND SPACE SCIENCES

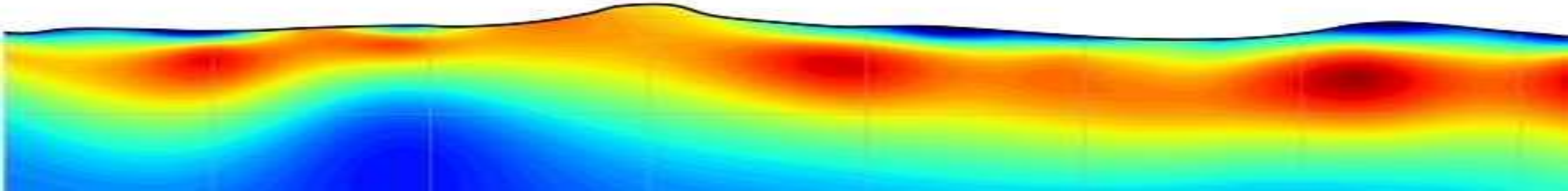
ESS302 Applied Geophysics II

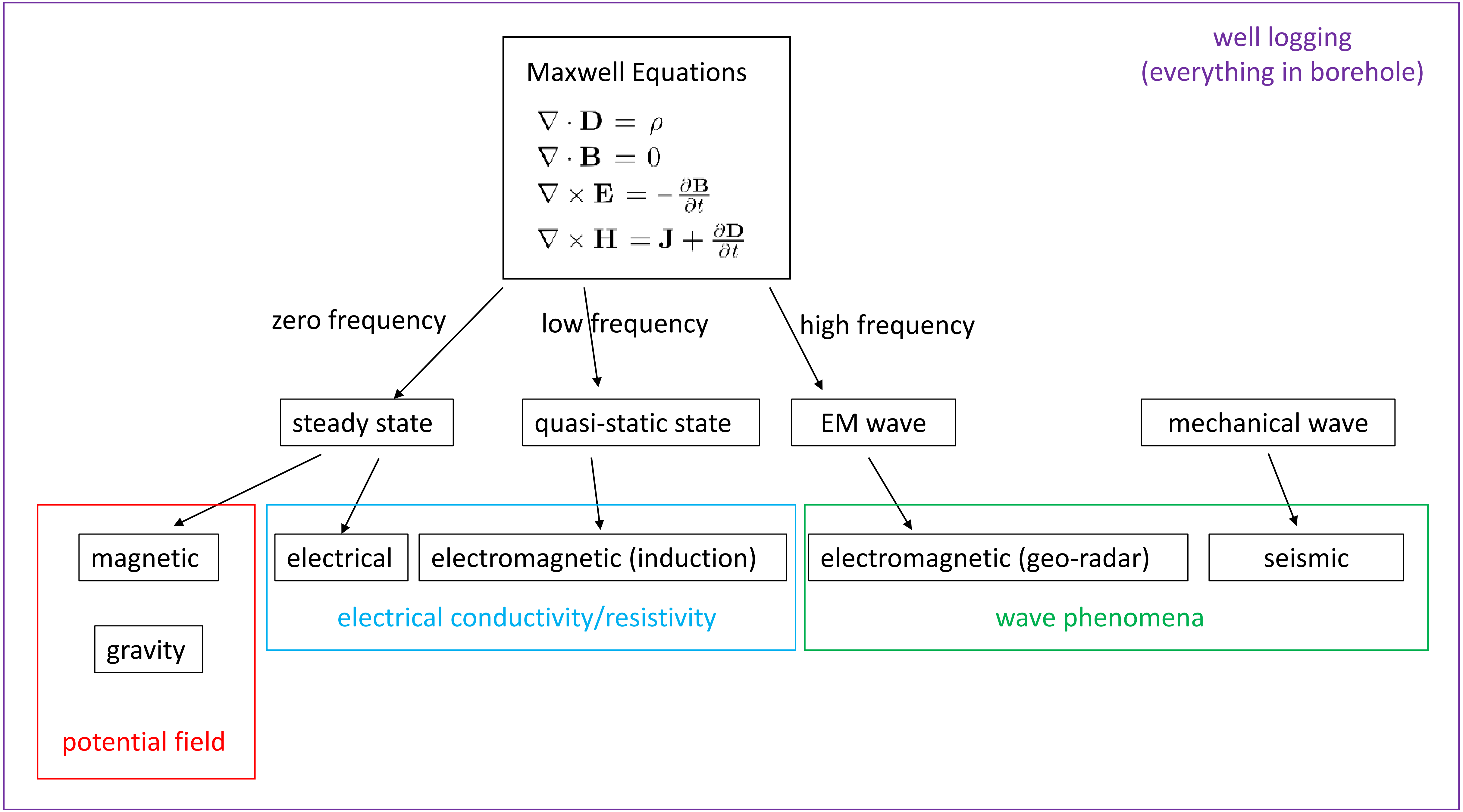
Gravity, Magnetic, Electrical, Electromagnetic and Well Logging

Electromagnetic 3: Induction Part A

Instructor: Dikun Yang

Feb – May, 2019




well logging
(everything in borehole)
Maxwell Equations
$\nabla \cdot \mathbf{D} = \rho$
$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$
zero frequency
low frequency
high frequency
steady state
quasi-static state
EM wave
mechanical wave
magnetic
gravity
potential field
electrical
electromagnetic (induction)
electrical conductivity/resistivity
electromagnetic (geo-radar)
seismic
wave phenomena

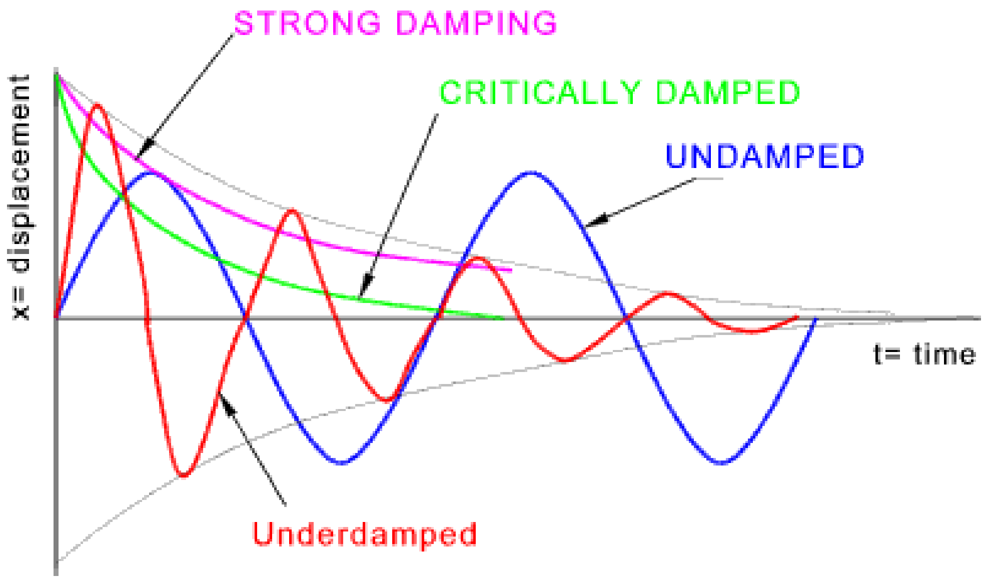
Quasi-static Maxwell's Equations

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \times H = \sigma E + \cancel{\epsilon \frac{\partial E}{\partial t}}$$

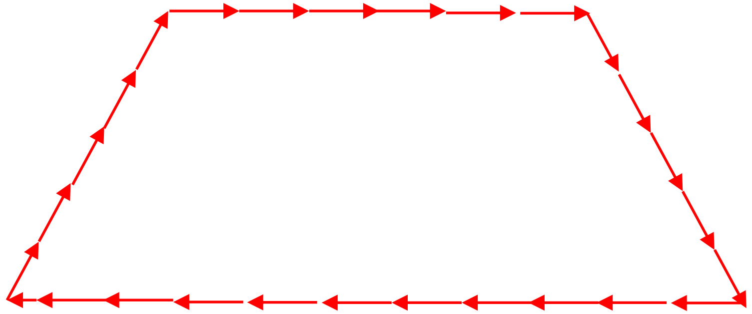
$$\nabla \times E = -i\omega\mu H$$

$$\nabla \times H = \sigma E + \cancel{i\omega\epsilon E}$$



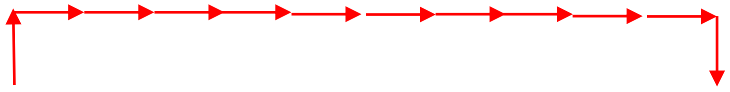
Wires and Loops

Electrical dipole
(a *small* piece of wire)



Closed loop

- Magnetic field (dB/dt)
- Non-contact (divergence free)
- Inductive coupling



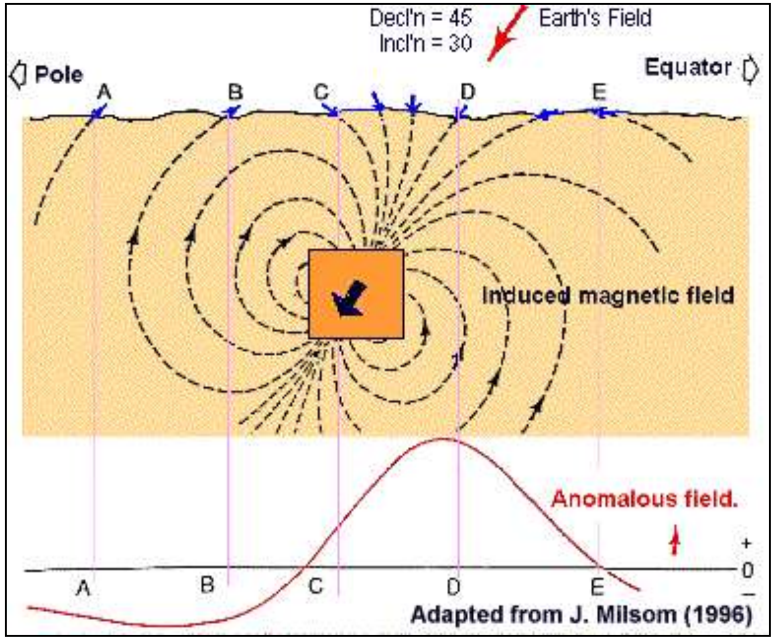
Grounded wire

- Electrical field (E)
- End points in contact with ground
- Galvanic and inductive coupling

Loop-loop System in Frequency Domain

EM =

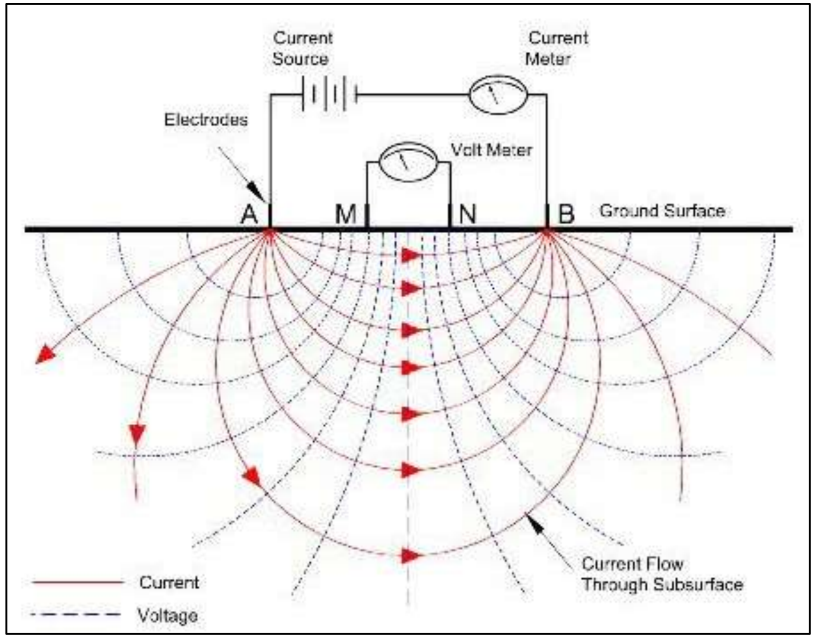
Magnetics



- Magnetic dipole
- Magnetic flux (B)

+

Electric Resistivity

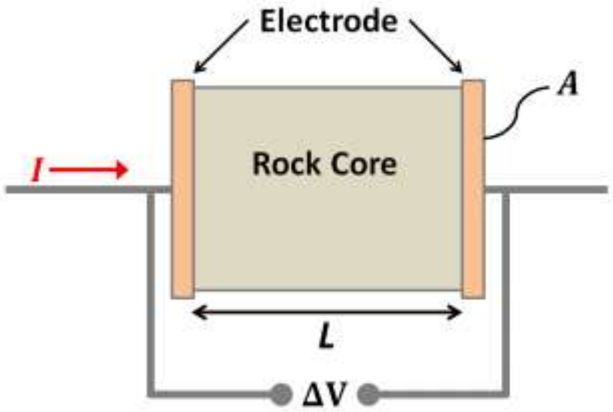


- Electric dipole
- Electric current (J)

Electrical energy transmission

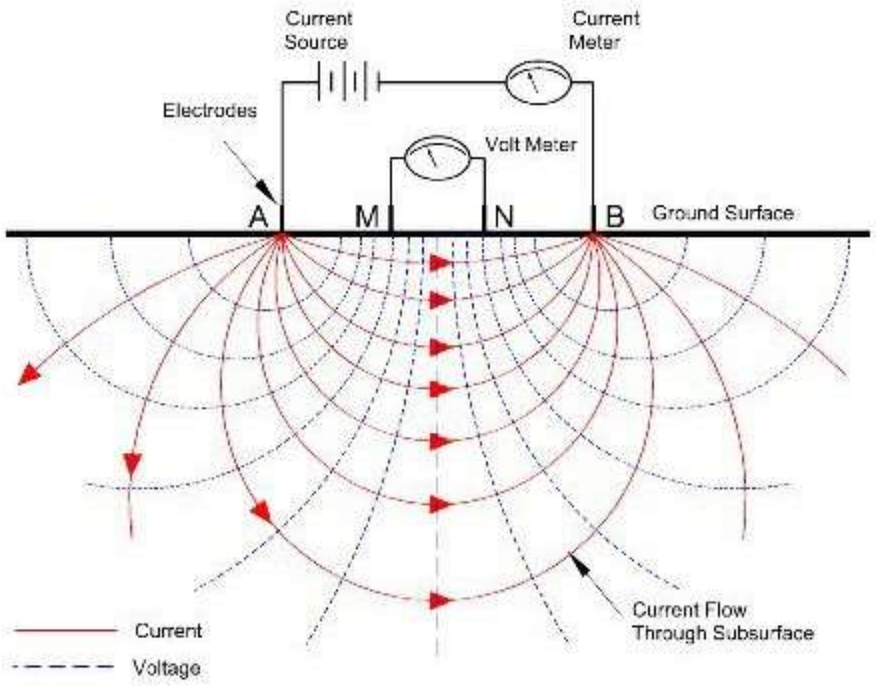
- Galvanic (electric current)

Ohm’s law



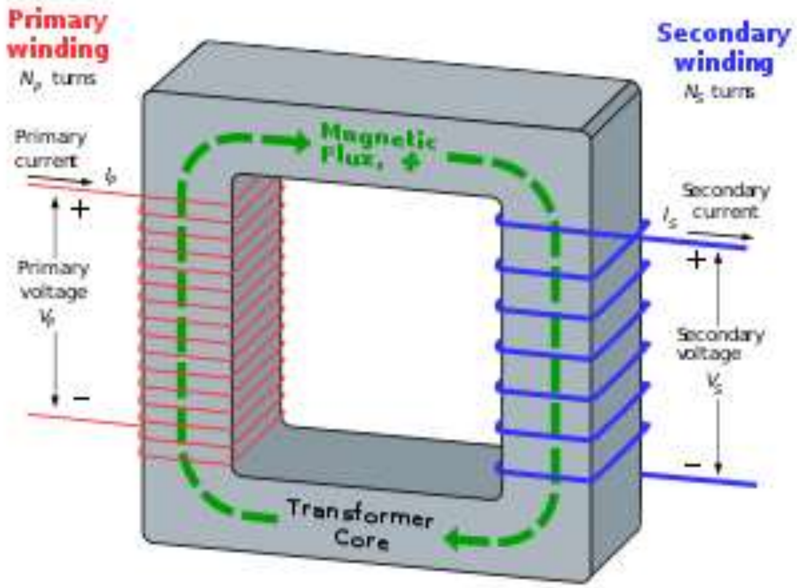
$$R = \frac{\Delta V}{I}$$

DC resistivity
(electric resistivity tomography)



Electrical energy transmission

- Inductive (magnetic flux B)



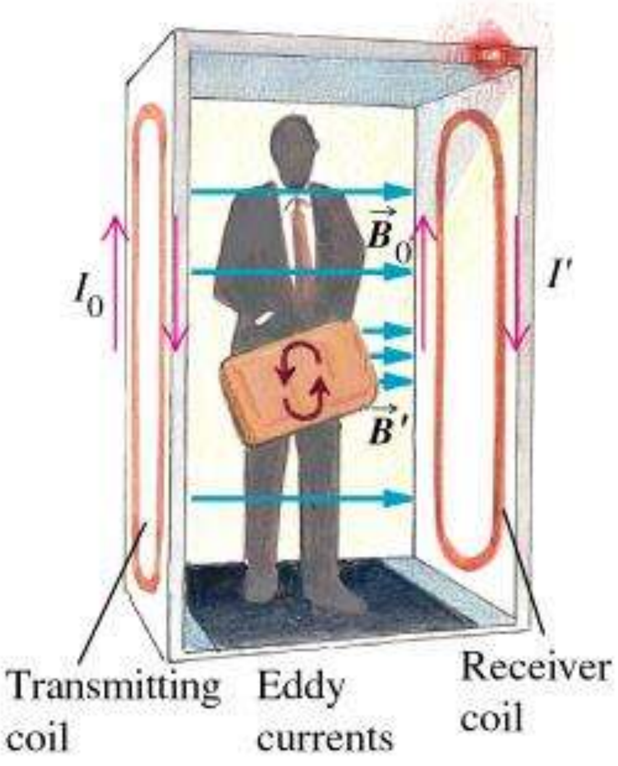
1. Change of current in the primary
2. Change of magnetic flux in the core
3. Induced current in the secondary

A transformer:

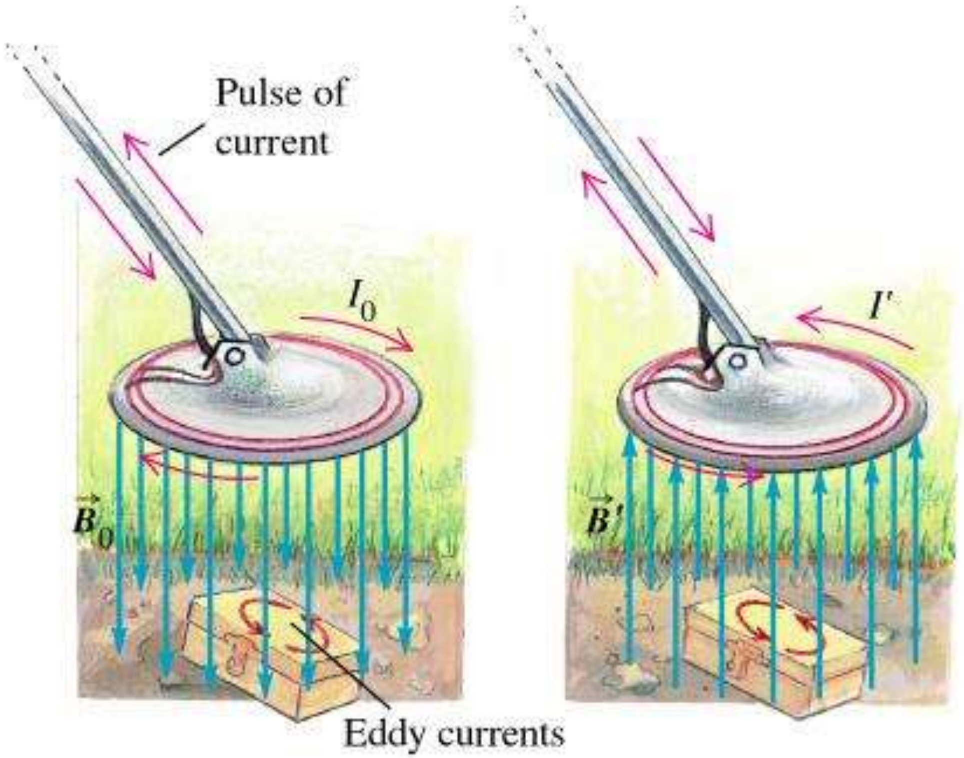
- No direct connection between primary and secondary windings
- Energy goes through in the forms of electric, magnetic then electric
- Magnetic flux linkage only in AC (requires non-stationary current)

Electrical energy transmission

- Inductive (magnetic flux B)



Security scan



Metal detector

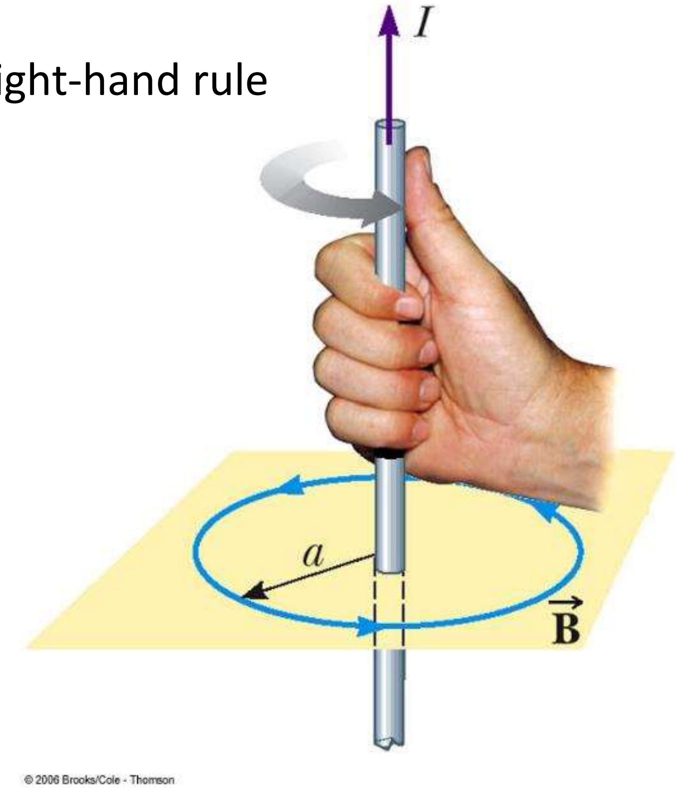
Ampere's law

J generates B

$$\nabla \times \mu^{-1}\mathbf{B} = \mathbf{J} = \sigma\mathbf{E}$$

Current in wire causes a magnetic field to surround it (iron filings).

Right-hand rule



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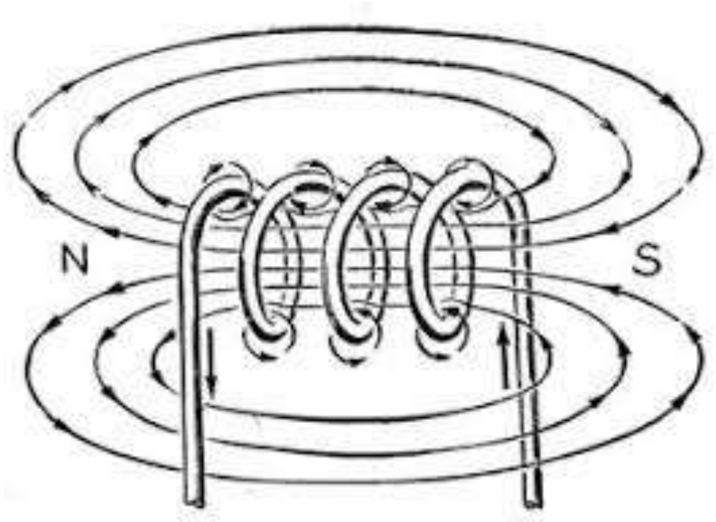
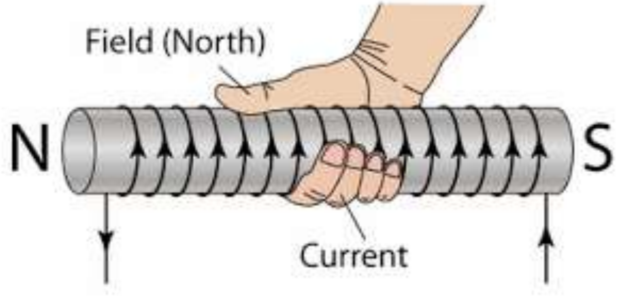
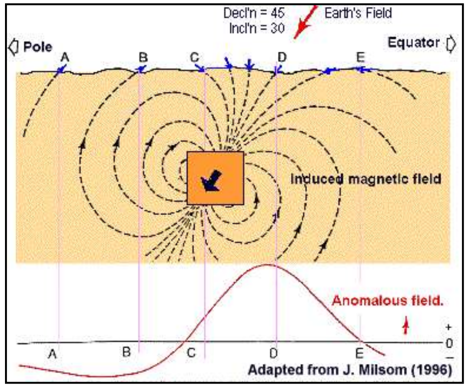




A small solenoid generates a magnetic field that can be approximated by a magnetic dipole (or a small bar magnet)

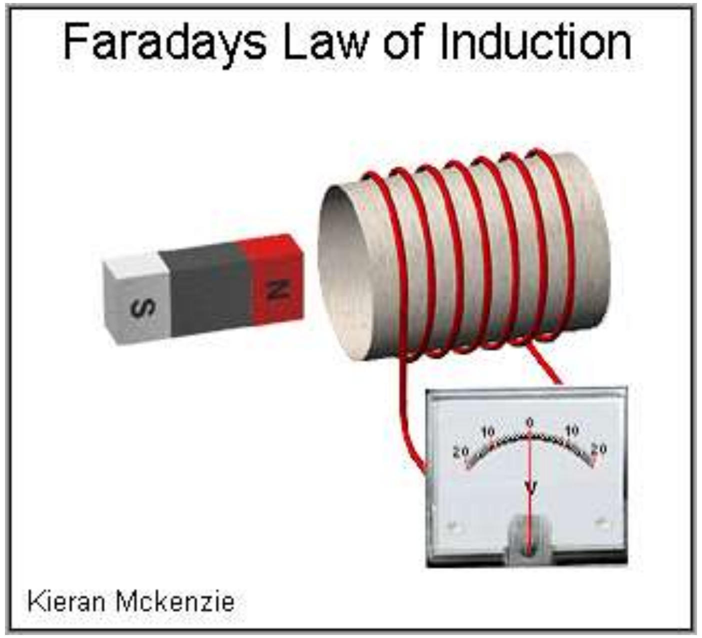


Still remember the magnetic dipole?

Faraday's law

<u>Change</u> of B generates J

$$\nabla \times \sigma^{-1}\mathbf{J} = \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

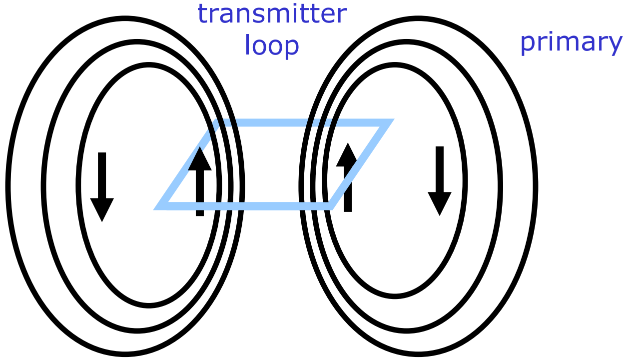


Induced current depends on

- How fast B changes
- How many B-field lines go through
- <u>How conductive the object is</u>

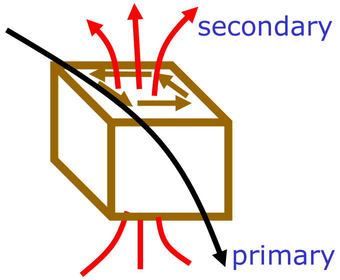
Communicate with the Earth without Contact

Transmitter loop



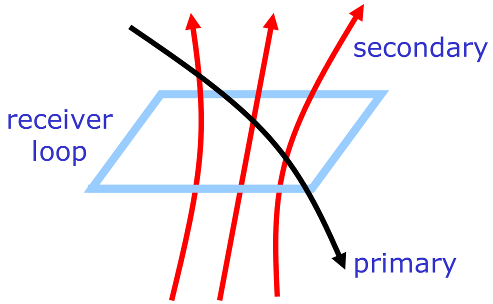
Ampere: time-varying current and changing primary magnetic field

Target/Ground



Faraday: current induced by the changing primary field; **Ampere**: induced current generates a secondary magnetic field

Receiver loop

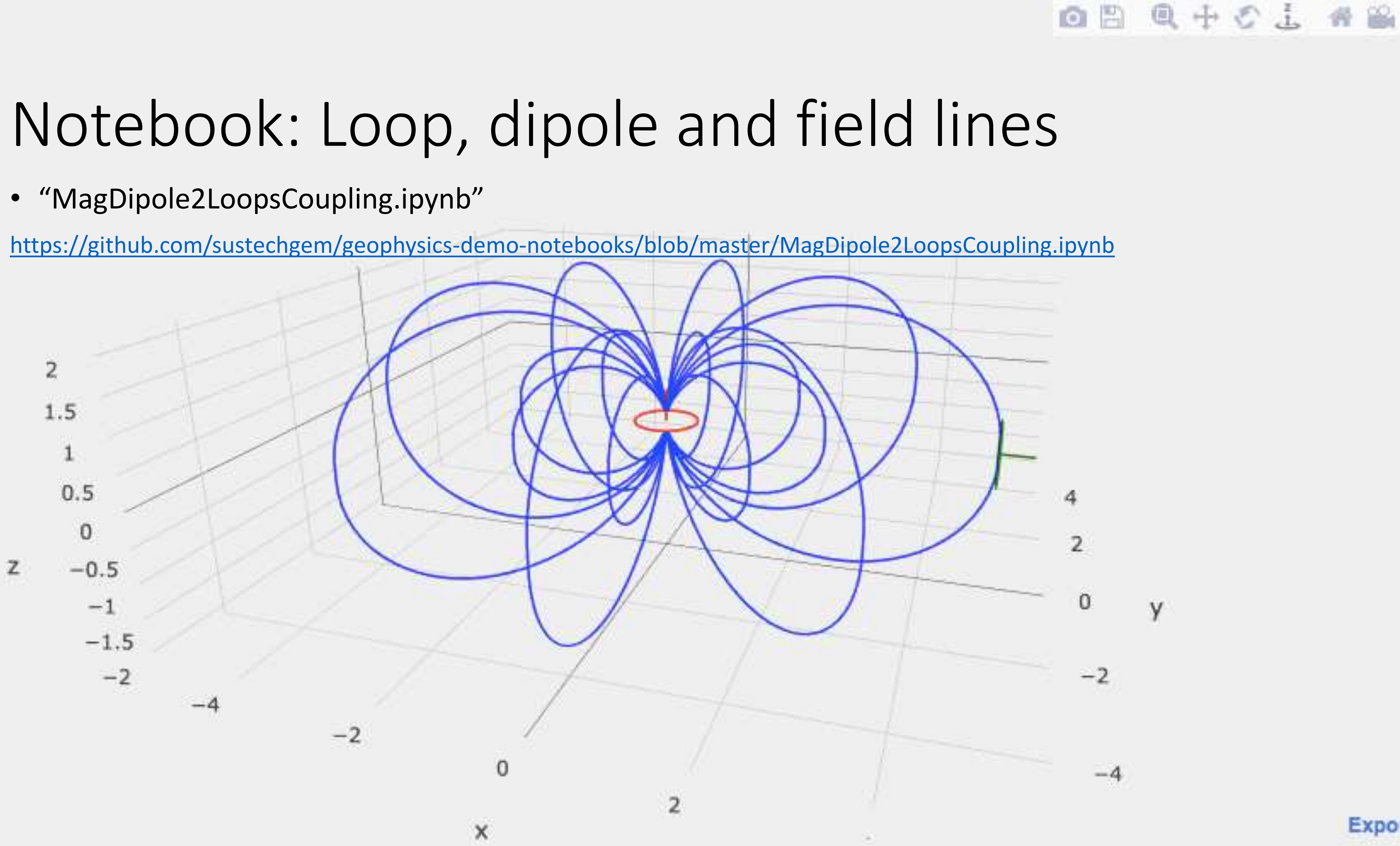


Faraday: measurable current induced in the loop by the changing secondary field

Notebook: Loop, dipole and field lines

- “MagDipole2LoopsCoupling.ipynb”

https://github.com/sustechgem/geophysics-demo-notebooks/blob/master/MagDipole2LoopsCoupling.ipynb



Task 1: Loop orientation

- “Run All” to get the default result
- Shown is a loop wire carrying an 1A current. The equivalent dipole moment is calculated as the product of loop area and current. Your estimate of the dipole moment is _________.
- The straight line perpendicular to the loop surface indicates the direction of the dipole moment. The plot you see represents a ***horizontal*** or ***vertical*** loop with a ***horizontal*** or ***vertical*** dipole moment. (circle one of the two choices)
- Adjust the declination and inclination of Loop 1, so the dipole moment points to the +x direction. Now the loop represents a ***horizontal*** or ***vertical*** dipole. (circle one of the two choices)

Task 2: Loop-loop coupling

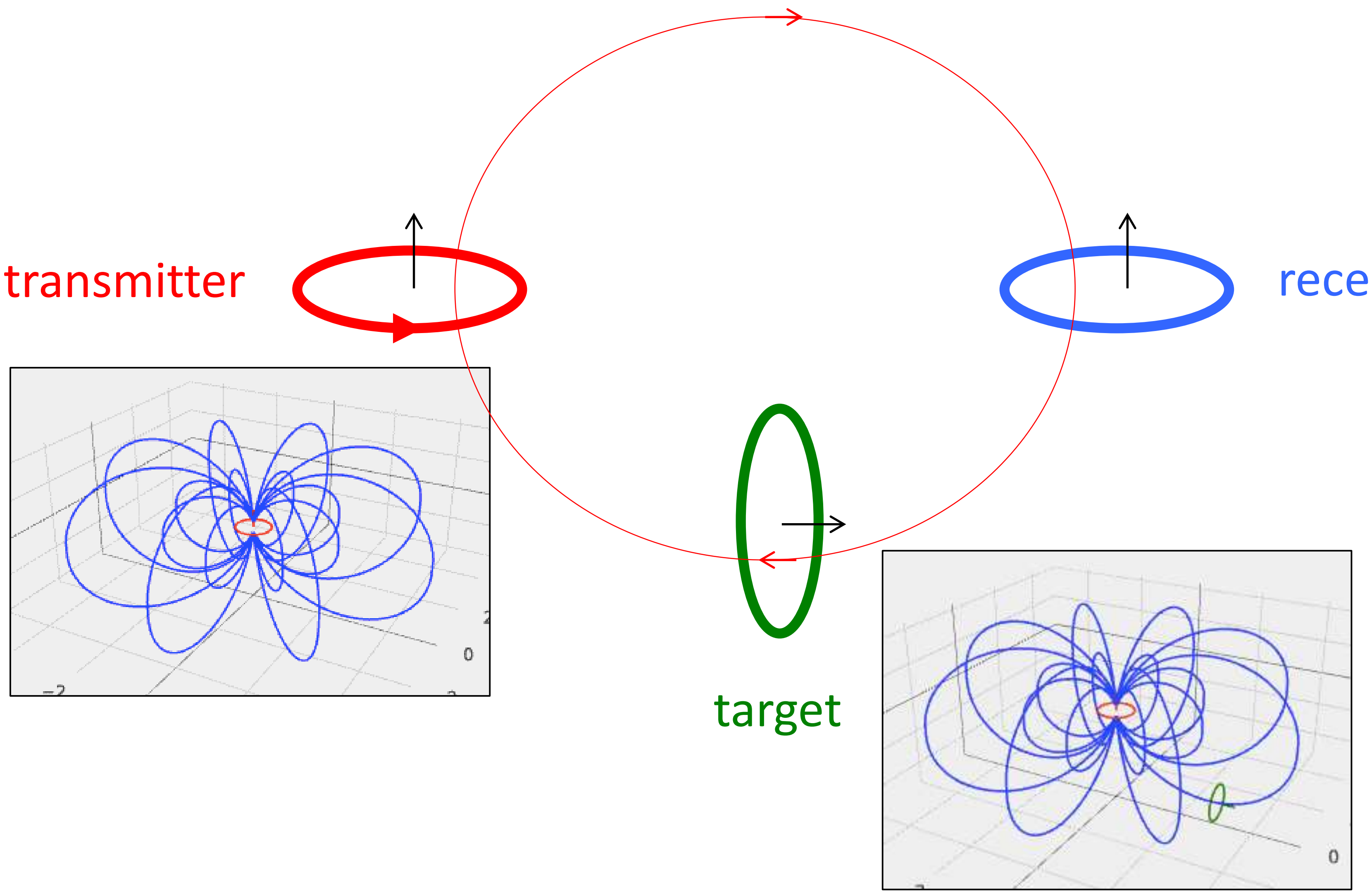
- Check “Show Loop 2” to turn on the second loop.
- Keep the orientation of Loop 2 unchanged and adjust the location of Loop 2 to a point where it has the best geometric coupling with Loop 1 through the magnetic flux linkage (tangential direction of field lines is parallel to the dipole moment direction of a loop).

- Change the orientation of Loop 2, so the dipole moment points to the +x direction.
- Move Loop 2 around to find two locations where (1) Loop 1 and 2 are fully coupled and (2) Loop 1 and Loop 2 are null coupled.

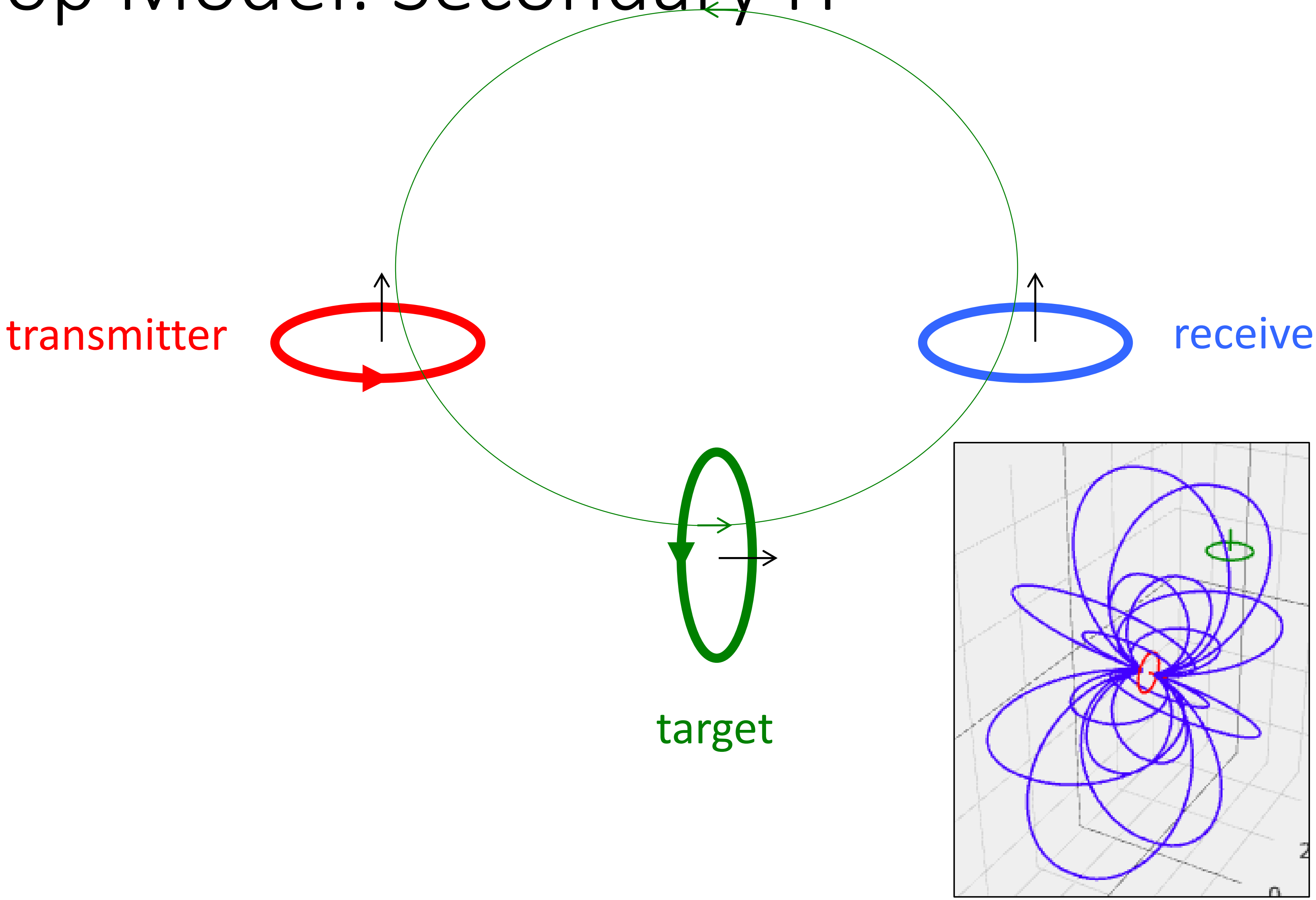
3-loop Model

Interactive 3D visualization in “MagDipoleLoops3D.ipynb”

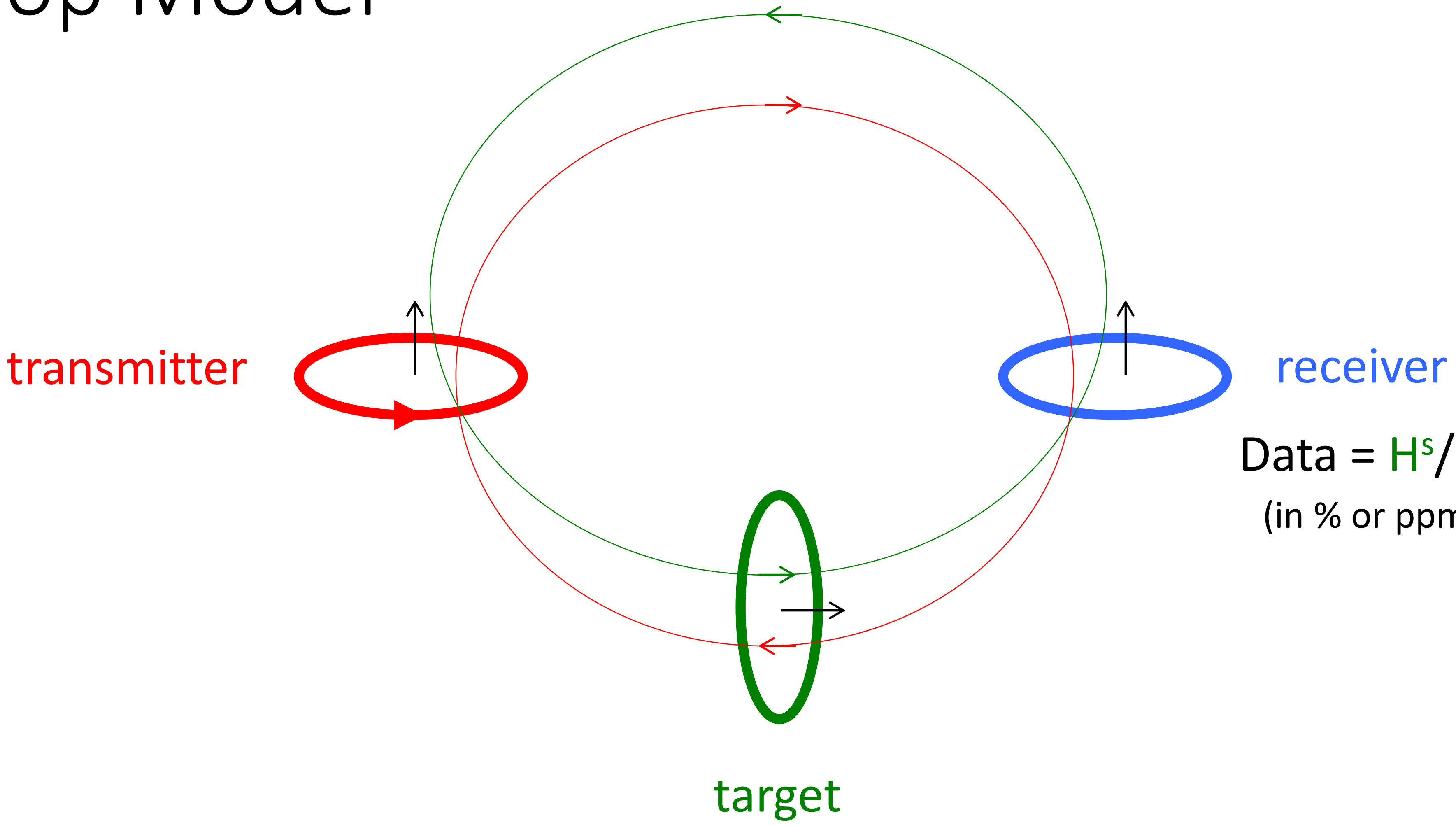
3-loop Model: Primary H^p



3-loop Model: Secondary H^s



3-loop Model



Data = H^s/H^p

(in % or ppm)

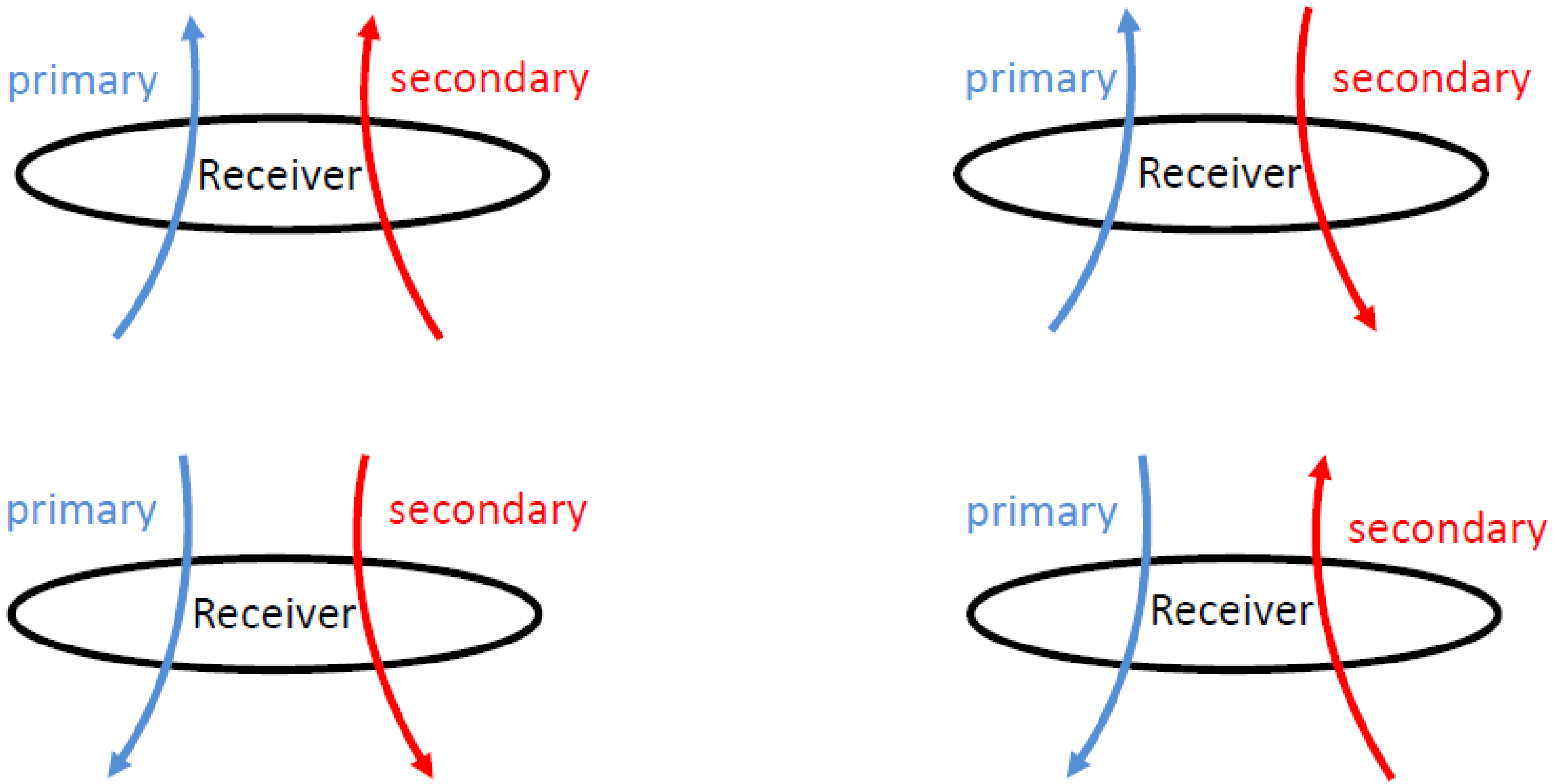
Question: Is the data positive or negative for the scenario on this page?

Hint: Think about the positive and negative anomalies in total field magnetics.

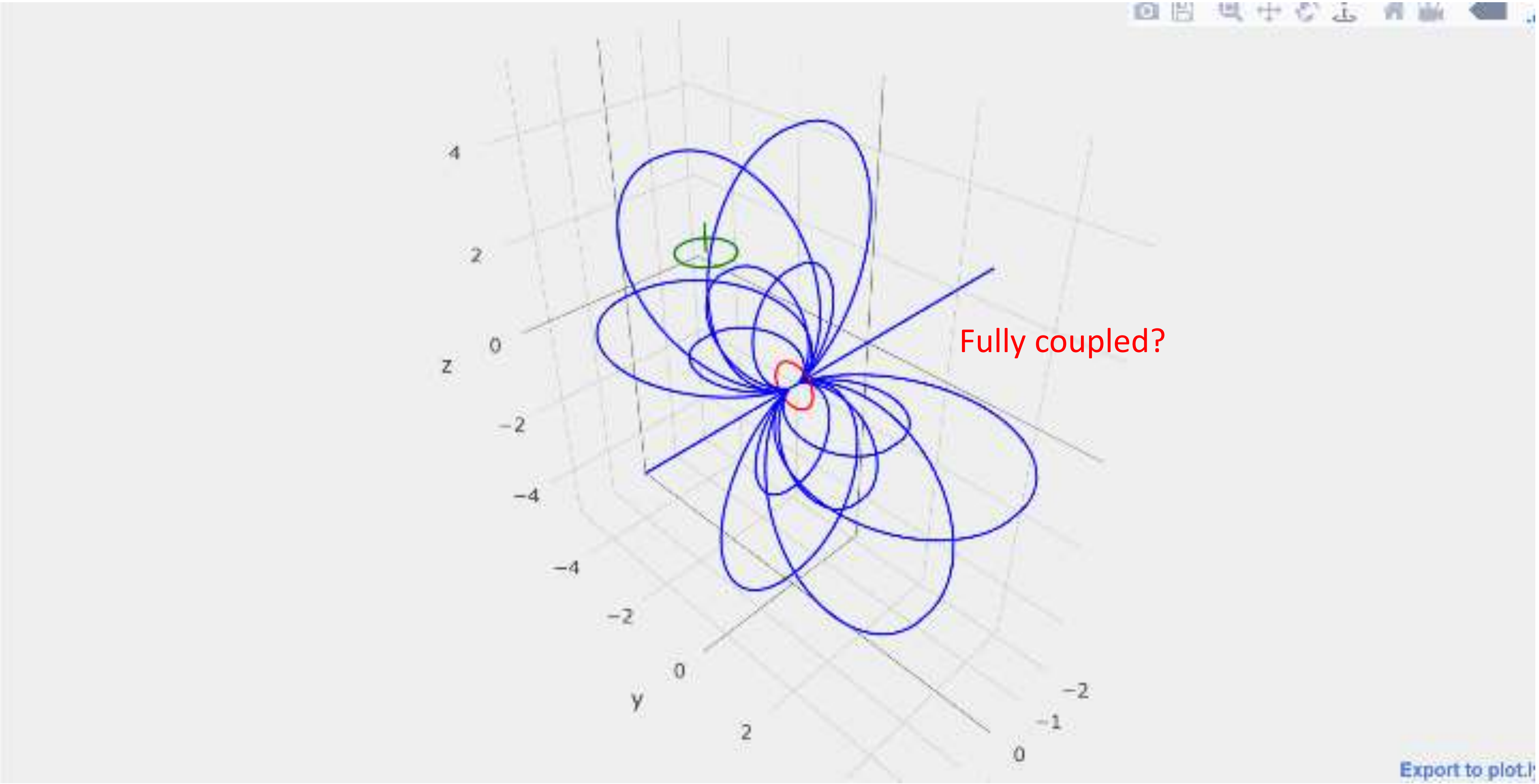
Data (Hs/Hp) Sign Convention

Positive
primary and secondary in same direction

Negative
primary and secondary in opposite directions

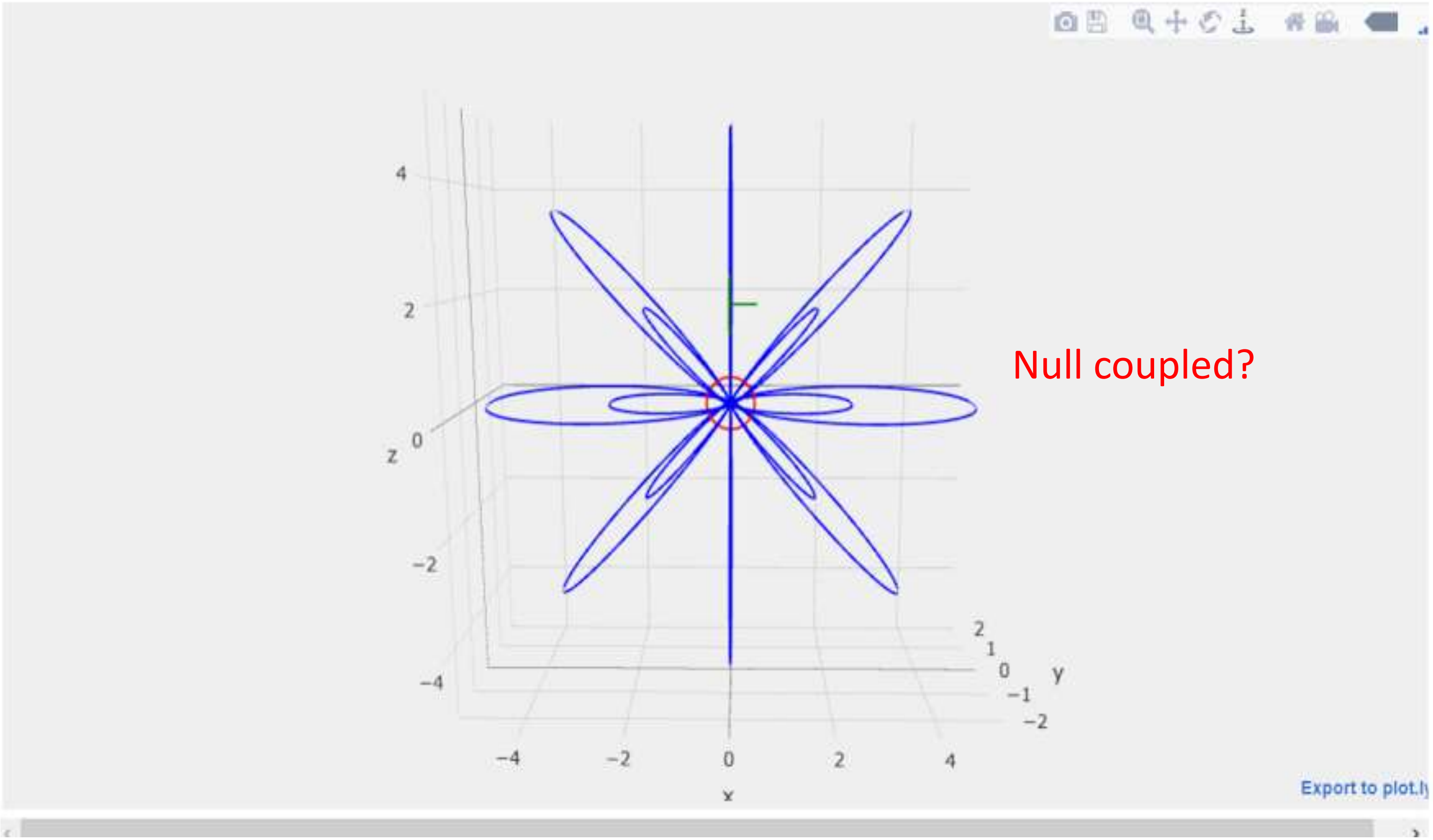


Coupling between Two Loops Through Magnetic Flux Linkage



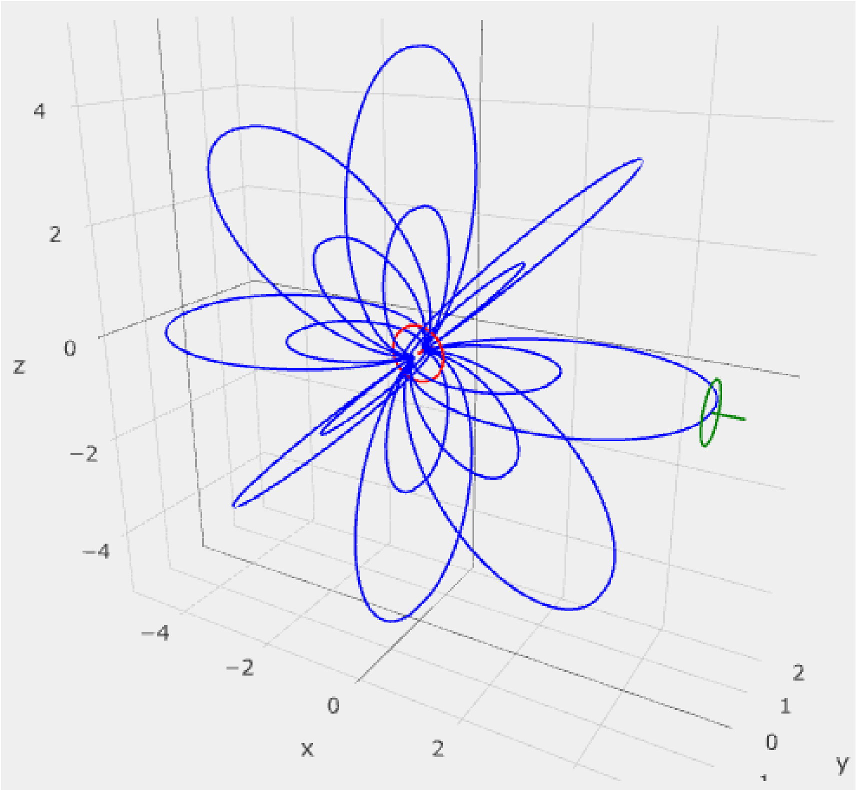
Fully coupled?

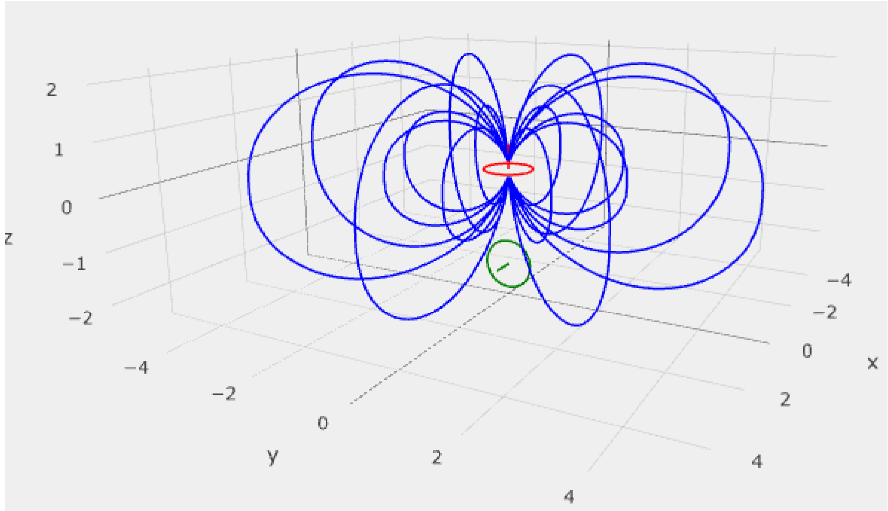
Coupling between Two Loops Through Magnetic Flux Linkage

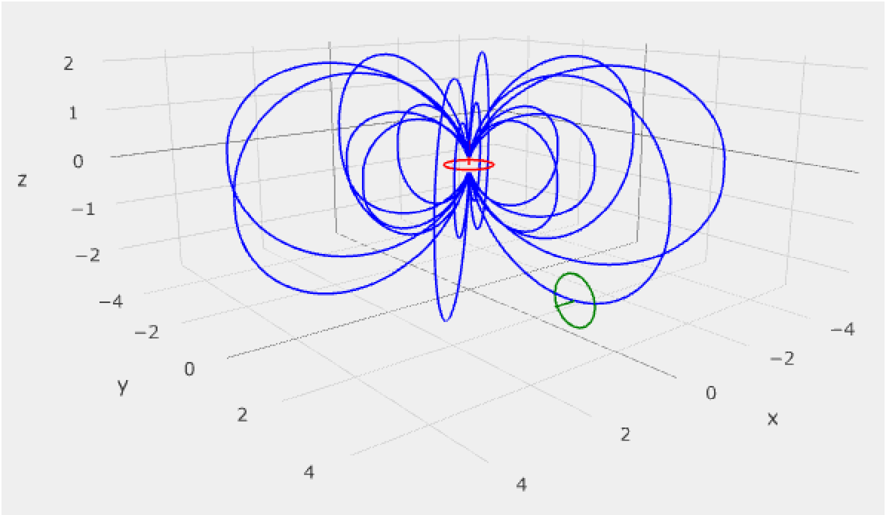


Null coupled?

Coupling between Two Loops Through Magnetic Flux Linkage

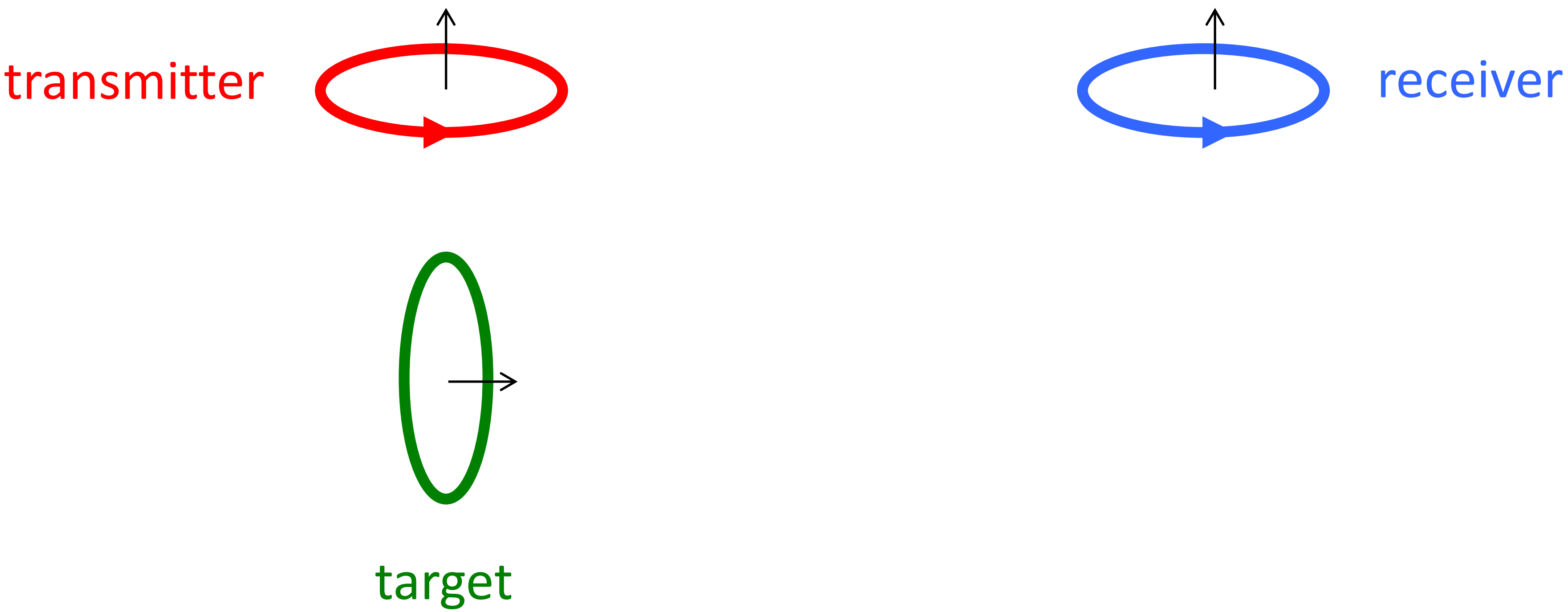




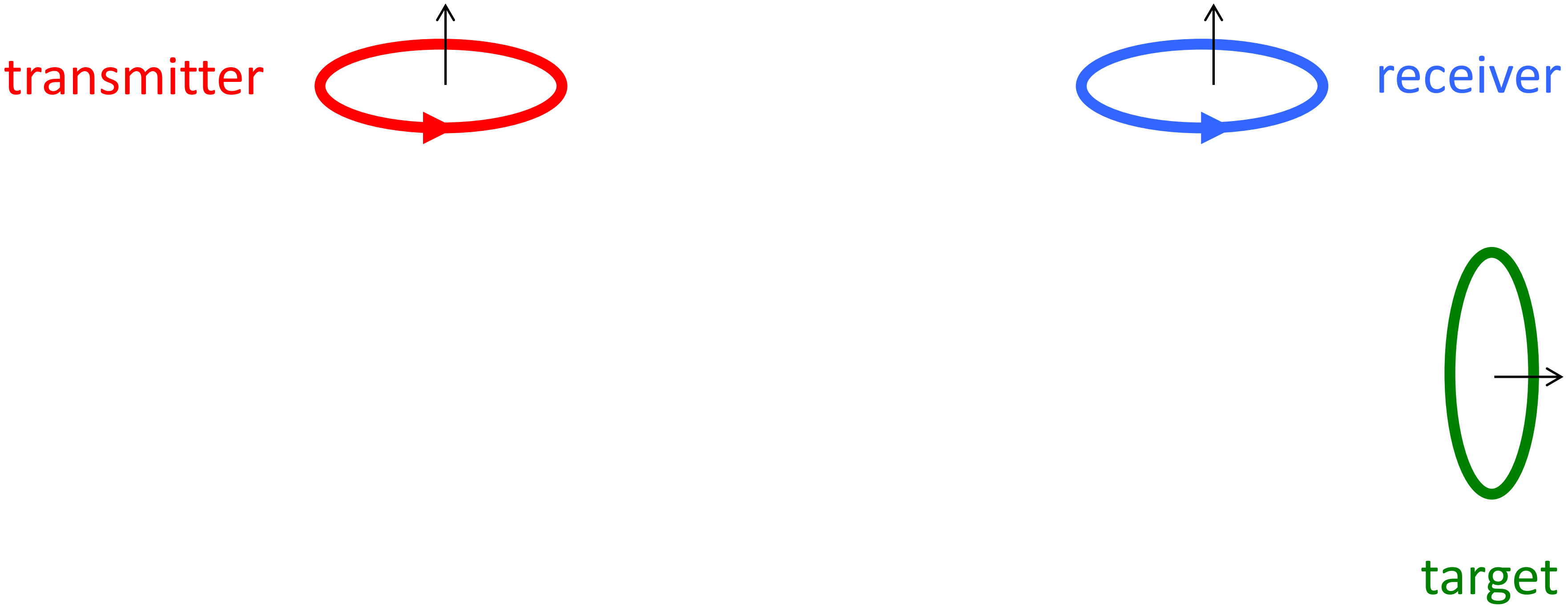


Null coupled

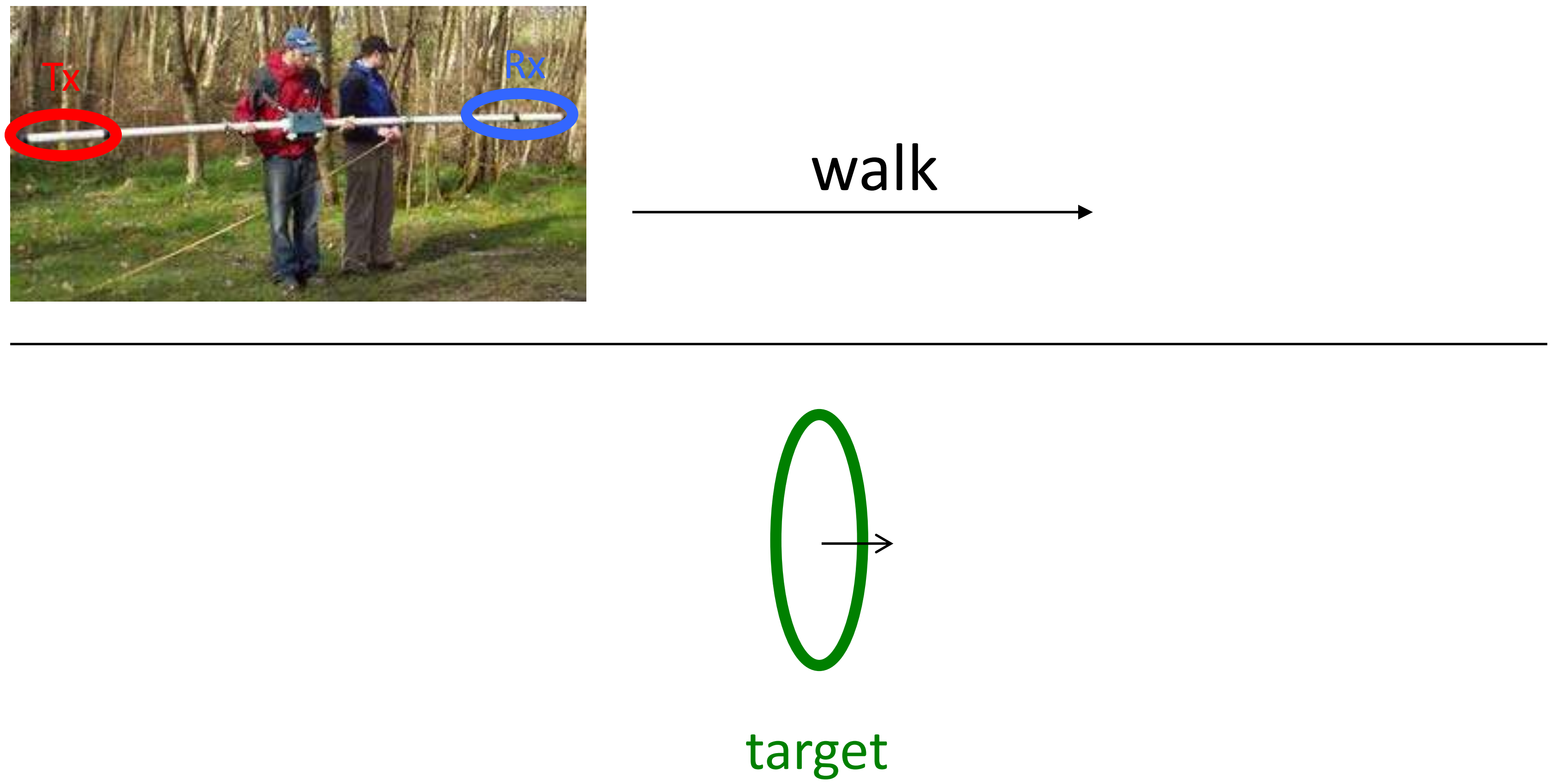
H^s/H^p: Positive or Negative?



H^s/H^p: Positive or Negative?



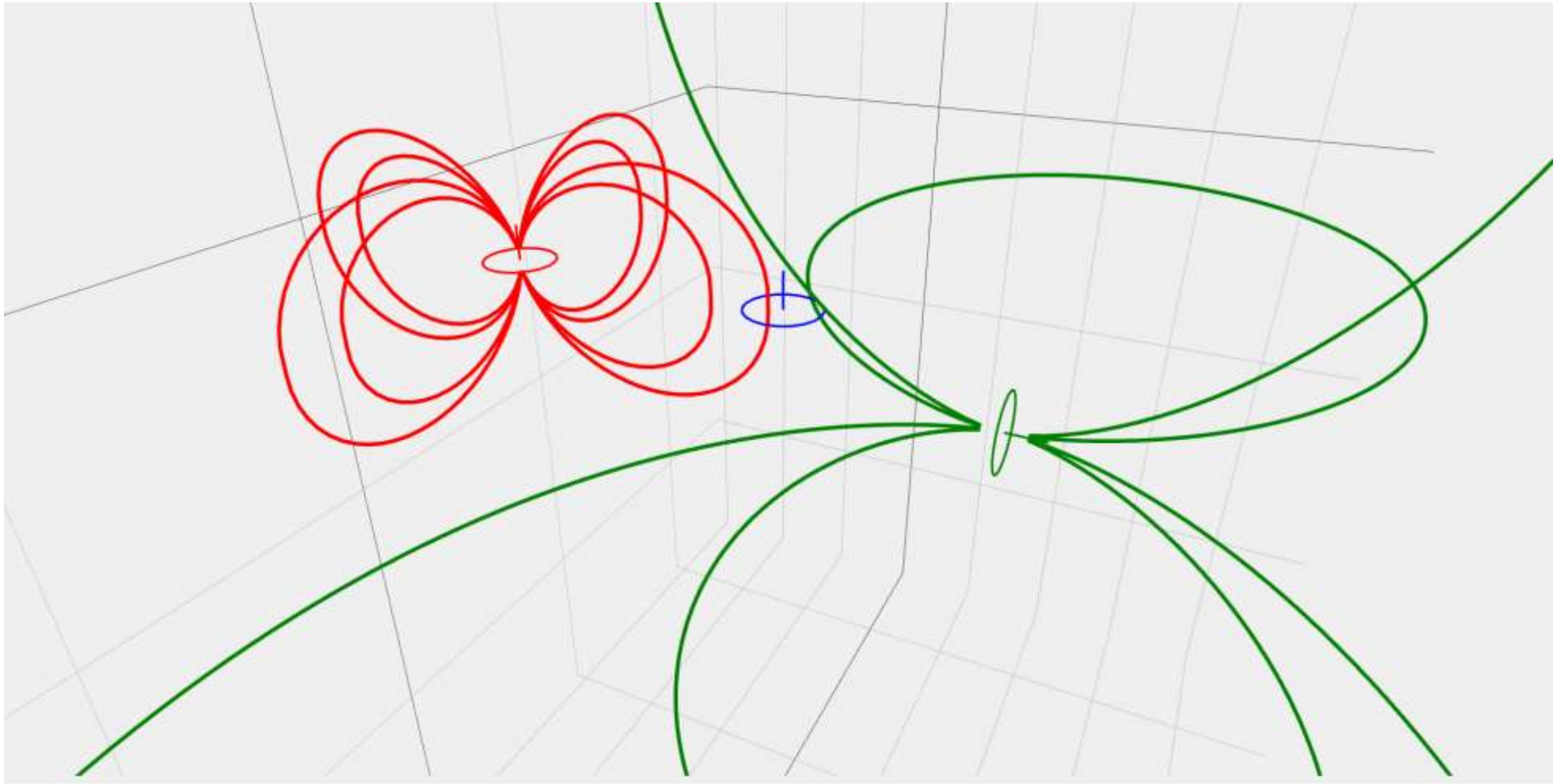
Hs/Hp Profile



Verify using Demo Notebook

- “MagDipole3LoopsCoupling.ipynb”

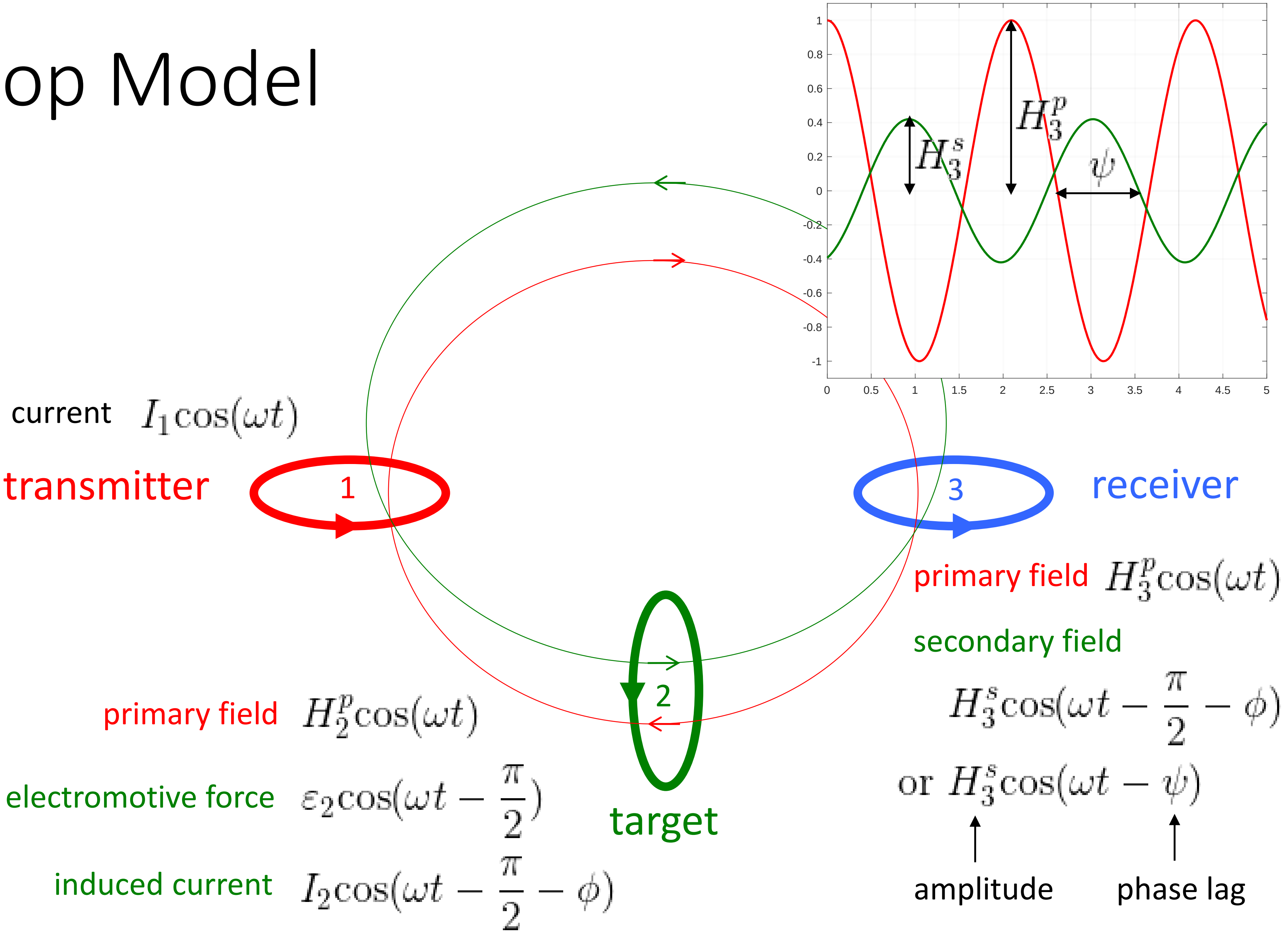
https://github.com/sustechgem/geophysics-demo-notebooks/blob/master/MagDipole3LoopsCoupling.ipynb



Drawing lines only helps qualitative understanding.

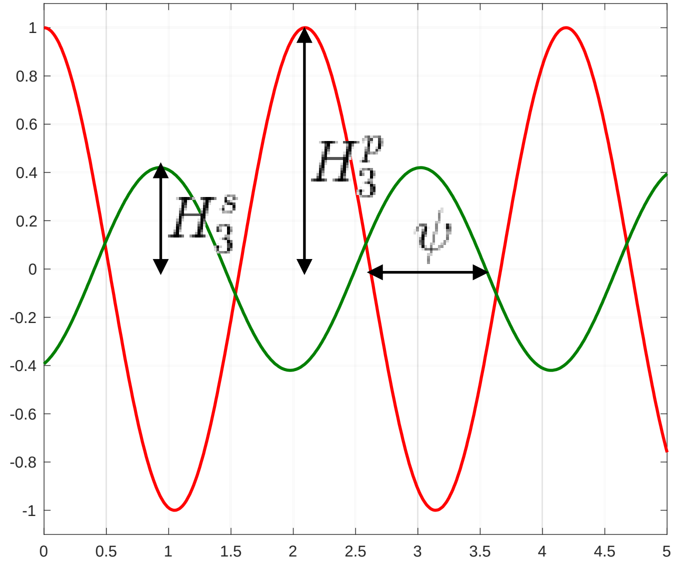
We need more math to do a quantitative interpretation.

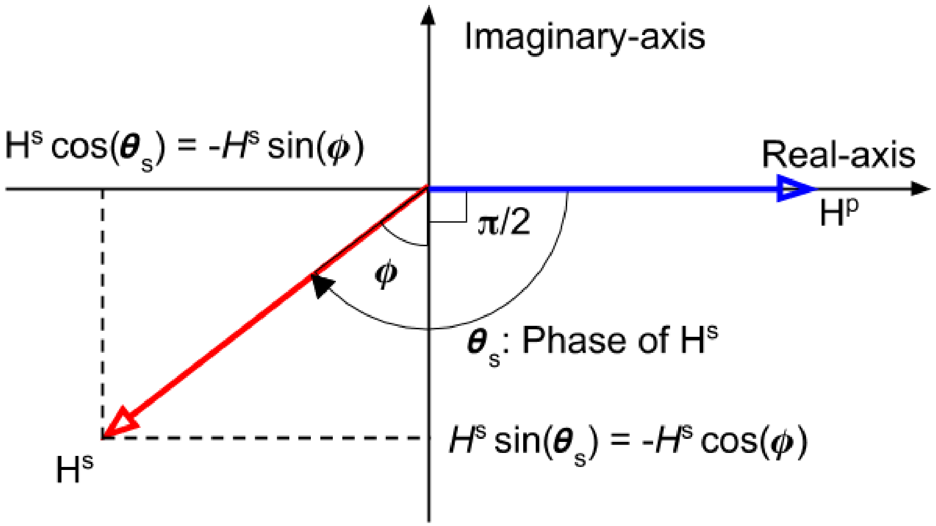
3-loop Model



current $I_1\cos(\omega t)$
transmitter
1
receiver
3
target
2
primary field $H_3^p\cos(\omega t)$
secondary field
$H_3^s\cos(\omega t - \frac{\pi}{2} - \phi)$
or $H_3^s\cos(\omega t - \psi)$
amplitude
phase lag
primary field $H_2^p\cos(\omega t)$
electromotive force $\varepsilon_2\cos(\omega t - \frac{\pi}{2})$
induced current $I_2\cos(\omega t - \frac{\pi}{2} - \phi)$
H_3^s
H_3^p
ψ
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Decompose Secondary Field





primary field $H_3^p \cos(\omega t)$

secondary field

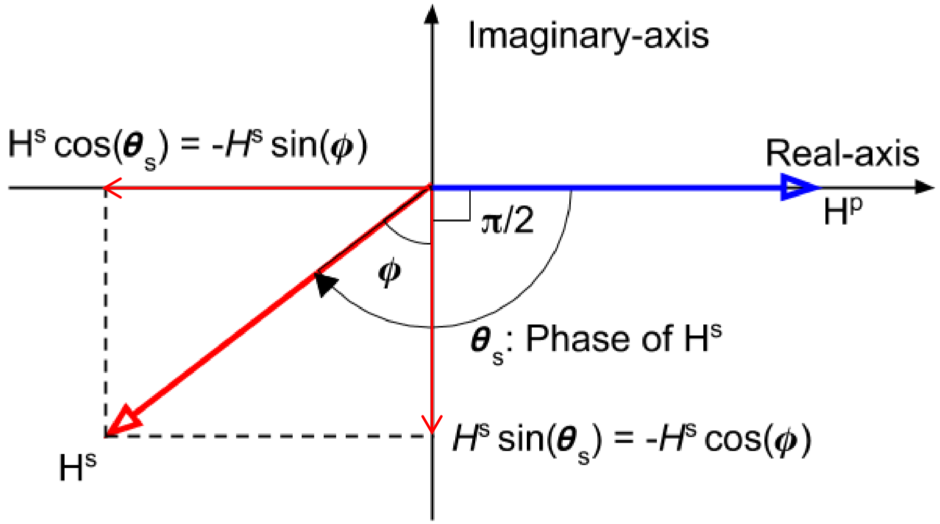
$$H_3^s \cos(\omega t - \frac{\pi}{2} - \phi)$$

or $H_3^s \cos(\omega t - \psi)$

$$\phi = tan^{-1}(\frac{\omega L}{R}) = tan^{-1}(\alpha)$$

- H^s swings in the third quadrant: $0 < \phi < 90°$
- ϕ depends on the induction number **α**
- **α** is a function of frequency **ω**, self inductance **L** and resistance **R** of Loop 2

Decompose Secondary Field



$$\phi = tan^{-1}(\frac{\omega L}{R}) = tan^{-1}(\alpha)$$

Question: What happens to the H^s (red arrow) for a very conductive or very resistive target?

Decompose H^s to two orthogonal components then normalize by H^p:

90° phase lag: called "out-of-phase", "quadrature", "imaginary"

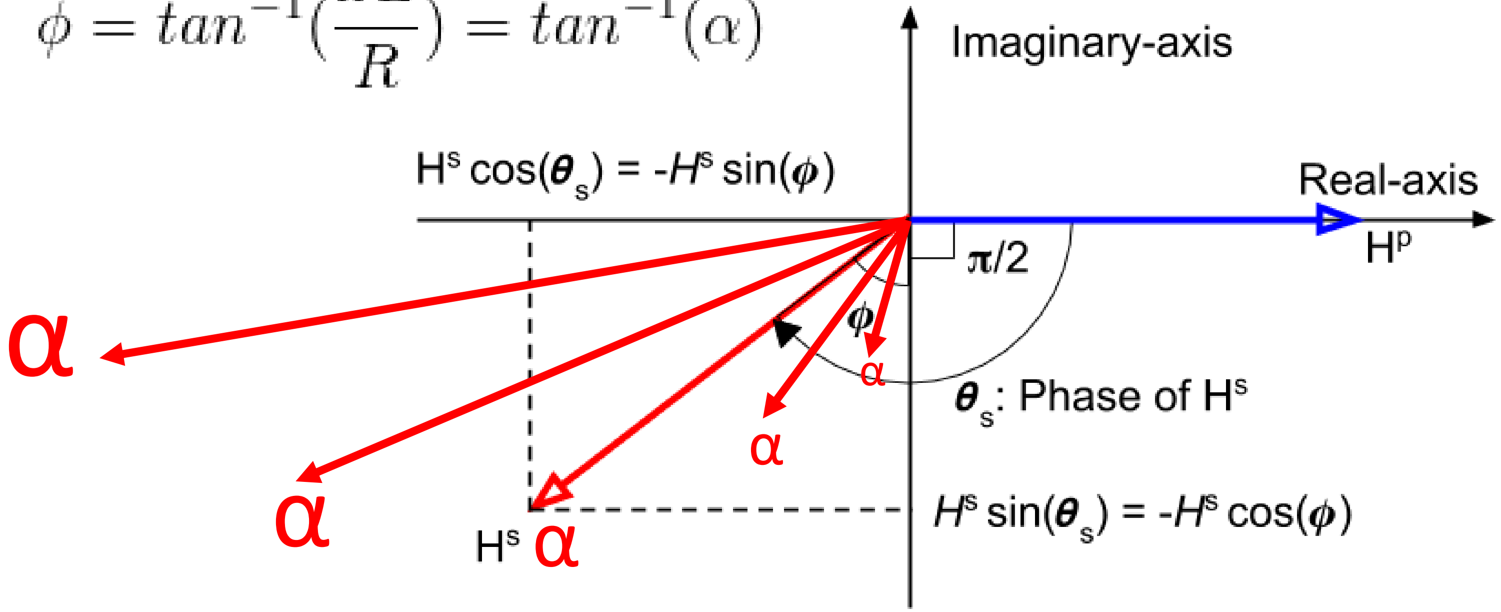
$$\frac{H^s \cos(\phi)}{H^p}$$

180° phase lag: called "in-phase", "real"

$$\frac{H^s \sin(\phi)}{H^p}$$

Response Function

$$\phi = tan^{-1}(\frac{\omega L}{R}) = tan^{-1}(\alpha)$$

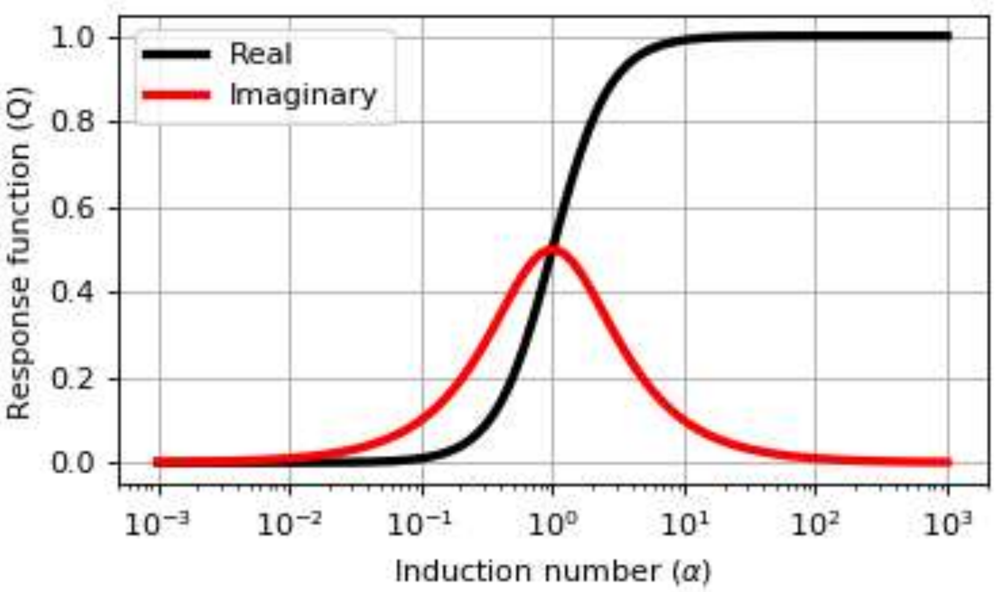


Question: How would the real and imaginary data change with the induction number α?

Response Function

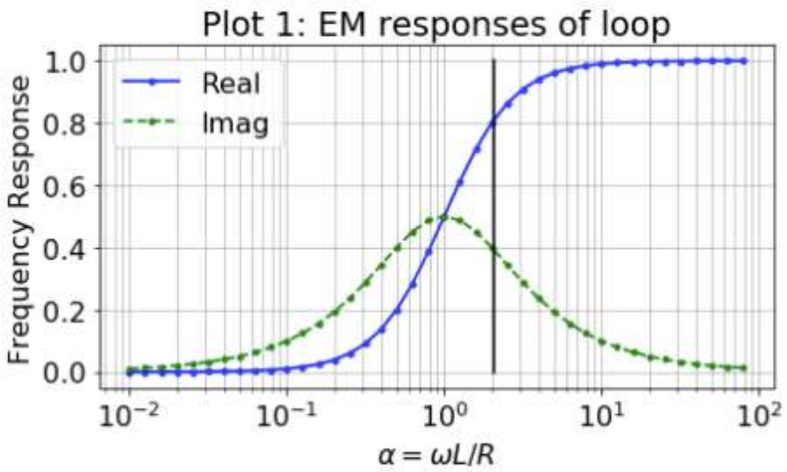
$$Q(\alpha) = \frac{i\alpha}{1 + i\alpha} = \frac{\alpha^2 + i\alpha}{1 + \alpha^2} \qquad \alpha = \frac{\omega L}{R}$$

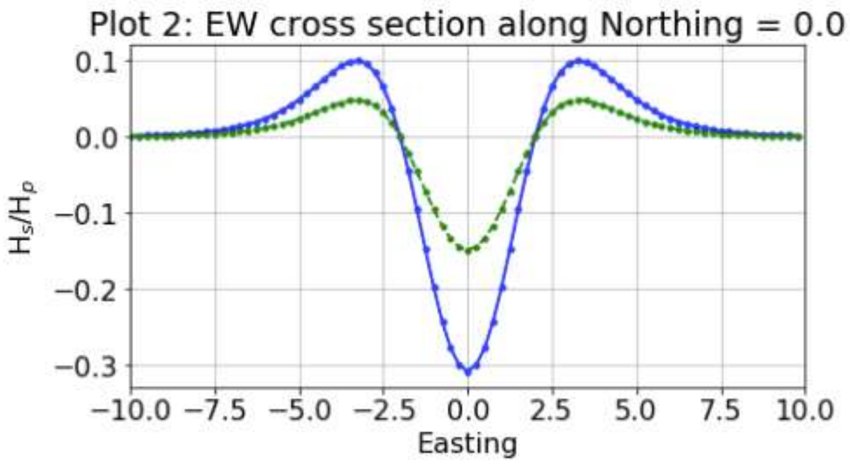
Resistive limit:
- low frequency
- low conductivity



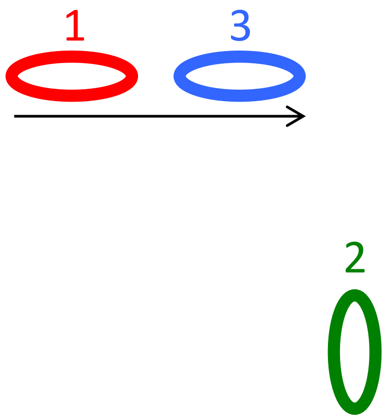
Inductive limit:
- high frequency
- high conductivity

Expected Data From a Loop Target





$$\frac{H_3^s}{H_3^p} = -\frac{M_{12}M_{23}}{M_{13}L}\left[\frac{\alpha^2 + i\alpha}{1+\alpha^2}\right]$$

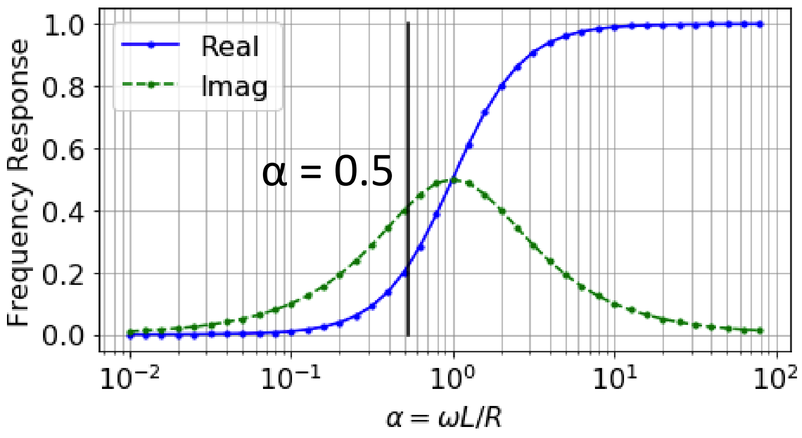


Coupling
- location, orientation
- overall magnitude

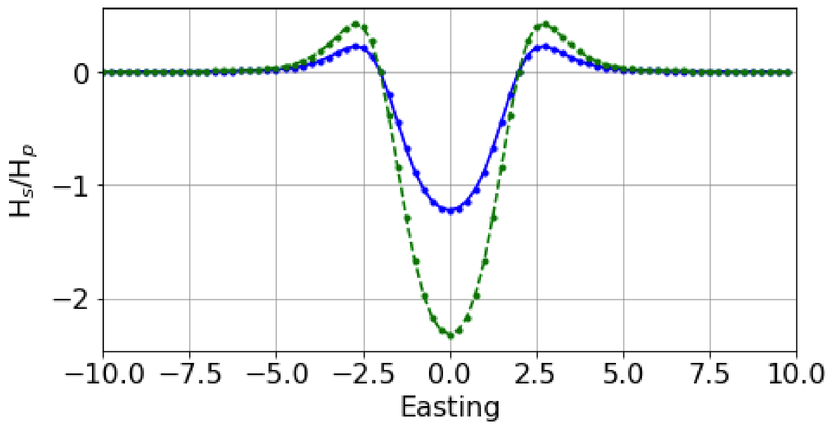
Induction
- properties of loop 2
- how much in Re & Im

A Smaller Induction Number

Response function of different α



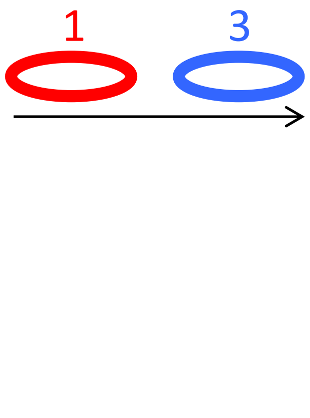
Data along a profile for α = 0.5



$$\frac{H_3^s}{H_3^p} = -\frac{M_{12}M_{23}}{M_{13}L}\left[\frac{\alpha^2 + i\alpha}{1+\alpha^2}\right]$$

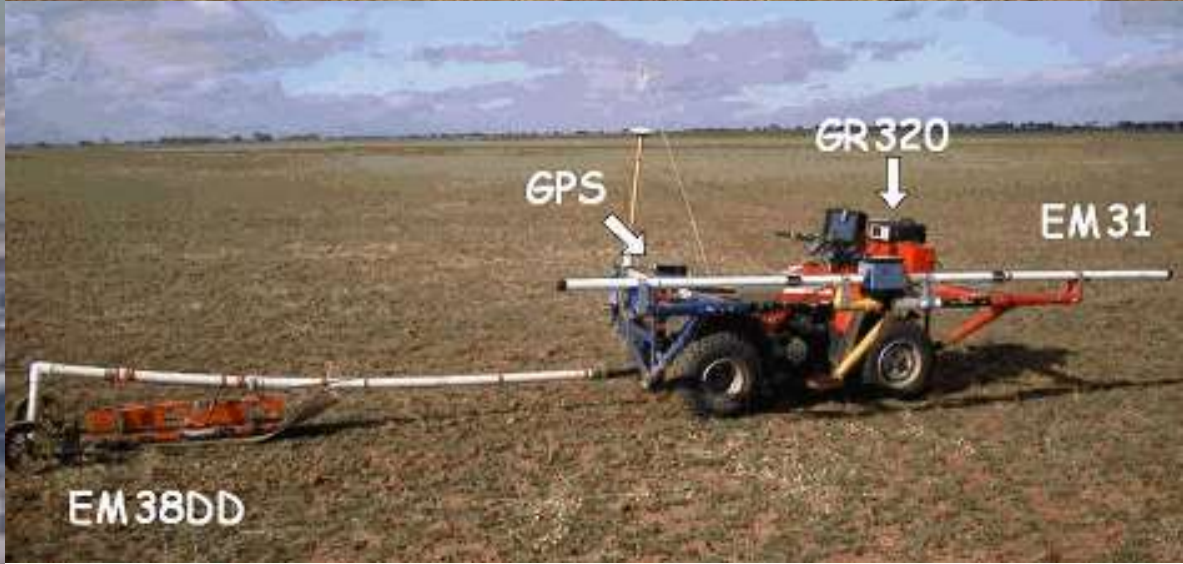
Coupling
- location, orientation
- overall magnitude

Induction
- properties of loop 2
- how much in Re & Im

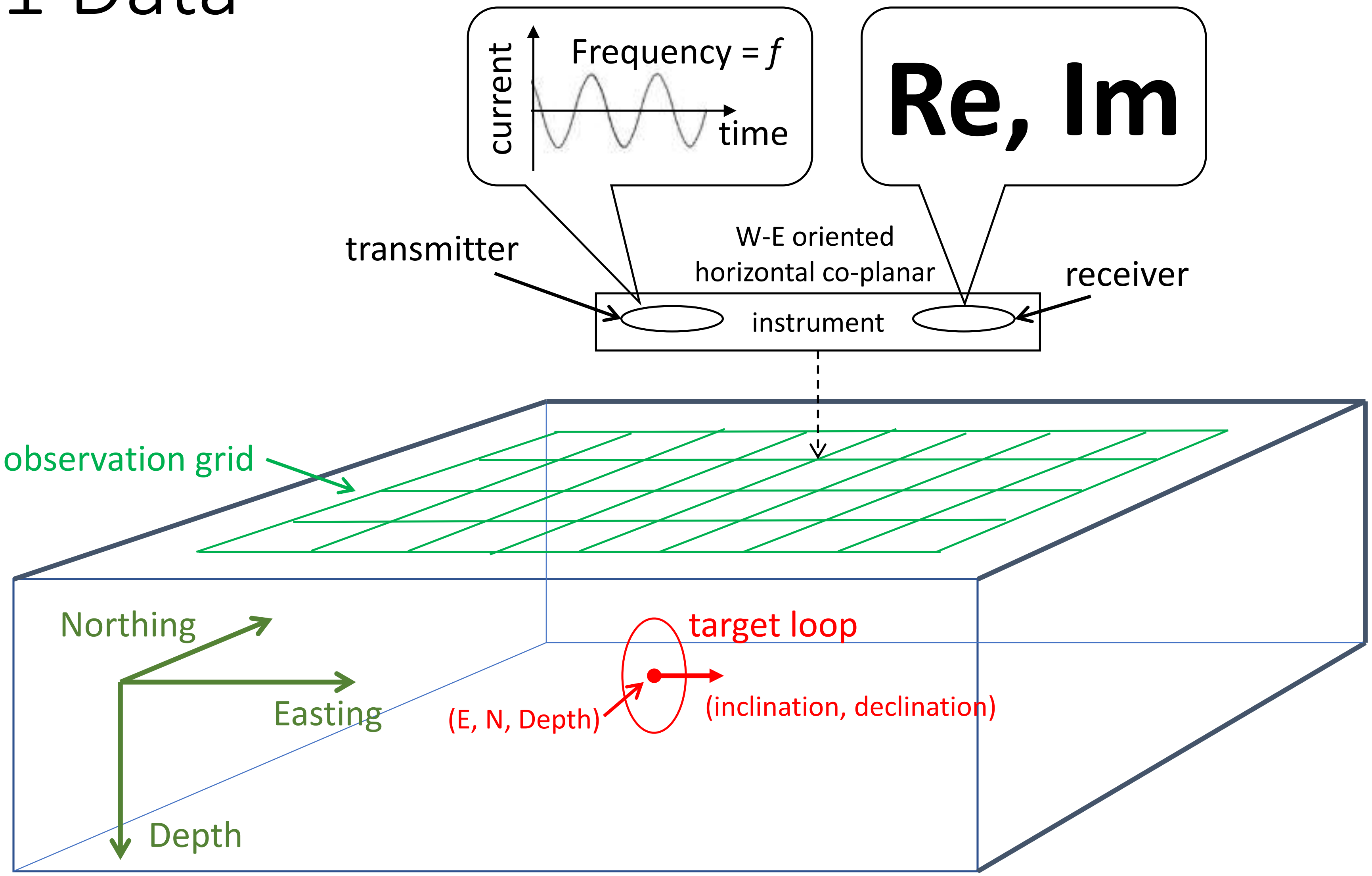


EM-31

- Frequency = 9.8 kHz
- Tx-Rx spacing = 3.66 m
- Horizontal or vertical coplanar
- “Ground conductivity meter”

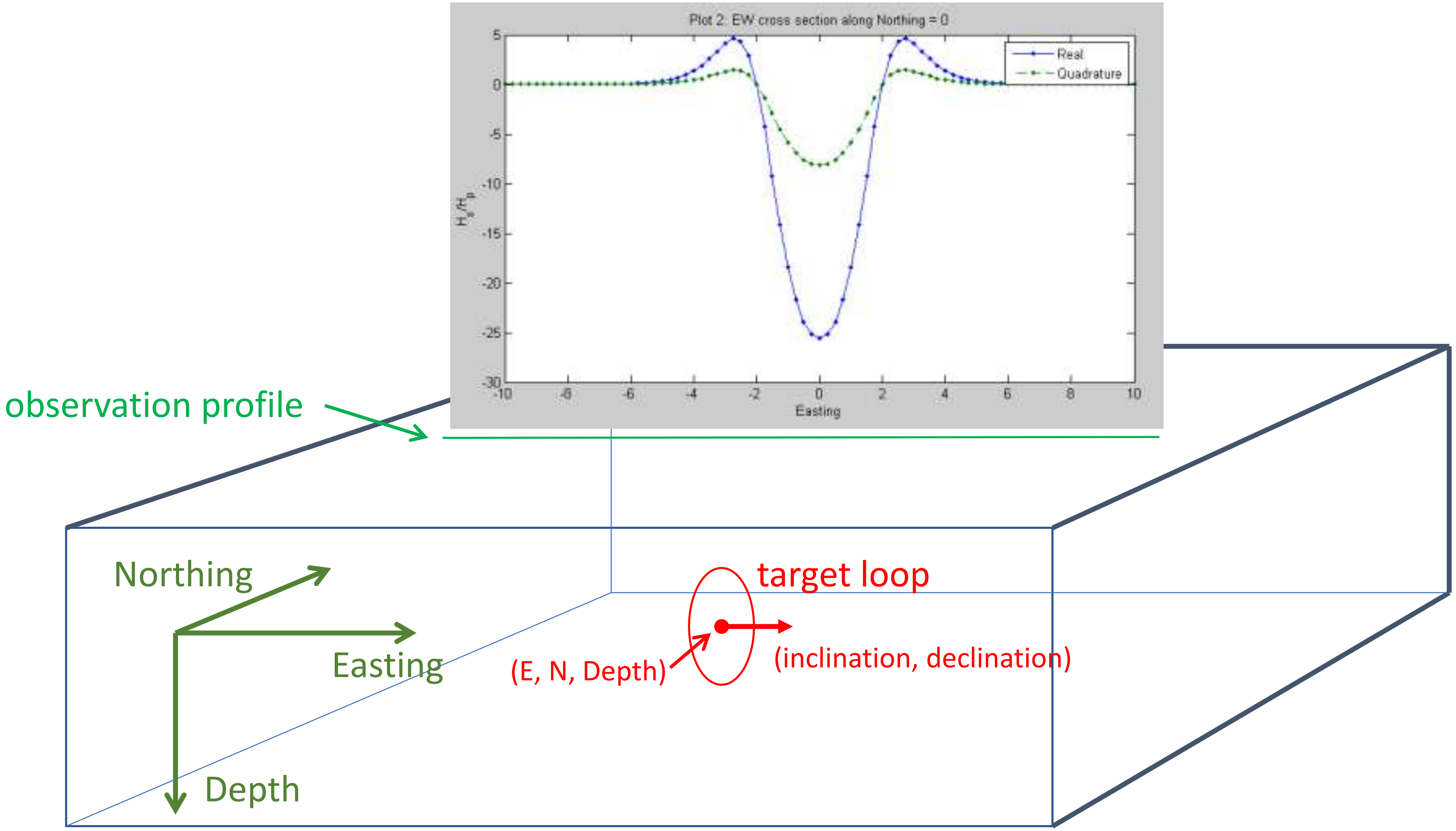


EM-31 Data

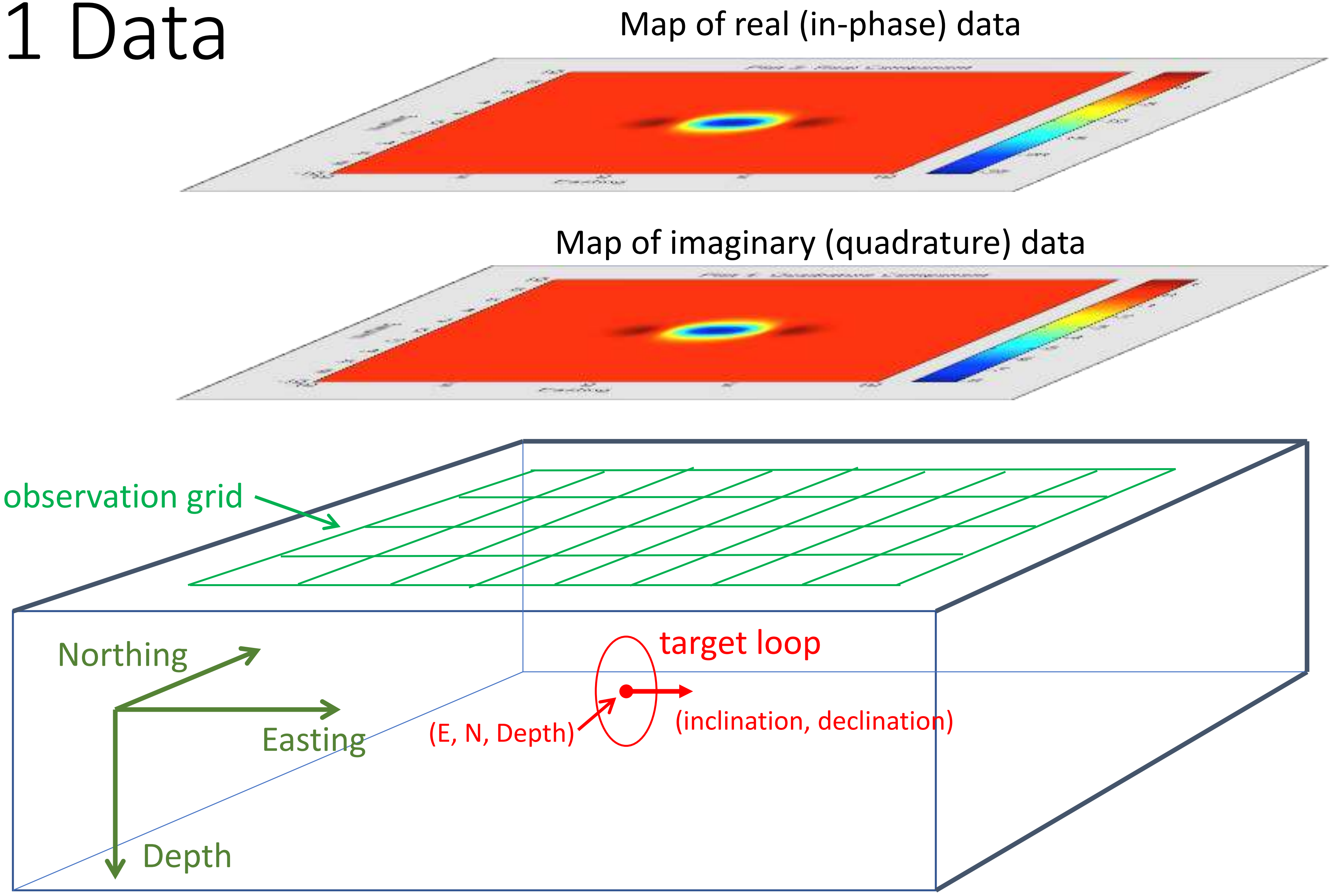


current
Frequency = f
time
Re, Im
transmitter
W-E oriented
horizontal co-planar
receiver
instrument
observation grid
Northing
Easting
Depth
target loop
(E, N, Depth)
(inclination, declination)

EM-31 Data

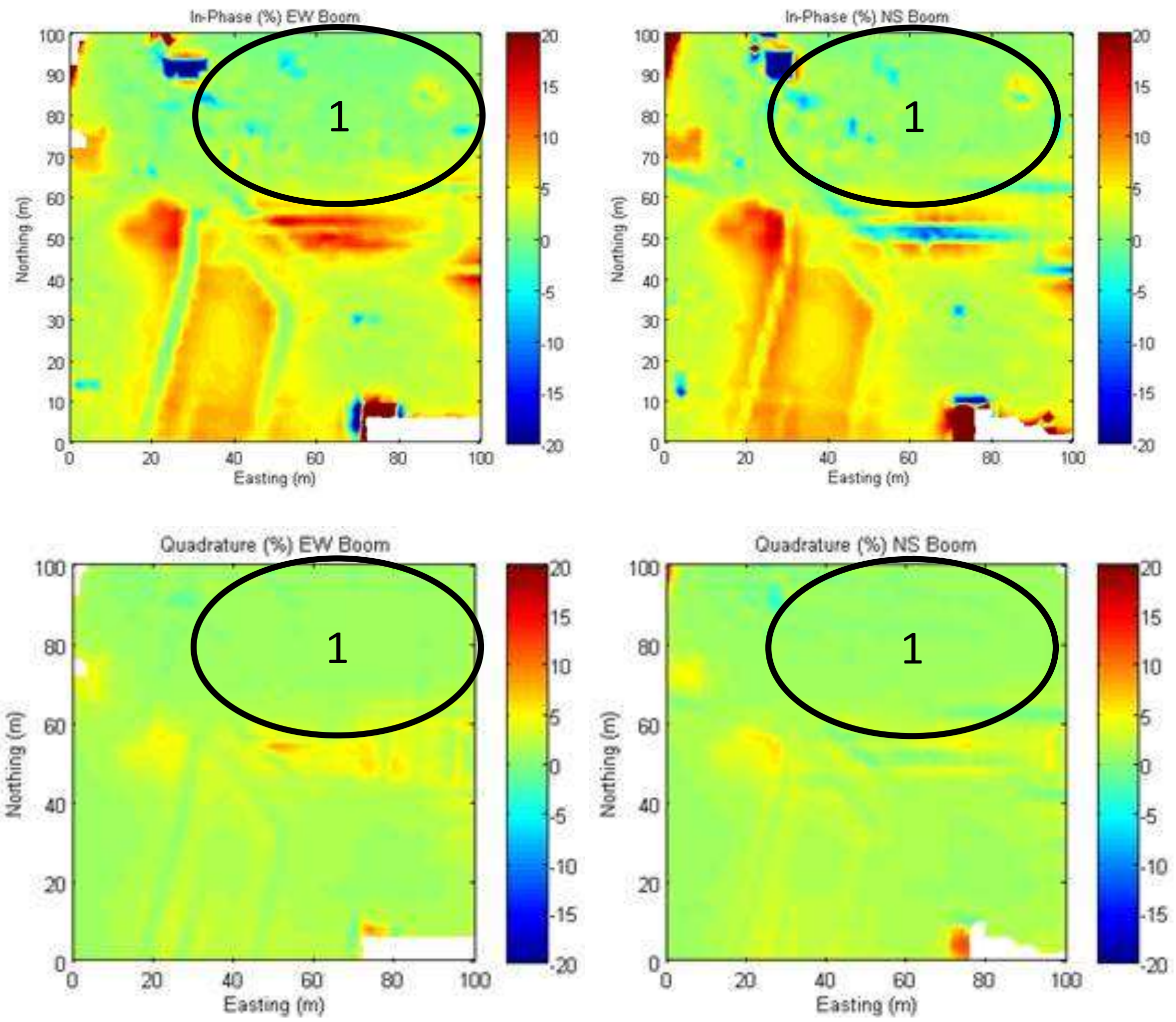

Plot 2: EW cross section along Northing = 0
Real
Quadrature
Easting
observation profile
Northing
Easting
Depth
target loop
(E, N, Depth)
(inclination, declination)

EM-31 Data



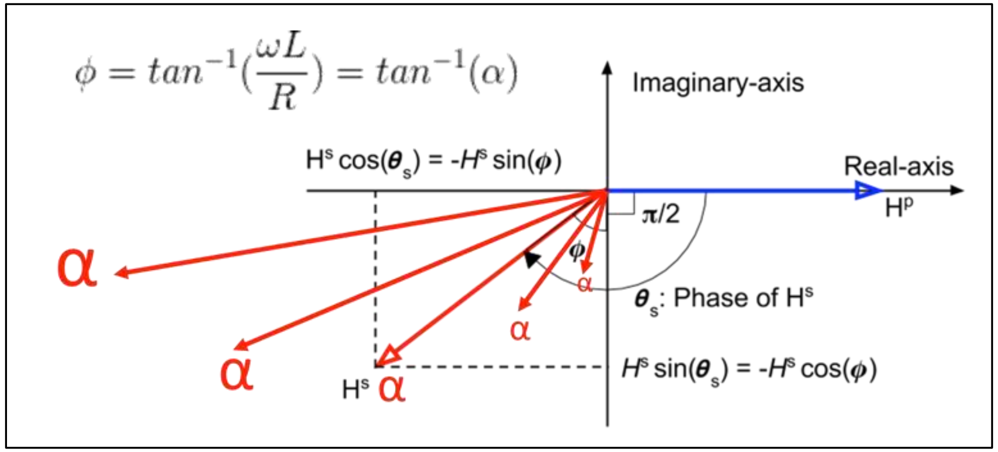
Map of real (in-phase) data
Map of imaginary (quadrature) data
observation grid
Northing
Easting
Depth
target loop
(E, N, Depth)
(inclination, declination)

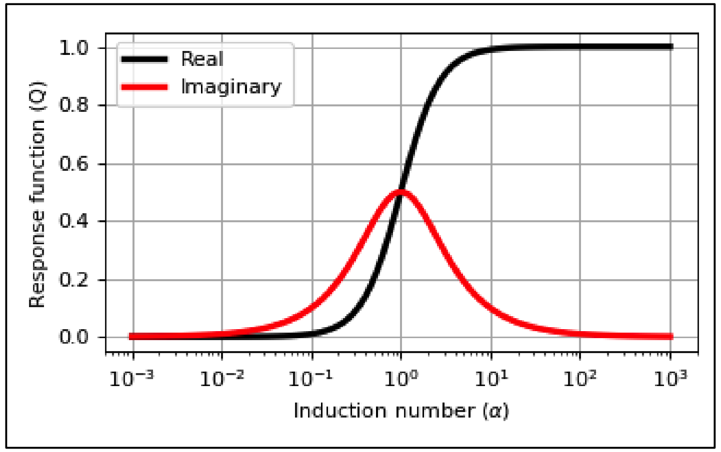
EM-31 Field Data



Data Feature 1:
Uniform, smooth and small

EM-31 Data at Low Induction





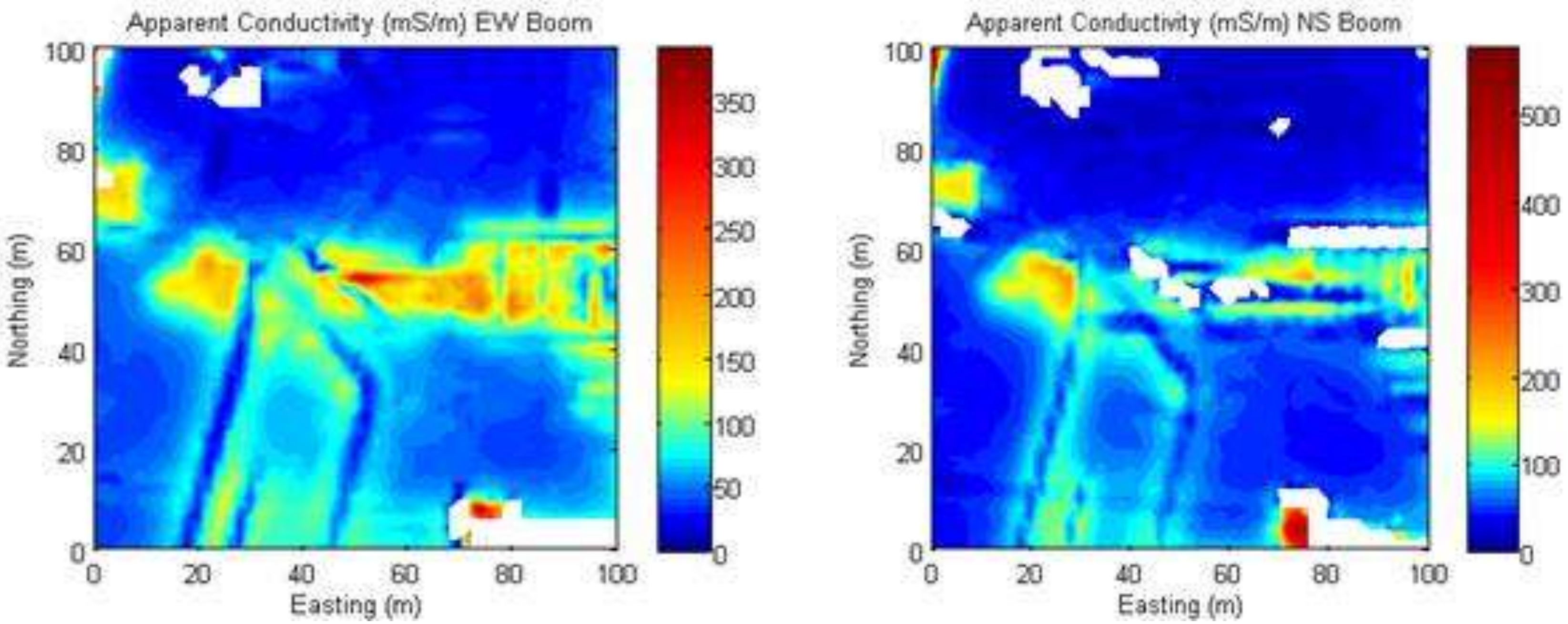
Small **Re** and small **Im** on the data maps, **α** big or small?

Low induction number:

- **H^s** data mostly in quadrature, **Im** > **Re** ≈ 0
- Very small induced current
- Subdivide the earth into many pieces; each piece interacts with Tx-Rx independently without interaction between any two pieces (**recall low induced magnetization in magnetics, easy calculation using superposition!**)

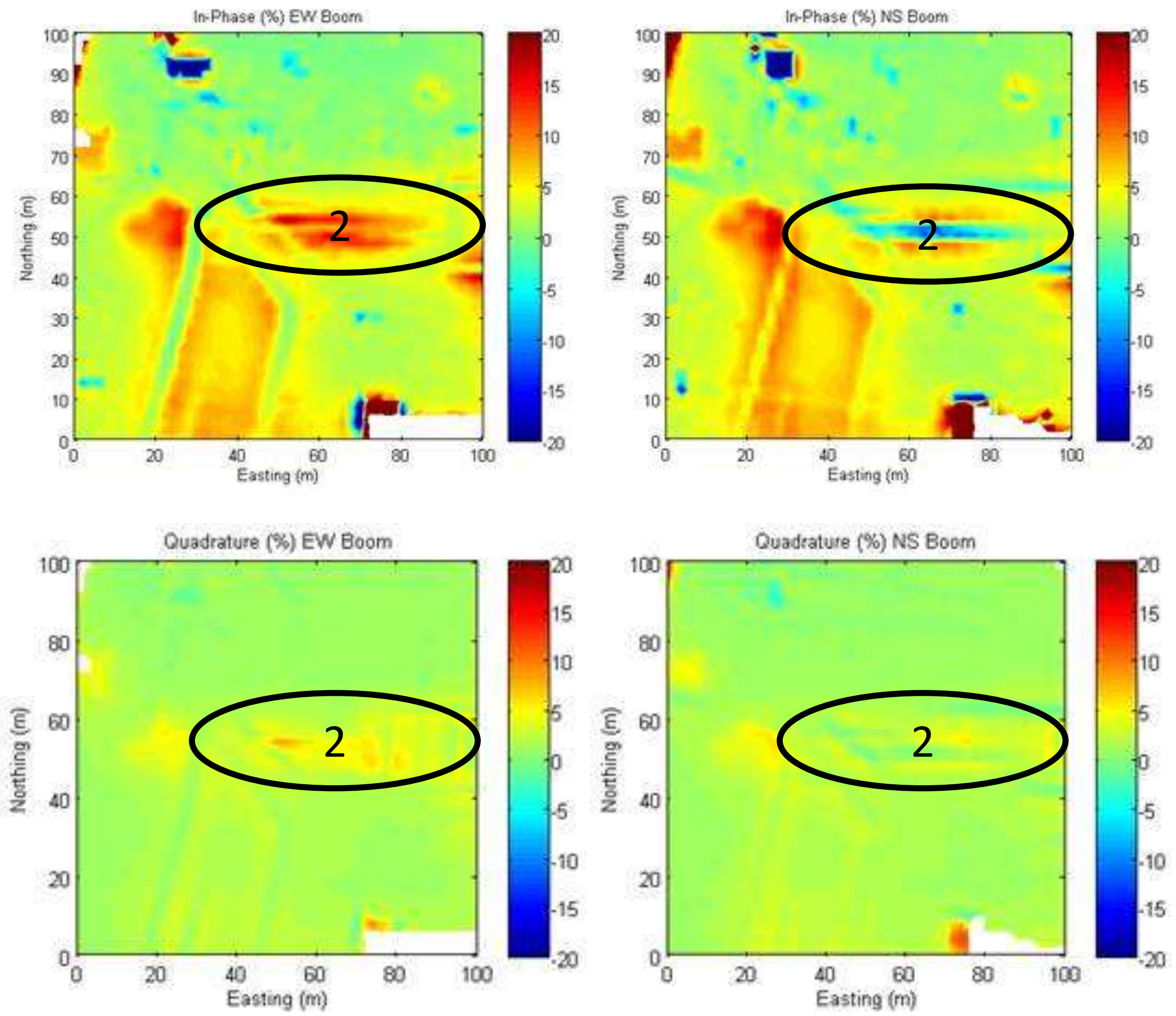
Apparent Conductivity

$$\sigma_a = \frac{4}{\omega \mu_0 s^2} \mathbf{Im}$$



Question: Which area on the maps is the most likely to have a reliable estimate of the ground conductivity?

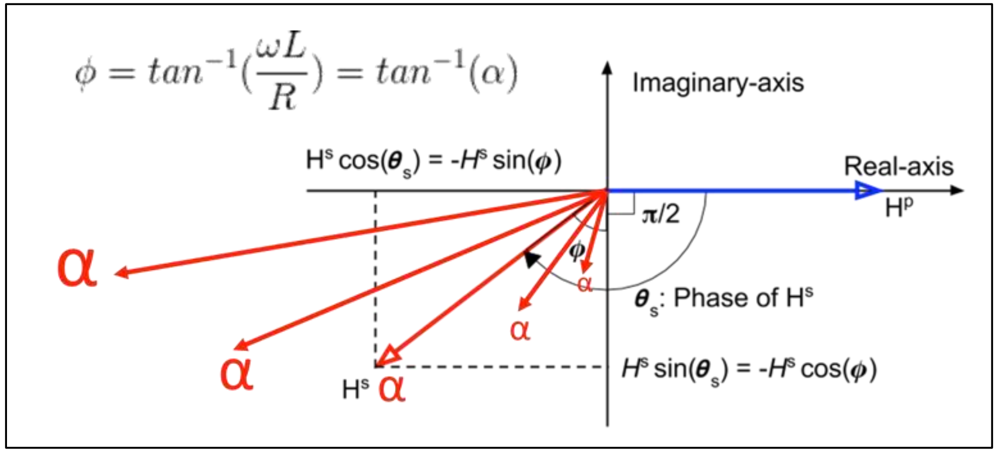
EM-31 Data Interpretation

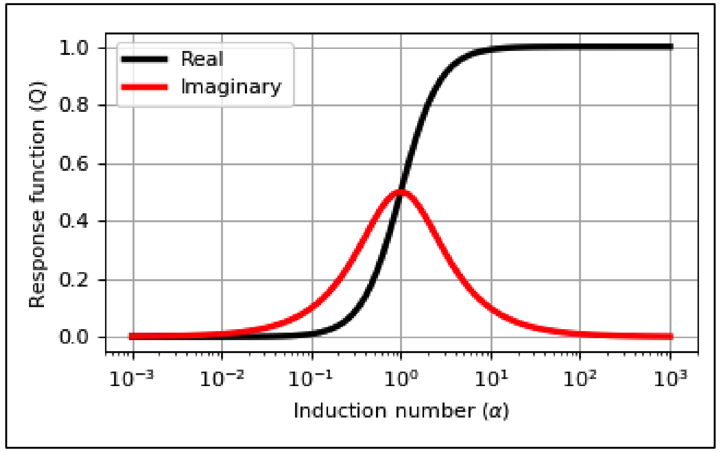


Data Feature 1:
Uniform, smooth and small

Data Feature 2:
Abrupt change
Positive and negative
Large **Re** and small **Im**

EM-31 Data at High Induction

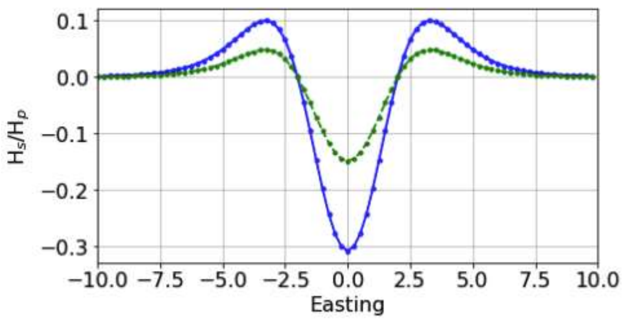




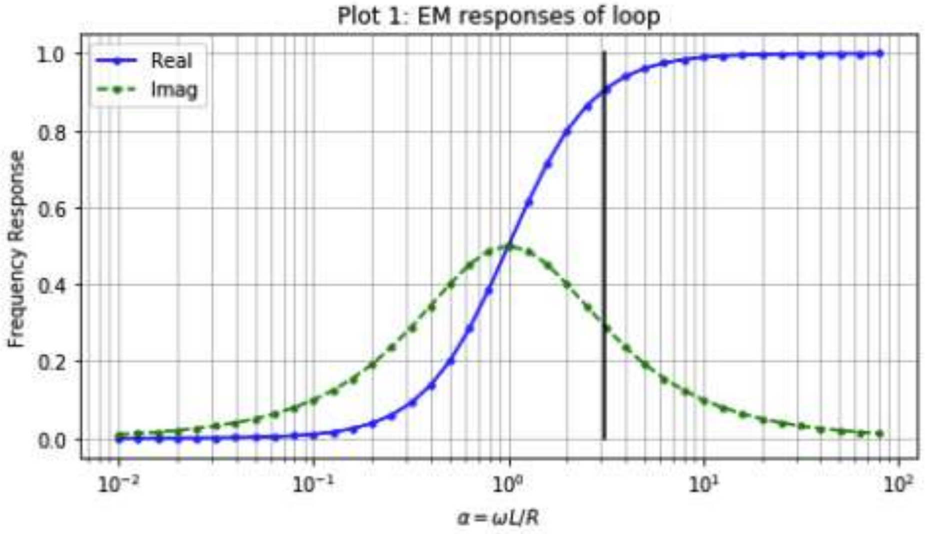
Large **Re** and small **Im** on the data maps, **α** big or small?

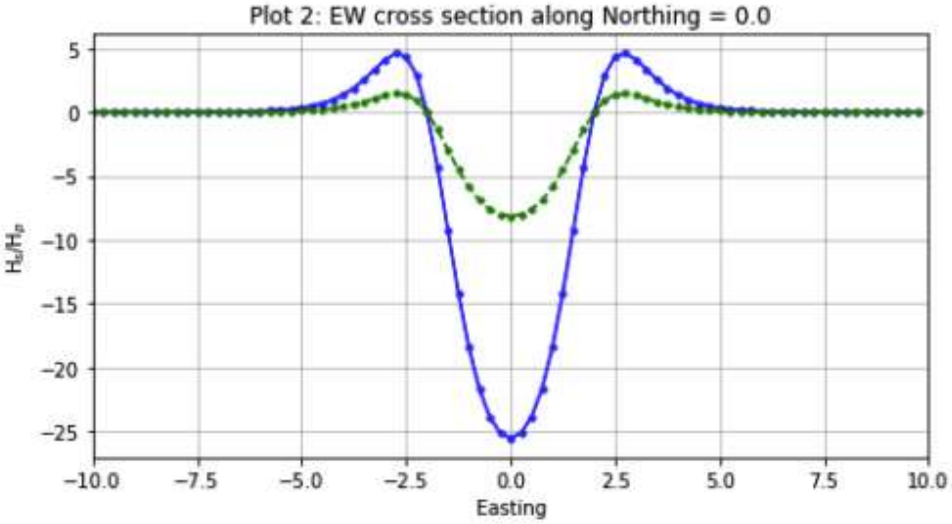
High induction number:

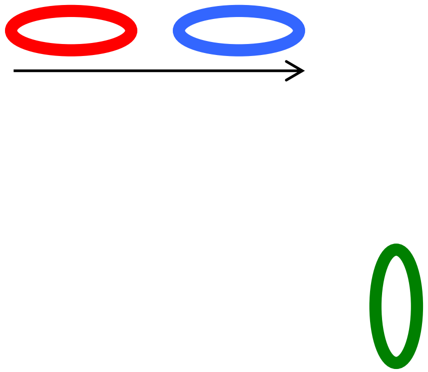
- **H^s** data mostly in in-phase, **Re** > **Im** ≈ 0
- Very strong induced current
- Cannot use apparent conductivity, but if the target is a good compact conductor, use the 3-loop model



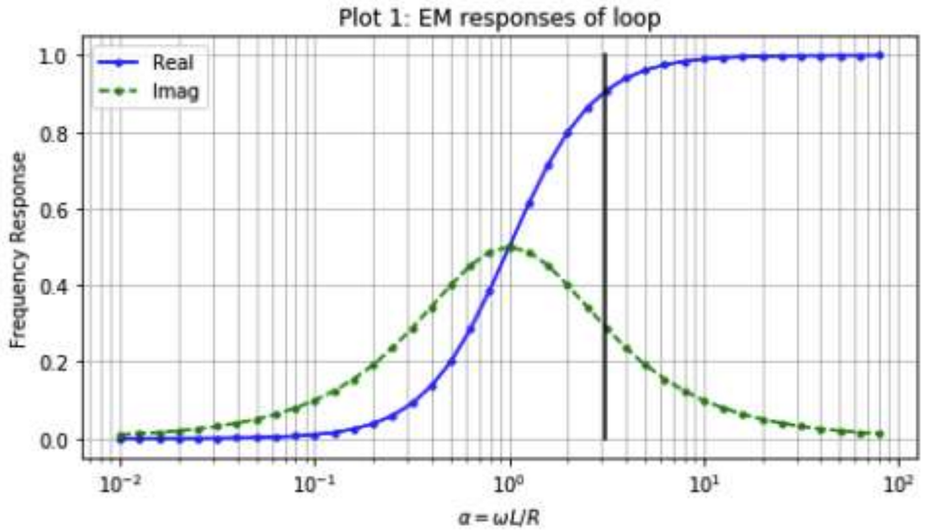
Vertical Target Loop

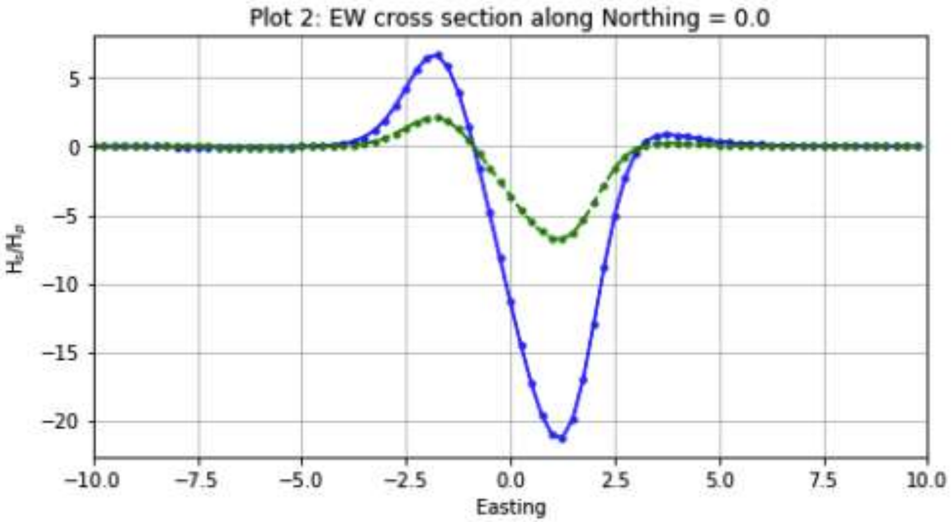


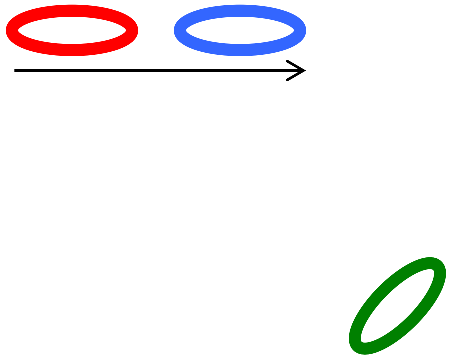




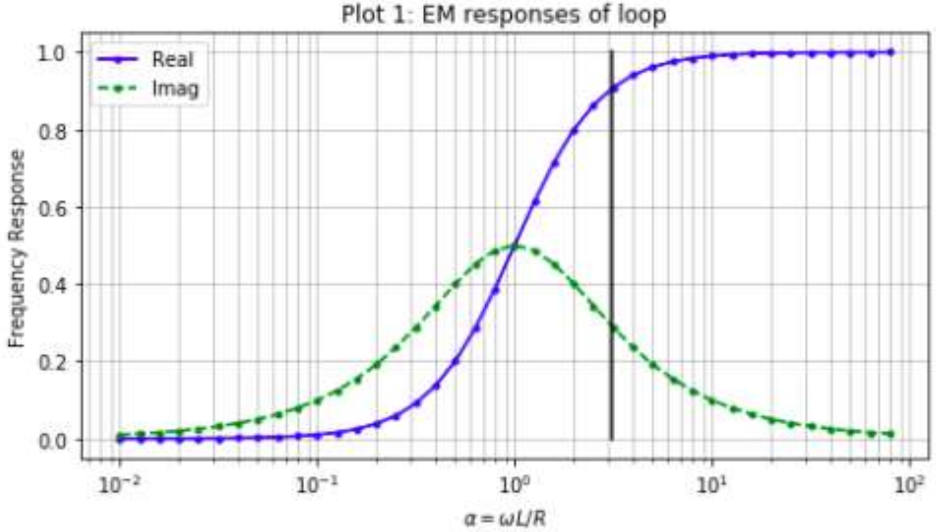
45 Degree Dipping Target Loop

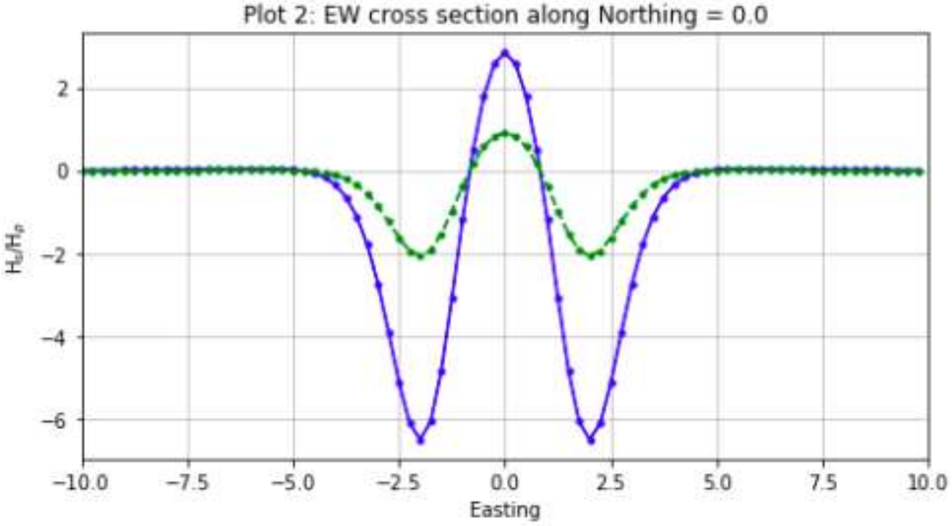


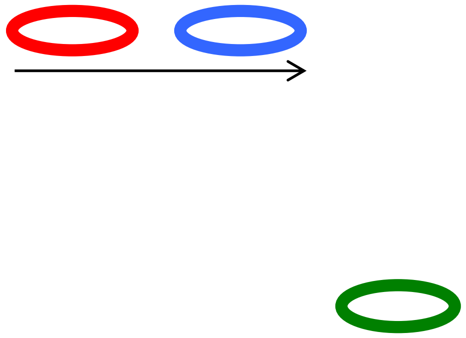




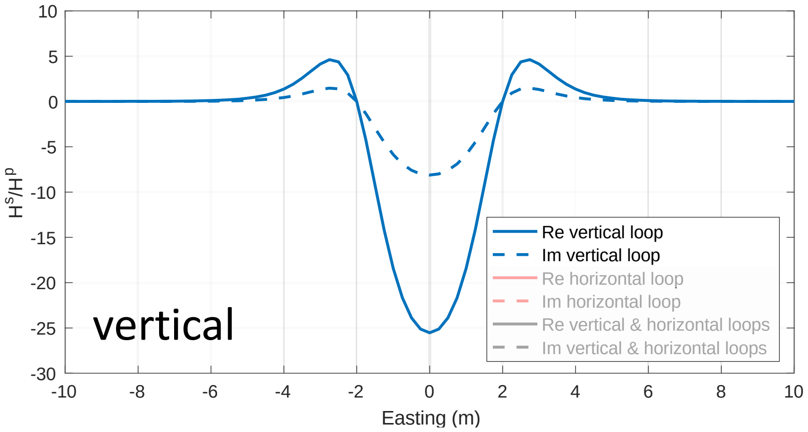
Horizontal Target Loop

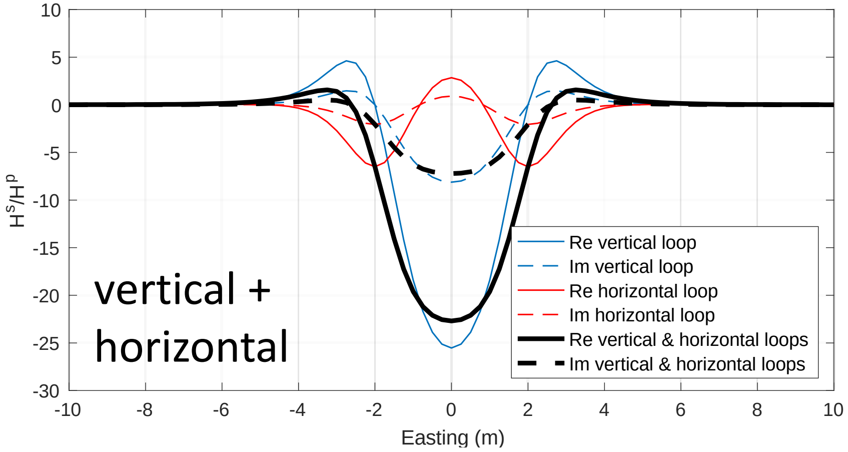


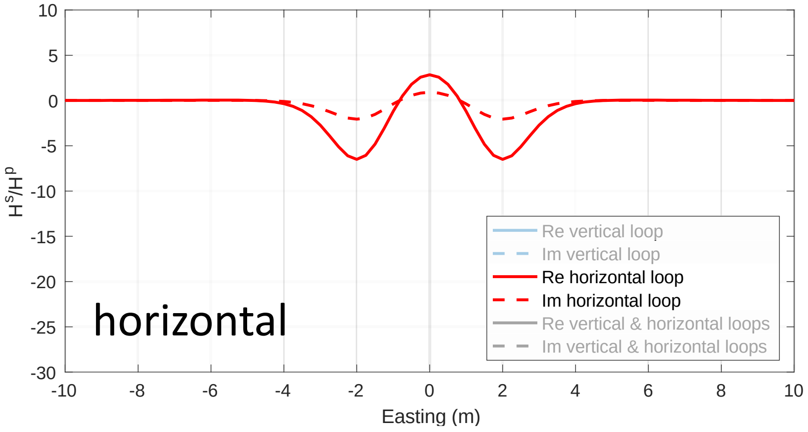


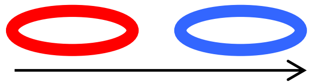


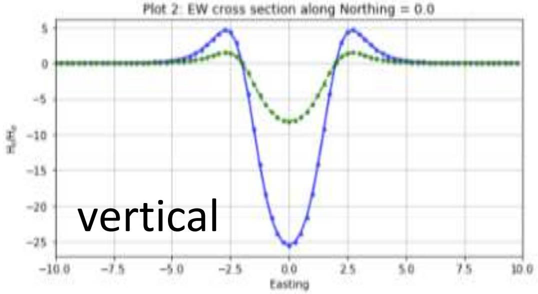
Equiaxed Target

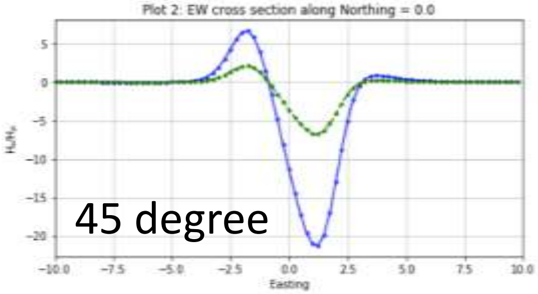


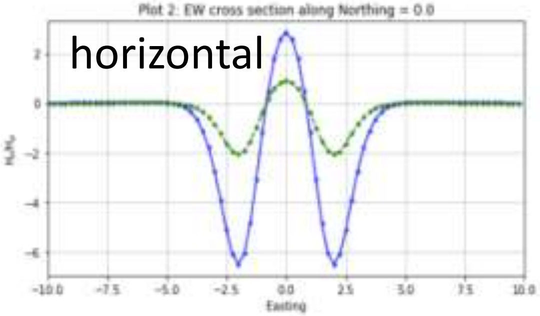


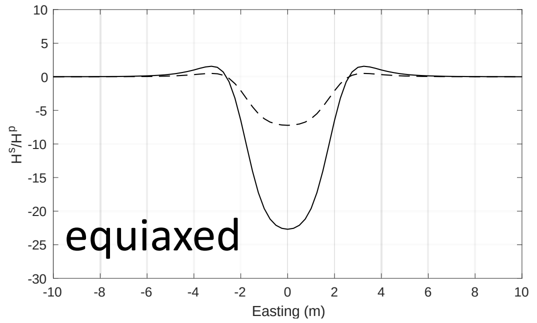


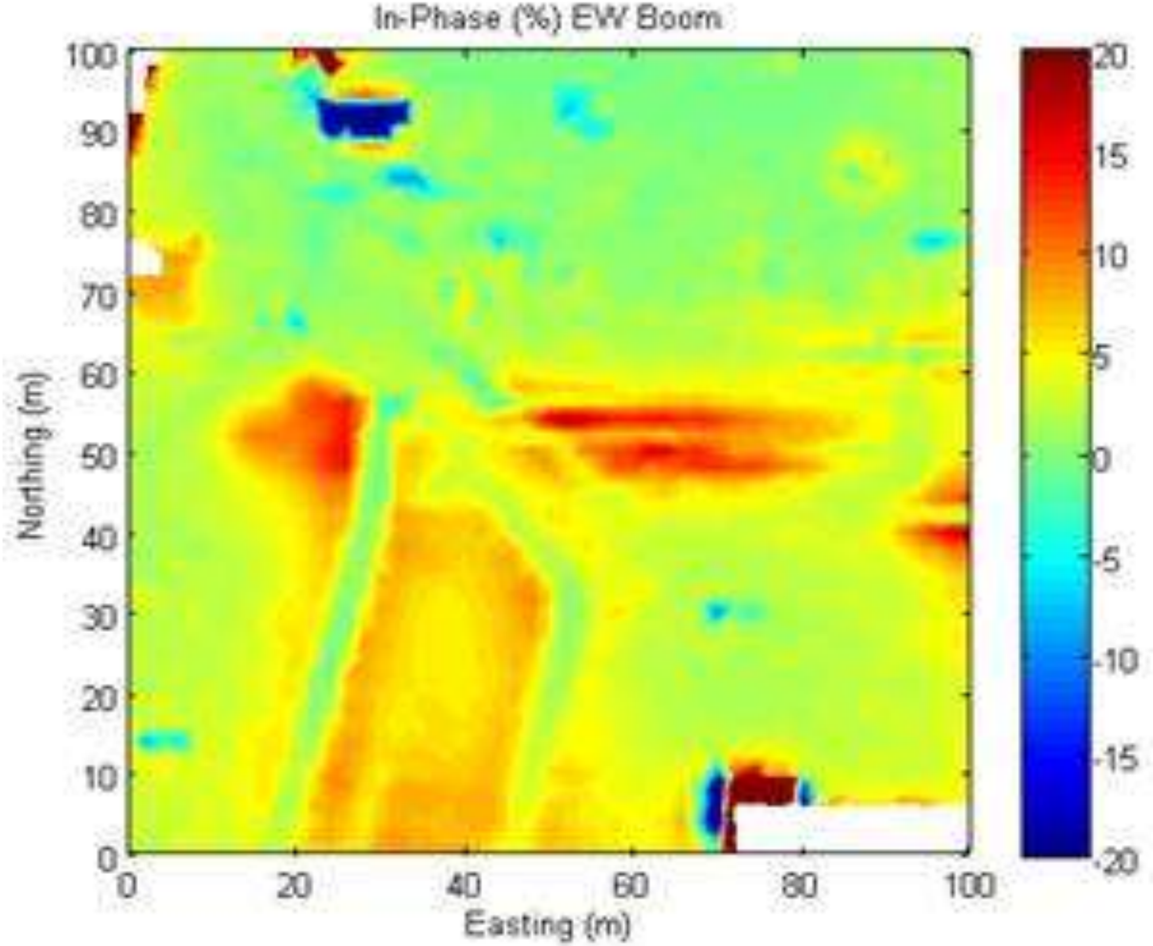












Question: Can you find those features on the data map and infer the geometry and orientations of the targets?

Summary

- EM induction: Quasi-static
- Loop-loop system in FD: Three loop model
  - Ampere’s Law and Faraday’s Law
  - Coupling
  - Induction number and response function
- EM-31 as an example
  - Positive or negative?
  - Compare in-phase with quadrature