

Observer design for discrete-time descriptor systems: An LMI approach

Zhenhua Wang^{*}, Yi Shen, Xiaolei Zhang, Qiang Wang

School of Astronautics, Harbin Institute of Technology, West Da-Zhi Street, 150001 Harbin, PR China

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ABSTRACT

In this paper, the observer design problem for discrete-time descriptor systems is considered. For the discrete-time linear descriptor systems, the necessary and sufficient conditions for the existence and convergence of the proposed observer are given and proved, and a systemic design approach is presented via the linear matrix inequalities formulation. Furthermore, an extension to a class of nonlinear descriptor systems with Lipschitz constraints is investigated. Simulation examples are given to illustrate the estimation performance of the proposed method.

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1. Introduction

Descriptor systems, which are also referred to as singular systems, differential–algebraic equation systems, or generalized state-space systems, can be used to describe a lot of practical systems such as electrical networks [1], constrained mechanical systems [2], biological systems [3], aircraft modeling [4], etc. Therefore, many control problems for descriptor systems have attracted considerable attention in the past several decades. Many studies in feedback control [5], robust control [6], fault-tolerant control [7], etc. have been presented in the literature.

The control of descriptor systems is often based on state feedback. Unfortunately, in many applications, the state of the system is not available for measurement. In this regard, the problem of state estimation for descriptor systems is of both theoretical and practical significance. Many significant results have been reported in observer design for descriptor systems; see e.g. [8–11] and the references therein. However, most of the existing works are concerned on continuous-time systems. Observer design for discrete-time descriptor systems has received little attention [12–15]. Moreover, only a few limited results have been published on the problem of designing discrete-time nonlinear descriptor observers [13–15]. As a result, there is a strong incentive for us to develop a novel observer design method for discrete-time descriptor systems.

Most recently, a novel observer design approach is presented in [16,17] to investigate sensor fault reconstruction and sensor compensation for state-space systems. Inspired by the results in [16,17], this paper considers the observer design problem for discrete-time descriptor systems. For the discrete-time descriptor linear systems, the necessary and sufficient condition for the existence and convergence of the proposed observer is given and proved. Moreover, the proposed observer design method is extended to a class of discrete-time nonlinear descriptor systems, where the nonlinearity is assumed to satisfy Lipschitz constraint. In the literature, many results have been proposed to design observers for discrete-time systems by using linear matrix inequality (LMI) techniques; see e.g. [18–20], just to name a few. Therefore, in this paper, the observer design is formulated as an LMI feasible problem, which is easily solved by standard convex optimization algorithms.

This paper is organized as follows. In Section 2, we briefly introduce the problem to be studied and the proposed observer. In Section 3, observer design for the discrete-time linear descriptor systems is presented and formulated as an LMI formulation. Linear observer is extended to nonlinear descriptor systems in Section 4. In Section 5, two simulation examples are given to demonstrate the validity of our results. Finally, several conclusions are drawn in Section 6.

2. Problem formulation and preliminaries

Consider the linear discrete-time descriptor system of the form

$$\begin{cases} Ex(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \quad (1)$$

^{*} Correspondence to: School of Astronautics, Harbin Institute of Technology, Room 602, Main Building, No. 92, West Da-Zhi Street, Harbin, Heilongjiang Province, PR China. Tel.: +86 451 86413411 8602; fax: +86 451 86418378.

E-mail address: zhwang1987@gmail.com (Z. Wang).

where $E \in \mathbb{R}^{n \times n}$ may be singular, i.e. $\text{rank}(E) = r \leq n$. $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^p$, and $y(k) \in \mathbb{R}^m$ denote the state, the known input and the output vectors, respectively. A , B , and C are known constant matrices with appropriate dimensions.

Without loss of generality, it is assumed that system (1) is observable [1,12], i.e.

$$\text{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n \quad (2)$$

and

$$\text{rank} \begin{bmatrix} zE - A \\ C \end{bmatrix} = n, \quad \forall z \in \mathbb{C}, |z| \geq 1, z \text{ finite}. \quad (3)$$

Since $\text{rank} \begin{bmatrix} E \\ C \end{bmatrix} = n$, there exists a full rank matrix $\begin{bmatrix} T & N \end{bmatrix}$ such that

$$TE + NC = I_n. \quad (4)$$

Our aim is to design an observer as the following form

$$\hat{x}(k+1) = TA\hat{x}(k) + TBu(k) + L(y(k) - C\hat{x}(k)) + Ny(k+1) \quad (5)$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the estimate of the descriptor state $x(k)$, and T , N , and L are matrices to be determined such that $\hat{x}(k)$ asymptotically converges to $x(k)$.

Remark 1. It is known that the descriptor system form contains the state-space form as a special case and thus can represent a much wider class of systems than its state-space counterpart [21]. The proposed method in this work is also valid for state-space systems, i.e., when the matrix $E = I$, the descriptor system (1) reduces to a discrete state-space system. In the case, by choosing $T = I_n$ and $N = 0$, the presented observer (5) reduces to the discrete-time Luenberger observer [22].

Remark 2. In the recent paper [14], the design parameters are subjected to three matrix equations, which requires complicated algebraic operation to transform the bilinear matrix inequalities problem to a standard LMI. Contrary to the observer form presented in [14], this paper proposes an observer with less parameter matrices and only requires solving one matrix equation (4). As a result, the design procedure in this paper is easy to follow.

The Schur complement Lemma is needed to deduce an LMI feasible problem.

Lemma 1 ([23]). *The LMI*

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0 \quad (6)$$

where $Q = Q^T$, $R = R^T$ is equivalent to

$$R > 0, \quad Q - SR^{-1}S^T > 0. \quad (7)$$

3. Linear descriptor observer design

In this section, observer design for discrete-time linear descriptor systems is formulated as an LMI feasibility problem. The necessary and sufficient conditions for the convergence of observer are presented by the following theorem.

Theorem 1. *For the plant (1), there is an asymptotic observer in the form of (5) if and only if there exist a positive matrix $P \in \mathbb{R}^{n \times n}$ and a matrix $W \in \mathbb{R}^{n \times m}$ satisfying the following linear matrix inequality*

$$\begin{bmatrix} -P & (TA)^T P - C^T W^T \\ P(TA) - WC & -P \end{bmatrix} < 0. \quad (8)$$

Proof. By using Eq. (4), the plant (1) is rewritten as

$$x(k+1) = TAx(k) + TBu(k) + Ny(k+1). \quad (9)$$

Define the estimation error $e(k)$ of the observer (5) as

$$e(k) = x(k) - \hat{x}(k). \quad (10)$$

From (5) and (9), the error dynamic equation is obtained as

$$e(k+1) = (TA - LC)e(k). \quad (11)$$

Taking a Lyapunov candidate as

$$V(k) = e^T(k)Pe(k). \quad (12)$$

The time difference of $V(k)$ is

$$\begin{aligned} \Delta V &= e^T(k+1)Pe^T(k+1) - e^T(k)Pe(k) \\ &= e^T(k)(TA - LC)^T P(TA - LC)e(k) - e^T(k)Pe(k). \end{aligned} \quad (13)$$

The inequality $\Delta V < 0$ holds for all $e(k) \neq 0$ if and only if

$$(TA - LC)^T P(TA - LC) - P < 0. \quad (14)$$

Using Lemma 1 for (14) gives

$$P > 0, \quad \begin{bmatrix} -P & (TA - LC)^T P \\ P(TA - LC) & -P \end{bmatrix} < 0. \quad (15)$$

Letting $W = PL$, it follows that (8) is equivalent to (15). From the Lyapunov stability theory, if (15) is satisfied, the error system (11) is asymptotically stable. \square

Remark 3. The matrix inequality (15) is transformed into the form of (8) since the latter inequality is an LMI, which is easily solved by standard convex optimization algorithms. If there exists P and W satisfying the linear matrix inequality (15), matrix L in observer (5) is determined by

$$L = P^{-1}W. \quad (16)$$

The following theorem infers that the existence of a linear descriptor observer under the assumptions (2) and (3).

Theorem 2. *Under the assumptions (2) and (3), the linear matrix equation (8) in Theorem 1 is solvable.*

Proof. Under the assumption (2), the general solution of (4) is

$$\begin{bmatrix} T & N \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^\dagger + Y \left(I_{n+m} - \begin{bmatrix} E \\ C \end{bmatrix} \begin{bmatrix} E \\ C \end{bmatrix}^\dagger \right) \quad (17)$$

where Y is an arbitrary matrix, which is designed such that T is of full rank [10]. Herein, X^\dagger denotes the pseudoinverse of a matrix X .

From the error dynamic equation (11), it can be seen that Eq. (8) is solvable if $TA - LC$ is Hurwitz. On the other hand, detectability of the pair (TA, C) means that there exists a gain matrix L so that $TA - LC$ is Hurwitz.

The detectability of the pair (TA, C) is characterized as

$$\text{rank} \begin{bmatrix} zI_n - TA \\ C \end{bmatrix} = n, \quad \forall z \in \mathbb{C}, |z| \geq 1, z \text{ finite} \quad (18)$$

which is equivalent to

$$\text{rank} \begin{bmatrix} zI_n - TA \\ zC \\ C \end{bmatrix} = n, \quad \forall z \in \mathbb{C}, |z| \geq 1, z \text{ finite}. \quad (19)$$

Substitute (4) into (19), it follows that

$$\text{rank} \begin{bmatrix} zI_n - TA \\ zC \\ C \end{bmatrix} = \text{rank} \left(\begin{bmatrix} T & N & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_m \end{bmatrix} \begin{bmatrix} zE - A \\ zC \\ C \end{bmatrix} \right). \quad (20)$$

$$\begin{bmatrix} -P + \eta\gamma_1^2 I_n & (TA)^T P - C^T W^T & (TA)^T P - C^T W^T \\ P(TA) - WC & P - \eta I_n & 0 \\ P(TA) - WC & 0 & -P \end{bmatrix} > 0 \quad (26)$$

Box I.

$$\begin{aligned} \Delta V &\leq e^T(k) [(TA - LC)^T P(TA - LC) - P] e(k) + 2e^T(k)(TA - LC)^T P \Delta\Phi + \Delta\Phi^T P \Delta\Phi + \eta\gamma_1^2 e^T(k)e(k) - \eta\Delta\Phi^T \Delta\Phi \\ &= \begin{bmatrix} e(k) \\ \Delta\Phi \end{bmatrix}^T \begin{bmatrix} (TA - LC)^T P(TA - LC) - P + \eta\gamma_1^2 e^T(k)e(k) & (TA - LC)^T P \\ P(TA - LC) & P - \eta I_n \end{bmatrix} \begin{bmatrix} e(k) \\ \Delta\Phi \end{bmatrix} \\ &= \begin{bmatrix} e(k) \\ \Delta\Phi \end{bmatrix}^T \Omega \begin{bmatrix} e(k) \\ \Delta\Phi \end{bmatrix} \end{aligned} \quad (31)$$

where

$$\Omega = \begin{bmatrix} (TA - LC)^T P(TA - LC) - P + \eta\gamma_1^2 e^T(k)e(k) & (TA - LC)^T P \\ P(TA - LC) & P - \eta I_n \end{bmatrix}.$$

Box II.

By using Sylvester's equality [10], we obtain

$$\text{rank} \begin{bmatrix} zI_n - TA \\ zC \\ C \end{bmatrix} = \text{rank} \begin{bmatrix} zE - A \\ zC \\ C \end{bmatrix}. \quad (21)$$

On the other hand, the assumption (3) is equivalent to

$$\text{rank} \begin{bmatrix} zE - A \\ zC \\ C \end{bmatrix} = n, \quad \forall z \in \mathbb{C}, \quad |z| \geq 1, \quad z \text{ finite}. \quad (22)$$

From (21) and (22), it follows that the detectability condition (18) is equivalent to (3). \square

4. Extension to discrete-time nonlinear descriptor systems

The results presented in Section 3 is based on a linear descriptor system model. Nevertheless, most practical systems are naturally described by nonlinear models. Therefore, the development of nonlinear descriptor observers plays a significant role in practical applications. Since many nonlinearities can be characterized (at least locally) by Lipschitz forms, Lipschitz nonlinear descriptor systems have been paid much attention [11,24,25]. However, few results have been reported in observer design for discrete-time Lipschitz nonlinear descriptor systems [14,15]. In this section, the observer design method presented in Section 3 is extended to a class of discrete-time nonlinear descriptor systems with Lipschitz constraints.

Consider the following nonlinear descriptor system

$$\begin{cases} Ex(k+1) = Ax(k) + Bu(k) + \Phi(x(k), u(k)) \\ y(k) = Cx(k) \end{cases} \quad (23)$$

where $\Phi : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ is a known nonlinear vector function, other symbols are as defined before.

It is assumed that $\Phi(x, u)$ is globally Lipschitz with respect to $x(k)$ with a Lipschitz constant γ , i.e.

$$\|\Phi(x_1, u(k)) - \Phi(x_2, u(k))\| \leq \gamma \|x_1 - x_2\| \quad (24)$$

for all $x_1, x_2 \in \mathbb{R}^n$, and $\gamma > 0$ is independent of $u(k)$.

The nonlinear descriptor observer is described as

$$\begin{aligned} \hat{x}(k+1) &= TA\hat{x}(k) + TBu(k) + L(y(k) - C\hat{x}(k)) \\ &\quad + T\Phi(\hat{x}(k), u(k)) + Ny(k+1) \end{aligned} \quad (25)$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the estimate of the descriptor state $x(k)$, T and N are determined by (4), L is the matrix to be further designed.

Remark 4. When the matrix $E = I$, i.e., the descriptor system (1) reduces to a discrete-time nonlinear state-space system, by choosing $T = I_n$ and $N = 0$, the presented observer (23) coincides with that in [20].

The following theorem extends Theorem 1 to deal with a class of discrete-time nonlinear descriptor systems in the form of (23).

Theorem 3. For a given scalar $\gamma > 0$, (25) is an asymptotic observer for the plant (23) if there exist a positive matrix $P \in \mathbb{R}^{n \times n}$, a matrix $W \in \mathbb{R}^{n \times m}$, and a scalar $\eta > 0$ satisfying the linear matrix inequality Eq. (26) which is given in Box I where $\gamma_1 = \gamma \lambda_{\max}(T)$, and $\lambda_{\max}(X)$ denotes the maximum eigenvalue of X . Moreover, $L = P^{-1}W$.

Proof. Considering (23) and (25), the error dynamic equation is given by

$$e(k+1) = (TA - LC)e(k) + \Delta\Phi \quad (27)$$

where $\Delta\Phi = T\Phi(x(k), u(k)) - T\Phi(\hat{x}(k), u(k))$.

Consider the following Lyapunov candidate function

$$V(k) = e^T(k)Pe(k). \quad (28)$$

Then, its time difference becomes

$$\begin{aligned} \Delta V &= V(k+1) - V(k) \\ &= e^T(k+1)Pe(k+1) - e^T(k)Pe(k) \\ &= [(TA - LC)e(k) + \Delta\Phi]^T P[(TA - LC)e(k) \\ &\quad + \Delta\Phi] - e^T(k)Pe(k) \\ &= e^T(k)(TA - LC)^T P(TA - LC)e(k) + 2e^T(k) \\ &\quad \times (TA - LC)^T P \Delta\Phi + \Delta\Phi^T P \Delta\Phi - e^T(k)Pe(k). \end{aligned} \quad (29)$$

As the nonlinear term $\Delta\Phi$ satisfies the Lipschitz condition, it is easily shown that

$$\|\Delta\Phi\| \leq \gamma_1 \|e(k)\|. \quad (30)$$

Then, for a scalar $\eta > 0$, we have Eq. (31) which is given in Box II.

Letting $W = PL$ and using Lemma 1, it is easy to show that (26) is equivalent to $P > 0$ and $\Omega < 0$, which gives

$$\Delta V < 0. \quad (32)$$

From the Lyapunov stability theory, the error system (27) is asymptotically stable. That is, (25) is an asymptotic observer for the plant (23). \square

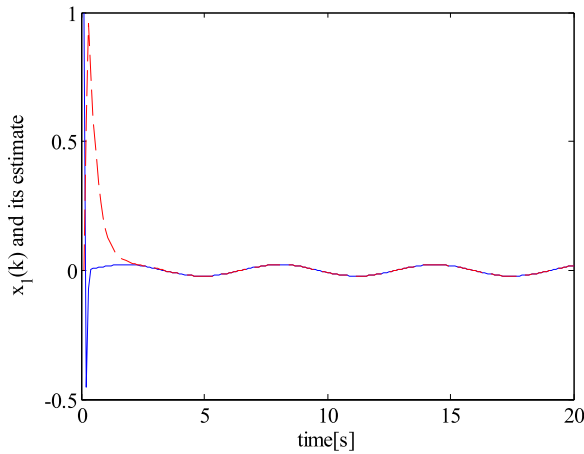


Fig. 1. State $x_1(k)$ (solid line) and its estimate $\hat{x}_1(k)$ (dashed line) in Example 1.

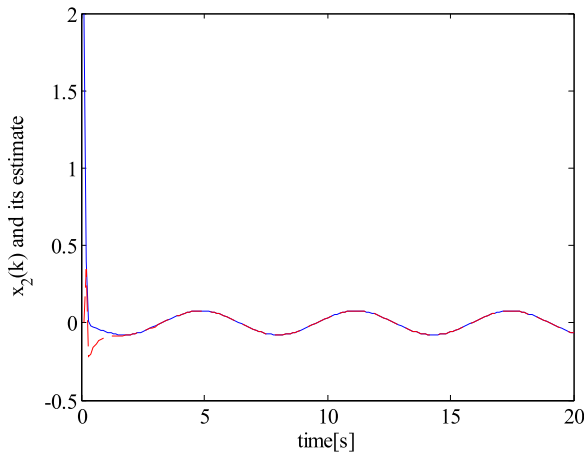


Fig. 2. State $x_2(k)$ (solid line) and its estimate $\hat{x}_2(k)$ (dashed line) in Example 1.

5. Simulations

In this section, two examples are given to show the effectiveness of the proposed method.

Example 1. Consider the following discrete-time descriptor system in the form of (1) with the following parameters [26]

$$E = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.153 & 0.045 & 0.069 \\ 0.156 & 0.252 & 0.156 \\ 0.135 & -0.171 & -0.636 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 1 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

A solution to (4) is obtained as

$$T = \begin{bmatrix} 0 & 0 & 1 \\ -0.4 & -0.4 & -0.2 \\ 0.6 & 0.4 & -0.8 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & 0 \\ -0.2 & 1 \\ 0.2 & -2 \end{bmatrix}.$$

Using the LMI toolbox to solve (8) and considering (16), we obtain

$$P = \begin{bmatrix} 1.0216 & 0.0319 & 0.2113 \\ 0.0319 & 1.2363 & -0.0307 \\ 0.2113 & -0.0307 & 1.0374 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.135 & -0.171 \\ -0.0282 & -0.0486 \\ 0.0462 & 0.2646 \end{bmatrix}.$$

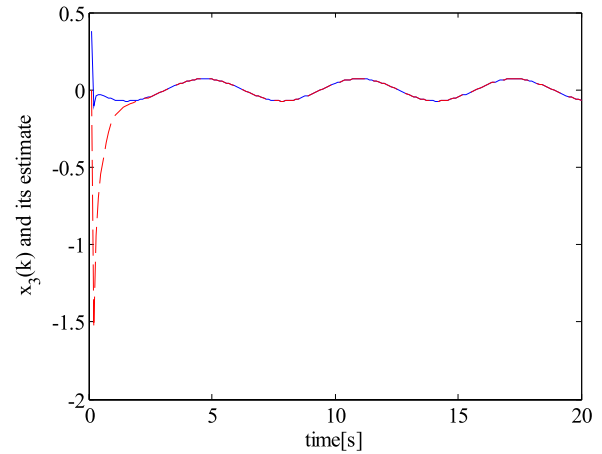


Fig. 3. State $x_3(k)$ (solid line) and its estimate $\hat{x}_3(k)$ (dashed line) in Example 1.

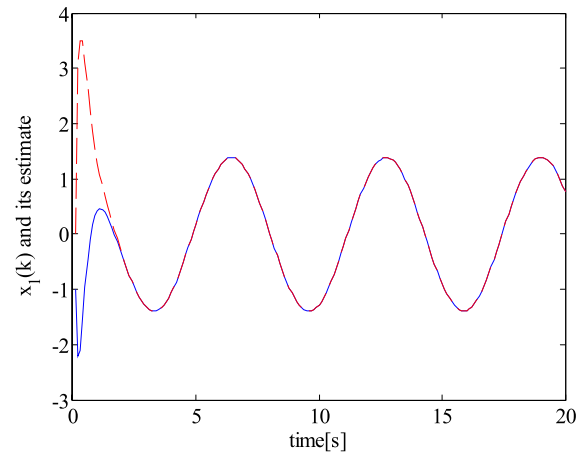


Fig. 4. State $x_1(k)$ (solid line) and its estimate $\hat{x}_1(k)$ (dashed line) in Example 2.

Figs. 1–3 exhibit the trajectories of the descriptor system states and their estimates, respectively. It is shown that the estimated states can closely track the states.

Example 2. We now illustrate the design for nonlinear discrete-time model. Consider the system described by (23) where

$$E = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0.7 & -1 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Phi(x(k), u(k)) = \begin{bmatrix} 0 \\ 0.05 \sin(x_3(k)) \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.$$

We can construct the observer in the form of (25) with

$$T = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad L = \begin{bmatrix} -0.3 \\ 0 \\ 0 \\ -0.3 \end{bmatrix}.$$

Simulation results are presented in Figs. 4–7, which show the convergence behavior of the proposed methods.

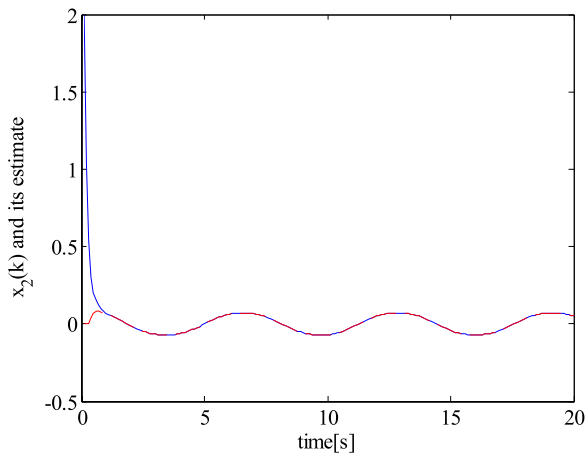


Fig. 5. State $x_2(k)$ (solid line) and its estimate $\hat{x}_2(k)$ (dashed line) in Example 2.

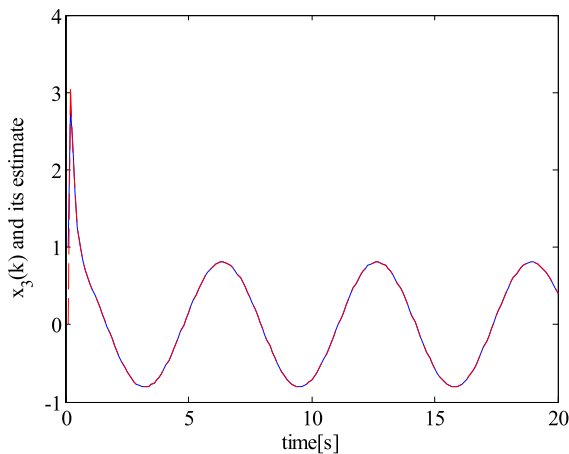


Fig. 6. State $x_3(k)$ (solid line) and its estimate $\hat{x}_3(k)$ (dashed line) in Example 2.

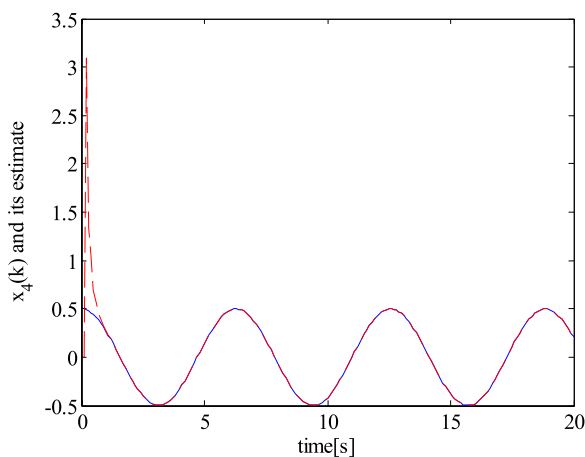


Fig. 7. State $x_4(k)$ (solid line) and its estimate $\hat{x}_4(k)$ (dashed line) in Example 2.

6. Conclusion

In this paper, a novel approach is presented to design observers for discrete-time descriptor systems. For discrete-time linear descriptor systems, the necessary and sufficient conditions for the convergence of the proposed observer are established and expressed in a linear matrix inequalities formulation. For a class of discrete-time nonlinear descriptor systems, the proposed method

is generalized to develop a nonlinear descriptor observer. The design procedure can be implemented systematically, and two simulation examples are used to demonstrate its effectiveness.

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