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PFIS Exposure Time Calculation

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Introduction

This document outlines the exposure time calculation for the imaging and spectroscopic modes of PFIS. The signal-to-noise ratio is derived as a function of exposure time. The function can be inverted to give the exposure time as a function of signal-to-noise.

The signal-to-noise calculation for any mode (imaging or spectroscopy) can be broken up into two pieces: determining how many photons arrive at the focal plane, and determining how these photons are distributed into the CCD readout bins.

We assume that the object and sky spectra have been calculated elsewhere, and are delivered to the focal plane in flux units of ergs/s/A for a point source object and ergs/s/A/arcsec^2 for a diffuse sky spectrum. We assume the area of the primary mirror has been applied to remove the cm^-2 dependence of the flux, and that the reflectivity losses of the primary mirror, the throughput losses of the spectrograph optics, and the quantum efficiency losses of the detector have been applied as well.

We will parameterize all the numerical inputs as constants, so that changes will flow easily through the calculation.

Constants

CCD pixel dimensions:

$$P_x = P_y = 15\,\mu$$

Focal length of the telescope:

$$f_{tel} = 46200\ mm$$

Focal length of the PFIS collimator:

$$f_{col} = 630\ mm$$

Focal length of the PFIS camera:

$$F_{cam} = 330\ mm$$

Readout noise in counts per readout bin:

$$\sigma_r = 6$$

Independent Variables

These are the inputs chosen by the user. The values shown are the defaults in the PFIS Exposure Time Calculator (ETC).

The user supplies either the exposure time, or the Signal-to-Noise ratio (which we call Q). Given one, we compute the other.

Binning factors in x and y. Pixels can be electronically binned together to make a larger integrating element.

$$B_x = B_y = 1$$

Total time on target (including readouts)

$$T = 3600\ s$$

Number of readouts:

$$n_r = 1$$

Time per readout:

$$T_r = 4\ s$$

Desired Signal-to-Noise ratio:

$$Q = 10$$

Full-width at half-maximum of the seeing disk, at the zenith, for Sutherland.

$$FWHM = 1.2''$$

Preparatory Calculations

These calculations produce basic values used later in the ETC.

The scale at the focal plane is

$$S_f = \frac{1}{f_{tel}} \cdot \frac{180 \cdot 3600}{\pi} \ ''/mm$$

The scale at the detector is

$$S_d = S_f \frac{f_{col}}{f_{cam}} \ ''/mm$$

The solid angle of the Point Spread Function is

$$\Omega = \pi \cdot FWHM^2 / 4 \ arc\sec^2$$

The area of a CCD pixel is

$$A_p = \frac{P_x}{1000} \cdot \frac{P_y}{1000} \ mm^2$$

The solid angle of a CCD pixel is

$$\Omega_p = A_p S_d^2 \ arc\sec^2$$

The area of a bin (pixels electronically binned together before readout) is

$$A_b = A_p B_x B_y \ mm^2$$

The solid angle of a bin is

$$\Omega_b = A_b S_d^2 \ arc\sec^2$$

The time spent counting photons is reduced by the time it takes to read out the CCD.

The net exposure time is

$$T_e = T - n_r T_r \ s$$

Spectrum Operations: Imaging

These operations specify how to determine how many photons per second are arriving at the focal plane. For a diffuse source or the sky background, the flux arrives at the focal plane from a solid angle subtended by the PSF at the focal plane.

For a diffuse spectrum, convert the flux per square arcsecond to flux:

$$F_{\lambda} = \Omega F_{\lambda,\Omega}$$

We convert the spectrum from ergs to counts by dividing the energy in each wavelength bin by the energy per photon at that wavelength. The energy per photon at frequency ν is $h\nu$; expressed in wavelength, the energy is hc/λ.

$$N_{\lambda} = \frac{\lambda F_{\lambda}}{hc} \, counts/s/A$$

where

$$hc = 6.626069 \times 10^{-27} erg\, s \cdot 2.997925 \times 10^{10} cm/s \times 10^{8} A/cm = 1.986445 \times 10^{-8} erg\, A$$

with length in units of Angstroms.

The count rate for a spectrum is

$$N = \int N_{\lambda} d\lambda \, counts/s$$

where $d\lambda$ is the wavelength step in Angstroms. Note that in the PFIS ETC java implementation, the wavelength step is constant, so it comes out of the integral:

$$N = d\lambda \sum N_{\lambda} \, counts/s$$

We now have the total count rate at the focal plane for any source.

Signal-to-Noise: Imaging

In this section we derive the signal-to-noise for imaging mode. In imaging mode, the photons gathered up in the PSF in the focal plane are distributed into the PSF as imaged onto the CCD readout bins. Subscript "o" refers to the object (target) spectrum and "s" to the sky (background) spectrum.

First compute the count rates for object and sky according to the previous section. Then distribute these counts into the PSF, as imaged by the spectrograph optics onto the detector:

$$N'_o = N_o \frac{\Omega_b}{\Omega} \; counts/s/bin$$

and

$$N'_s = N_s \frac{\Omega_b}{\Omega} \; counts/s/bin$$

where the primes indicate that the count rate is per readout bin. Note that for a diffuse spectrum, the Ω we put in, in the previous section, we take out here.

The noise for each component is given by

$$\sigma_o = \sqrt{N'_o T_e} \; counts$$

and

$$\sigma_s = \sqrt{N'_s T_e} \; counts$$

The total signal in counts per second per bin is given by

$$N' = N'_o + N'_s \, counts/s/bin$$

The noise per bin comes from three sources: object, sky, and readout. The total noise per bin is given by

$$\begin{aligned} \sigma^2 &= \sigma_o^2 + \sigma_s^2 + n_r \sigma_r^2 \\ &= N'_o T_e + N'_s T_e + n_r \sigma_r^2 \\ &= N' T_e + n_r \sigma_r^2 \end{aligned}$$

The signal-to-noise ratio is

$$\begin{aligned} Q &= \frac{N' T_e}{\sigma} \\ &= \frac{N' T_e}{\sqrt{N' T_e + n_r \sigma_r^2}} \end{aligned}$$

We may want to choose the SNR, and derive the required exposure time. We invert the expression for Q to get

$$N\square T_e = \frac{Q^2 \pm Q\sqrt{Q^2 + 4n_r\square_r^2}}{2}$$

We take the plus sign for meaningful results, and note that in the absence of readout noise,

$$Q = \sqrt{N\square T_e}$$

and

$$T_e = \frac{Q^2}{N\square}s$$

as expected.

Spectrum Operations: Spectroscopy

In this section we derive the signal to noise (as a function of resolution element) for spectroscopy.

Slit Losses

The spectrum must be scaled for slit losses, which we treat as a gray (wavelength-independent) scale factor. To compute the slit loss (actually the slit throughput), we start with the seeing at the zenith, and the slit width, both in arcseconds. We will use 3 standard zenith-seeing values, Good (0.8”), Median (1.0”), and Poor (1.2”).

Scale the fwhm from the zenith:

$$FWHM_{Z=37} = FWHM_{Z=0}\sec^{0.6}(37^\circ)$$

Add this in quadrature with the 0.6 arcsecond telescope PSF:

$$FWHM = \sqrt{FWHM * FWHM + 0.6 * 0.6}$$

Now derive the standard deviation σ of this PSF from the FWHM:

$$\sigma = \frac{FWHM}{2\sqrt{2\ln(2)}}$$

The slit throughput is the integral of a Gaussian over the slit:

$$S = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\square_s/2}^{\square_s/2} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} dx$$

where $\square_s$ is the width of the slit in arcseconds.

It is convenient to convert the integral above into the Error Function with a change in variables. Doing so yields

$$S = \frac{2}{\sqrt{\pi}}\int_0^{\square/2} e^{-t^2} dt = erf(\square/2)$$

where

$$\square = \frac{\square_s}{\sqrt{2}\sigma}.$$

Kogelnik Efficiency

In this section we calculate the grating throughput as a function of wavelength using the Kogelnik approximation.

Kogelnik Inputs

The independent variables in this treatment (and their defaults) are:

The grating angle is

$$\theta = \pi / 4$$

The groove frequency is

$$\nu_g = 3000 \ mm^{-1}$$

The grating period in Angstroms is

$$\Lambda = \frac{10^7}{\nu_g} \ Angstroms$$

The VPH grating index of refraction is

$$n_g = 1.48$$

The VPH grating index modulation is

$$\Delta n = 0.1$$

The VPH grating thickness is

$$d = 2 \times 10^4 \ \ Angstroms$$

Kogelnik Calculations

The angle of incidence inside the grating is

$$\sin\square_g = \sin\square / n_g .$$

Define the obliquity factors as

$$c_r = c_s = \cos\square_g .$$

Note that the obliquity factors can differ from each other under conditions that do not apply to PFIS; we distinguish them here, even though they are equal for PFIS, just to be able to track them in the Kogelnik paper.

The dephasing measure is given by

$$\square_\square = K\sin\square_g \,\square\, \frac{K^2\square}{4\square n_g}$$

where

$$K = \frac{2\square}{\square}.$$

The distance from Littrow is given by

$$\square_\square = \square_\square d / 2c_s .$$

The grating equation tells us that

$$\sin\square + \sin\square = \square/\square$$

where $\square$ is the angle of incidence of the light and $\square$ is the angle of diffraction. This equation sets a limit on how large $\square$ can be. We see that

$$\square/\square \,\square\, \sin\square < 1$$

or

$$\square < \square(1 + \sin\square).$$

This defines the red-most wavelength for which the efficiency calculation should be done.

The angle of diffraction inside the grating is

$$\begin{aligned} \sin\square_g &= \sin\square / n_g \\ &= (\square/\square \,\square\, \sin\square)/n_g \end{aligned}$$

(Note that this equation also sets a red-most limit on the wavelength $\square$, but for $n_g > 1$ this limit is redder than that imposed by the grating equation in air. We choose the bluer of the two limits.)

The superblaze efficiencies for the S and P polarizations are given by

$$\square_s = \frac{\square\,\square n\, d}{\square\sqrt{c_r c_s}}$$

$$\square_p = \square_s \cos(\square_g + \square_g)$$

The S and P efficiencies are given by

$$\square_s = \frac{\sin^2\sqrt{\square_s^2 + \square^2}}{1 + \square^2/\square_s^2}$$

and

$$\square_p = \frac{\sin^2\sqrt{\square_p^2 + \square^2}}{1 + \square^2/\square_p^2}.$$

Finally, the net unpolarized efficiency is obtained from the average of the S and P efficiencies:

$$\square = \frac{\square_s + \square_p}{2}$$

Signal-to Noise: Spectroscopy

In spectroscopy mode, we compute the signal-to-noise as a function of wavelength. We know our input spectra to much higher spectral resolution that we will achieve on the detector. The resolution element is determined by (among other things) the width of the slit, which is dispersed and reimaged by the spectrograph optics onto the detector. The resolution element affects us in both parts of the SNR calculation. First, we do not integrate the incoming spectra over all wavelengths at the focal plane to get the incoming count rate; we integrate them only over the resolution element, thereby preserving some wavelength dependence at the focal plane. Second, we disperse these photons over a geometrical area on the detector whose x-direction (in the direction of dispersion) is determined by the geometrical extent of the resolution element in pixel space.

We modify the section on Imaging Spectrum Operations by defining

$$N_{re} = \int_{\Delta\lambda_{re}} N_\lambda d\lambda \; counts/s/re$$

Eric Burgh can easily show that the resolution element is given by

$$\Delta\lambda_{re} = \frac{\theta_s \pi}{180 \cdot 3600} \sigma \cos\alpha \frac{f_{tel}}{f_{col}} \; Angstroms$$

This is constant over wavelength.

We get the angular dispersion from the grating equation:

$$\frac{d\beta}{d\lambda} = \frac{1}{\sigma \cos\beta} \; radians/A$$

The linear dispersion is

$$\frac{dx}{d\lambda} = f_{cam} \frac{d\beta}{d\lambda} \; mm/A$$

The area of the resolution element is the size of the resolution element in the dispersion direction multiplied by the width of the PSF in the cross-dispersion direction:

$$A_{re} = \frac{dx}{d\lambda} \Delta\lambda_{re} \cdot \frac{FWHM}{S_d} \; mm^2$$

The counts per bin:

$$N'_{o,re} = N_{o,re} \frac{A_b}{A_{re}} \; counts/s/bin$$

and

$$N'_{s,re} = N_{s,re} \frac{A_b}{A_{re}} \; counts/s/bin$$

The SNR calculation now proceeds as given in the imaging section.