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Chapter 18

Learning → Online (feedback থাকবে, তার উপর ভিত্তি করে learn করতে হয়)

Learning → offline (Pre-observed data থেকে learning)

- Label may or may not present in the input data. If not, agent needs to define which is what.

- Label is good for learning, but label needs to be set manually which is time consuming and there may be errors. So, expensive.

- Unlabeled is less expensive, because we can collect data directly from sensors, no ~~manual~~ manual labeling.

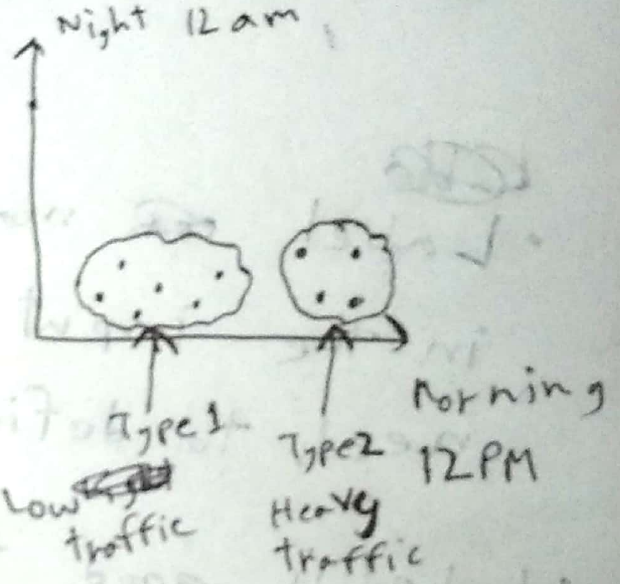
good or bad বলেনি নিজেও learn করছে]

[~~হচ্ছে, বলবেনা~~ input good নাকি bad হচ্ছে নিজেও learn করছে

Learning →

→ Unsupervised (Data do not have labels)

Clustering Algorithms



[~~[illegible]~~ learning agent data থেকে grouping গুলো detect করবে] ~~[illegible]~~

→ এরপর চাইলে grp গুলোয় label দিয়ে ~~[illegible]~~ supervised learning করা ~~[illegible]~~ যায়।

→ Supervised learning (input-output pairs) that means labelled.

যদি আগে থেকে data থাকে

Online/offline ~~[illegible]~~ ~~both~~ either হতে পারে।

↳ Reinforcement

(কিছুটা online learning এর মত। feedback থেকে learn করে।)

Supervised Learning

Generalization → একটা agent unseen data থেকে যত ভালো predict/classify করতে পারে ...

→ ~~Unit~~ Activities

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— Select the simplest hypothesis consistent with the data.

Over-fitting → অতিমাত্রায় training set dependent হয়ে যায় (Generalized হতে না।)

$$h^* = \underset{h \in \mathcal{H}}{\operatorname{argmax}} \; P(h \mid data)$$

By Bayes' rule this is equivalent to

$$h^* = \underset{h \in \mathcal{H}}{\operatorname{argmax}} \; P(data \mid h) \; \cancel{P(h)} \; P(h)$$

$/p(data)$

[illegible] দি কারণ হচ্ছে const.

$$P(B_1 \mid x) = \frac{P(x \mid B_1)\, P(B_1)}{P(x)}$$

$$y = \max f(x) \longleftarrow$$ $f(x)$ এর max

$$x^* = \underset{x}{\operatorname{argmax}} \, f(x) \longleftarrow$$ যেখানে x এর জন্য $f(x)$ max

যেহেতু max দরকার ~~এই~~ নাই, কোনো point-এ max হওয়া লাগবে, তাই variable part max করলেই হবে।

$$P(D \mid BS, BP) = 1.5d \quad , \quad P(\neg D \mid BS, BP) = 0.9d$$

$$P(D) + P(\neg D) = 1$$

$$1.5d + 0.9d = 1$$

$$\therefore d = \ldots$$

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Supervised learning
- → Classification (Discrete output)
- → Regression (output is a real number)

Decision Tree

ID3 → Decision tree making algo.

→ Any boolean function can be expressed using decision tree.

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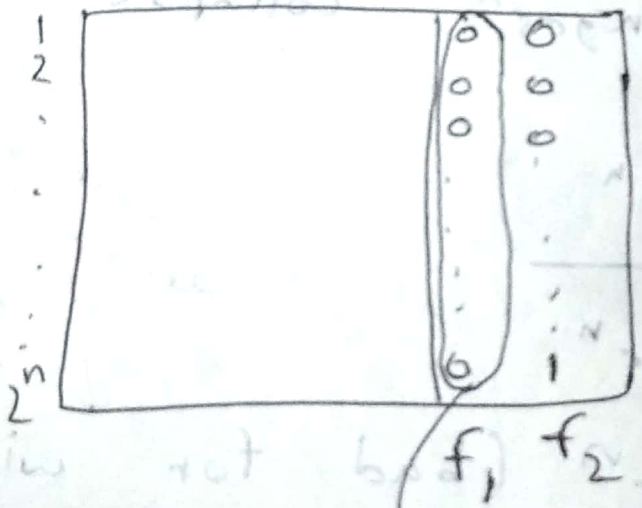
~~# Congestion~~

n variables

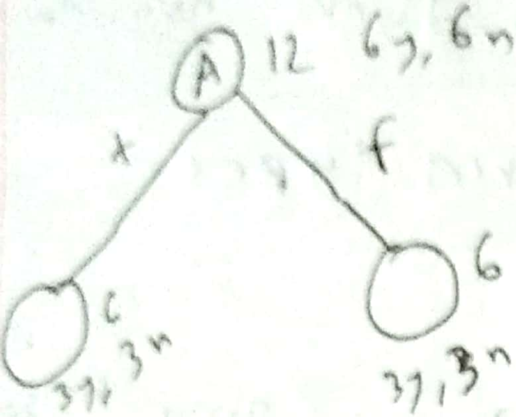
\# boolean functions = 2^{2^n}

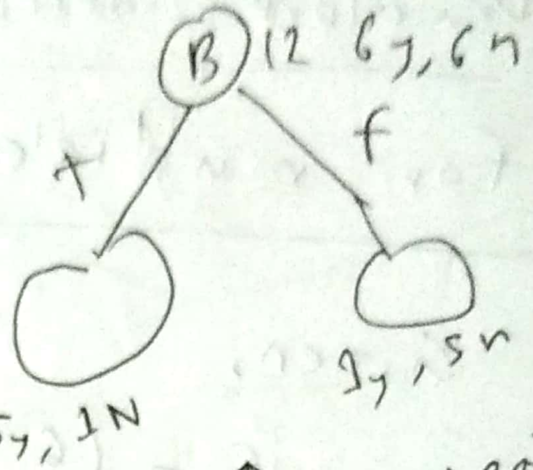
Truth Table



→ number of bits = 2^n

Input same, output different, causes →
Noisy data, missing some important
attributes.





↑ এটা leaf ও Probability পাওয়ার বেশি।

Heuristic measure → Information gain

→ কোনো attribute-এর ~~একটি~~ কোনো কোনো value এর জন্য ট্রেইন কোনো কোনো example না থাকলে, তাহলে কোনো কোনো parent-এর যেসব value বেশি আছে, সেটা দিয়ে হবে।

→ Review অন্য ব্যক্তিকে দিয়ে করাতে হবে।

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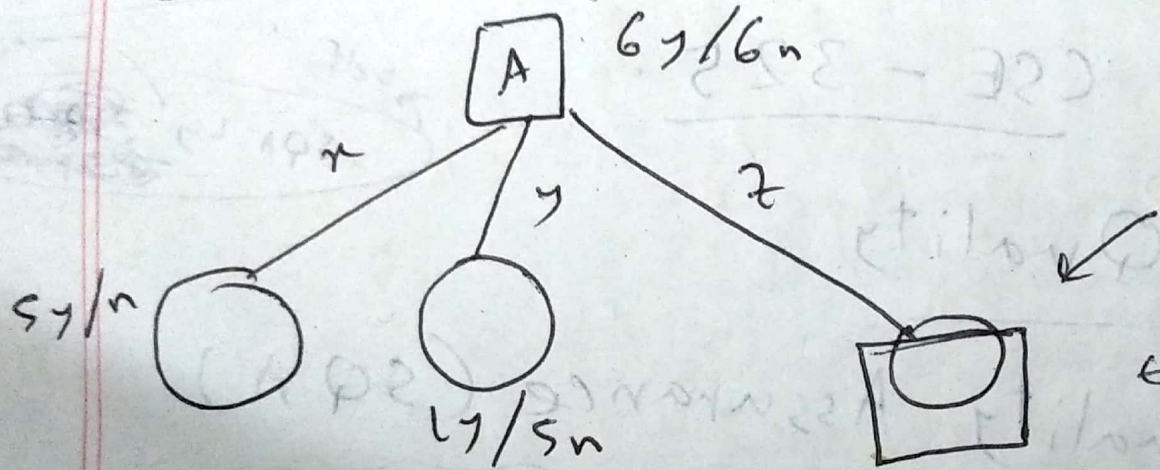
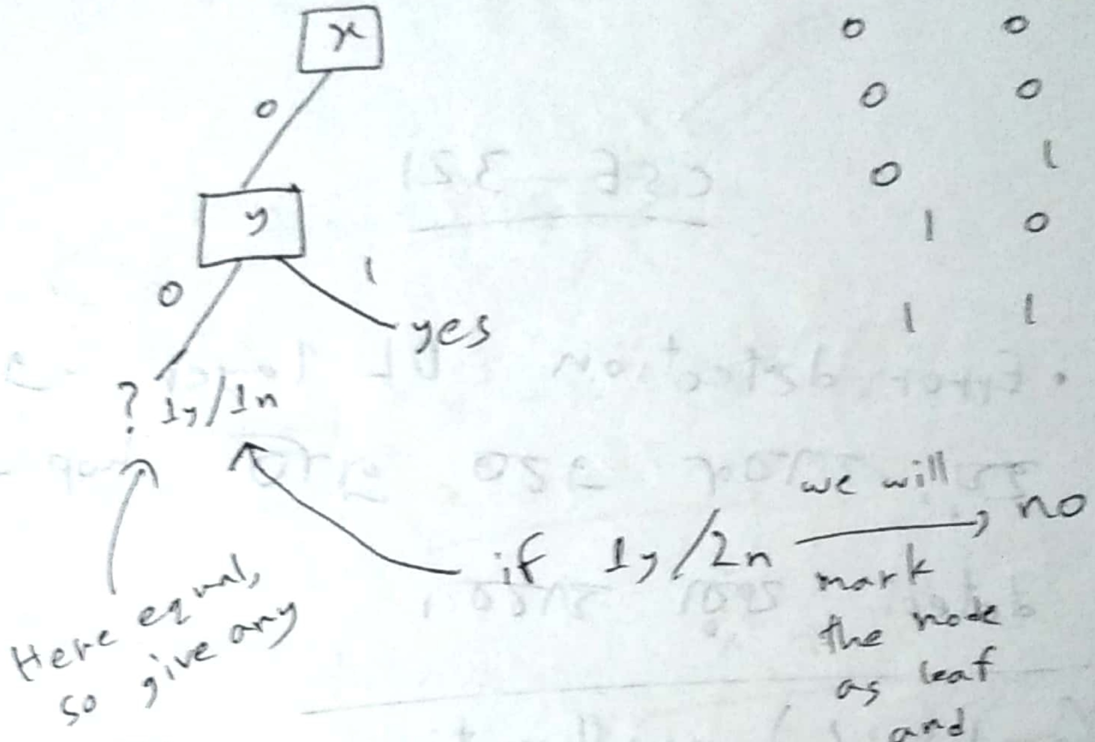
case 3 :



No example (make it leaf)

Decision will be taken based on the majority outcome of parent node. Here, y and n equal, so give any.

case 4:



x	y	Z
0	0	0
0	0	1
0	1	1
1	0	1
1	1	1

→ Entropy of random coin is 100% or maximum.
(All outcomes are equally likely)

random variable → V, Probability of $v_k \rightarrow P(v_k)$

$$H(V) = -\sum_k P(v_k) \log_2 P(v_k)$$

for coin toss: $H(V) = P(v_1)\log_2 \frac{1}{P(v_1)} + P(v_2)\log_2 \frac{1}{P(v_2)}$

$$= \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2 = 1$$

এরে base ২ use করা হয়, value change হলেও comparison same থাকছে। Base vary করে maxima হতে scale করা হয়।

22. Jump

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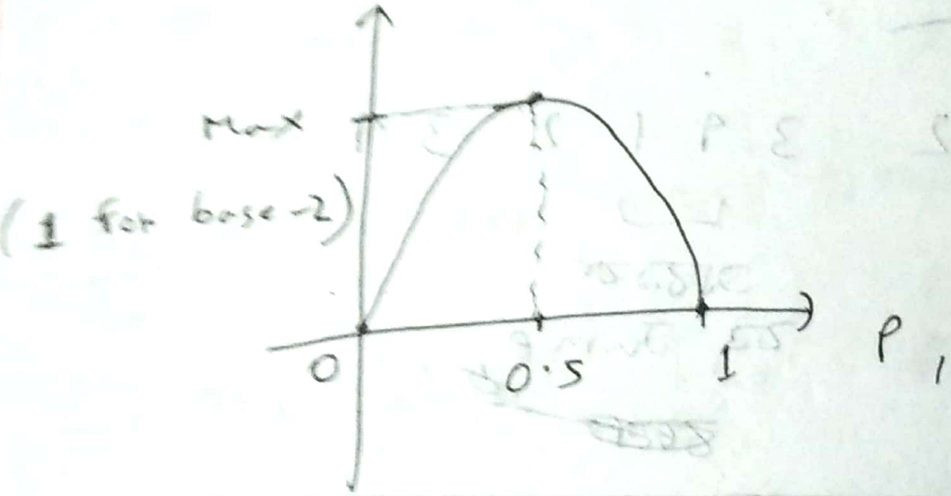
Information Gain:

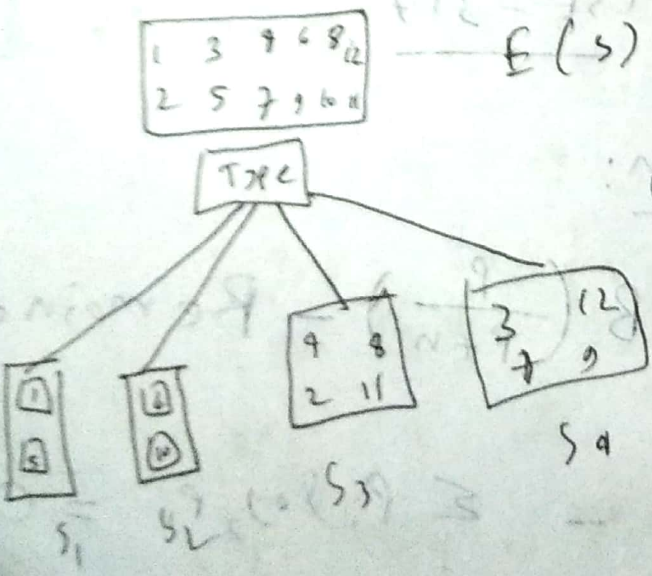
$$Gain(A) = B\left(\frac{p}{p+n}\right) - Remainder(A)$$

$$entropy = -\sum_i p_i \log_2 p_i = 0.97$$

for binary, $P_1 = \frac{6}{10}$, $P_2 = 1 - P_1$

$$B(q) = -q\log q - (1-q)\log(1-q)$$





$E(S) = 1$, $E(S_1) = 1$, $E(S_2) = 1$, $E(S_3) = 1$, $E(S_4) = 1$

Average (weighted)

$$\frac{\text{example in the set}}{\text{example in the parent}} \times E$$

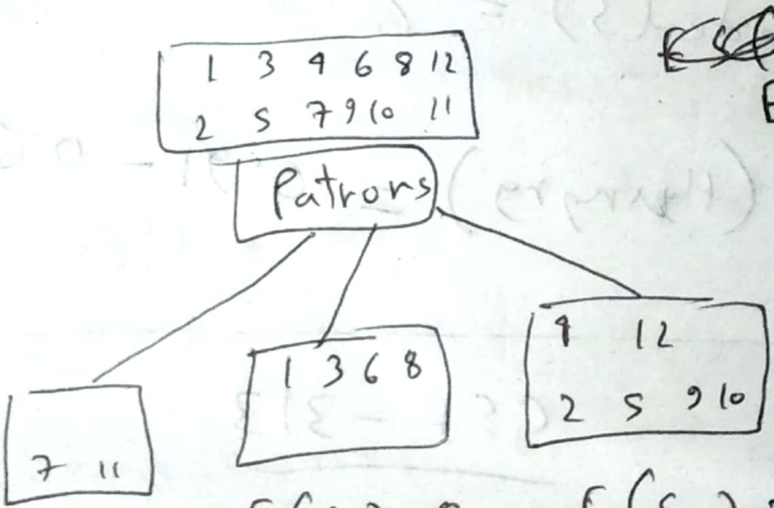
$$\frac{2}{12}\times 1 + \frac{2}{12}\times 1 + \frac{4}{12}\times 1 + \frac{4}{12}\times 1 = 1$$

So, previous E = Next E

Rem(S)

$$Gain(Type) = 1 - 1 = 0$$

$E(S) = 1$



$E(S_2) = 0$ $\quad E(S_3) = 0.918$

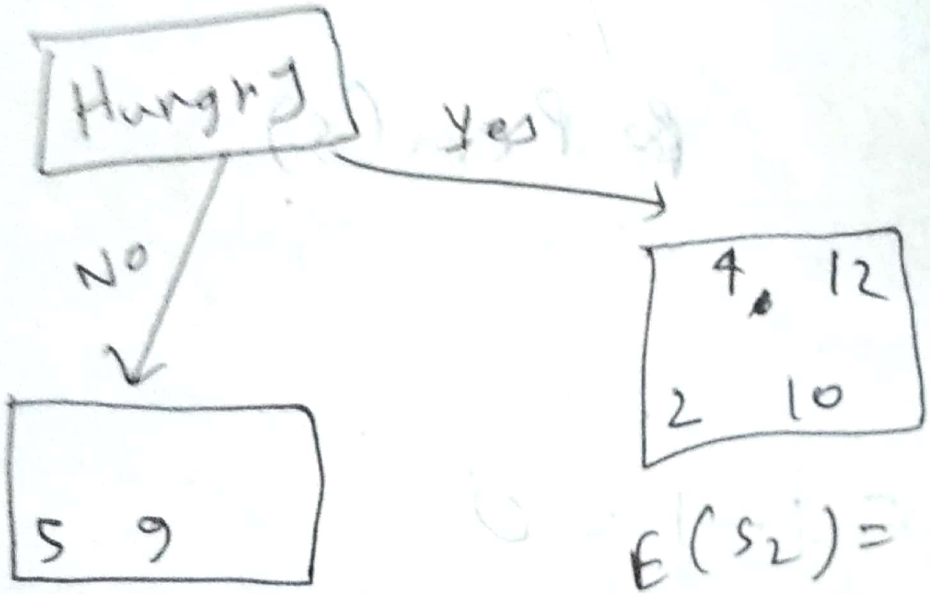
$E(S_1) = 0$

$$Rem(S) = 0.918 \times \frac{6}{12} = 0.46$$

$$Gain(Patrons) = 1 - 0.46 = 0.54$$

$P \rightarrow 4, 12$ $\quad$ $N \rightarrow 2, 5, 9, 10$

$E(S) = 0.91$



$E(S_2) = 1$

$E(S_1) = 0$

$$\text{Remainder}(S) = \frac{4}{6} \times 1 = 0.67$$

$$\text{Gain}(\text{Hungry}) = 0.91 - 0.67 = 0.24$$

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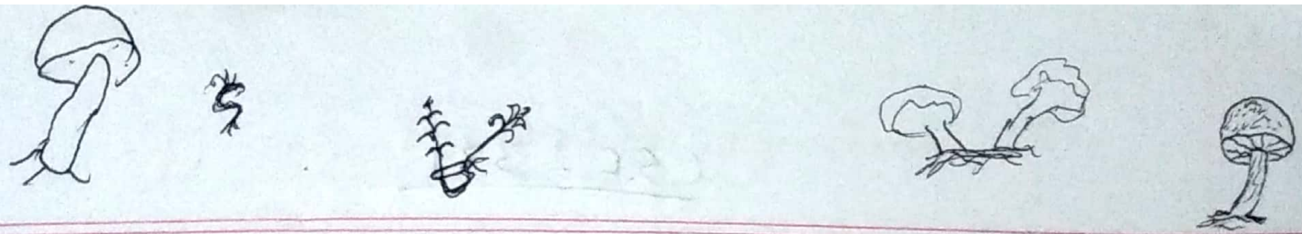
- যেসব leaf এ এসেছে example গুলো most likely specific path-এ থাকবে, এগুলো post-pruning এ cut-off করা হতে পারে (যদি accuracy না কমে)

- কোনো example এর কোনো attribute missing থাকলে -
  - → ওই row টা বাদ দেয় (এটা সবসময় সম্ভব হয় না, কারণ examples কম থাকলে কিছু row ফেলে দিলে, performance কমে যাবে।)
  - → ওই attribute এর যত value examples এ আছে তার mean টা নিয়ে ওখানে বসানো।
  - → অন্যান্য complex techniques আছে।



Probabilistic Reasoning (chapter - 14)

Russel Norwig

Bayesian Network

$$\cancel{P(x=5 \mid y=6}$$

$$P(x=5 \mid y=6) = \frac{P(x=5,\ y=6)}{P(y=6)}$$

$$= \frac{1}{6}$$

$$P(x) = \sum_{y} P(x, y)$$

$\boxed{\text{Marginal Probability}}$

$$P(y=6) = \sum_{x} P(X=x, Y=6)$$

$$= P(X=1, Y=6) + P(X=2, Y=6) +$$

$$\cdots + P(X=6, Y=6$$

$$= \frac{1}{36} + \frac{1}{36} + \cdots + \frac{1}{36} = \frac{1}{6}$$

Posterior Probability → Symptoms দেখে cause-এর **probability** বের করার চেষ্টা

Likelihood Probability → Cause given, symptoms এর **probability**

In bayesian network, cause → Symptoms

P(B)
.001

Prior probability

B	E	P(A)
t	t	.95
t	f	.94
f	t	.29
f	f	.001

← Likelihood probability (conditional probability)

$P(A|B,C) = P(A|B)$ → [A and C are conditionally independent given B]

(C → independent)

{B is more significant than C, for A}

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$P(A|B,C) = P(A)$; So, A is not dependent on B and C.

$P(B|A,E) \neq P(B|A)$ But $\boxed{P(B|A) = P(B)}$

$$P(x_1, \ldots, x_n) = P(x_n \mid x_{n-1}, \ldots, x_1)\, P(x_{n-1}, \ldots x_1)$$

$$= P(x_n \mid x_{n-1}, \ldots, x_1)\, P(x_{n-1} \mid x_{n-2}, \ldots, x_1) \ldots$$

$$P(x_2 \mid x_1)\, P(x_1)$$

$$= \prod_{i=1}^{n} P(x_i \mid x_{i-1} \ldots x_1)$$

For Bayesian Network

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{Parent}(x_i))$$

[with topo-sort]

if direct parents are given, x_i will be independent of others.

Inference :

$P(A \mid B = \text{true}) \rightarrow$ A → এর সব outcome এর Probability বের করতে হবে।

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$$P(B \mid j, m) = \frac{P(B, j, m)}{P(j, m)} = \alpha P(B, j, m)$$

Marginal from joint $P(A) = \sum_b P(A, b)$

Variable Elimination Algorithm:

$$P(m, j, a, \neg b) = \sum_e P(m, j, a, \neg b, e)$$

$$= P(m, j, a, \neg b, e) + P(m, j, a, \neg b, \neg e)$$

$$= \cancel{P(m \mid a)\, P(j \mid a)\, P(a \mid \neg b)\, P(a \mid e)} \ldots$$

$$= P(m \mid a)\, P(j \mid a)\, P(a \mid \neg b, e)\, P(\neg b)\, P(e)$$

$$+ P(m \mid a)\, P(j \mid a)\, P(a \mid \neg b, \neg e)\, P(\neg b)\, P(\neg e)$$

$$= P(m \mid a)\, P(j \mid a)\, P(\neg b) \left[P(a \mid \neg b, e)\, P(e) + P(a \mid \neg b, \neg e)\, P(\neg e) \right]$$

$$= 0.9 \times 0.7 \times 0.999 \left[0.29 \times .002 + .001 \times 0.998 \right]$$

$$= 9.93 \times 10^{-4}$$

$P(B \mid j, m) := P(B)\, P(E)\, P(A \mid B, E)\, P(J \mid A)\, P(M \mid A)$

$f_1(B) = P(B)$, $f_2(E) = P(E)$, $f_3(A, B, E) =$

$P(A \mid B, E)$.

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$$P(B, j, m) = \sum_a \sum_e P(B, j, m, a, e)$$

$$= \sum_a \sum_e P(j|a) \cdot P(m|a) \cdot P(a|B, e) \cdot P(B) \cdot P(e)$$

$$= P(B) \sum_a P(j|a) P(m|a) \sum_e P(a|B, e) P(e)$$

A	B	P
T	T	0.2
T	F	0.3
F	T	0.6
F	F	0.8

B	C	P
T	T	0.3
T	F	0.4
F	T	0.8
F	F	0.2

A	B	C	P
T	T	T	0.06
T	T	F	0.08
T	F	T	0.29
T	F	F	0.06
F	T	T	0.18
F	T	F	0.29
F	F	T	0.69
F	F	F	0.16

A	C	P_1
T	T	0.3
T	F	0.14
F	T	0.82
F	F	0.4

Normalization means যাতে sum 1 হয়.

So, $\frac{x_i}{\Sigma x}$

$$\sum_a P(a|b) = 1$$

$$\sum_a P(a,b) \neq 1$$

$$\sum_a \sum_b P(a,b) = 1$$

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$$= (I_0 \ldots$$

$$= I_0 + \frac{r_0}{2}$$

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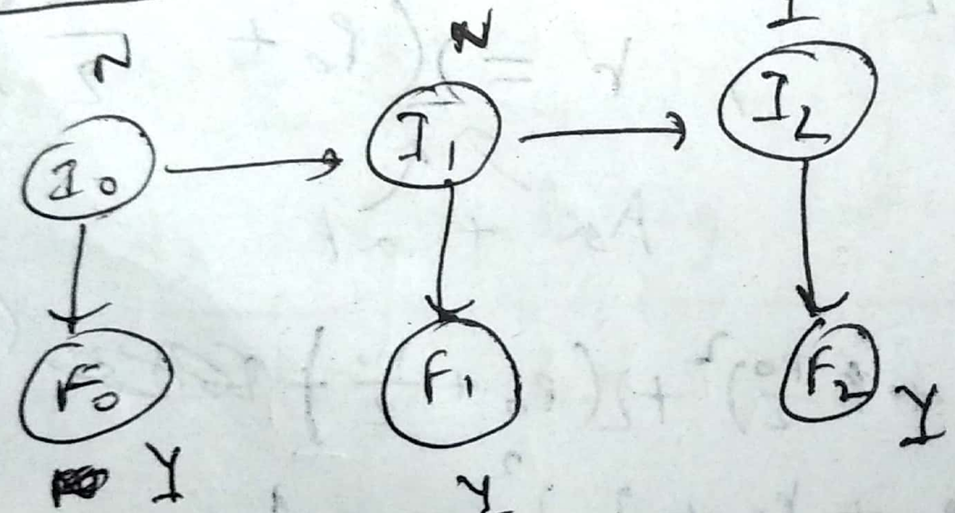
Temporal Probabilistic Models

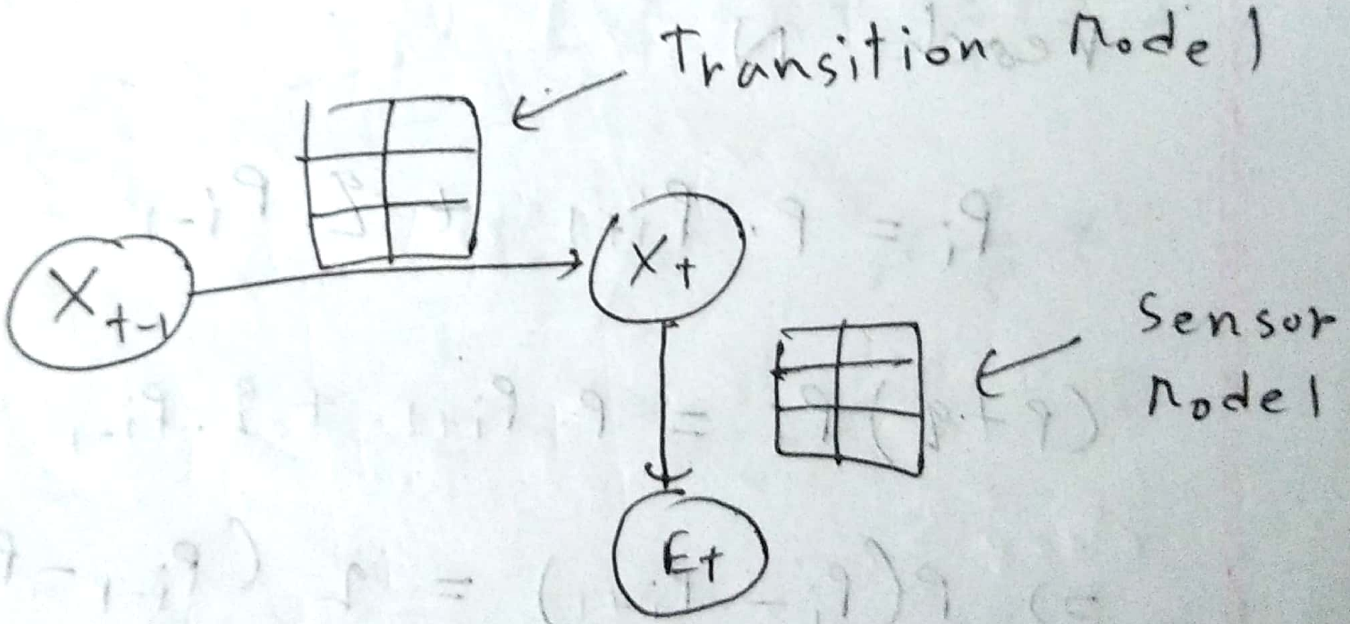


Hidden Markov Models

→ Only one RV in the state.

Stationary Process → Transition model do not change over time.

$$P(X_2 | X_1) = P(X_3 | X_2) = \dots$$



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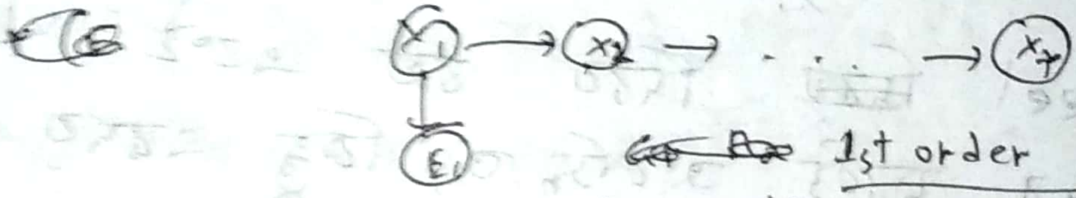
2^t combination of values (if binary)

$$P(X_{1:t}, E_{1:t}) = P(E_{1:t} \mid X_{1:t})\, P(X_{1:t})$$



1st order Markov assumption

$$P(E_1 \mid X_1, X_2, \ldots) = P(E_1 \mid X_1) \qquad P(X_2 \mid X_1, \ldots) = P(X_2 \mid X_1)$$

$P(A,B \mid C) = P(A \mid B,C)\, P(B \mid C)$

$$= P(E_t \mid E_{1:t-1}, X_{1:t})\, P(E_{1:t-1} \mid X_{1:t})\, P(X_t \mid X_{1:t-1})\, P(X_{1:t-1})$$

$$\boxed{P(E_{t-1} \mid E_{1:t-2}, X_{1:t})\, P(E_{1:t-1} \mid X)}$$

$$= P(E_t : X_t)\, P(E_{1:t-1} \mid X_{1:t})\, P(X_t \mid X_{1:t-1})\, P(X_{t-1} \mid X_{1:t-2})\, P(X_{1:t-2})$$

$$= \prod_{i=1}^{t} P(E_i \mid X_i) \cdot \prod_{i=1}^{t} P(X_i \mid X_{i-1})$$

$$= \prod_{i=1}^{t} \left[P(E_i \mid X_i)\, P(X_i \mid X_{i-1}) \right]$$

sensor model/observation model/Emission model

Filtering Query:

$$P(X_t \mid e_{1:t})$$

Prediction Query:

$$P(X_{t+k} \mid e_{1:t})$$

Smoothing Query:

$$P(X_k \mid e_{1:t}), \quad 0 \le k < t$$

Most Likely Explanation Query:

$$P(X_{1:t} \mid e_{1:t})$$

Filtering

$$P(X_{t+1} \mid e_{1:t+1}) = P(\underbrace{X_{t+1}}_{A} \mid e_{1:t}, \underbrace{e_{t+1}}_{B})$$

$$= \frac{P(e_{t+1} \mid X_{t+1}, e_{1:t})\, P(X_{t+1} \mid e_{1:t})}{P(e_{t+1} \mid e_{1:t})}$$

(Sensor model)

$$= \alpha \cdot P(e_{t+1} | X_{t+1}) \, P(X_{t+1} | e_{1:t})$$

marginalize over X_t

$$P(X_{t+1} | e_{1:t}) = \sum_{x_t} (X_{t+1}, x_t | e_{1:t})$$

$$= \sum_{x_t} P(X_{t+1} | x_t, e_{1:t}) \, P(x_t | e_{1:t})$$

$$= \sum_{x_t} P(X_{t+1} | x_t) \, P(x_t | e_{1:t})$$

(Transition model)

base case

$$P(x_1 | e_1)$$

$$= \alpha P(e_1 | x_1) P(x_1)$$

for every value (n)

$$P(X_{t+1} | e_{1:t+1}) = \alpha P(e_{t+1} | X_{t+1}) \sum_{x_t}^{n} P(X_{t+1} | x_t) \, P(x_t | e_{1:t})$$

e.g. $\rightarrow$ True, false, ...

$n \rightarrow$ number of states.

Complexity $O(tn^2)$

if $n =$ some number of states (like 2, 4 ...)

$O(t \cdot 4)$

$\hookrightarrow O(t)$

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Let, $P(X_t \mid e_{1:t}) = f_{1:t}$

$\therefore P(X_t = x_i \mid e_{1:t}) = f_{1:t}[i]$

So,

$$f_{1:t+1}[i] = \alpha P(e_{t+1} \mid X_{t+1} = x_i) \sum_j P(X_{t+1} = x_i \mid X_t = x_j) P(X_t = x_j \mid e_{1:t})$$

$$= \alpha (O_{t+1,i}) \sum_j (a_{i,j})(f_{1:t}[j])$$

R_{t-1}	$P(R_t)$
t	0.7
f	0.3

R_t	$P(U_t)$
t	0.9
f	0.2

$P(R_2 \mid U_{1:2})$

	t= 1	2	3	
r	0.818	0.883		
¬r	0.182	0.117		

$$P(R_1 = r \mid U_1) = \alpha\, P(U_1 \mid R_1 = r)\, P(R_1 = r)$$

$$= \alpha \times 0.9 \times 0.5 \leftarrow \text{assume given}$$

$$= .45\alpha = 0.818$$

$$P(R_1 = \neg r \mid U_1) = \alpha\, P(U_1 \mid R_1 = \neg r)\, P(R_1 = \neg r)$$

$$= \alpha \times 0.2 \times .5 = .1\alpha$$

$$= 0.182$$

$$\alpha = \frac{20}{11}$$

$$P(R_2 \mid U_{1:2}) = \alpha\, P(U_2 \mid R_2) \sum_{r_1} P(R_2 \mid r_1)\, P(r_1 \mid u_1)$$

Forward Algorithm

$$P(R_2 = r \mid u_{1:2}) = \alpha\, P(u_2 \mid R_2 = r) \sum_{r_1} P(R_2 = r \mid r_1)\, P(r_1 \mid u_1)$$

$$= \alpha \times 0.9 \times (0.7 \times 0.818 + 0.3 \times 0.182)$$

$$= 0.56448\, \alpha = 0.883$$

$$P(R_2 = \neg r \mid u_{1:2}) = 0.075\, \alpha$$

$$= 0.117$$

Prediction

$$P(X_{t+k} \mid e_{1:t})$$

$k \geq 1$

$$P(X_{t+1} \mid e_{1:t}) = \sum_{x_t} P(X_{t+1}, x_t \mid e_{1:t})$$

$$= \sum_{x_t} P(X_{t+1} \mid x_t, e_{1:t})\, P(x_t \mid e_{1:t})$$

$$= \sum_{x_t} P(X_{t+1} \mid x_t)\; P(x_t \mid e_{1:t})$$

Table

Forward algorithm

Filtering gives better estimate, as have already observed that days event. (observation)

Likelihood of Evidence Sequence

$$P(e_{1:t}) = \sum_{x_t} P(e_{1:t}, x_t)$$

$$P(e_{1:t}, x_t) = P(e_{1:t-1}, e_t, x_t)$$

$$= P(e_t \mid x_t, e_{1:t-1})\, P(e_{1:t-1}, x_t)$$

$$= P(e_t \mid x_t) \sum_{x_{t-1}} P(x_t, x_{t-1}, e_{1:t-1})$$

$$= P(e_t \mid x_t) \sum_{x_{t-1}} P(x_t \mid x_{t-1})\, P(x_{t-1}, e_{1:t-1})$$

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Smoothing:

$P(X_k \mid e_{1:t})$ where $1 \le k < t$

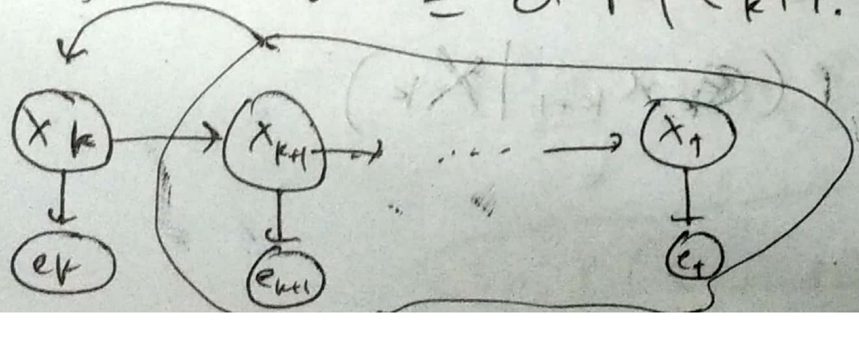
→ It will improve the filtered probability. Improving probability of past states may help in some decision making.

$$P(X_k \mid e_{1:t}) = P(X_k \mid e_{1:k}, e_{k+1:t})$$

$$= \alpha P(e_{k+1:t} \mid e_{1:k}, X_k) \cdot \underbrace{P(X_k \mid e_{1:k})}_{\text{Filtering}}$$

এটা থাকলে আগের e এর উপর depend করবে না।

$$= \alpha P(e_{k+1:t} \mid X_k) P(X_k \mid e_{1:k})$$



$$P(e_{k+1:t} \mid X_k) = \cancel{\sum_{x_{k+1}} P(e_{k+1:t} \mid X_k, X_{k+1})\, P(}$$

$$= \sum_{x_{k+1}} P(e_{k+1:t}, x_{k+1} \mid X_k)$$

$$= \sum_{x_{k+1}} P(e_{k+1:t} \mid X_k, x_{k+1})\, P(x_{k+1} \mid X_k)$$

$$= \sum_{x_{k+1}} P(e_{k+2:t}, e_{k+1} \mid X_{k+1})\, P(x_{k+1} \mid X_k)$$

$$= \sum_{x_{k+1}} P(e_{k+1} \mid e_{k+2:t}, X_{k+1})\, P(x_{k+1} \mid X_k)\, P(e_{k+2:t} \mid X_{k+1})$$

$$= \sum_{x_{k+1}} P(e_{k+1} \mid x_{k+1})\, P(e_{k+2:t} \mid x_{k+1})\, P(x_{k+1} \mid X_k)$$

$$P(e_{k+1:t} \mid X_k) = b_{k+1,t}$$

$$b_{k+1,t}[i] = P(e_{k+1:t} \mid X_k = x_i)$$

$$= \sum_{x_j} P(e_{k+1} \mid X_{k+1} = x_j) P(X_{k+1} = x_j \mid X_k = x_i)\, b_{k+2,t}[j]$$

$$= \sum_{x_j} (O_{k+1,j})(a_{ij})(b_{k+2,t}[j])$$

base condition:

$$P(e_{t+1:t} \mid X_t) = 1$$

empty seq.

Probability of empty seq. is 1.

$$P(e_{t:t} \mid X_{t-1}) = P(e_t \mid X_{t-1})$$

$$= \sum_{x_t} P(e_t, x_t \mid X_{t-1})$$

$$= \sum_{x_t} P(e_t \mid x_t) P(x_t \mid X_{t-1})$$

So, $P(e_{t+1:t} \mid X_t) = 1$

$$P(X_k = x_i \mid e_{1:t}) = \alpha \, f_{1:k}[i] \times b_{k+1,t}[i]$$

Backward Probability table

Backward $\rightarrow O(tn^2)$

forward $\rightarrow O(tn^2)$

point-wise multiplication $\rightarrow O(\underline{tn^2})$?

$O(n)$ হওয়ার কথা

Overall $\rightarrow O(tn^2)$

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Chapter-16 এর last part :
online class

CSE – 313

CSE — ~~[illegible]~~ 317

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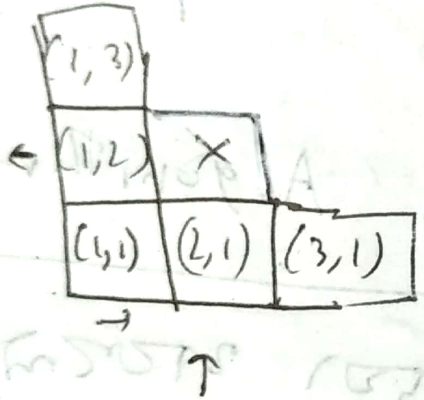
Expected Utility of a policy



$S_1 = s$

$S_2 = \{(1,2), (2,1), (1,1)\}$

P: 0.1 0.8 0.1

π

$(1,1) \longrightarrow R$

$S_3 \longrightarrow \{(1,3), (1,2), (1,1), (2,1), (3,1)\}$

P :

$$U^{\pi}(s) = E\left[\sum_{t=0}^{\infty} \gamma^{t} R(S_t)\right] ; \; S_0 = s$$

Optimal policy

$$\pi_s^{*} = \arg\max_{\pi} U^{\pi}(s)$$

$$\pi^{*}(s) = \arg\max_{a \in A(s)} \boxed{\sum_{s'} U(s') P(s' \mid s, a)}$$

Expected utility of neighbouring ~~policy~~ states.

Value Iteration Algorithm

• ~~ae~~ U(s) জানা থাকলে policy বের করা easy. Policy বের করে U(s) বের করা কঠিন

$$U(s) = R(s) + \gamma \max_{a} \sum_{s'} P(s' | s, a) U(s')$$

এটি থাকায়
non-linear eq^n

neighbours of current state

$$U(s) = R(s) + \gamma R(s_1) + \gamma^2 R(s_2) + \ldots$$

$$= \sum_{t=0}^{\infty} \gamma^t R(s_t)$$

$$= R(s) + \gamma \{ R(s_1) + \gamma R(s_2) + \ldots \}$$

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Utility, $U(s) = R(s) + \gamma . R(s_1) + \gamma^2 . R(s_2) + \dots$

Utility is total discounted rewards of over all possible sequence states.

start এ 0 থাকলে দ্রুত converge হবে। [Random দিয়ে start করলে বেশি time লাগে।]

Let, $U_i(s) = 0$

$$U_{i+1}(s) = R(s) + \gamma \max_a \sum_{s'} P(s'|s,a) . U_i(s')$$

i - is the number of iteration.

$|S| = n$, $|actions| = m$

$O(n \times m \times t)$ ← number of iteration value iteration

Policy iteration algorithm:

$$\pi_{i+1}(s) = \underset{a \in A}{\operatorname{argmax}} \sum_{s'} U(s') \, P(s' | s, a)$$

If policy is given,

$$U_i(s) = R(s) + \gamma \sum_{s'} P(s' | s, \pi_i(s)) \, U_i(s')$$

[Linear]

Policy-evaluation $\longrightarrow n \times t$

Optimal policy calc $\longrightarrow n \times m$

assume T times

$$= n \times t \times T + n \times m \times T$$

$$\approx n \times m \times T$$