Section A

Sets

$\boldsymbol{ID}$	Set of Import Depots (ID)
$\boldsymbol{LVC}$	Set of Local Vaccination Centres (LVC)
$\boldsymbol{CCD}$	Set of Census Collection Districts (CCD)
$\boldsymbol{D}$	Set of Vaccine Delivery Arcs from ID to LVC
$\boldsymbol{R}$	Set of routes a citizen can travel along between CCDs and LVCs
$\boldsymbol{T}$	Set of Weeks

Data

$\boldsymbol{ImportC_i}$	Cost to import vaccines to $\boldsymbol{i} \in \boldsymbol{ID}$
$\boldsymbol{P_c}$	Population of $c \in C$
$\boldsymbol{DelC}$	Cost to deliver vaccines from an ID to an LVC per kilometre per dose
$\boldsymbol{DelL_d}$	Length in kilometres of $d \in D$
$\boldsymbol{o_d} \in \boldsymbol{ID}$	Origin ID of vaccine travelling along $d \in D$
$\boldsymbol{v_d} \in \boldsymbol{LVC}$	Destination of vaccine travelling along $d \in D$
$\boldsymbol{DriveL_r}$	Length in kilometres of $r \in R$
$\boldsymbol{DriveC}$	Cost per kilometre for a citizen to travel
$\boldsymbol{t_r} \in \boldsymbol{LCD}$	LVC that a citizen is travelling too along $r \in R$
$\boldsymbol{f_r} \in \boldsymbol{CCD}$	CCD that a citizen is travelling from along $r \in R$
$\boldsymbol{Cap}$	Capacity of Vaccines which can be stored at an ID
$\boldsymbol{TMax}$	Maximum number of vaccine doses which can be administered at an LVC
$\boldsymbol{WMax}$	Maximum number of vaccine doses that can be delivered per week at an LVC
$\boldsymbol{DelayC}$	Cost in delaying vaccines per person per week
$\boldsymbol{VaxDiff}$	Maximum allowable difference in the fraction of population vaccinated at CCDs

Variables

$\boldsymbol{x_{d,t}}$	Number of vaccine doses being delivered along $\boldsymbol{d} \in \boldsymbol{D}$ at $\boldsymbol{t} \in \boldsymbol{T}$
$\boldsymbol{y_i}$	Number of vaccine doses being imported to $i \in ID$
$\boldsymbol{z_{r,t}}$	Number of citizens travelling to get their vaccine along $r \in R$ at $t \in T$
$\boldsymbol{s_{l,t}}$	Number of vaccine doses stored at $l \in LVC$ at $t \in T$
$\boldsymbol{unvax_{c,t}}$	Number of citizens from $c \in CCD$ that are unvaccinated at $t \in T$

Objective

$$\min \sum_{i \in I} ImportC_i \times y_i + \sum_{d \in D} \sum_{t \in T} DelC \times DelL_d \times x_{d,t} + \sum_{r \in R} \sum_{t \in T} DriveC \times DriveL_r \times z_{r,t} + \sum_{c \in C} \sum_{t \in T} DelayC \times unvax_{c,t}$$

Constraints

Ensuring the number of vaccine doses imported to each ID does not exceed capacity:

$$\boldsymbol{y_i \le Cap} \qquad \forall\, i \in ID$$

Ensuring the number of vaccines doses leaving an ID does not exceed the number of vaccines there:

$$\sum_{\substack{t \in T}} \sum_{\substack{d \in D \\ s.t\ o_d = i}} x_{d,t} \le y_i \qquad \forall\, i \in ID$$

Ensuring the number of vaccines stored at an LCD is equal to the number of vaccines stored from the previous week (none if it is the first week) plus the incoming vaccines from IDs minus the number of vaccines administered that week:

$$s_{l,t} = s_{l,t-1} + \sum_{\substack{d \in D \\ s.t\ v_d = l}} x_{d,t} - \sum_{\substack{r \in R \\ s.t\ t_r = l}} z_{r,t} \qquad \forall\, l \in LVC, \forall\, t > 0$$

$$s_{l,t} = \sum_{\substack{d \in D \\ s.t\ v_d = l}} x_{d,t} - \sum_{\substack{r \in R \\ s.t\ t_r = l}} z_{r,t} \qquad \forall\, l \in LVC, t = 0$$

Ensuring the number of vaccines administered at an LCD over the 6 weeks does not exceed capacity:

$$\sum_{\substack{r \in R \\ s.t\ t_r = l}} \sum_{t \in T} z_{r,t} \le \boldsymbol{TMax} \qquad \forall\, l \in LVC$$

Ensuring the number of vaccines delivered at each LVC per week does not exceed the maximum amount:

$$\sum_{\substack{r \in R \\ s.t\ t_r = l}} z_{r,t} \le \boldsymbol{WMax} \qquad \forall\, l \in LVC, \forall\, t \in T$$

Ensuring the entire population of a CCD is vaccinated:

$$\sum_{\substack{r \in R \\ s.t\ f_r = c}} \sum_{t \in T} z_{r,t} = \boldsymbol{P_c} \qquad \forall\, c \in CCD$$

Ensuring that the number of people unvaccinated is equal to the number of people unvaccinated from the previous week (the entire population if it’s the first week) minus the number of vaccines administered to that CCD in the given week:

$$unvax_{c,t} = unvax_{c,t-1} - \sum_{\substack{r \in R \\ s.t\ f_r = c}} z_{r,t} \qquad \forall\, c \in CCD, \forall\, t > 0$$

$$unvax_{c,t} = p_c - \sum_{\substack{r \in R \\ s.t\ f_r = c}} z_{r,t} \qquad \forall\, c \in CCD, t = 0$$

Ensuring that the difference in the fraction of people unvaccinated at any given CCD at a particular time does not exceed 0.1:

$$-\mathbf{0.1} \leq \frac{\boldsymbol{unvax}_{\boldsymbol{c_1,t}}}{\boldsymbol{p}_{\boldsymbol{c_1}}} - \frac{\boldsymbol{unvax}_{\boldsymbol{c_2,t}}}{\boldsymbol{p}_{\boldsymbol{c_2}}} \leq \mathbf{0.1} \qquad \forall\, c_1 \in CCD, \forall\, c_2 \in CCD, \forall\, t \in T$$

Non-Negativity Constraints:

$$\boldsymbol{x}_{\boldsymbol{d,t}} \geq \mathbf{0} \qquad \forall\, d \in D, \forall\, t \in T$$

$$\boldsymbol{y}_{\boldsymbol{i}} \geq \mathbf{0} \qquad \forall\, i \in ID$$

$$\boldsymbol{z}_{\boldsymbol{r,t}} \geq \mathbf{0} \qquad \forall\, r \in R, \forall\, t \in T$$

$$\boldsymbol{s}_{\boldsymbol{l,t}} \geq \mathbf{0} \qquad \forall\, l \in LVC, \forall\, t \in T$$

$$\boldsymbol{unvax}_{\boldsymbol{c,t}} \geq \mathbf{0} \qquad \forall\, c \in CCD, \forall\, t \in T$$

Section B

Thank you for reaching out in regards to planning your vaccination distribution strategy (VDS) in Pacific Paradise. The following report provides advice and the cost of your optimal distribution based upon the information provided to our firm in each one of your communications. Our findings are summarised below.

Communication 1

Using the information supplied in your first communication, the optimal cost of the vaccination distribution would be:

$14992146

You would be required to import the following number of vaccines to each import depot (ID):

Import Depot	Vaccine Doses
ID-A	0
ID-B	90023
ID-C	0

Since there is no maximum capacity for the number of imported vaccines at each ID, and it is cheaper to import vaccines to ID-B ($142/dose), all vaccines are imported to this location. Vaccines would be distributed from ID-B to the local vaccination centres (LVCs) as follows:

	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
Vaccines Delivered	11320	18846	12148	12442	7398	12911	8877	6081

Note that the number of vaccines administered at each LVC is the same as the number delivered.

Communication 2

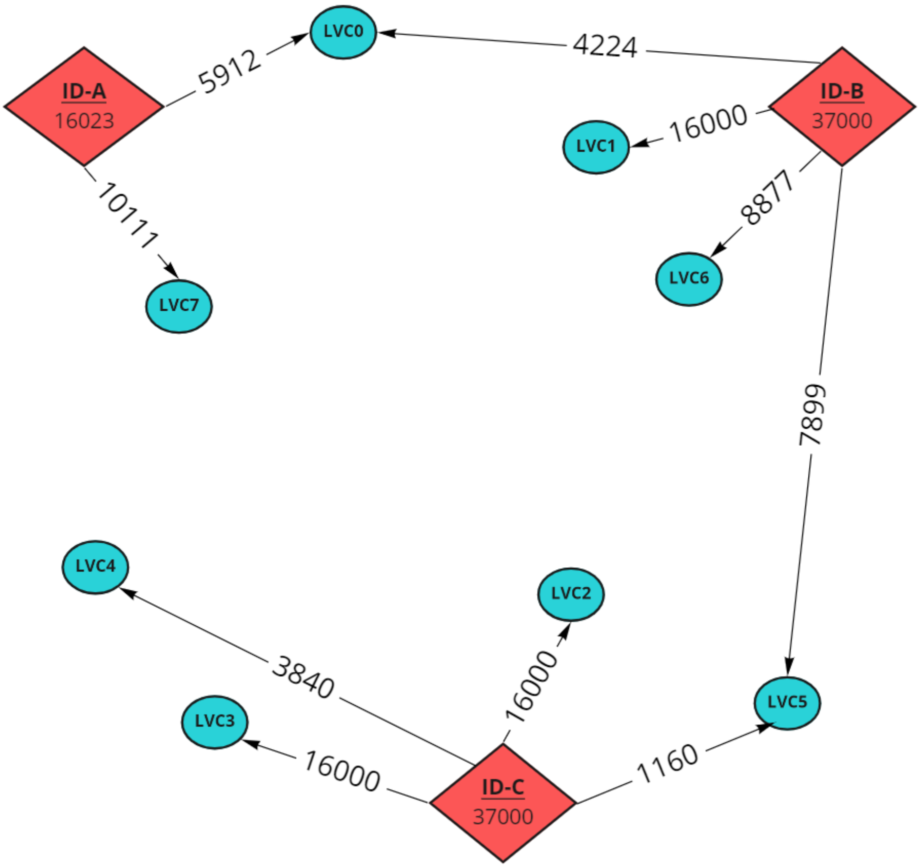
During your second communication, you indicated that each ID and LVC has a maximum capacity. Accounting for these maximum capacities, the optimal cost would be raised to:

$15550355

The primary reason for increased costs is that vaccines will now need to be imported to all three IDs to ensure none of them go over capacity:

Import Depot	Vaccine Doses
ID-A	16023
ID-B	37000
ID-C	37000

ID-B and IB-C import to their maximum capacity as they have a cheaper importation cost when compared with ID-A. It's worth noting that importing the vaccines to all three IDs will reduce the vaccine delivery cost slightly. This is because delivery can now occur between IDs and LVCs that neighbour each other as shown in the below diagram:



The number of vaccines that would be administered from each LVC is also shown in the below table:

	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
Vaccines Administered	10136	16000	16000	16000	3840	9059	8877	10111

Communication 3

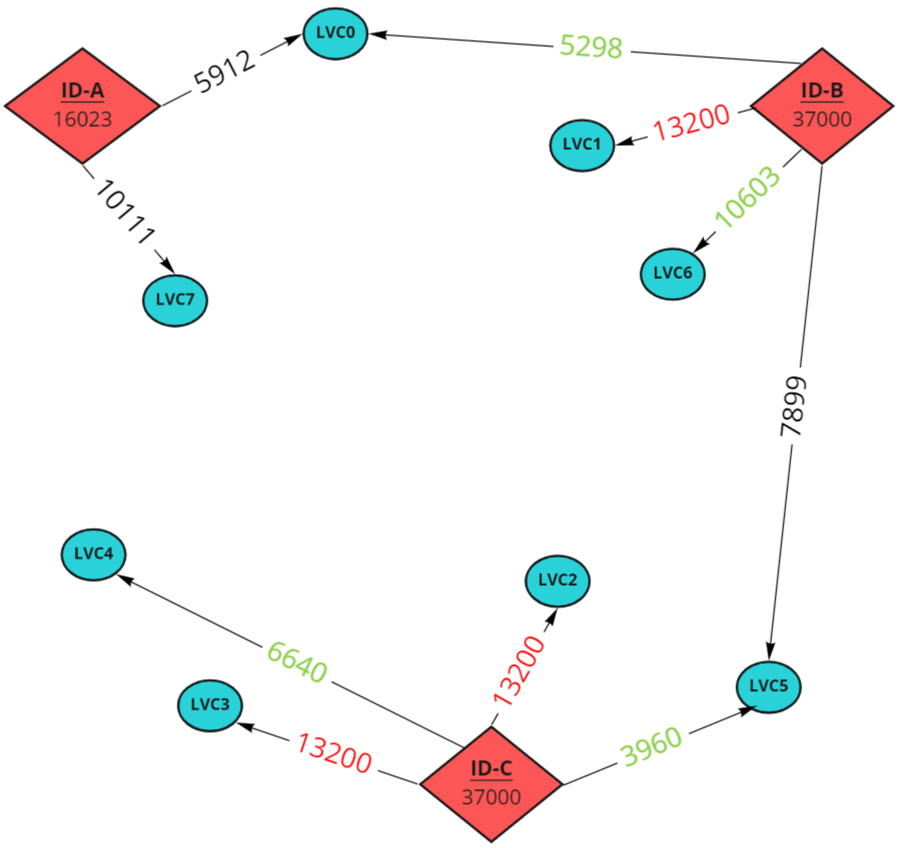
In communication 3 you asked our firm to provide a 6-week plan for distribution. You also indicated that your LVCs could only administer 2200 vaccine doses per week. The optimal cost under these constraints would be:

$15569386

These new requirements did not change the importation of vaccines:

Import Depot	Vaccine Doses
ID-A	16023
ID-B	37000
ID-C	37000

The distribution of vaccines from IDs to LVCs, however, did change:



This is because the 2200 vaccine/per week capacity only allows a maximum of 13200 vaccines to be administered from each LVC over the 6-week period (in comparison to the previously allowed 16000). This reduced the number of vaccines being sent along the red arrows (shown above) and increased the number of vaccines being sent along the green arrows. Hence, the number of vaccines Pacific Paradise would be required to administer at each LVC would be:

	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
Vaccines Administered	11210	13200	13200	13200	6640	11859	10603	10111

Communication 4

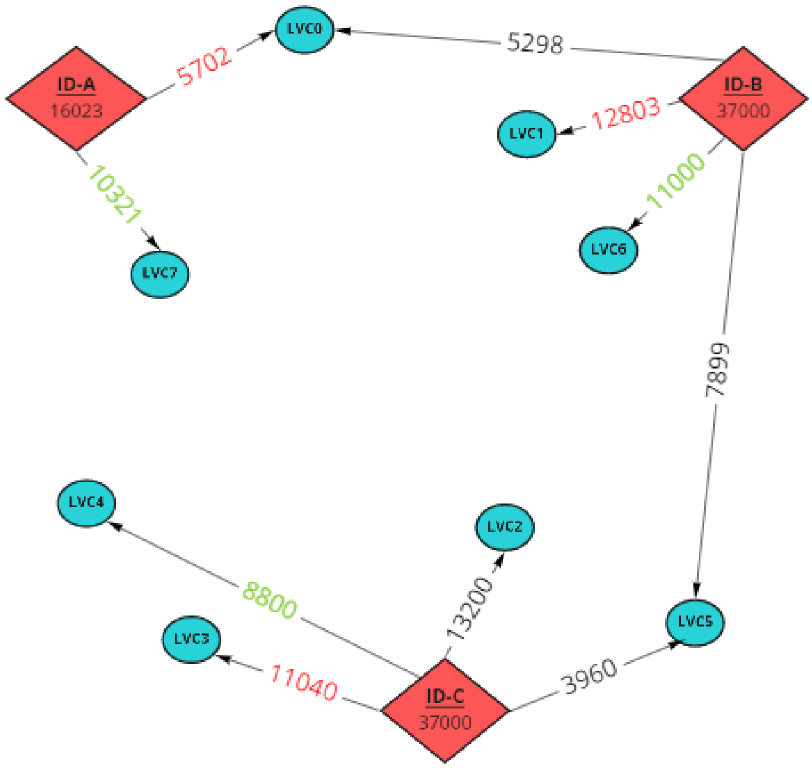
In communication 4 you asked the firm to consider the cost of delaying vaccines to certain census collection districts (CCDs). Under these conditions, the optimal cost for the Pacific Paradise vaccine distribution would increase to:

$17479538

These requirements did not change the importation of vaccines:

Import Depot	Vaccine Doses
ID-A	16023
ID-B	37000
ID-C	37000

However, delivery from each ID to LVC has changed slightly:



This has changed as some citizens will now be getting their vaccine administered from a different CCD; this is because it is more cost effective to send a citizen to a LVC slightly further away to get their vaccines administered sooner. The weekly delivery of vaccines (and totals) for each LVC is shown below:

Vaccines Delivered	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
Week 1	2200	2200	2200	2200	2200	2200	2200	2200
Week 2	2200	2200	2200	2200	2200	2200	2200	2200
Week 3	2200	2200	2200	2200	2200	2200	2200	2200
Week 4	2200	2200	2200	2200	2200	2200	2200	2200
Week 5	2200	2200	2200	2200	0	2200	2200	1521
Week 6	0	1803	2200	40	0	859	0	0
Total	11000	12803	13200	11040	8800	11859	11000	10321

Communication 5 – Final Model

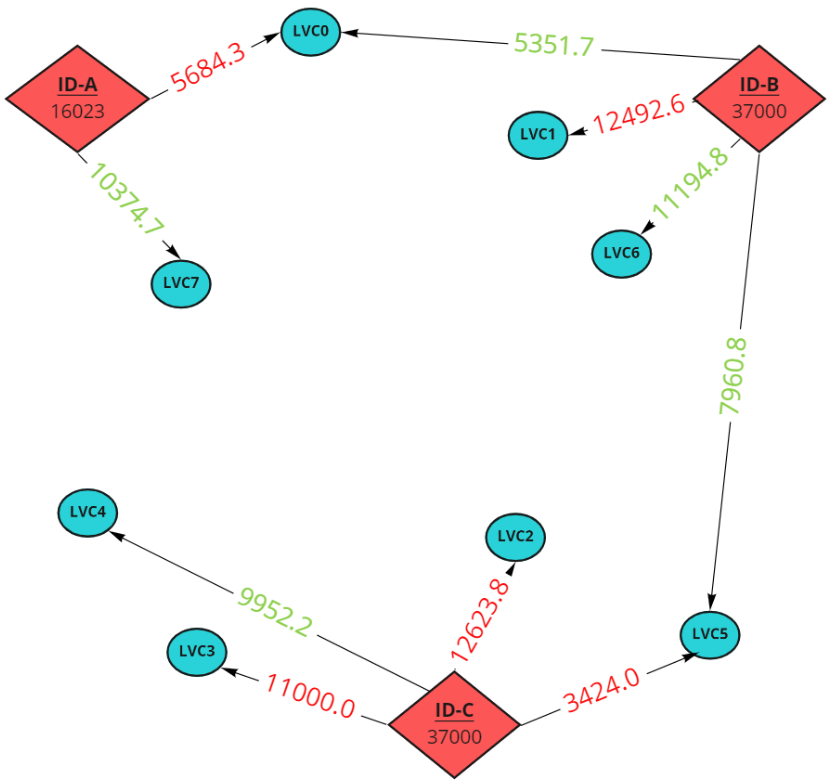
Communication 5 added the requirement for a fair distribution of vaccines across CCDs. This raises the optimal cost of the Pacific Paradise VDS to our firm's final estimate of:

$17495906

Under this model, vaccine importation to IDs would be:

Import Depot	Vaccine Doses
ID-A	16023
ID-B	37000
ID-C	37000

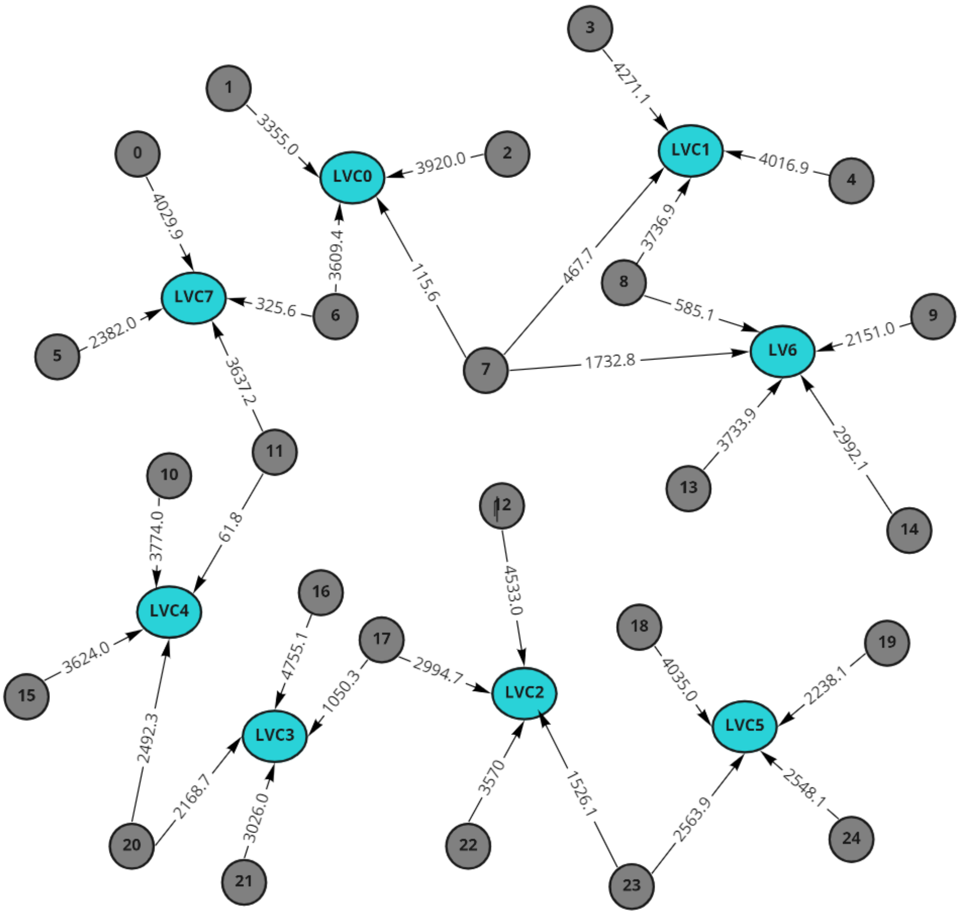
Total vaccine distribution across the 6 weeks would be:



This has changed as now some citizens may need to get their vaccines administered at different LVCs. It may cost the citizen more to travel to this LVC, however, it ensures they receive their vaccine in a similar timeline to citizens of other CCDs.

The weekly delivery of vaccines (and totals) for each LVC as well as how many citizens need to travel to each LVC is shown below:

Vaccines Delivered	LVC0	LVC1	LVC2	LVC3	LVC4	LVC5	LVC6	LVC7
Week 1	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0
Week 2	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0
Week 3	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0
Week 4	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0	2200.0
Week 5	2200.0	2200.0	2200.0	2200.0	1152.2	2200.0	2200.0	1574.7
Week 6	0.0	1492.6	1623.8	0.0	0.0	384.9	194.8	0.0
Total	11000.0	12492.6	12623.8	11000.0	9952.2	11384.9	11194.8	10374.7



Recommendations

For the benefit of Pacific Paradise, our firm has identified some areas which could help reduce costs for the VDS. A key area to improve the overall cost would be increasing the amount of vaccines each LVC can administer each week. The below table shows the possible savings for vaccine distribution if the weekly vaccine administering capacity is increased by 1:

LVC	Savings per Vaccine ($)
LVC0	111
LVC1	184
LVC2	167
LVC3	114
LVC4	64
LVC5	158
LVC6	152
LVC7	88
Total	1039

Thus, improving LVC1s weekly capacity would provide the best value for money, reducing the overall cost by $184 for every extra vaccine it can administer. This is because in the current VDS, LVC1 administers the most vaccines, and hence improving its capacity would see the greatest cost reduction.

Overall cost can also be reduced by improving the capacity of IDs. The below table shows the cost improvement when the vaccine capacity of each ID is increased by 1.

ID	Savings per Vaccine
ID-A	0
ID-B	29
ID-C	21
Total	49

Thus, improving the capacity of ID-B would give the most value improving the cost by $29 per vaccine. This is because importation is cheapest at ID-B. No money should be invested into improving ID-A as extra capacity would not reduce the overall cost.

Considerations

Our firm has also identified some further areas you may need to consider when distributing vaccines. It is unlikely that your vaccination distribution will be 100% efficient; there will likely be loss of vaccines through expiry, breakage etc. Hence, we recommend you import and deliver a further 5% of vaccine doses to each LVC as a precaution.

This would raise your total cost to:

$18281441

Our firm wishes you the best of luck in the vaccination of Pacific Paradise and we hope this report has been of assistance. Please contact us if you require any further information on how to implement your VDS.

Kind Regards,

William Barker

Operations Research Manager