

**Base case:**  $h = 0$

**Prove:**  $g_0 = F_{(0+2)} - 1$

$$\text{LHS} = g_{(0)}$$

$$= 0$$

$$\text{RHS} = F_{(0+2)} - 1$$

$$= F_2 - 1 \quad (\text{Fibonacci definition})$$

$$= 1 - 1$$

$$= 0$$

**Induction case:**  $g_h = g_{(h-1)} + g_{(h-2)} + 1$

**IH:** Assume  $g_{(h-1)} = F_{(h-1+2)} - 1$

Assume  $g_{(h-2)} = F_{(h-2+2)} - 1$

$$g_{(h)} = G_{(h-1)} + G_{(h-2)} + 1$$

$$= (F_{(h-1+2)} - 1) + (F_{(h-2+2)} - 1) + 1$$

$$= (F_{(h+1)} - 1) + (F_h - 1) + 1 \quad (\text{IH definition})$$

$$= F_{(h+1)} + F_h - 1$$

$$= F_{(h+2)} - 1 \quad (\text{Fibonacci definition})$$

$$= g_h$$

**Done.**