

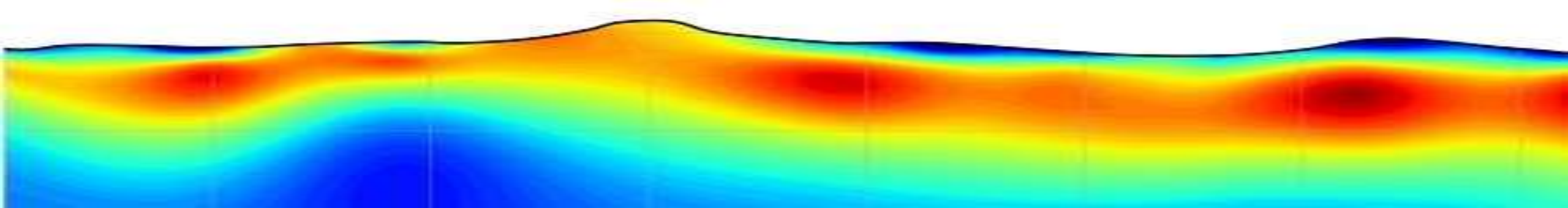
ESS302 Applied Geophysics II

Gravity, Magnetic, Electrical, Electromagnetic and Well Logging

Electrical 1: Theory

Instructor: Dikun Yang

Feb – May, 2019



well logging
(everything in borehole)

Maxwell Equations

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

zero frequency

low frequency

high frequency

steady state

quasi-static state

EM wave

mechanical wave

magnetic

gravity

potential field

electrical

electromagnetic (induction)

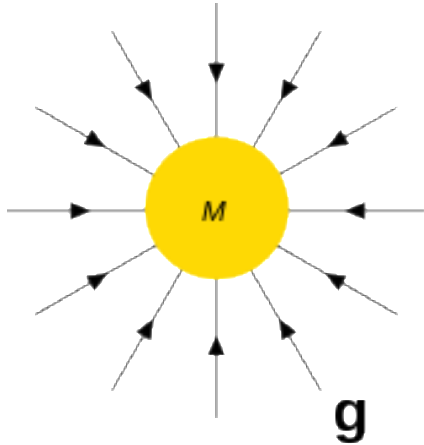
electrical conductivity/resistivity

electromagnetic (geo-radar)

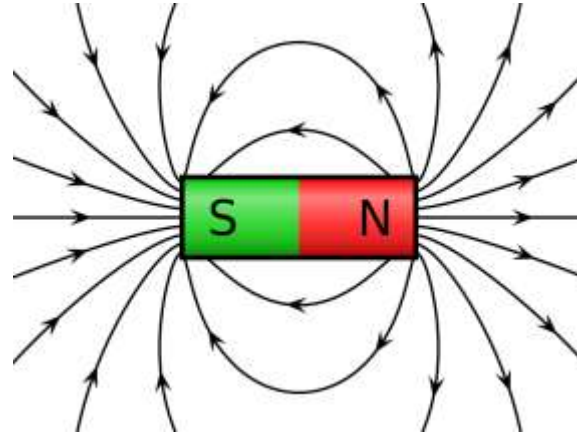
seismic

wave phenomena

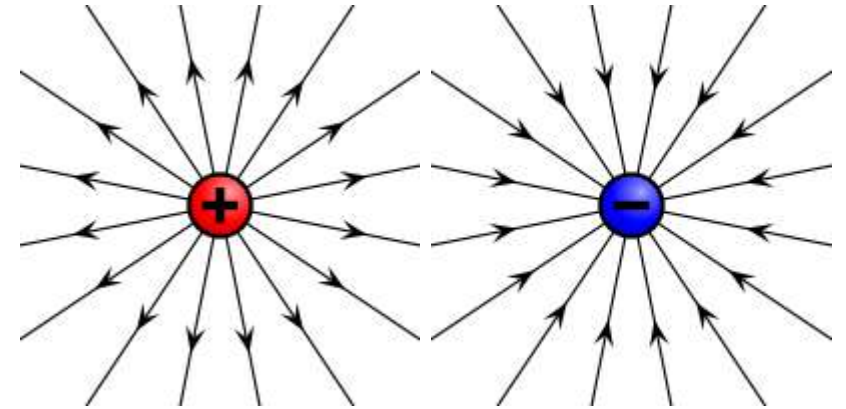
Charge, Force, Field, Potential



- Only positive charge (mass)
- Measure field to infer charge distribution
- External excitation: None (passive)

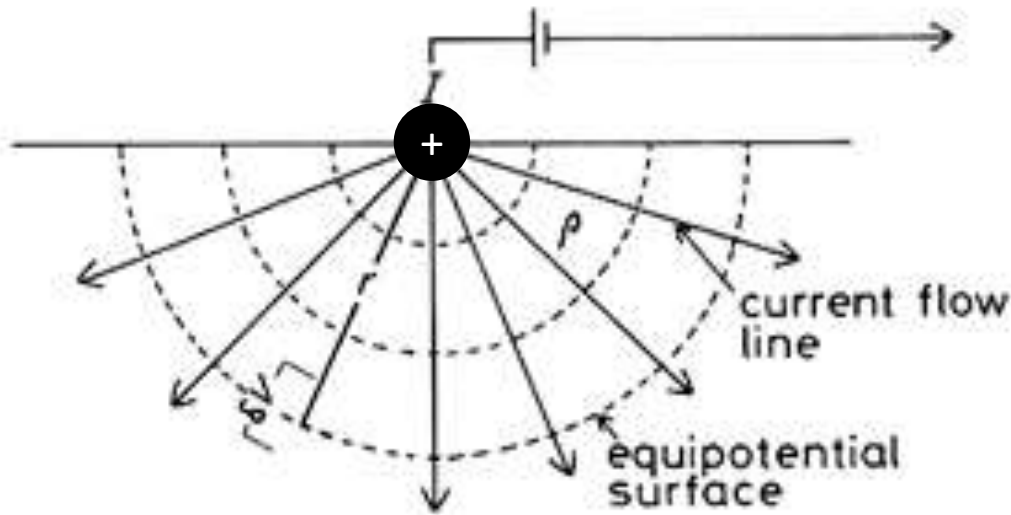


- Positive and negative charge but they have to be bounded as dipole (no monopole)
- Measure field to infer dipole distribution or susceptibility that gives rise to dipole distribution
- External excitation: geomagnetic field (passive)

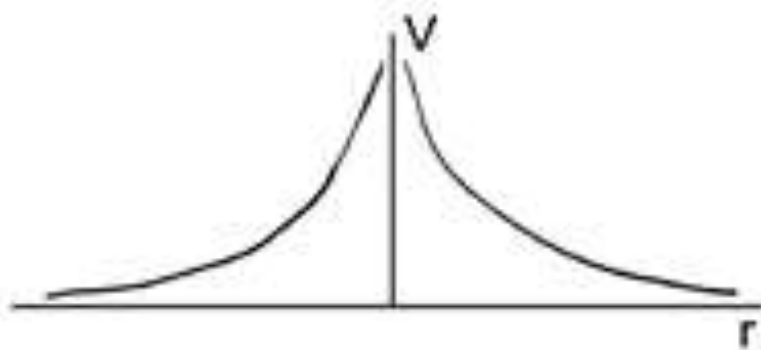


- Positive and negative charge that can be arbitrarily located
- Measure field/potential to infer dipole distribution or resistivity that gives rise to charge distribution
- External excitation: artificially injected electrical injection (active)

Electrical Potential



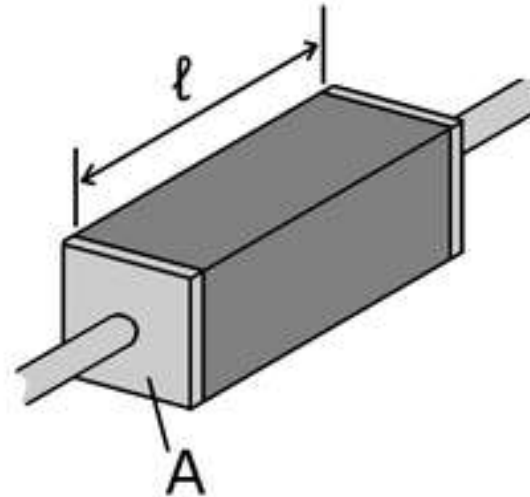
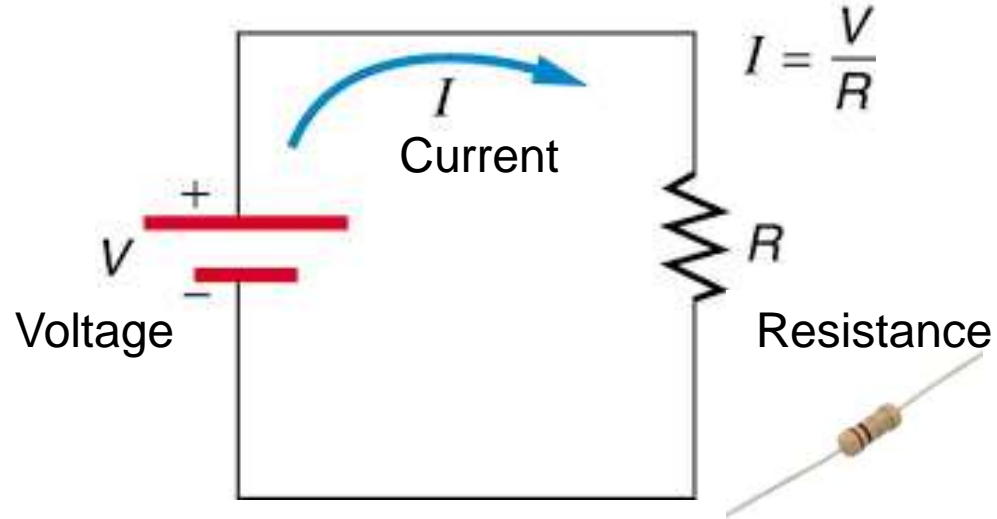
- Current flows radially outwards
- A positive charge at injection point
- Electrical potential decays as $1/r$



$$V(r) = \frac{I}{2\pi\sigma r} = \frac{I\rho}{2\pi r}$$

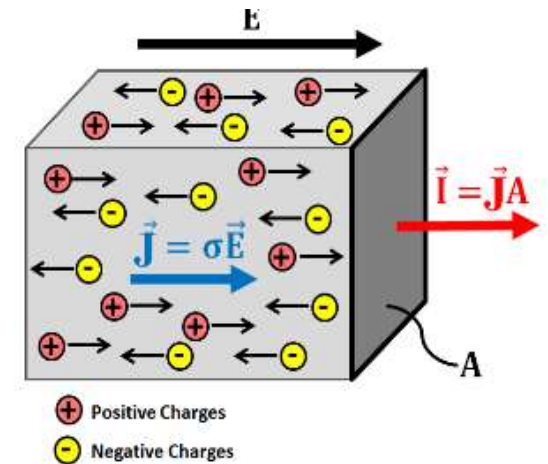
Electrical Resistivity or Conductivity

Ohm's law



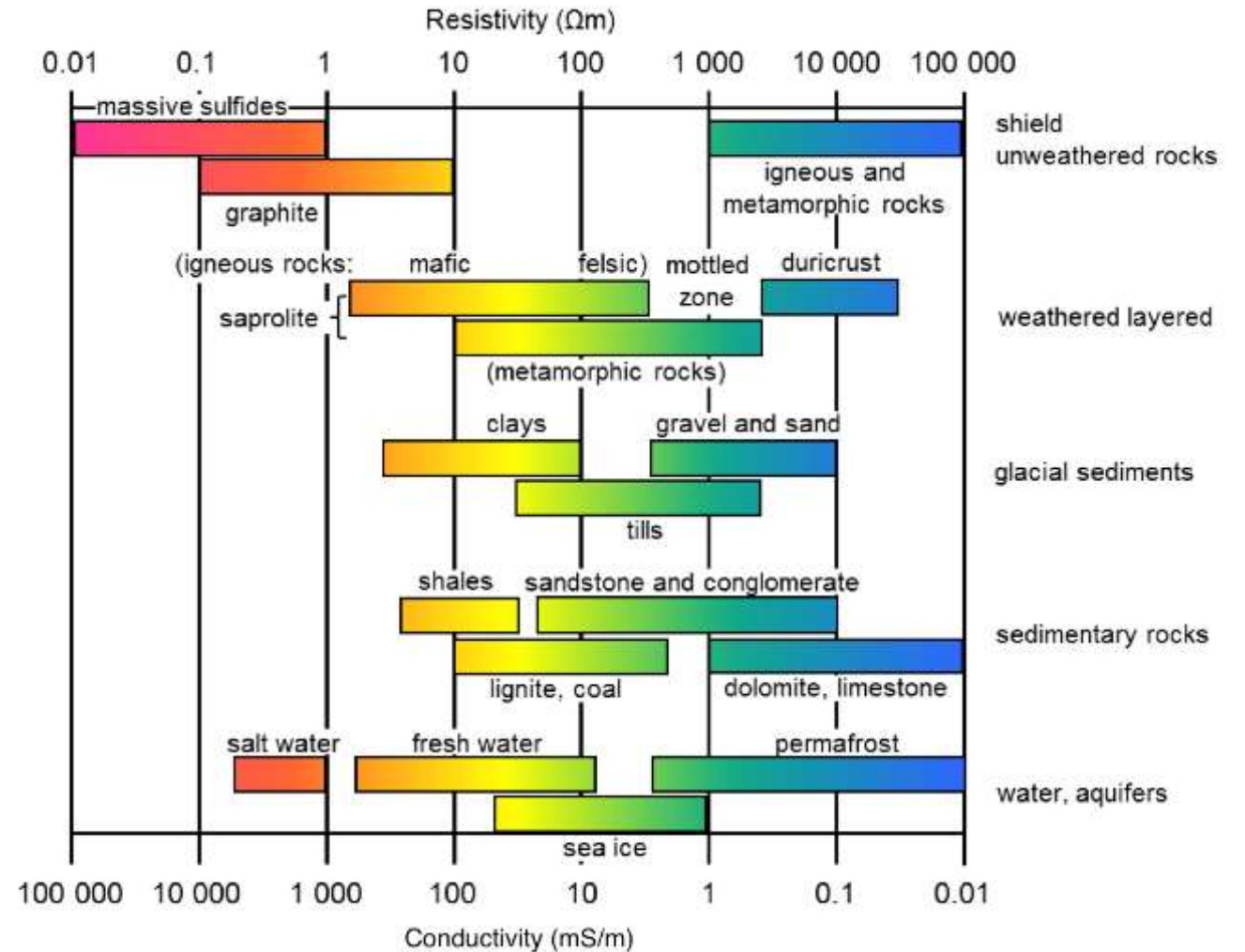
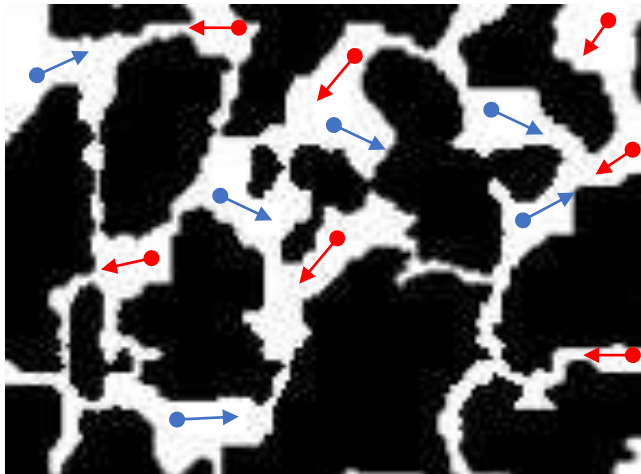
Resistivity (in Ωm) $\rho = R \frac{A}{\ell}$,

or conductivity (in S/m) $\sigma = \frac{1}{\rho}$.



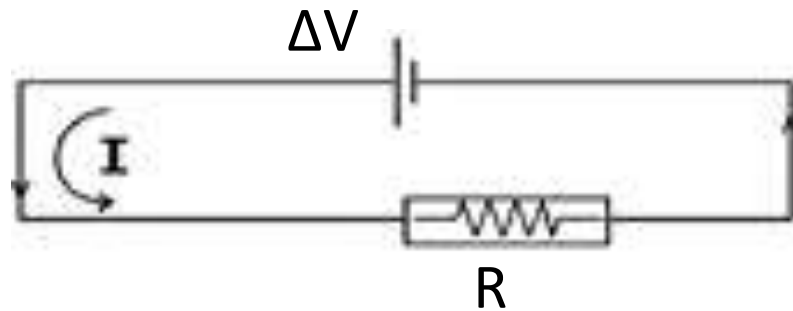
- Electrons
- Ions

Earth's Resistivity



Measurement of Resistance or Resistivity

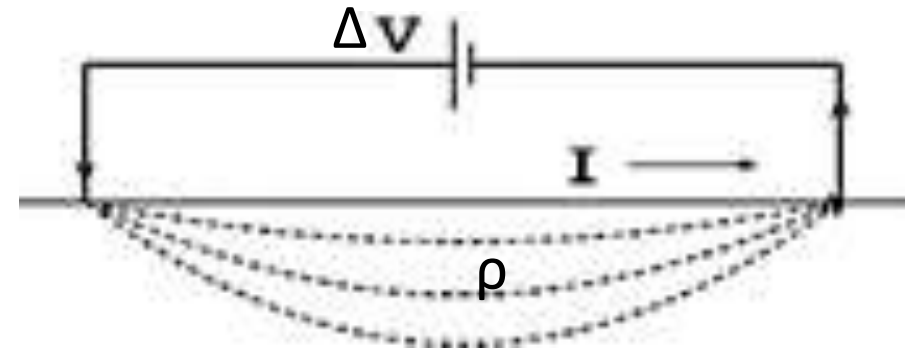
Electrical Circuit



Ohm's Law

$$\Delta V = IR$$

Earth Circuit



For the Earth:

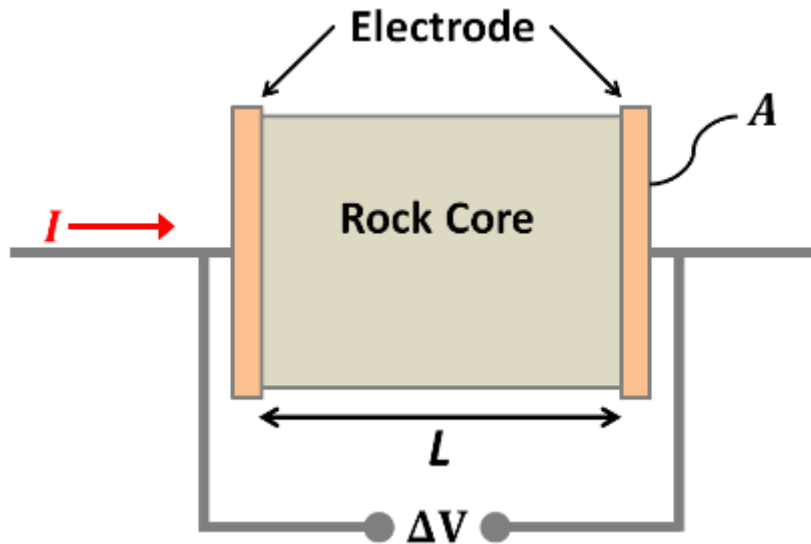
$$\Delta V = I\rho G$$

Depends on:

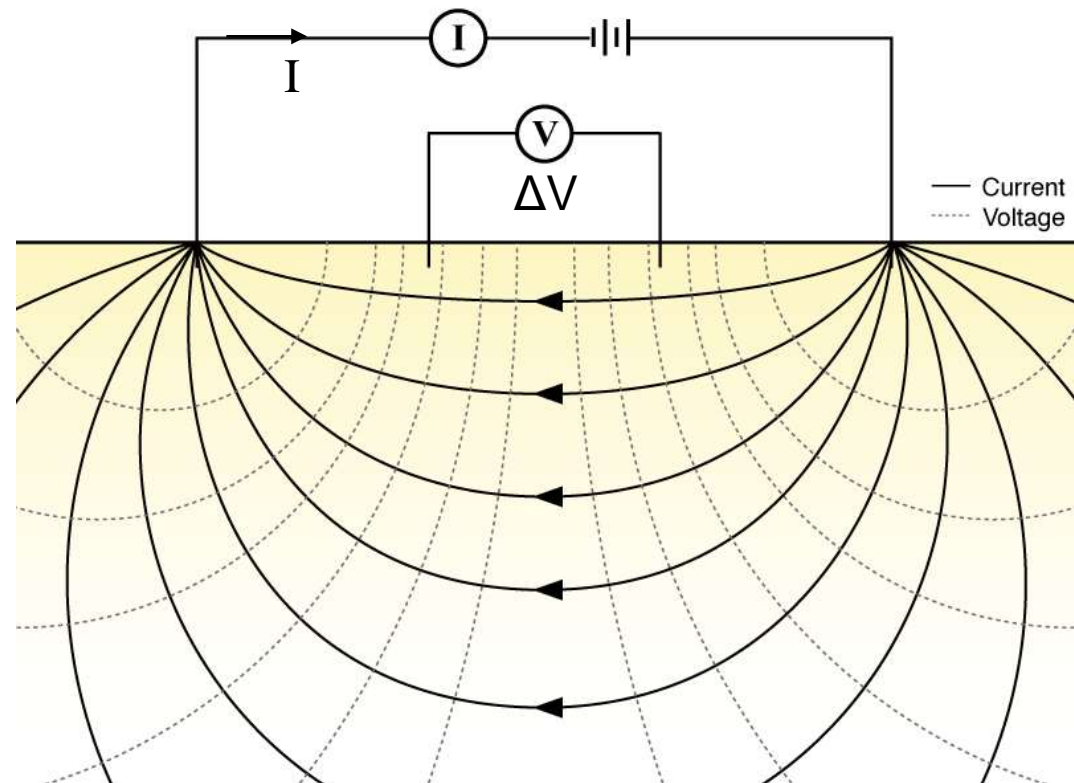
- 1) Earth's resistivity (ρ)
- 2) Geometry of electrodes (G)

Measuring Earth's Materials

Resistivity measurement in the lab

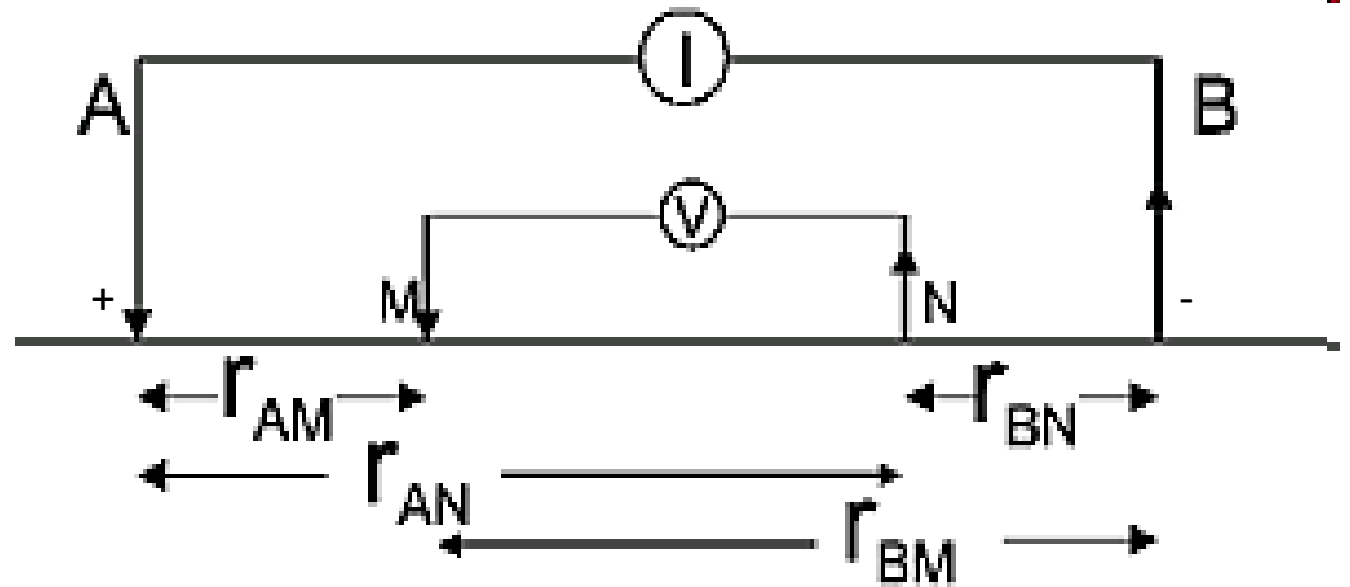
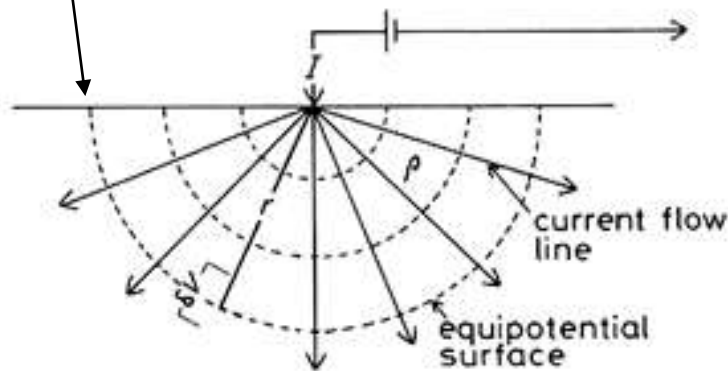


Resistivity survey in the field



Four-electrode Array

$$V(r) = \frac{I}{2\pi\sigma r} = \frac{I\rho}{2\pi r}$$



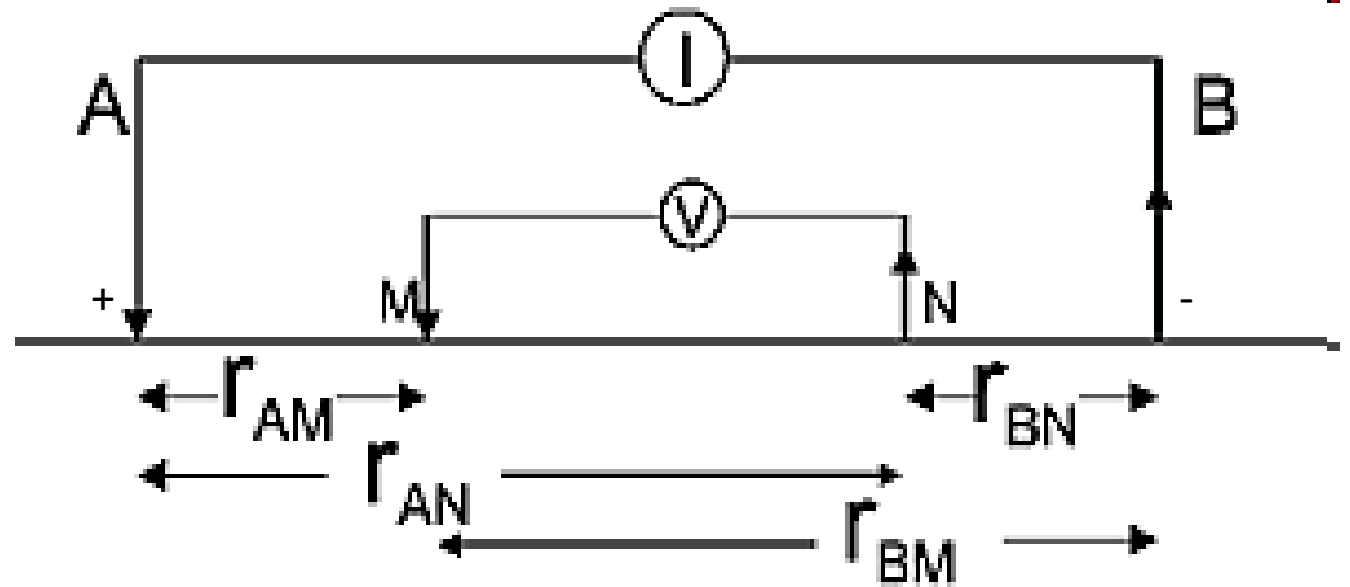
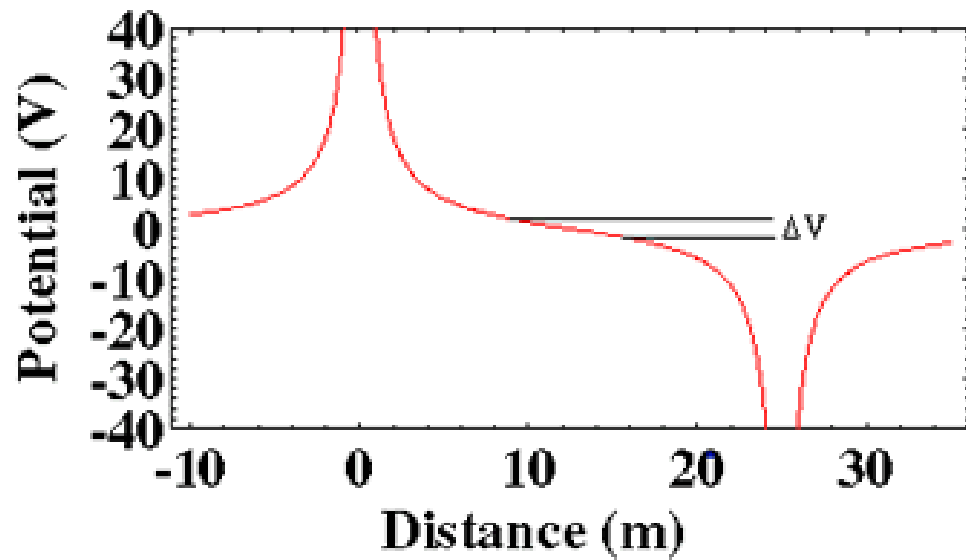
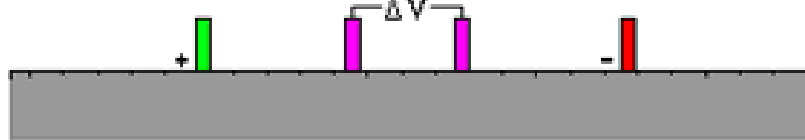
$$\Delta V = I\rho G = \frac{I\rho}{2\pi} \left\{ \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right\}$$

$$\rho = \frac{\Delta V}{IG}$$

Calculated earth's resistivity

Four-electrode Array

current electrode potential electrodes current electrode



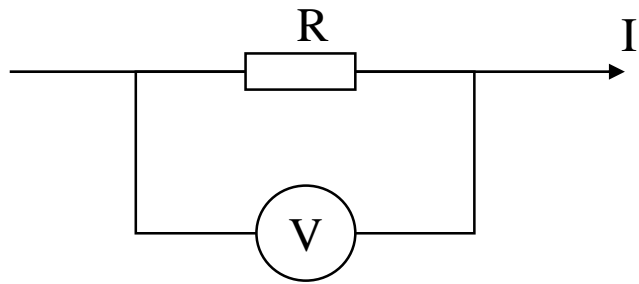
$$\Delta V = I\rho G = \frac{I\rho}{2\pi} \left\{ \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right\}$$

$$\rho = \frac{\Delta V}{IG}$$

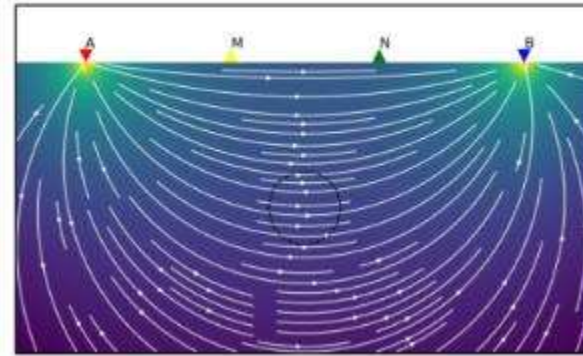
Calculated earth's resistivity

Inhomogeneous Earth

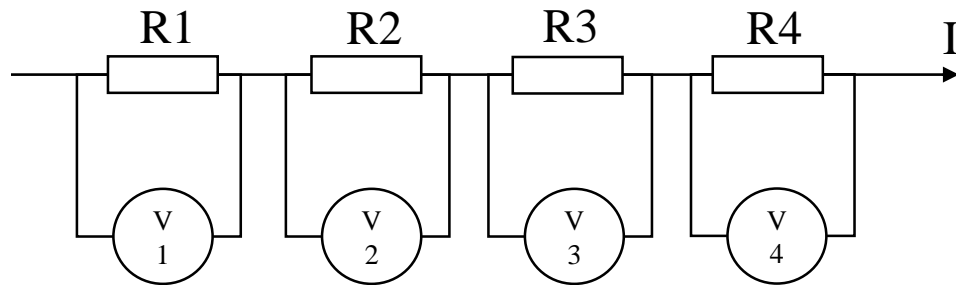
Uniform sample



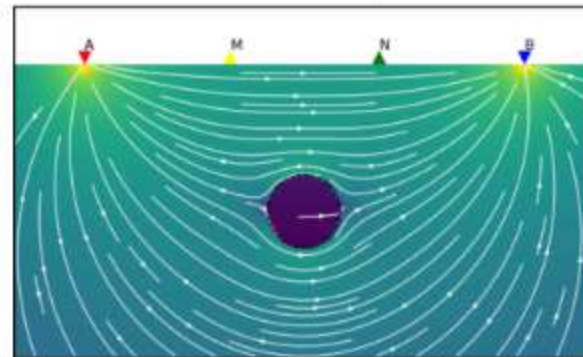
Homogenous earth



Non-uniform sample



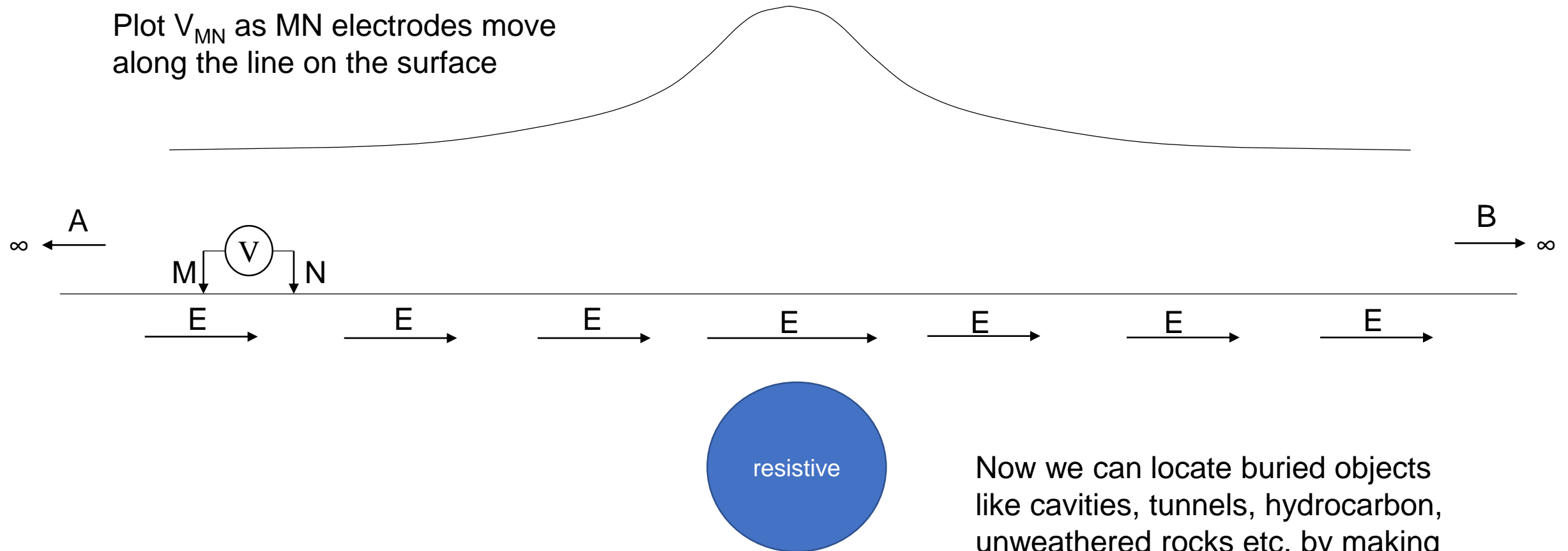
Resistive sphere



How would V_{MN} change if a resistor exists?

Finding a Sphere

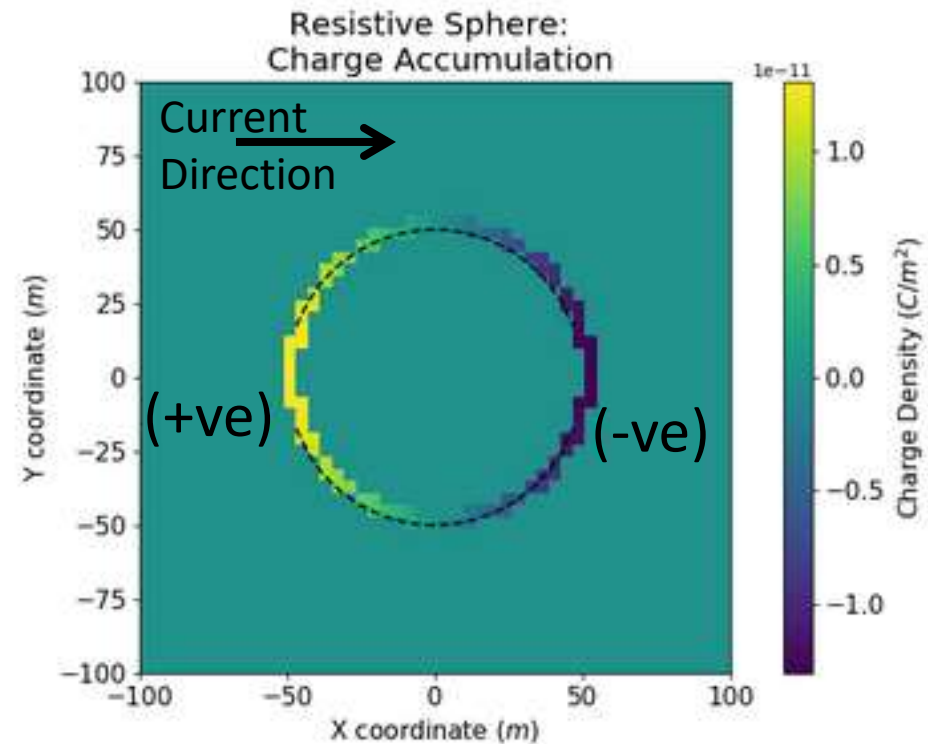
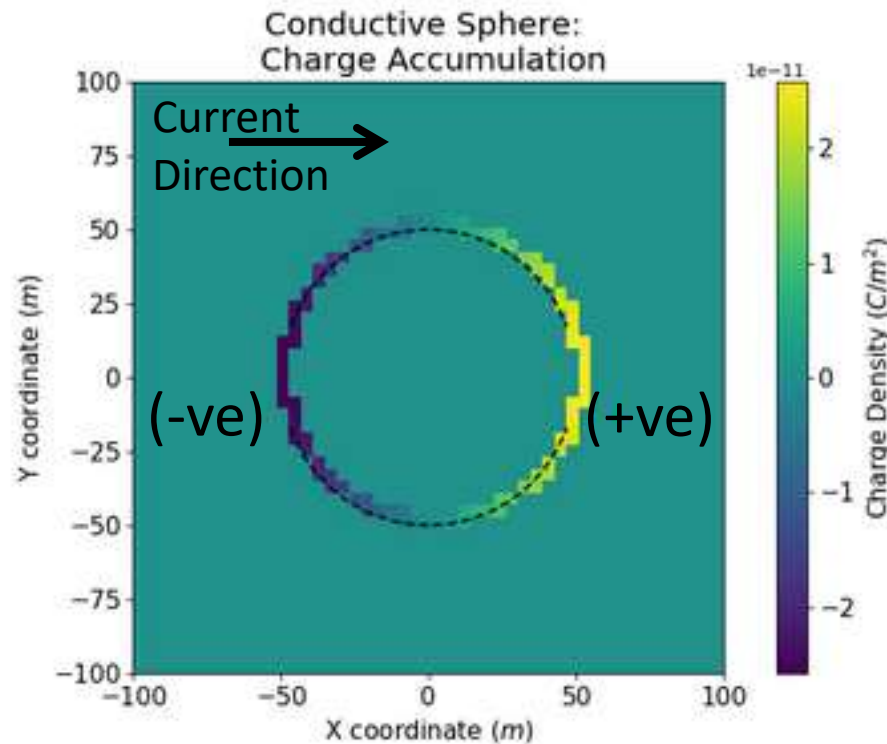
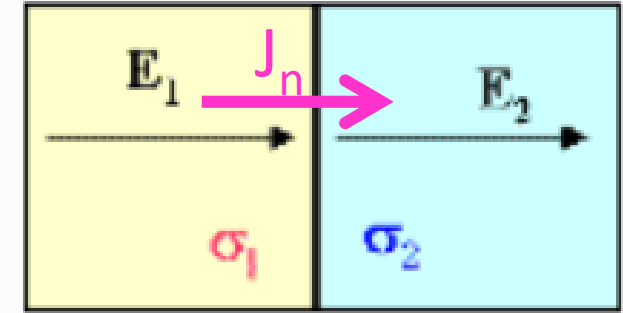
Plot V_{MN} as MN electrodes move along the line on the surface



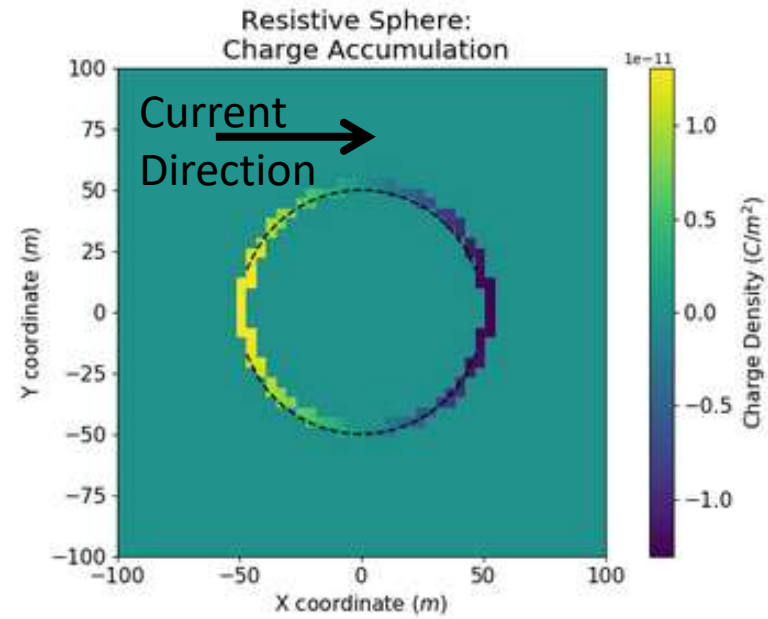
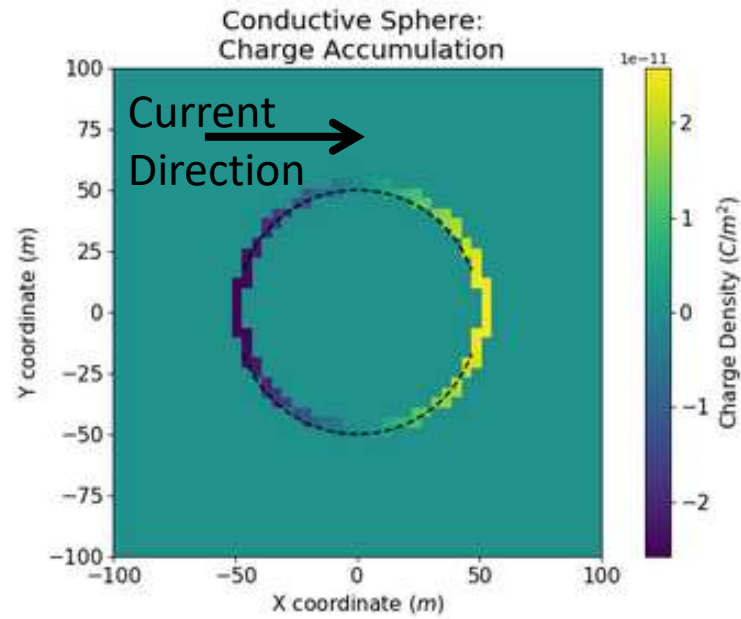
Now we can locate buried objects like cavities, tunnels, hydrocarbon, unweathered rocks etc. by making measurements on the surface!

In Terms of Charges

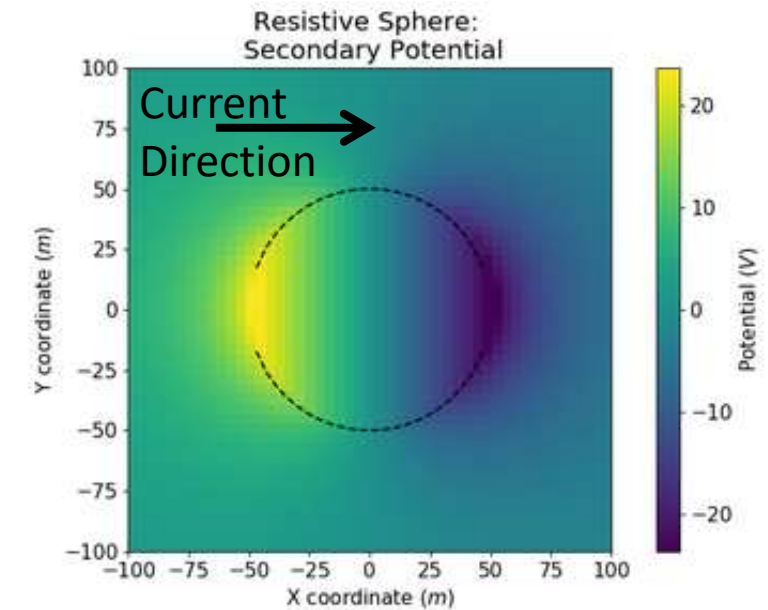
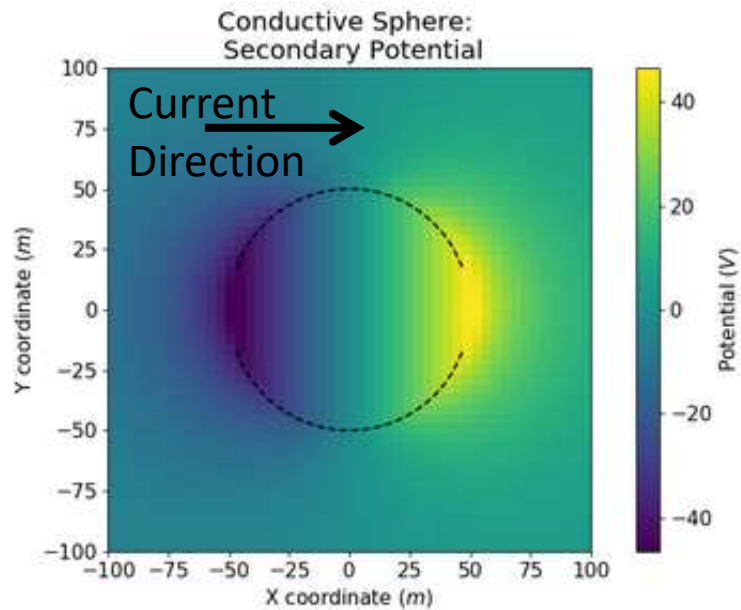
- Charges build-up on boundaries
From resistor into conductor \rightarrow negative charges build-up
From conductor into a resistor \rightarrow positive charges build-up



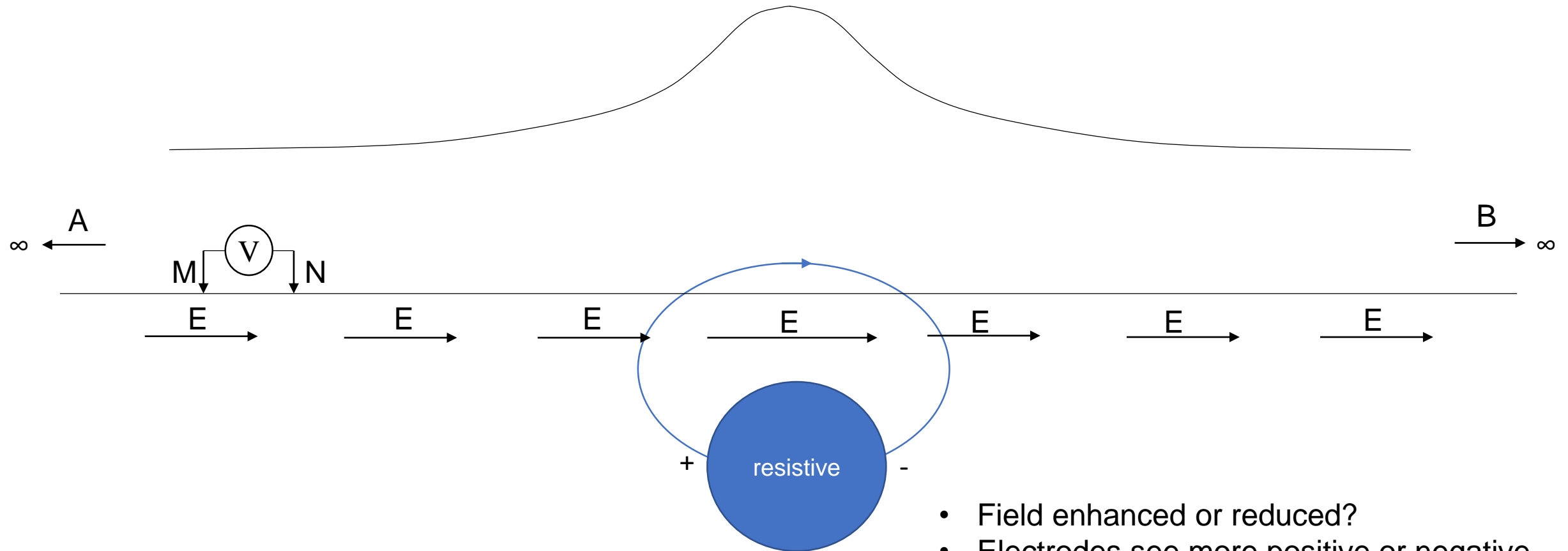
- Charges



- Secondary Potential



Finding a Sphere



- Field enhanced or reduced?
- Electrodes see more positive or negative charge?

File Edit View Insert Cell Kernel Widgets Help

Trusted

Python 3



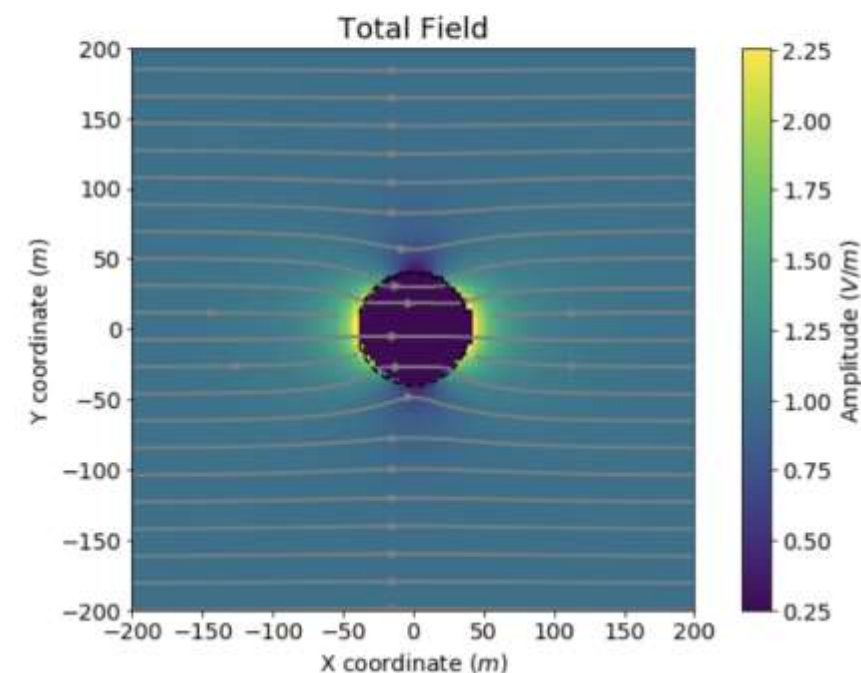
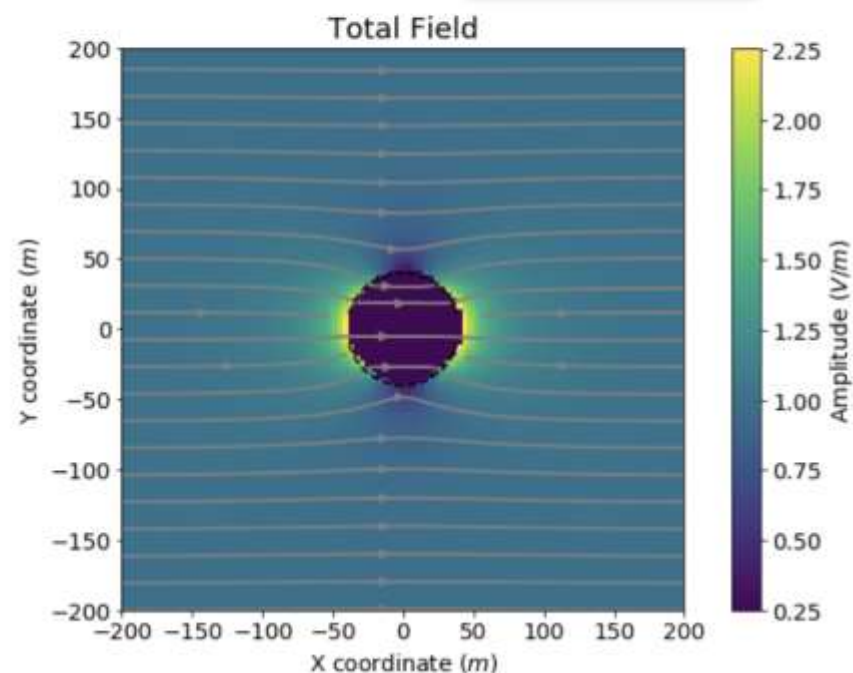
```
Figure2a=ToggleButtons(options=['Total','Secondary'],value = 'Secondary'),  
Figure2b=ToggleButtons(options=['Potential','ElectricField','CurrentDensity','ChargesDensity'],value = 'ElectricField')
```

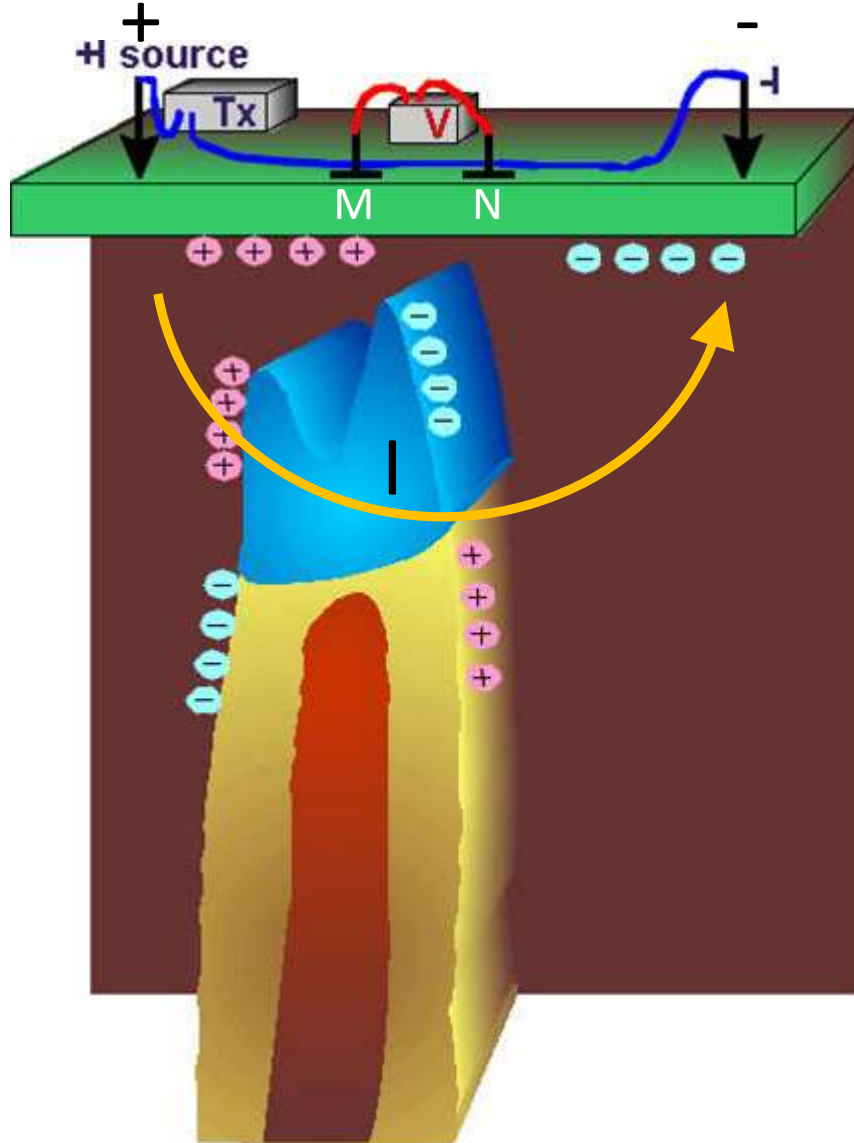
R 40.00

log_sig0 -3.00

log_sig1 -2.00

Figure1a	Configuration	Total	Secondary	
Figure1b	Potential	ElectricField	CurrentDensity	ChargesDensity
Figure2a	Total	Secondary		
Figure2b	Potential	ElectricField	CurrentDensity	ChargesDensity



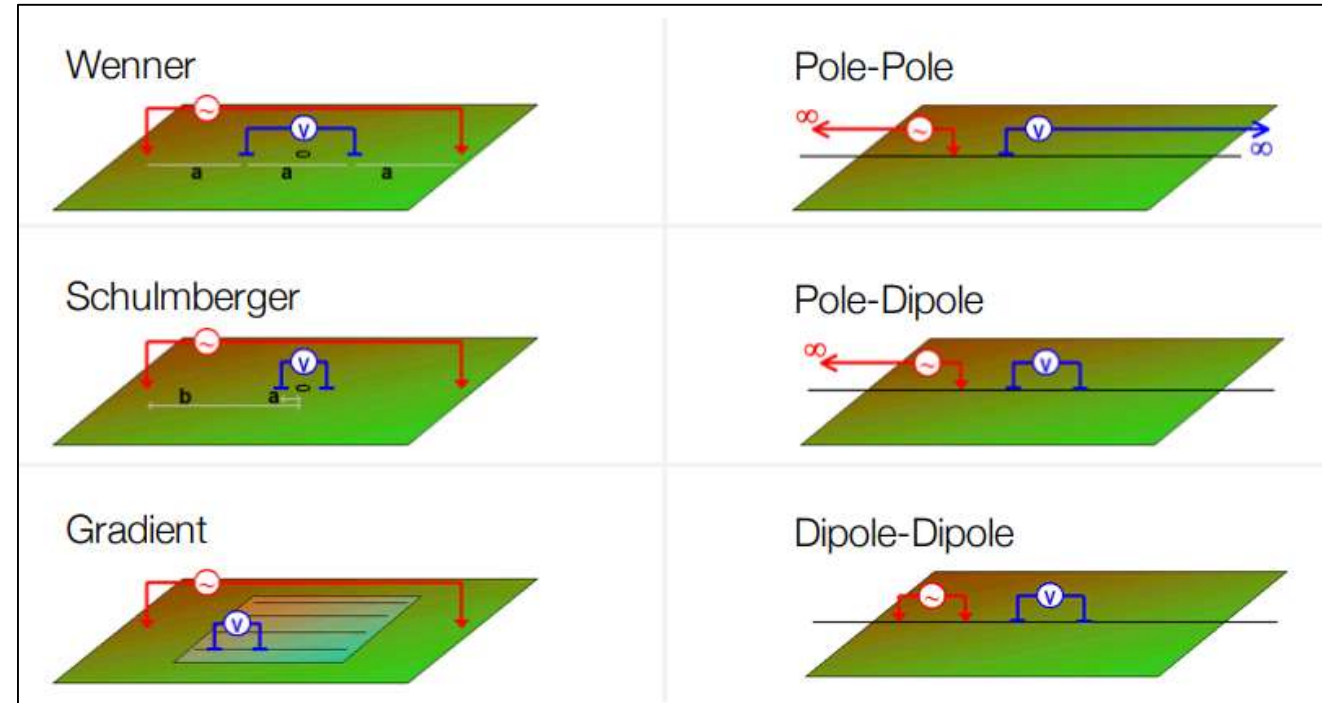
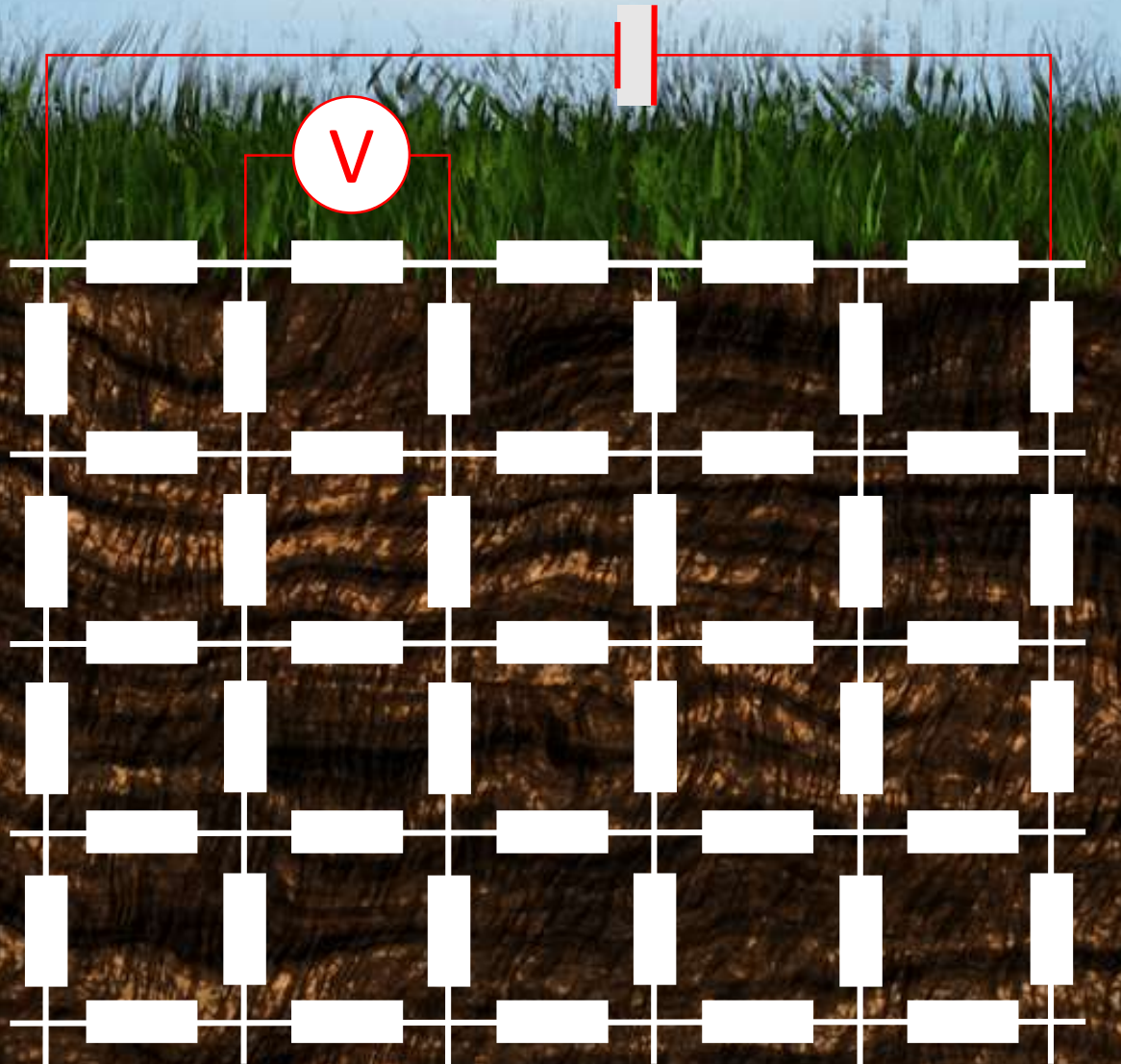


Physical Properties

Rock type	Ohm-m
Overburden	12
Host rock	200
Gossan	420
Mineralization (pyritic)	0.6
Mineralization (pyrrhotite)	0.6

- Is anomalous potential +ve or -ve at location N?
- Is $\Delta V = V_N - V_M$ +ve or -ve?

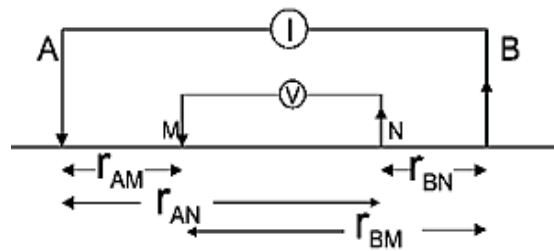
Electrode Arrays: A Circuit Perspective



How to gain:

- Lateral resolution - Profiling
- Depth (vertical) resolution - Sounding

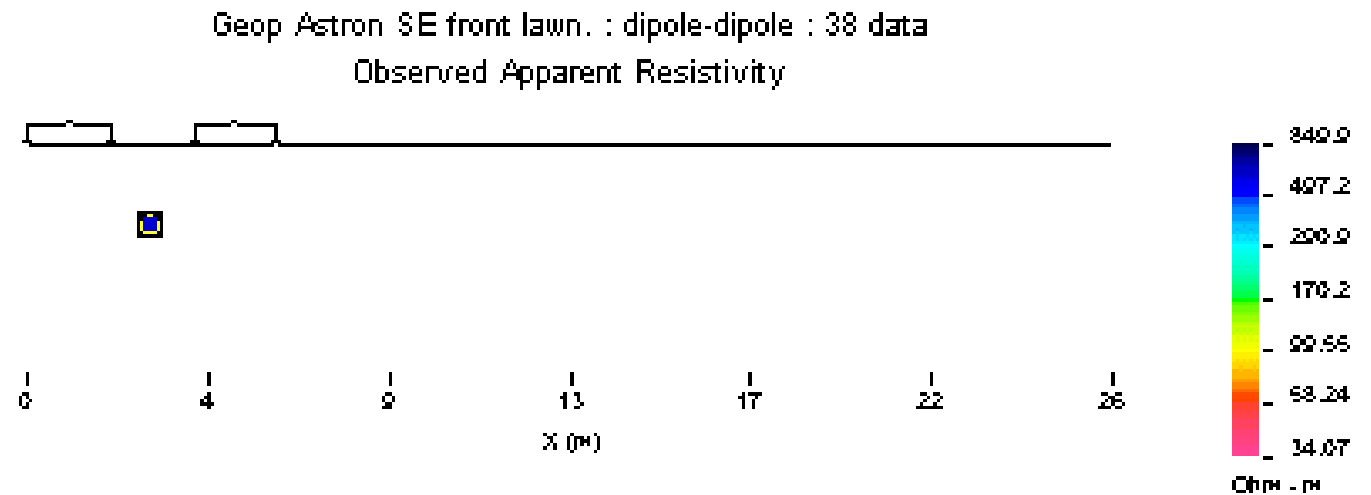
Apparent Resistivity on Pseudo-section



$$\Delta V = I\rho G = \frac{I\rho}{2\pi} \left\{ \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AN}} + \frac{1}{r_{BN}} \right\}$$

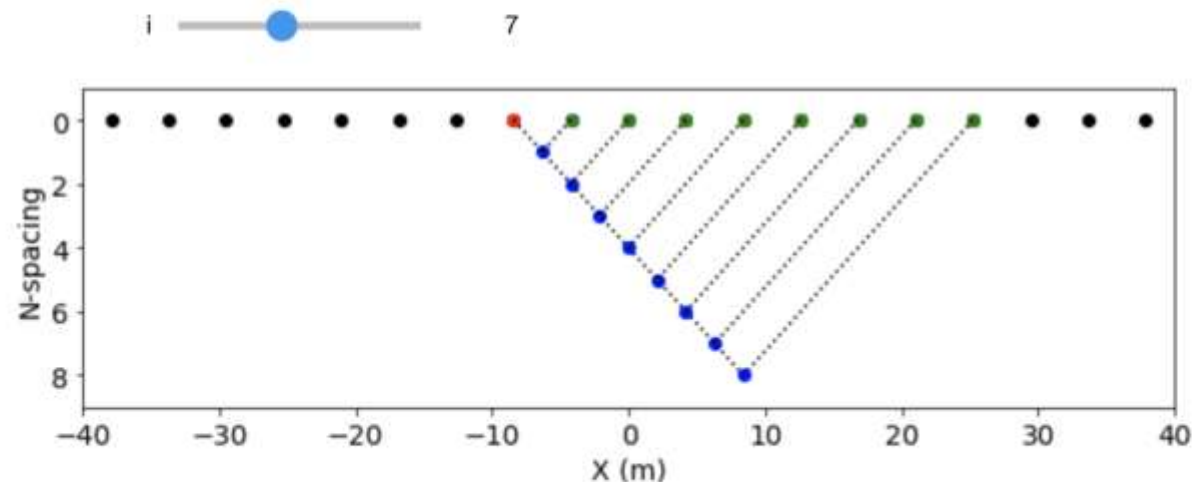
$$\rho = \frac{\Delta V}{IG}$$

True resistivity or apparent resistivity



Useful in revealing lateral and vertical variation in resistivity
Transform of data – Unit in Ωm but still data!

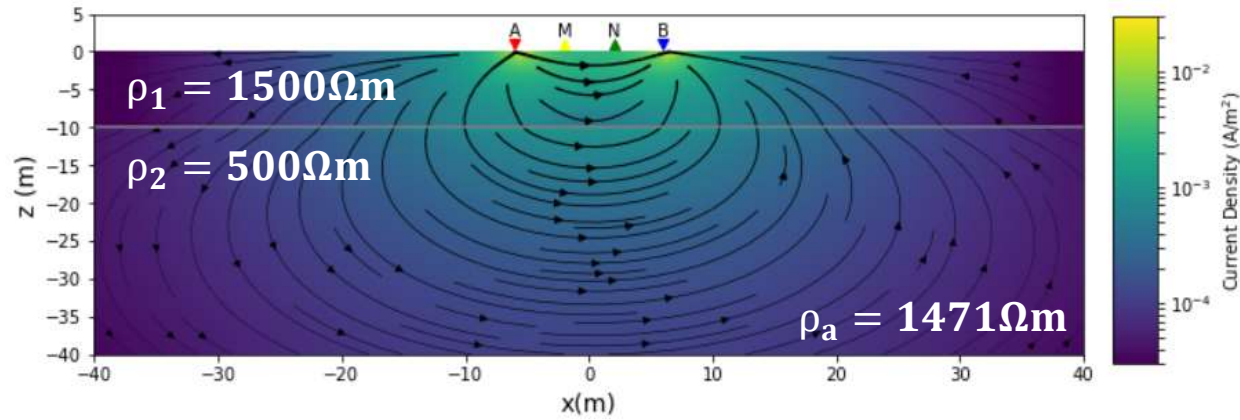

```
In [5]: out = MidpointPseudoSectionWidget()
display(out)
```



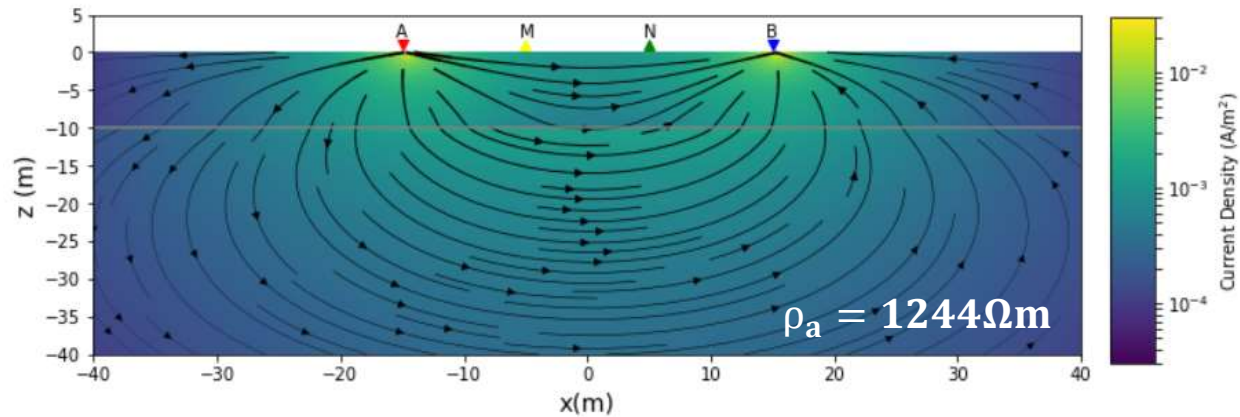
- ρ_1 : Resistivity of the halfspace
- ρ_2 : Resistivity of the cylinder
- **xc**: x location of cylinder center
- **zc**: z location of cylinder center
- **r**: radius of cylinder
- **surveyType**: Type of survey
- **Run Interact**: Use this button to update your plot

Note: The numerical results shown in this plot are generated from a 2d code such that the source is a line of current. This greatly speeds up the computation. Accurate potentials obtained from point current sources require the 2.5D code.

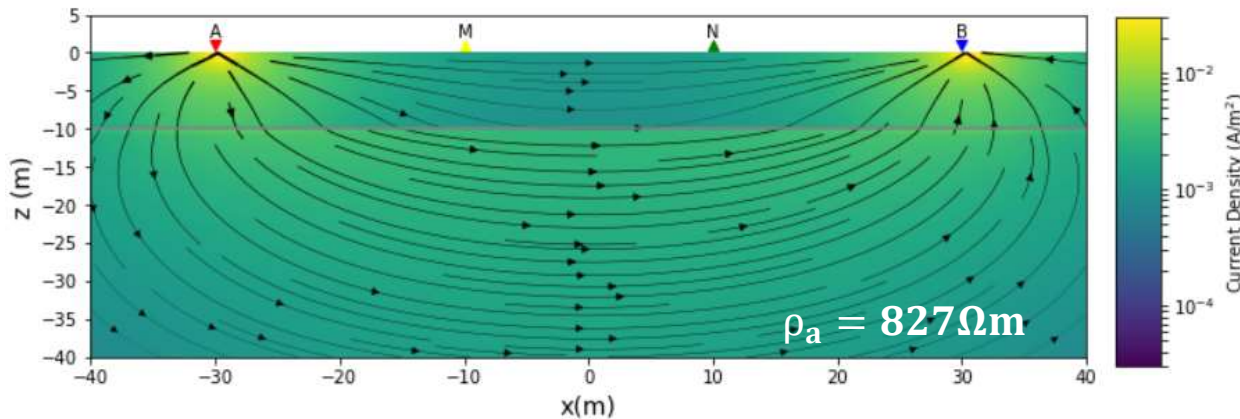
```
In [6]: out = DC2DPseudoWidget()
display(out)
```

- Most currents near electrodes
- Only sees top layer



- Currents in lower layer
- Lower apparent resistivity



- Apparent resistivity sensitive to lower layer

Summary

- Electrical: Charge, field, force, potential
- Electrical resistivity/conductivity
- Ohm's law applied to the Earth: Four-electrode array (ABMN)
- Understanding DC resistivity data
 - Charges build-up
 - Circuit
- Electrode arrays and sensitivity
- Apparent resistivity