

Synthetic Power System, Generator Cost Curves

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1. Introduction

The Optimal Power Flow (OPF) determines the optimal operating point of the power system, considering the desired cost function, subject to the network constraints. The OPF can have different objective functions such as minimizing the generation cost, minimizing the loss in the network or tuning the voltage profiles (optimizing the reactive power). In most cases, the total generation cost is chosen as the objective function.

$$\min Cost(x, u)$$

$$\text{s.t. } M(x, u) = 0$$

$$N(x, u) \leq 0$$

Where, $Cost(x, u)$ is the cost function of the power system. “ x ” is a vector of dependent variables such bus voltages and angles and “ u ” is a vector of control variables such as active and reactive power. $M(x, u)$ represents equality constraints and $N(x, u)$ represents inequality constraints in the network.

The cost function $Cost(x, u)$ represents the generation cost in the power system to meet a specific amount of load (demand). The generation cost based on the unit output can be defined into different forms of cubic, quadratic or piecewise linear. The quadratic cost function is used to represent units’ cost curves in this study.

$$Cost = a + bP_G + cP_G^2$$

To come up with proper constant, linear and quadratic coefficients for each unit in the synthetic power system, different approaches are used which are explained in part 2.

2. Problem Formulation and available samples

For fossil steam units, typical heat rate data, which is shown in the table below (EPRI report 1977), were used along with EIA fuel cost data for each type of fuel, to develop representative \$/hour versus MW output.

TYPICAL FOSSIL GENERATION UNIT NET HEAT RATES

Fossil Unit-Description	Unit Rating (MW)	100 % Output (BTU/KWH)	80 % Output (BTU/KWH)	60 % Output (BTU/KWH)	40 % Output (BTU/KWH)	25 % Output (BTU/KWH)
Steam--Coal	50	11000	11088	11429	12166	13409*
Steam--Oil	50	11500	11592	11949	12719	14019*
Steam--Gas	50	11700	11794	12156	12940	14262*
Steam--Coal	200	9500	9576	9871	10507	11581*
Steam--Oil	200	9900	9979	10286	10949	12068*
Steam--Gas	200	10050	10130	10442	11115	12251*
Steam--Coal	400	9000	9045	9252	9783	10674*
Steam--Oil	400	9400	9447	9663	10218	11148*
Steam--Gas	400	9500	9548	9766	10327	11267*
Steam--Coal	600	8900	8989	9205	9843	10814*
Steam--Oil	600	9300	9393	9681	10286	11300*
Steam--Gas	600	9400	9494	9785	10396	11421*
Steam--Coal	800-1200	8750	8803	9048	9625*	
Steam--Oil	800-1200	9100	9155	9409	10010*	
Steam--Gas	800-1200	9200	9255	9513	10120*	

*For study purposes, units should not be loaded below the points shown.

For the capacities other than what is represented in the above table, the heat rates are calculated by averaging the boundaries, which include the capacity.

Different fuel type prices are obtained from EIA website. For those fuel types such as Landfill Gas which no report was available at EIA website, other references as named below were used. For other fuel type which are represented in the excel file but are not shown in below table, the price of the most similar fuel type has been applied.

Fuel type	\$/MBTU	Reference
Coal	1.78	EIA
Natural Gas	2.68	EIA
Residual Oil	9.25	EIA
Wood/Waste Wood	3.72	EIA
Petroleum Coke	2.56	EIA
Landfill Gas	6.18	EPA

To come up with the proper a, b and c coefficients, the \$/hour cost has been calculated corresponding to 40%, 60%, 80% and 100% of the unit capacity. Using these value and the cost function represented above the a, b and c coefficients have been calculated.

$$\begin{bmatrix} 1 & p & p^2 \\ 1 & 0.8p & (0.8p)^2 \\ 1 & 0.6p & (0.6p)^2 \\ 1 & 0.4p & (0.4p)^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \cong \begin{bmatrix} \$/h \\ \$/h \\ \$/h \\ \$/h \end{bmatrix}$$

If we consider:

$$\begin{bmatrix} 1 & p & p^2 \\ 1 & 0.8p & (0.8p)^2 \\ 1 & 0.6p & (0.6p)^2 \\ 1 & 0.4p & (0.4p)^2 \end{bmatrix} = A$$

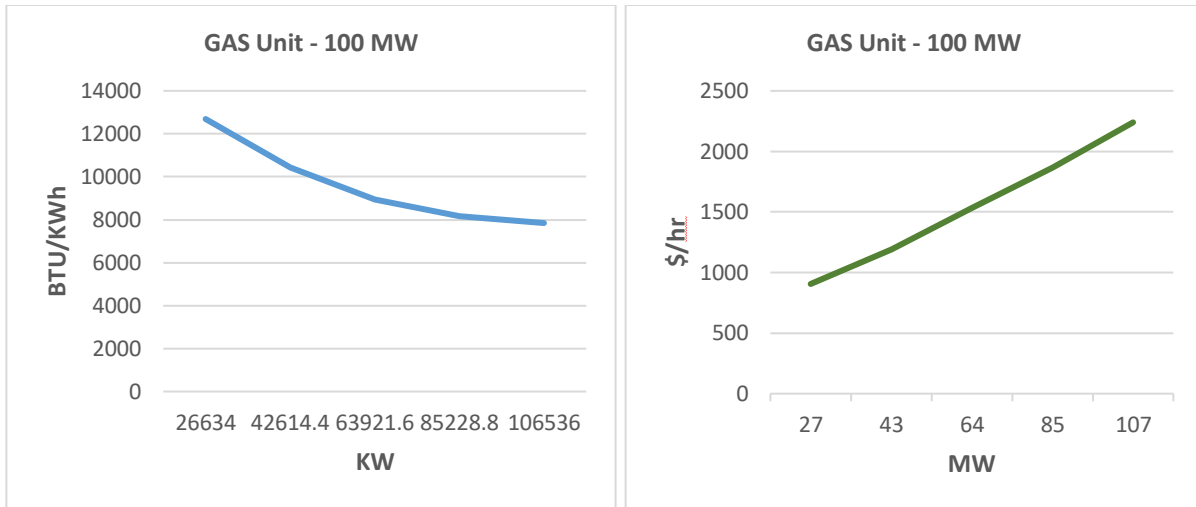
$$\begin{bmatrix} \$/h \\ \$/h \\ \$/h \\ \$/h \end{bmatrix} = Y$$

Then:

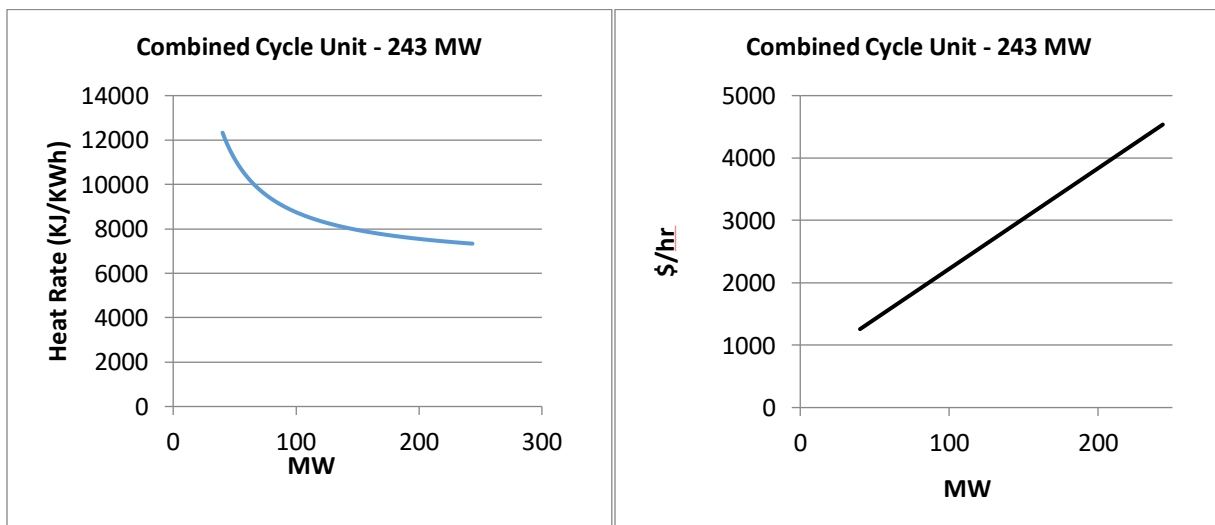
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = [A^T \cdot A]^{-1} \cdot A^T \cdot Y$$

For Gas units, typical heat rate data was obtained from the GE website. For the units with 17 MW to 112 MW with both Gas and Liquid fuels, heat rates were obtained from the GE online simulator (As an example, one sample is represented below). For, units with higher capacities, \$/hour values were obtained by combining two or three of the available samples. Same fuel prices as represented in previous table are used for Gas units.

LMS 100 GAS - 100 MW		
KW	BTU/KWH	\$/h
26634	12689	905.7297
42614.4	10421	1190.147
63921.6	8940	1531.51
85228.8	8177	1867.735
106536	7845	2239.877



For Combine Cycle units, physical samples from CCGT units were used. Heat rate samples for CCGT units with 168MW to 721 MW were available. Same fuel price as represented before, is used for Combined Cycle units. For the Combined Cycle units with the gas turbine as the prime mover (CT units) the CCGT heat rates were used. For the Combined Cycle units with steam turbine as the prime mover (CA units) the average of the heat rate of a CCGT unit and a similar ST unit has been used. For the units with capacities other than the available samples, the most similar sample has been scaled up or scaled down to come up with the proper value for corresponding \$/h values.



For Hydro units, since their cost curves is similar to CCGT units, the same samples have been used to come up with proper a, b and c values.

For internal combustion units, since we did not have access to physical data, market data is used to calculate the coefficient. Among different ISO's, ERCOT was chosen since in its reports of generator bids, the type of the generation units is also included.

For other types of units including Wind, Solar, Geothermal, Battery and also nuclear steam turbines and biomass units such as municipal solid waste units (any biomass other than wood/waste wood and landfill gas) zero values have been assigned to a, b and c coefficients since they are non-dispatchable.

3. Clustering

There is around 6700 units in the US power system. To come up with proper a, b and c coefficients for each of the units, using the approaches explained above, is a very time consuming process. To accelerate the process, units have been divided into multiple clusters. To come up with accurate clustering, at first units have been divided into 9 categories based on the unit type, including Steam, Gas, Hydro, Combined Cycle, Wind, Solar, Geothermal, Battery and internal combustion. Then each category has been divided into number of clusters based on unit's capacity, Fuel type, Longitude and latitude and the state that the unit is installed in.

Numerical values have been assigned to the narrative explanation of the units such as Fuel type (Technology), prime mover and state. 50 states have been divided into 9 districts of North, North West, North East, Central, West, East, South, South West and South East and weights of 1 to 9 have assigned to the states based on their districts. For the fuel type, 32 different categories have been identified and values between 1 to 32 have been assigned. For the prime movers (unit types), numerical values between 1 to 9 have been assigned.

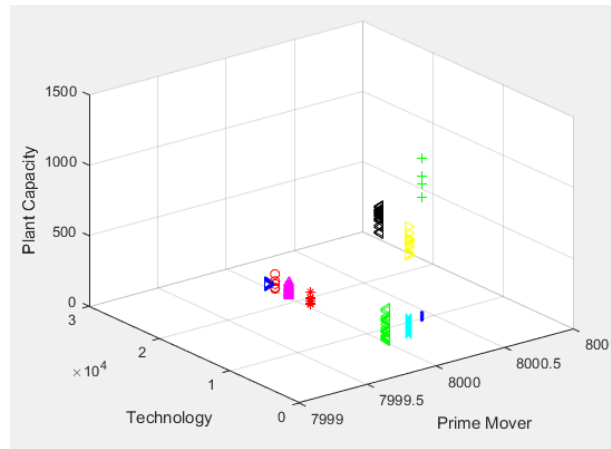
To equalize the numerical values of prime mover and fuel type with longitude and latitude values, they have been scaled up by the weight of 1000.

The two most common methods of clustering are K-mean and hierarchical algorithms. The hierarchical approach starts by assigning each data point into one cluster. In the next step, it combines the most similar clusters and continues doing that until it comes up with only one cluster at the end. Therefore, the output of the step which matches the optimum number of clusters for the case study must be used for further analysis. Although the hierarchical algorithm is very accurate, due to its computational complexity, it is not a proper approach for large data sets. The K-mean algorithm does the clustering in four steps.

1. Based on the predefined number of clusters, it assigns random centroid to each cluster.
2. In the next step, it computes the distance of each data point to the centroids and assign the data point to the centroid, which has the lowest distance.

3. Then it assigns new centroids to the clusters based on the average distance of the data points in the clusters.
4. The K-mean algorithm repeats step 2 and 3 until no data point migrates to another cluster.

K-mean algorithm is very fast for the large data sets, therefore this approach is used to do the clustering. As it is shown in the figure below, K-mean algorithm assigns generators with same prime mover, technology and more or less same capacity into one cluster.



There are few methods, which can be used to determine the optimum number of clusters, but none of them can guarantee the accurate results. Therefore, the optimum number of clusters for each category is determined by trial and error in this study. Based on above explanation, the 6700 units have been categorized into 102 clusters. Table below includes the details of clustering:

Unit type	Number of clusters
Steam	40
Gas	20
Combined Cycle	20
Hydro and Pump Storage	10
Internal Combustion	7
Wind	1
Solar	1
Battery	1
Geothermal	1
Other	1

In each cluster, proper a, b and c coefficient have been assigned to the units which could be cross-matched with available samples. For the other units in the same cluster random a, b and c coefficients have been assigned matching the units in the boundaries.

Results have been summarized in below table:

Unit Type	Fuel Type	Capacity(MW)	A(\$/h)	B(\$/MWh)	C(\$/MWh ²)
Steam	Coal	20-1425	40-2316	12-16	0.001-0.077
	Oil	23-978	252-5896	67-87	0.008-0.363
	Gas	20-895	65-2305	19-25	0.0020-0.126
	Landfill Gas	23-50	173-379	58	0.11-0.25
	Petroleum Coke	27-703	78-1582	18-23	0.002-0.08
	Wood & Waste Wood	20-107	88-440	32-34	0.02-0.16
	Nuclear/Geothermal/Solar/Municipal Solid Waste	---	0	0	0
Gas	Natural Gas	20-435	135-3400	2-17	0.0005-0.39
	Petroleum Liquids	20-296	416-3473	21-70	0.001-0.96
	Biomass	---	0	0	0
Combined Cycle	Coal	133	992	9	0.01
	Natural Gas	20-1848	230-4606	13-17	0.001-0.11
	Petroleum Liquids	20-105	792-4105	52-55	0.07-0.4
	Landfill Gas	20-30	530-804	37	0.18-0.27
	Petroleum Coke	112	1207	14	0.018
	Wood & Waste Wood	33	554	22	0.09
Hydro	Conventional and Pump storage	20-6495	22-7248	14	0.0003-0.1
Internal Combustion	Natural Gas	20-200	9-100	6-7	0.01-0.13
	Petroleum Liquids	20-534	23-846	25-26	0.01-0.3
	Landfill Gas	26	53	25	0.3
Wind/Solar/Battery	---	---	0	0	0