CSE - 317

2020-21

5. b) $P(M|d)$, $P(R|d)$ [Changing question according to our syllabus]

$$P(M|d) = \alpha P(M,d)$$

$$= \alpha \sum_r \sum_a P(M,d,r,a)$$

$$= \alpha \sum_r \sum_a P(a|d) P(d|M,r) P(M|r) P(r)$$

$$= \alpha \sum_r P(d|M,r) P(M|r) P(r) \sum_a P(a|d)$$

$$= \alpha \sum_r P(d|M,r) P(M|r) P(r)$$

$f_1(R)$

R	P
l	0.8
h	0.2

$f_2(R,M)$

R	M	P
l	y	0.6
l	n	0.4
h	y	0.1
h	n	0.9

$f_3(R,M,T)$

R	M	T	P
l	y	t	.6
l	y	d	.4
l	n	t	.7
l	n	d	.3
h	y	t	.4
h	y	d	.6
h	n	t	.5
h	n	d	.5

CS CamScanner

$f_3(R,M,T) \longrightarrow f_4(R,M)$

R	M	P
l	y	.4
l	n	.3
h	y	.6
h	n	.5

$f_5(R,M) = f_1(R) \times f_2(R,M)$

R	M	P
l	y	.48
l	n	.32
h	y	.02
h	n	.18

$f_6(R,n) = f_5(R,n) \times \cancel{B}\, f_1(R,n)$

R	n	P
l	y	·192
l	n	·096
h	y	·012
h	n	·09

Summing out R,

n	P
y	0·204
n	0·186

Normalizing,

n	P(n|d)
y	·523
n	·477

$$P(R|d) = \alpha P(R, d)$$

$$= \alpha \sum_m P(d|R, m) P(m|R) P(R)$$

Summing out M, (from f_6)

R	P
l	·288
h	·102

Normalizing,

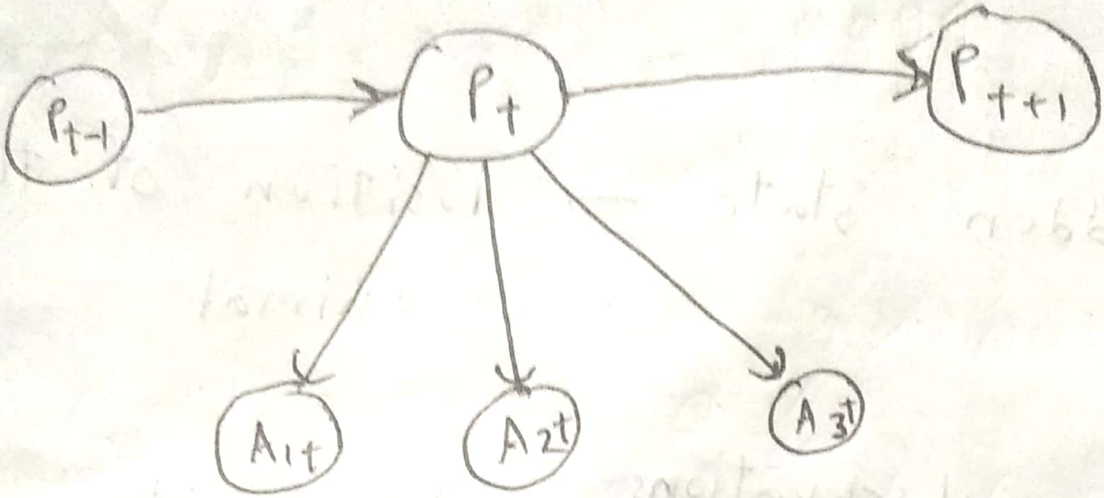
R	P(R|d)
l	·738
h	0·262

6.b)

Hidden state → Position of the animal

Possible observations :

A_1	A_2	A_3
yes	yes	yes
yes	yes	no
yes	no	yes
yes	no	no
no	yes	yes
no	yes	no
no	no	yes
no	no	no



Transition model

P_{t-1}	C_1	C_2	C_3	m
C_1	·8	·05	·05	·1
C_2	·05	·8	·05	·1
C_3	·05	·05	·8	·1
m	0·1	0·1	0·1	0·7

C_1 = Corner 1

C_2 = Corner 2

C_3 = Corner 3

m = middle

Observation model

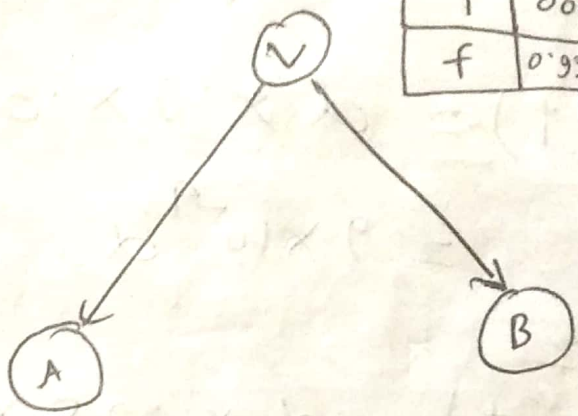
P_t	A_{1t}
C_1	·6
C_2	·1
C_3	·1
M	·4

P_t	A_{2t}
C_1	·1
C_2	·6
C_3	·1
m	·4

P_t	A_{3t}
C_1	·1
C_2	·1
C_3	·6
m	·4

5. a)

V	P
t	.001
f	0.999



A V	P
t	.95
f	.1

B	P
t	.9
f	.05

$$P(V=t \mid A=t) = \frac{P(A=t, V=t)}{P(A=t)}$$

$$= \alpha\, P(A=t \mid V=t)\, P(V=t)$$

$$= \alpha \times .95 \times .001$$

$$= 9.5 \times 10^{-4} . \alpha$$

$$P(V=f \mid A=t) = \alpha\, P(A=t \mid V=f)\, P(V=f)$$

$$= \alpha \times .1 \times .999$$

$$= .0999\alpha$$

$\therefore P(V=t \mid A=t) = .0094$

$P(V=t \mid B=t) = \alpha \times .9 \times .001$

$= 9 \times 10^{-4} \alpha$

$P(V=f \mid B=t) = \alpha \times .05 \times .999$

$= 0.04995 \alpha$

$\therefore P(V=t \mid B=t) = 0.018$

So, B returning positive is more indicative.

5.b)

$$P(w|dry) = \alpha P(w, dry)$$
$$= \alpha P(dry|w) \cdot P(w)$$

instantiating,

$f_3(w) =$

w	P
sun	.9
rain	.25

$$f_4(w) = f_1(w) \times f_3(w)$$

w	P
sun	.765
rain	.0375

Normalizing,

w	P(w|dry)
sun	0.953
rain	0.047

6.a)

$$P(B \mid -e, +j) = \alpha P(B, -e, +j)$$

$$= \alpha \sum_a \sum_m P(B, -e, +j, a, m)$$

$$= \alpha \sum_a \sum_m P(B) P(-e) P(a \mid B, -e) P(m \mid a) P(+j \mid a)$$

$$= \alpha P(B) P(-e) \sum_a P(a \mid B, -e) \cancel{P(m \mid a)} P(+j \mid a)$$

$P(-e)$

E	P
$-e$	.998

$P(+j \mid a)$

a	P
$+a$	.9
$-a$	.05

$P(a \mid B, -e)$

B	A	P
$+b$	$+a$	.94
$+b$	$-a$	.06
$-b$	$+a$	.001
$-b$	$-a$	.999

$P(a|B,-e) \times P(+j|a)$

B	A	P
+b	+a	.846
+b	-a	.003
-b	+a	.0009
-b	-a	.04995

Summing out,

B	P
+b	.849
-b	.05085

Multiplying P(B) and P(-e)

B	P
+b	.000847
-b	.0507

Normalizing,

B	P
+b	.0164
-b	0.9836

b)

$$P(E \mid +m, +j) = \alpha P(E, +m, +j)$$

$$= \alpha \sum_b \sum_a P(E, +m, +j, b, a)$$

$$= \alpha P(E) \sum_a \cancel{P(a \mid +m, +j)} \; P(+m \mid a)$$

$$P(+j \mid a) \sum_b P(a \mid E, b)\, P(b)$$

$$f_1(E, B, A) = P(a \mid E, b) \times P(b)$$

B	E	A	P
+b	+e	+a	9.5×10^{-4}
+b	+e	-a	5×10^{-5}
+b	-e	+a	9.4×10^{-4}
+b	-e	-a	6×10^{-5}
-b	+e	+a	.28971
-b	+e	-a	.70929
-b	-e	+a	9.99×10^{-4}
-b	-e	-a	.998001

Summing out b,

E	A	P
+e	+a	.29066
+e	−a	.70934
−e	+a	1.939×10^{-3}
−e	−a	.998061

$P(+m \mid a) = f_2(A)$

A	P
+a	.7
−a	.01

$P(+j \mid a) = f_3(A)$

A	P
+a	0.9
−a	.05

$\cancel{= f_2(A) \times f_3(A) \times f_4}$

$f_3(E, A)$

Multiplicating these,

E	A	P
+e	+a	.1831
+e	−a	3.5467×10^{-4}
−e	+a	1.222×10^{-3}
−e	−a	4.99×10^{-4}

Summing out A

E	P
+e	0.1835
−e	1.721×10^{-3}

Multiplying P(E)

E	P
+e	3.67×10^{-4}
−e	1.718×10^{-3}

Normalizing,

E	P
+e	0.176
−e	0.829

7.b)

$$\pi(+r) = 0.6 \times 0.7 + 0.4 \times 0.3$$
$$= 0.54$$

$$\pi(-r) = 0.6 \times 0.3 + 0.4 \times 0.7$$
$$= 0.46$$

CamScanner

	1	2	
+r	·84	~~0.887~~	·179
−r	·16	~~0.[illegible]~~	·821

At 1:

$\cancel{P(r_1) = d \times}$

$$P(+r) = d \times .9 \times \pi(+r)$$

$$= 0.486d$$

$$P(-r) = d \times .2 \times \pi(-r)$$

$$= 0.092d$$

$$\therefore \langle 0.486d, 0.092d \rangle = \langle 0.84, 0.16 \rangle$$

At 2:

$$P(+r) = d \times .1 \times (.84 \times .7 + .16 \times .3)$$

$$= \cancel{0.5724d} = 0.0636d$$

$P(-t) = \alpha \times .8 \times (.84 \times .3 + .16 \times .7)$

$\cancel{0.0728\alpha} = 0.2912\alpha$

$\cancel{\langle 0.5824\alpha, 0.0728\alpha \rangle = \langle 0.887, .113 \rangle}$

$\therefore \langle 0.0636\alpha, 0.2912\alpha \rangle =$

$\langle 0.179, 0.821 \rangle$

8. a)

$$H(\text{initial}) = \frac{3}{5}\log_2 \frac{5}{3} + \frac{2}{5}\log_2 \frac{5}{2}$$

$$= 0.971$$

$$R(A_1) = \frac{4}{5}\left(\frac{2}{4}\log_2 \frac{4}{2} + \frac{2}{4}\log_2 \frac{4}{2}\right) + \frac{1}{5}\left(1\log_2 1\right) = 0.8$$

$$R(A_2) = \frac{3}{5}\left(\frac{1}{3}\log_2\frac{3}{1} + \frac{2}{3}\log_2\frac{3}{2}\right)$$

$$+ \frac{2}{5}\left(1\log_2 1\right)$$

$$\cancel{= 0.918} = 0.55$$

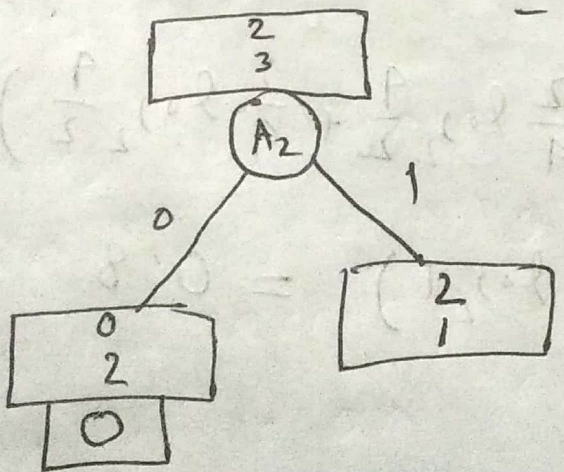
$$R(A_3) = \frac{2}{5}\left(\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2\right)$$

$$+ \frac{3}{5}\left(\frac{2}{3}\log_2\frac{3}{2} + \frac{1}{3}\log_2\frac{3}{1}\right)$$

$$= 0.951$$

max gain = gain $(A_2) = 0.971 - 0.55$

$$= 0.421$$



Now,

$$H_{EBS} = \frac{2}{3}\log_2\frac{3}{2} + \frac{1}{3}\log_2\frac{3}{1}$$

$$= 0.918$$

$$R(A_1) = \frac{1}{3}\left(1\log_2 1\right) + \frac{2}{3}\left(1\log_2 1\right)$$

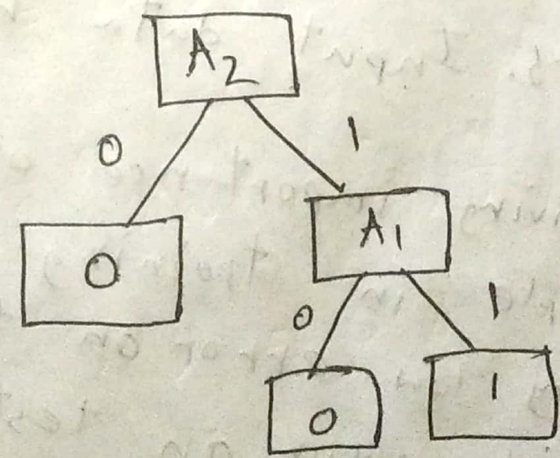
$$= 0$$

$$R(A_3) = \frac{1}{3}\left(1\log_2 1\right) + \frac{2}{3}\left(\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2\right)$$

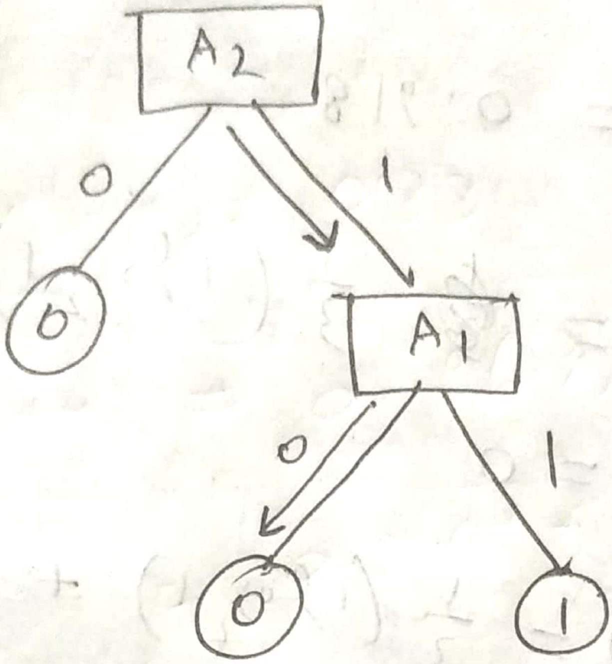
$$= 0.67$$

max gain = gain $(A_1) = 0.918$

So, final tree



ii) For (0, 1, 1)



Output will be 0.

8.c)

i) Agent learns from ~~Exp~~ input-output pairs. Input data have labels.

ii) Giving importance on every example in training set. It results in very low error on training data but very high error on test data. Hurts generalization.

2018-19

5. a)

i) $P(+x \mid -y) = \alpha\, P(+x, -y)$

$= \alpha \times 0.3$

$P(-x \mid -y) = \alpha\, P(-x, -y)$

$= \alpha \times 0.1$

$\therefore P(+x \mid -y) = 0.75$

ii) $P(-y \mid +x) = \alpha\, P(-y, +x)$

$= \alpha \times 0.3$

$P(+y \mid +x) = \alpha\, P(+y, +x)$

$= \alpha \times 0.2$

$\therefore P(-y \mid +x) = 0.6$

iii) $P(+x \mid +y) = \alpha\, P(+x, +y)$

$= \alpha \times 0.2$

$$P(-x \mid +y) = \alpha . P(-x, +y)$$

$$= \alpha \times 0.4$$

$$\therefore P(+x \mid +y) = 0.33$$

iv) $P(-x \mid +y) = \alpha . P(-x, +y)$

$$= \alpha \times 0.4$$

$$P(+x \mid +y) = \alpha . P(+x, +y)$$

$$= \alpha \times 0.2$$

$$\therefore P(-x \mid +y) = 0.67$$

5.b) $P(W \mid summer) = \alpha P(W, summer)$

$\cancel{= \alpha P(W}$

$= \alpha \sum_{t} P(W, summer, t)$

Instantiating,

T	W	P
hot	sun	.2
hot	rain	.15
cold	sun	.10
cold	rain	0.05

Summing out,

W	P
Sun	.30
rain	.20

Normalizing,

W	P
Sun	0.6
Rain	0.4

6. a)

$$H_{EBS} = \frac{85}{185} \log_2 \frac{185}{85} + \frac{100}{185} \times \log_2 \frac{185}{100}$$

$$= 0.995$$

$$R(A) = \frac{75}{185}\left(1 \log_2 1\right) + \frac{110}{185}\left(\frac{10}{110} \log_2 \frac{110}{10} + \frac{100}{110} \log_2 \frac{110}{100}\right)$$

$$= 0.261$$

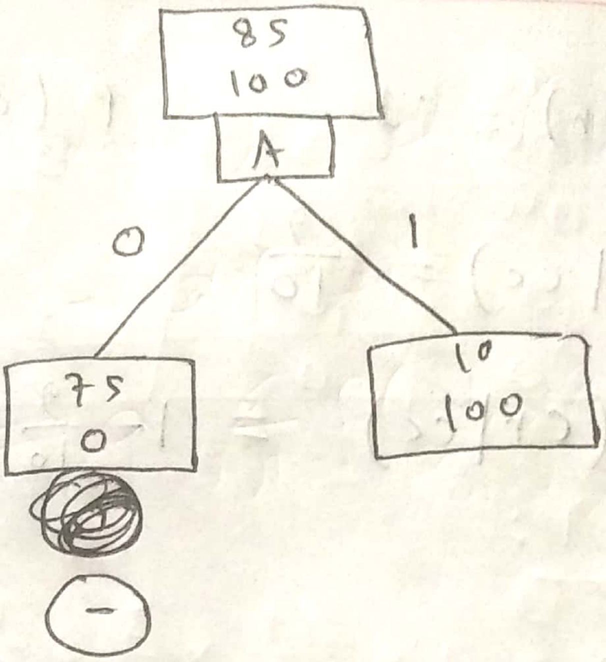
$$R(B) = \frac{60}{185} \times \left(1 \log_2 1\right) + \frac{125}{185}\left(\frac{25}{125} \times \log_2 \frac{125}{25} + \frac{100}{125} \times \log_2 \frac{125}{100}\right)$$

$$= 0.488$$

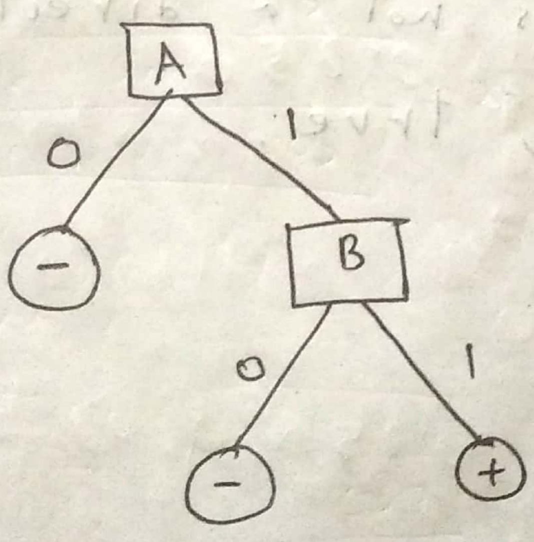
$\therefore$ Max gain = gain (A) = $.995 - .261$

$$= 0.734$$

CS CamScanner



Now, only B is remaining. So, the final tree will be:



6.b) $P(cr) = .1$ $P(cc|cr) = .4$

$P(\neg cr|cc) = \frac{9}{10}$

$\therefore P(cr|cc) = 1 - \frac{9}{10} = \frac{1}{10}$

7.b) i) As, T is a direct parent of T', and L is not a direct parent of T', true.

[Assuming $\perp\!\!\!\perp$ means independence]

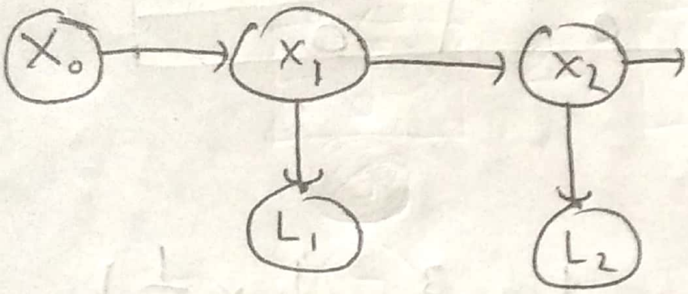
ii) False

iii) False

iv) False

~~v) False~~ v) False

2.a) Register states are state variable and left bit is evidence variable.



Transition model:

X_{t-1}	00	01	10	11
00	$\frac{1}{2}+\frac{1}{4}$	$\frac{1}{4}$	0	0
01	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
10	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
11	0	0	$\frac{1}{4}$	$\frac{1}{4}+\frac{1}{2}$

Observation model

X_t	0	1
00	1	0
01	1	0
10	0	1
11	0	1

b)

	0	1	2	3
00	$\frac{1}{4}$	0.5	0.57	0
01	$\frac{1}{4}$	0.5	0.43	0
10	$\frac{1}{4}$	0	0	1
11	$\frac{1}{4}$	0	0	0

For day 1:

$$P(00) = \alpha \times 1 \times \left(\frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}\right)$$

$$= 0.25\alpha$$

$$P(01) = \alpha \times 1 \times \left(\frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4}\right)$$

$$= 0.25\alpha$$

$P(10) = 0$

$P(11) = 0$

$\therefore \langle 0.25\alpha, 0.25\alpha, 0, 0\rangle = \langle 0.5, 0.5, 0, 0\rangle$

For day 2:

$$P(00) = \alpha \times 1 \times \left(0.5 \times \frac{3}{4} + \frac{1}{4} \times 0.5\right)$$

$$= \cancel{0.792\alpha} = 0.5\alpha$$

$$P(01) = \alpha \times 1 \times \left(0{\cdot}5 \times \frac{1}{4} + 0{\cdot}5 \times \frac{1}{2}\right)$$

$$= 0{\cdot}375\alpha$$

$$P(10) = 0$$

$$P(11) = 0$$

$$\cancel{\langle 0{\cdot}792\alpha, 0{\cdot}375\alpha, 0, 0 \rangle}$$

$$\therefore \langle 0{\cdot}5\alpha, 0{\cdot}375\alpha, 0, 0 \rangle = \langle 0{\cdot}57, 0{\cdot}43, 0, 0 \rangle$$

<u>For day 3:</u>

$$P(00) = 0$$

$$P(01) = 0$$

$$P(10) = \alpha \times 1 \times \left(\frac{1}{4} \times 0{\cdot}43\right)$$

$$= 0{\cdot}1075\alpha$$

$$P(11) = \alpha \times 1 \times 0$$

$$= 0$$

$$\langle 0, 0, 0{\cdot}1075\alpha, 0 \rangle = \langle 0, 0, 1, 0 \rangle$$

c)

	0	1	2	3
00	$\frac{5}{64}$	$\frac{1}{16}$	0	1
01	$\frac{5}{64}$	$\frac{1}{8}$	$\frac{1}{4}$	1
10	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{3}{4}$	1
11	0	0	1	1

<u>For $t=2$:</u>

$P(0,0) = 0$

$P(0,1) = \frac{1}{4} \times 1 \times 1 = \frac{1}{4}$

$P(1,0) = \frac{1}{2} \times 1 \times 1 + \frac{1}{4} \times 1 \times 1$

$= \frac{3}{4}$

$P(1,1) = \frac{1}{4} \times 1 \times 1 + \frac{3}{4} \times 1 \times 1$

$= 1$

<u>For $t=1$</u>

$P(0,0) = \frac{1}{4} \times \frac{1}{4} \times 1 = \frac{1}{16}$

$P(0,1) = \frac{1}{2} \times \frac{1}{4} \times 1 = \frac{1}{8}$

$P(1,0) = \frac{1}{4} \times \frac{1}{4} \times 1 = \frac{1}{16}$

$P(1,1) = 0$

<u>for $t = 0$</u>

$P(0,0) = \frac{3}{4} \times \frac{1}{16} \times 1 + \frac{1}{4} \times \frac{1}{8} \times 1$

$= \frac{5}{64}$

$P(0,1) = \frac{1}{4} \times \frac{1}{16} + \frac{1}{2} \times \frac{1}{8}$

$= \frac{5}{64}$

$P(1,0) = \frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$

$P(1,1) = 0$

So, using forward and backward table:

	0	1	2	3
00	0.0195α	$\frac{\alpha}{32}$	0	0
01	0.0195α	$\frac{\alpha}{16}$	0.1075α	0
10	$\frac{1}{128}\alpha$	0	0	α
11	0	0	0	0

$\xrightarrow{\text{Normalizing}}$

	0	1	2	3
00	0.42	0.33	0	0
01	0.42	0.67	1	0
10	0.16	0	0	1
11	0	0	0	0

d)

	0	1	2	3
00	0·25 →	0·1875 →	0·14	0
01	0·25 →	0·125 →	0·0625	0
10	0·25	0	0	0·015625
11	0·25	0	0	0

For $t = 1$

$$p(00) = 1 \times \max\left\{0{\cdot}25 \times \frac{3}{4},\ {\cdot}25 \times \frac{1}{4}\right\}$$

$$= 0{\cdot}1875$$

$$p(01) = 1 \times \max\left\{\frac{1}{4} \times 0{\cdot}25,\ \frac{1}{2} \times 0{\cdot}25 \times \frac{1}{4} \times 0{\cdot}25\right\}$$

$$= 0{\cdot}125$$

$p(10) = 0$, $p(11) = 0$

For $t = 2$

$$p(00) = 1 \times \max\left\{0{\cdot}1875 \times \frac{3}{4},\ 0{\cdot}125 \times \frac{1}{4}\right\}$$

$$= 0{\cdot}14$$

$$P(01) = 1 \times \max\left\{\frac{1}{4} \times 0.1875, \frac{1}{2}\, 0.125\right\}$$

$$= 0.0625$$

$P(10) = 0$, $P(11) = 0$

<u>for $t = 3$</u>

$P(00) = 0$, $P(01) = 0$

$$P(10) = 1 \times \max\left\{\frac{1}{4} \times 0.0625\right\}$$

$$= 0.015625$$

$$P(11) = 1 \times \max\{0\} = 0$$

So, most likely sequence,

$$\underset{\overline{0}}{01} \longrightarrow \underset{\overline{1}}{01} \longrightarrow \underset{\overline{2}}{01} \longrightarrow \underset{\overline{3}}{10}$$

4.a):

<u>1st step:</u>

$$R(O) = \frac{4}{14}(1\log_2 1) + \frac{5}{14}\left(\frac{2}{5}\log_2\frac{5}{2} + \frac{3}{5}\log_2\frac{5}{3}\right) + \frac{5}{14}\left(\frac{4}{5}\log_2\frac{5}{4} + \frac{1}{5}\log_2\frac{5}{1}\right)$$

$$= 0.605$$

$$R(T) = \frac{4}{14}\left(\frac{2}{4}\log_2\frac{4}{2} + \frac{2}{4}\log_2\frac{4}{2}\right) + \frac{6}{14}\left(\frac{5}{6}\log_2\frac{6}{5} + \frac{1}{6}\log_2\frac{6}{1}\right) + \frac{4}{14}\left(\frac{3}{4}\log_2\frac{4}{3} + \frac{1}{4}\log_2\frac{4}{1}\right)$$

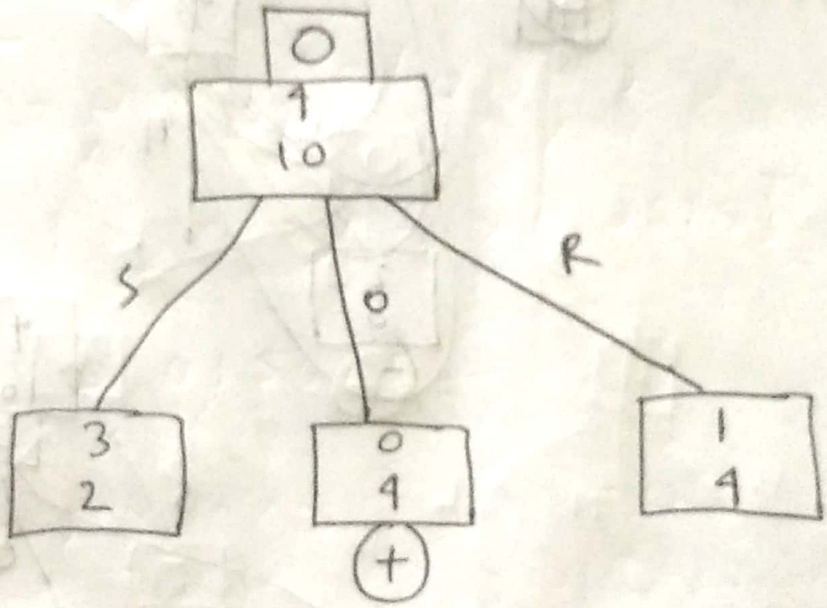
$$= 0.796$$

$$R(H) = \frac{7}{14}\left(\frac{3}{7}\log_2\frac{7}{3} + \frac{4}{7}\log_2\frac{7}{4}\right) + \frac{7}{14}\left(\frac{6}{7}\log_2\frac{7}{6} + \frac{1}{7}\log_2\frac{7}{1}\right) = 0.788$$

$$R(W) = \frac{8}{14} \times \left(\frac{6}{8}\log_2\frac{8}{6} + \frac{2}{8}\log_2\frac{8}{2}\right)$$

$$+ \frac{6}{14} \times \left(\frac{4}{6}\log_2\frac{6}{4} + \frac{2}{6}\log_2\frac{6}{2}\right)$$

$$= 0.857$$



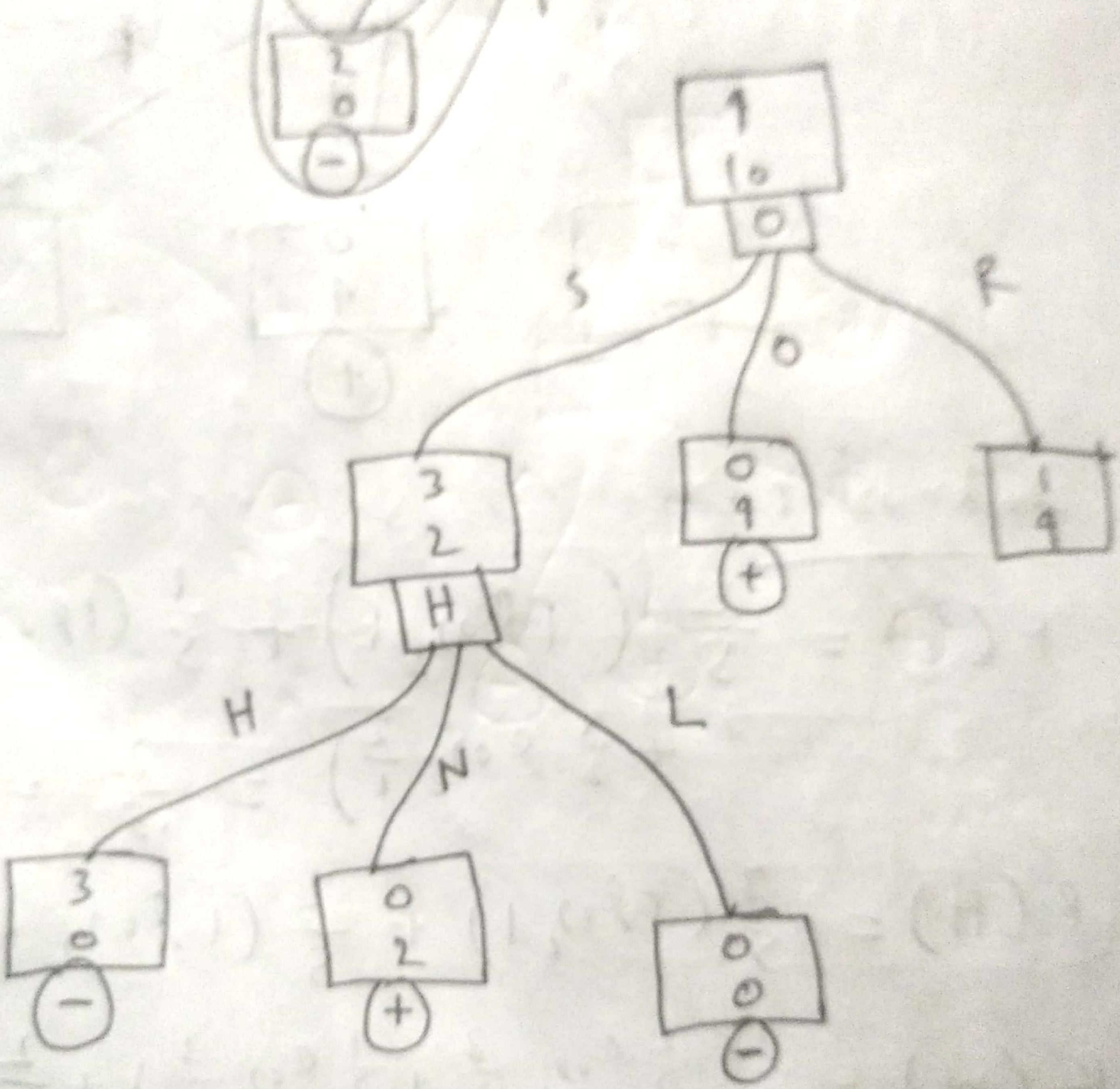
2nd step:

$$R(T) = \frac{2}{5}\left(1\log_2 1\right) + \frac{1}{5}\left(1\log_2 1\right) + \frac{2}{5}\left(\frac{1}{2}\log_2\frac{2}{1} + \frac{1}{2}\log_2\frac{2}{1}\right) = \frac{2}{5} = 0.4$$

$$R(H) = \frac{3}{5}\left(1\log_2 1\right) + \frac{2}{5}\left(1\log_2 1\right) = 0$$

$$R(W) = \frac{3}{5}\left(\frac{2}{3}\log_2\frac{3}{2} + \frac{1}{3}\log_2\frac{3}{1}\right) + \frac{2}{5}\left(\frac{1}{2}\log_2\frac{2}{1} + \frac{1}{2}\log_2\frac{2}{1}\right)$$

$$= 0.95$$


2
0
−
9
10
O
S
O
R
3
2
H
0
9
+
1
9
H
N
L
3
0
−
0
2
+
0
0
−

3rd step:

$$R(T) = \frac{3}{5}\left(1 \log_2 1\right) + \frac{2}{5}\left(\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2\right)$$

$$= 0.4$$

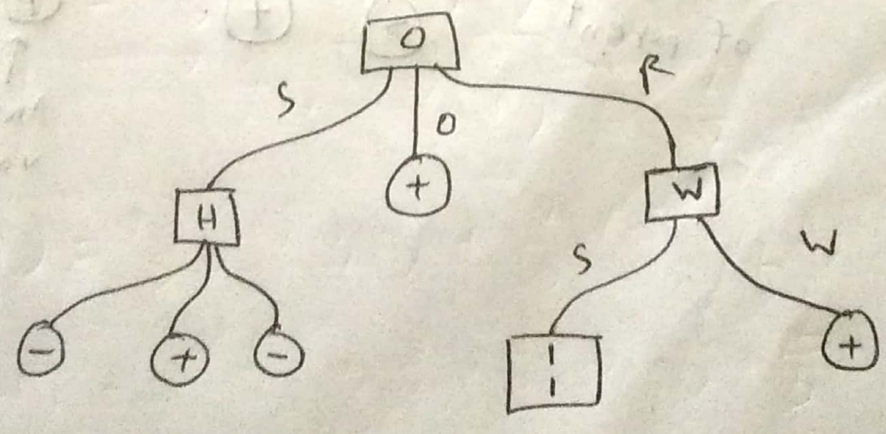
$$R(H) = \frac{2}{5}\left(1\log_2 1\right) + \frac{3}{5}\left(\frac{2}{3}\log_2 \frac{3}{2} + \frac{1}{3}\log_2 \frac{3}{1}\right)$$

$$= 0.55$$

$$R(W) = \frac{3}{5}\left(1\log_2 1\right) + \frac{2}{5}\left(\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2\right)$$

$$= 0.4$$

As, T has missing value, choosing W.



4th step:

$R(T) = 0$ and $R(H) = 0$

So, any one can be used.

Final tree