

5. b)  $p(n|d), p(r|d)$  [ changing question according to our syllabus ]

$$p(n|d) = d p(n,d)$$

$$= d \sum_r \sum_a p(n,d,r,a)$$

$$= d \sum_r \sum_a p(a|d) p(d|r,n) p(n|r)$$

$$= d \sum_r p(d|m,r) p(n|r) p(r) \sum_a p(a|d)$$

$$= d \sum_r p(d|n,r) p(n|r) p(r)$$

$f_3(R, n, T)$

$f_1(R)$

R	P
I	0.8
H	0.2

$f_2(R, n)$

R	n	P
I	Y	0.6
I	N	0.4
H	Y	0.1
H	N	0.9

R	n	T	P
I	Y	T	0.6
I	Y	D	0.4
I	N	T	0.7
I	N	D	0.3
H	Y	T	0.9
H	Y	D	0.6
H	N	T	0.5
H	N	D	0.5

$$f_3(R, n, T) \rightarrow f_4(R, n)$$

R	n	P
g	γ	.9
g	n	.13
h	γ	.6
h	n	.5

$$f_5(R, n) = f_1(R) \times f_2(R, n)$$

R	n	P
g	γ	.98
g	n	.32
h	γ	.02
h	n	.18

$$f_6(r, n) = f_5(r, n) \times \Phi f_7(r, n)$$

r	n	P
g	y	.192
g	n	.096
h	y	.012
h	n	.09

Summing out r,

n	P
y	0.204
n	0.186

Normalizing,

n	P(n d)
y	.523
n	.477

$$P(R|d) = d P(R, d)$$

$$= d \sum_m P(d|R, m) P(m|R) P(R)$$

Summing out  $m$ , (from  $f_6$ )

R	P
g	·288
h	·102

Normalizing,

R	$P(R d)$
g	·738
h	0·262

(SIN)	1
(SAR)	0
(FFF)	1

6.b)

Hidden state → Position of the animal

Possible observations

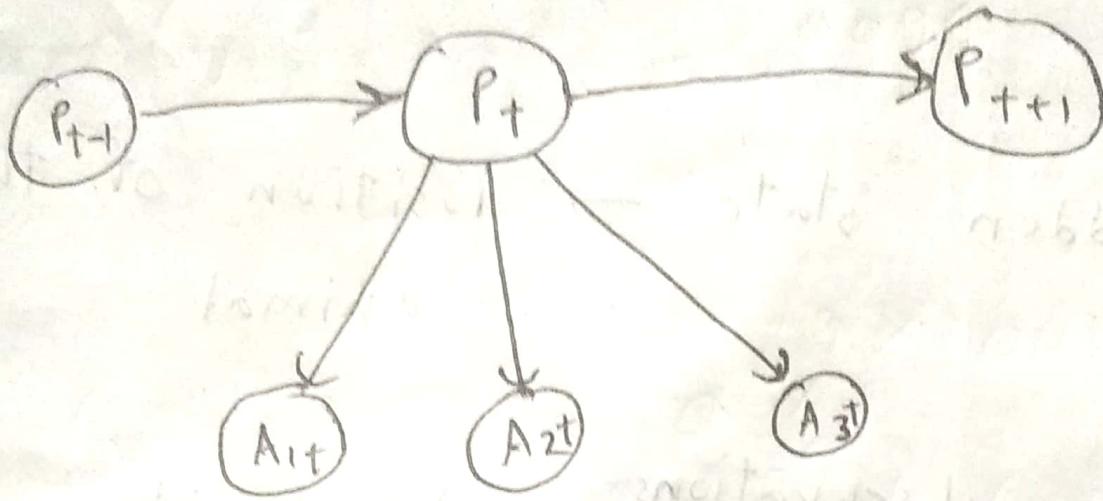
~~A1 A2 A3~~

	$A_1$	$A_2$	$A_3$
1.	yes	yes	yes
2.	yes	yes	no
3.	yes	no	yes
4.	yes	no	no
5.	no	yes	yes
6.	no	yes	no
7.	no	no	yes
8.	no	no	no

1.	1	1
2.	1	2
3.	1	3
4.	2	1
5.	2	2

1.	1	1
2.	1	2
3.	1	3
4.	2	1
5.	2	2

1.	1	1
2.	1	2
3.	1	3
4.	2	1
5.	2	2



Transition model

$P_{t-1}$	$C_1$	$C_2$	$C_3$	M
$C_1$	.8	.05	.05	.1
$C_2$	.05	.8	.05	.1
$C_3$	.05	.05	.8	.1
M	0.1	0.1	0.1	0.7

$c_1$  = Corner 1

$c_2$  = Corner 2

$c_3$  = Corner 3

~~Corner 4~~

n = middle

Observation model

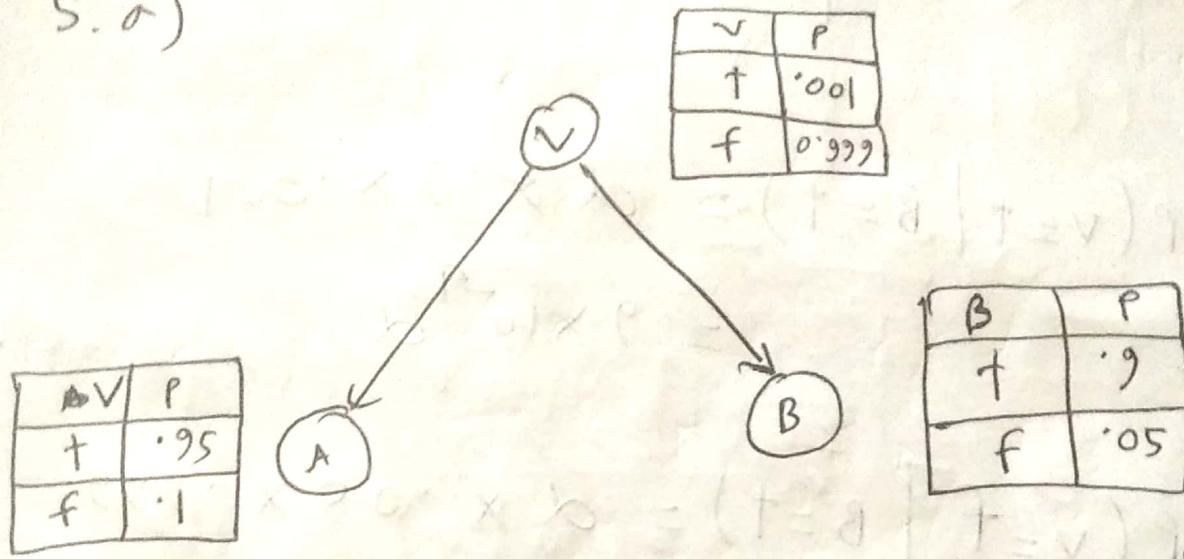
$P_t$	$A_{1t}$
$C_1$	.6
$C_2$	.1
$C_3$	.1
M	.9

$P_t$	$A_{2t}$
$C_1$	.1
$C_2$	.6
$C_3$	.1
M	.9

$P_t$	$A_{3t}$
$C_1$	.1
$C_2$	.1
$C_3$	.6
M	.4

2019-20

5. a)



$$\cancel{P(V=t | A=t)} = \frac{P(A=t, V=t)}{P(A=t)}$$

$$\begin{aligned}
 &= d \cdot P(A=t | V=t) \cdot P(V=t) \\
 &= d \times .95 \times .001 \\
 &= 9.5 \times 10^{-4} \cdot d
 \end{aligned}$$

$$\begin{aligned}
 P(V=f | A=t) &= d \cdot P(A=t | V=f) \cdot P(V=f) \\
 &= d \times .1 \times .999 \\
 &= .999d
 \end{aligned}$$

$$\therefore P(V=t \mid A=t) = .0099$$

$$P(V=t \mid B=t) = d \times .9 \times .001 \\ = 9 \times 10^{-9} d$$

$$P(V=f \mid B=t) = d \times .05 \times .999 \\ = 0.04995 d$$

$$\therefore P(V=t \mid B=t) = 0.018$$

So, B returning positive is more indicative.

5.b)

$$P(w|dry) = \alpha P(w, dry)$$
$$= \alpha P(dry|w) \cdot P(w)$$

instantiating,

$$f_3(w) =$$

w	P
sun	.9
rain	.25

$$f_4(w) = f_1(w) \times f_3(w)$$

w	P
sun	.765
rain	.0375

Normalizing,

w	$P(w dry)$
sun	0.953
rain	0.047

6.a)

$$\begin{aligned} p(B| -e, +j) &= d p(B, -e, +j) \\ &= d \sum_a \sum_m p(B, -e, +j, a, m) \\ &= d \sum_a \sum_m p(B) p(-e) p(a|B, -e) \\ &\quad p(m|a) p(+j|a) \end{aligned}$$

$$= d p(B) p(-e) \sum_a p(a|B, -e) \cancel{p(m|a)} \\ \cancel{p(+j|a)}$$

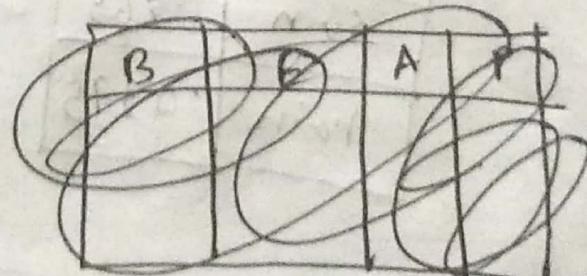
$p(-e)$

E	P
-e	.998

$p(+j|a)$

a	P
+a	.9
-a	.05

$p(a|B, -e)$



B	A	P
+b	+a	.99
+b	-a	.06
-b	+a	.001
-b	-a	.999

$$P(\alpha | \beta, -e) \times P(+j | \alpha)$$

B	A	P
+b	+a	.896
+b	-a	.003
-b	+a	.0009
-b	-a	.09995

Summing out,

B	P
+b	.899
-b	.05085

Multiplying  $P(B)$  and  $P(-e)$

B	P
+b	.000897
-b	.0507

Normalizing,

B	P
+b	.0169
-b	.9836

b)

$$P(E | +m, +j) = \alpha P(E, +m, +j)$$

$$= \alpha \sum_b \sum_a P(E, +m, +j, b, a)$$

$$= \alpha P(E) \sum_a p(a | +m, +j) p(+m | a)$$

$$P(+j | a) \sum_b p(a | E, b) P(b)$$

$$f_1(E, B, A) = p(a | E, b) \times p(b)$$

B	E	A	P
+b	+e	+a	$9.5 \times 10^{-9}$
+b	+e	-a	$5 \times 10^{-5}$
+b	-e	+a	$9.4 \times 10^{-9}$
+b	-e	-a	$6 \times 10^{-5}$
-b	+e	+a	28971
-b	+e	-a	70929
-b	-e	+a	$9.99 \times 10^{-9}$
-b	-e	-a	998001

Summing out b,

E	A	P	
+e	+a	0.009	29066
+e	-a	0.009	70934
-e	+a	1.939	$\times 10^{-3}$
-e	-a	0.998061	

$$P(+m|a) = f_2(A) \quad | \quad P(+n|a) = f_3(A)$$

A	P
+a	0.7
-a	0.01

A	P
+a	0.9
-a	0.05

~~$f_2(E, A) \times f_3(A) \times f_4$~~

Multiplicating these,

E	A	P
+e	+a	0.1831
+e	-a	$3.5967 \times 10^{-9}$
-e	+a	$1.222 \times 10^{-3}$
-e	-a	$4.99 \times 10^{-9}$

Summing out A

E	P
+e	0.1835
-e	$1.721 \times 10^{-3}$

Multiplying P(E)

E	P
+e	$3.67 \times 10^{-9}$
-e	$1.718 \times 10^{-3}$

Normalizing,

E	P
+e	0.176
-e	0.829

$$7-b) \quad \pi(+r) = 0.6 \times 0.7 + 0.4 \times 0.3 \\ = 0.54$$

$$\pi(-r) = 0.6 \times 0.3 + 0.4 \times 0.7 \\ = 0.46$$

5 TA

8 TA

11 TA

14 TA

17 TA

20 TA

23 TA

26 TA

29 TA

32 TA

35 TA

38 TA

41 TA

44 TA

47 TA

50 TA

53 TA

56 TA

59 TA

62 TA

65 TA

68 TA

71 TA

74 TA

77 TA

80 TA

83 TA

86 TA

89 TA

92 TA

95 TA

98 TA

101 TA

104 TA

107 TA

110 TA

113 TA

116 TA

119 TA

122 TA

125 TA

128 TA

131 TA

134 TA

137 TA

140 TA

143 TA

146 TA

149 TA

152 TA

155 TA

158 TA

161 TA

164 TA

167 TA

170 TA

173 TA

176 TA

179 TA

182 TA

185 TA

188 TA

191 TA

194 TA

197 TA

200 TA

203 TA

206 TA

209 TA

212 TA

215 TA

218 TA

221 TA

224 TA

227 TA

230 TA

233 TA

236 TA

239 TA

242 TA

245 TA

248 TA

251 TA

254 TA

257 TA

260 TA

263 TA

266 TA

269 TA

272 TA

275 TA

278 TA

281 TA

284 TA

287 TA

290 TA

293 TA

296 TA

299 TA

302 TA

305 TA

308 TA

311 TA

314 TA

317 TA

320 TA

323 TA

326 TA

329 TA

332 TA

335 TA

338 TA

341 TA

344 TA

347 TA

350 TA

353 TA

356 TA

359 TA

362 TA

365 TA

368 TA

371 TA

374 TA

377 TA

380 TA

383 TA

386 TA

389 TA

392 TA

395 TA

398 TA

401 TA

404 TA

407 TA

410 TA

413 TA

416 TA

419 TA

422 TA

425 TA

428 TA

431 TA

434 TA

437 TA

440 TA

443 TA

446 TA

449 TA

452 TA

455 TA

458 TA

461 TA

464 TA

467 TA

470 TA

473 TA

476 TA

479 TA

482 TA

485 TA

488 TA

491 TA

494 TA

497 TA

500 TA

503 TA

506 TA

509 TA

512 TA

515 TA

518 TA

521 TA

524 TA

527 TA

530 TA

533 TA

536 TA

539 TA

542 TA

545 TA

548 TA

551 TA

554 TA

557 TA

560 TA

563 TA

566 TA

569 TA

572 TA

575 TA

578 TA

581 TA

584 TA

587 TA

590 TA

593 TA

596 TA

599 TA

602 TA

605 TA

608 TA

611 TA

614 TA

617 TA

620 TA

623 TA

626 TA

629 TA

632 TA

635 TA

638 TA

641 TA

644 TA

647 TA

650 TA

653 TA

656 TA

659 TA

662 TA

665 TA

668 TA

671 TA

674 TA

677 TA

680 TA

683 TA

686 TA

689 TA

692 TA

695 TA

698 TA

701 TA

704 TA

707 TA

710 TA

713 TA

716 TA

719 TA

722 TA

725 TA

728 TA

731 TA

734 TA

737 TA

740 TA

743 TA

746 TA

749 TA

752 TA

755 TA

758 TA

761 TA

764 TA

767 TA

770 TA

773 TA

776 TA

779 TA

782 TA

785 TA

788 TA

791 TA

	1	2	
+r	• 84	0.88	• 179
-r	• 16	0.82	• 821

At 1 :

$$\rho(+r) = \cancel{0.88}$$

$$\begin{aligned} \rho(+r) &= d \times 0.9 \times \pi (+r) \\ &= 0.986d \end{aligned}$$

$$\rho(-r) = d \times 0.2 \times \pi (-r)$$

$$= 0.092d$$

$$\therefore \langle 0.986d, 0.092d \rangle = \langle 0.84, 0.16 \rangle$$

At 2 :

$$\begin{aligned} \rho(+r) &= d \times 1 \times (0.84 \times 7 + 0.16 \times 3) \\ &= \cancel{0.5228} = 0.0636d \end{aligned}$$

$$P(-t) = d \times .8 \times (.89 \times .3 + .16 \times .7)$$

~~$$0.0728d = 0.2912d$$~~

~~$$\langle 0.5774d, 0.0728d \rangle \rightarrow \langle 0.887, 0.13 \rangle$$~~

$$\therefore \langle 0.0636d, 0.2912d \rangle = \langle 0.179, 0.821 \rangle$$

8-a)

$$H(\text{initial}) = \frac{3}{5} \vartheta_0 \vartheta_2 \frac{5}{3} + \frac{2}{5} \vartheta_0 \vartheta_2 \frac{5}{2}$$
$$= 0.971$$

$$R(A_1) = \frac{4}{5} \left( \frac{2}{4} \vartheta_0 \vartheta_2 \frac{9}{2} + \frac{2}{4} \vartheta_0 \vartheta_2 \frac{9}{2} \right) +$$
$$\frac{1}{5} (\vartheta_0 \vartheta_2 1) = 0.8$$

$$R(A_2) = \frac{3}{5} \left( \frac{1}{3} \vartheta_2 \frac{3}{1} + \frac{2}{3} \vartheta_2 \frac{3}{2} \right) \\ + \frac{2}{5} (1 \vartheta_2 1)$$

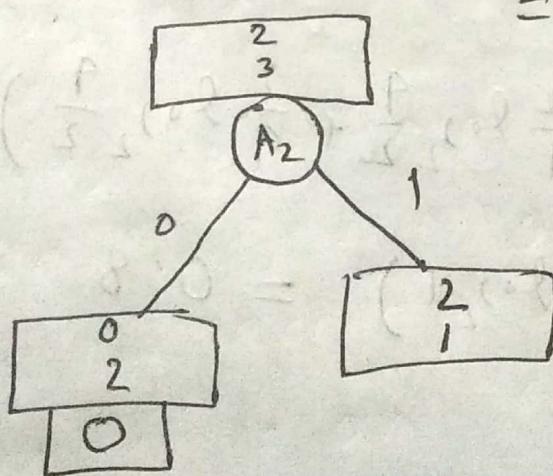
~~0.971~~ = 0.55

$$R(A_3) = \frac{2}{5} \left( \frac{1}{2} \vartheta_2 2 + \frac{1}{2} \vartheta_2 2 \right)$$

$$+ \frac{3}{5} \left( \frac{2}{3} \vartheta_2 \frac{3}{2} + \frac{1}{3} \vartheta_2 \frac{3}{1} \right)$$

= 0.951

max gain = gain  $(A_2) = 0.971 - 0.55$   
 $= 0.421$



Now,

$$H_{EBS} = \frac{2}{3} \log_2 \frac{3}{2} + \left( \frac{1}{3} \log_2 \frac{3}{1} \right)$$

$$= 0.918$$

$$R(A_1) = \frac{1}{3} \left( (\log_2 1) \right) + \frac{2}{3} \left( (\log_2 1) \right)$$

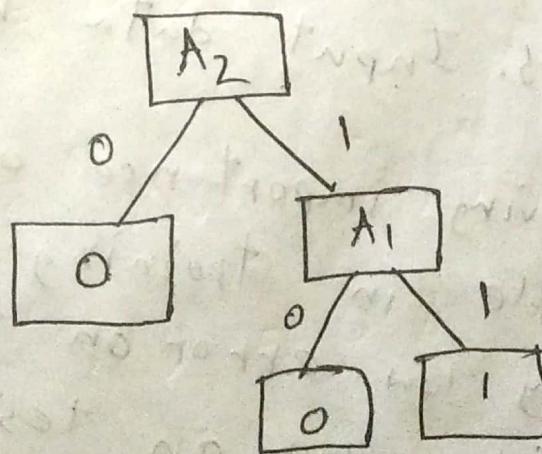
$$= 0$$

$$R(A_3) = \frac{1}{3} \left( (\log_2 1) \right) + \frac{2}{3} \left( \frac{1}{2} \log_2^2 \frac{1}{2} + \frac{1}{2} \log_2^2 \frac{1}{1} \right)$$

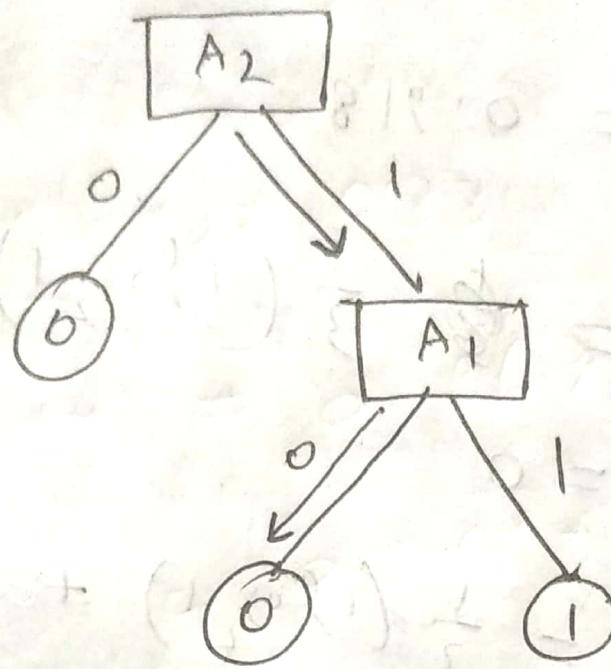
$$= 0.67$$

$$\text{max gain} = \text{gain}(A_1) = 0.918$$

So, final tree



ii) For  $(0, 1, 1)$



Output will be 0.

8.c)

i) Agent learns from ~~exp~~ input-output pairs.

Input data have labels.

ii) Giving importance on every example in training set. It results in very low error on training data but very high error on test data. Hurts generalization.

2018-19

5. a) i)  $P(+x | -y) = d P(+x, -y)$   
 $= d \times 0.3$

$P(-x | -y) = d P(-x, -y)$   
 $= d \times 0.1$

$\therefore P(+x | -y) = 0.75$

ii)  $P(-y | +x) = d P(-y, +x)$   
 $= d \times 0.3$

$P(+y | +x) = d P(+y, +x)$   
 $= d \times 0.2$

$\therefore P(-y | +x) = 0.6$

iii)  $P(+x | +y) = d P(+x, +y)$   
 $= d \times 0.2$

$$P(-x|+y) = d \cdot P(-x, +y)$$
$$= d \times 0.4$$

$$\therefore P(+x|+y) = 0.33$$

$$iv) P(-x|+y) = d \cdot P(-x, +y)$$
$$= d \times 0.4$$

$$P(+x|+y) = d \cdot P(+x, +y)$$
$$= d \times 0.2$$

$$\therefore P(-x|+y) = 0.67$$

$$5.b) P(W \mid \text{summer}) = d P(W, \text{summer})$$

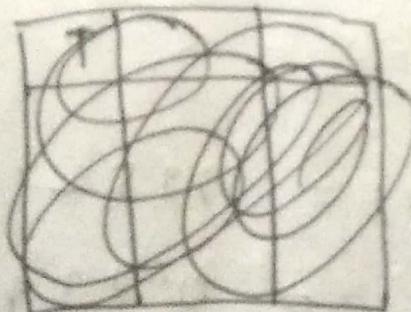
~~softmax~~

$$= d \sum_t P(W, \text{summer}, t)$$

Instantiating,

T	W	P
hot	sun	0.2
hot	rain	0.15
cold	sun	0.10
cold	rain	0.05

Summing out,



W	P
Sun	0.30
Rain	0.20

Normalizing,

W	P
Sun	0.6
Rain	0.4

6.a)

$$H_{EBS} = \frac{85}{185} \log_2 \frac{185}{85} + \frac{100}{185} \times \log_2 \frac{185}{100}$$
$$= 0.995$$

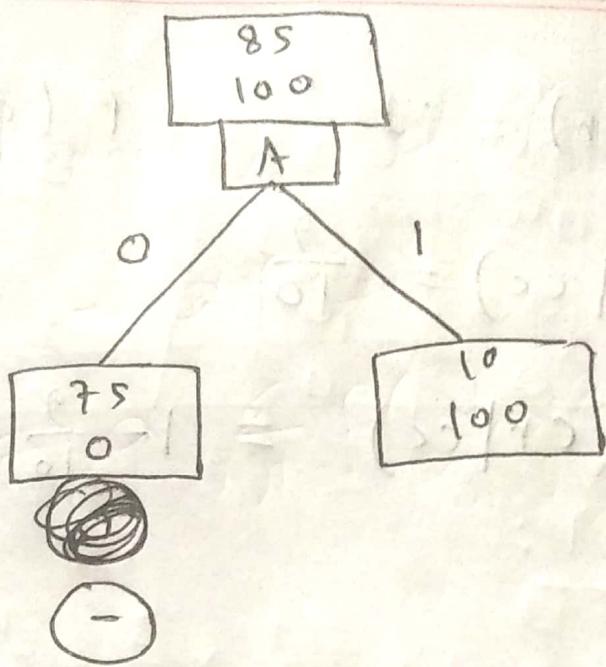
$$R(A) = \frac{75}{185} \left( 1 \log_2 1 \right) + \frac{110}{185} \left( \frac{10}{110} \log_2 \frac{110}{10} + \frac{100}{110} \log_2 \frac{110}{100} \right)$$

$$= 0.261$$

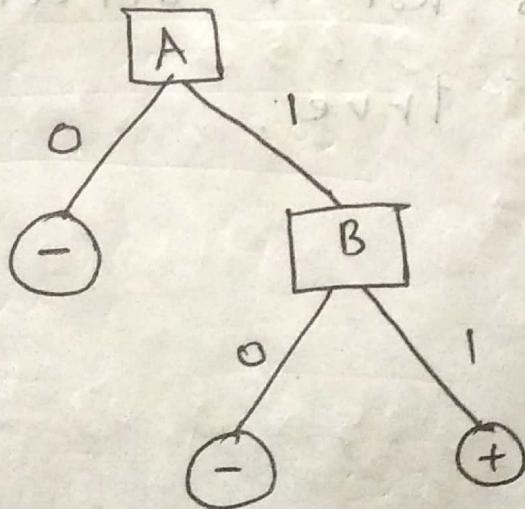
$$R(B) = \frac{60}{185} \times \left( 1 \log_2 1 \right) + \frac{125}{185} \left( \frac{25}{125} \times \log_2 \frac{125}{25} + \frac{100}{125} \times \log_2 \frac{125}{100} \right)$$

$$= 0.488$$

$$\therefore \text{Max gain} = \text{Gain}(A) = 0.995 - 0.261$$
$$= 0.734$$



Now, only B is remaining. So, the final tree will be:



$$6.b) P(\text{cr}) = 1 \quad P(\text{cc}|\text{cr}) = 4$$

$$P(\neg\text{cr}|\text{cc}) = \frac{9}{10}$$

$$\therefore P(\text{cr}|\text{cc}) = 1 - \frac{9}{10} = \frac{1}{10}$$

7.b)

i) As, T is

[Assuming

$\perp\!\!\!\perp$  means

a direct parent of  $T'$ , [new independence]

and L is not a direct parent

of  $T'$ , true.

ii) False

iii) False

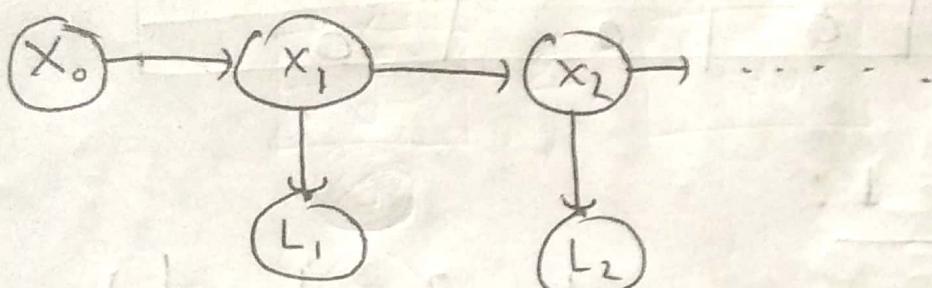
iv) False

~~v) False~~

v) False

2017-18

2.a) Register states are state variable  
and left bit is evidence variable.



Transition model:

$x_{t+1}$	00	01	10	11
00	$\frac{1}{2} + \frac{1}{4}$	$\frac{1}{4}$	0	0
01	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
10	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
11	0	0	$\frac{1}{4}$	$\frac{1}{4} + \frac{1}{2}$

Observation model

$x_t$	0	1
00	1	0
01	1	0
10	0	1
11	0	1

b)

	0	1	2	3
00	$\frac{1}{4}$	0.5	0.57	0
01	$\frac{1}{4}$	0.5	0.43	0
10	$\frac{1}{4}$	0	0	1
11	$\frac{1}{4}$	0	0	0

for day 1:

$$P(00) = d \times 1 \times \left( \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4} \right)$$
$$= 0.25d$$

$$P(01) = d \times 1 \times \left( \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} \right)$$
$$= 0.25d$$

$$P(10) = 0$$

$$P(11) = 0$$

$$\therefore \langle 0.25d, 0.25d, 0, 0 \rangle = \langle 0.5, 0.5, 0, 0 \rangle$$

for day 2:

$$P(00) = d \times 1 \times \left( 0.5 \times \frac{3}{4} + \frac{1}{4} \times 0.5 \right)$$
$$= \cancel{0.75d} = 0.5d$$

$$P(01) = d \times 1 \times \left(0.5 \times \frac{1}{4} + 0.5 \times \frac{1}{2}\right)$$
$$= 0.375d$$

$$P(10) = 0$$

$$P(11) = 0$$

~~$$\langle 0.375d, 0.375d, 0, 0 \rangle$$~~

$$\therefore \langle 0.5d, 0.375d, 0, 0 \rangle = \langle 0.57, 0.43, 0, 0 \rangle$$
$$F = 1 \times 1 \times \frac{1}{P} = (1, 0)$$

for day 3:

$$P(00) = 0$$

$$P(01) = 0$$

$$P(10) = d \times 1 \times \left(\frac{1}{4} \times 0.43\right)$$

$$= 0.1075d$$

$$P(11) = d \times 1 \times 0$$

$$= 0 = 1 \times \frac{1}{P} \times \frac{1}{P} = (0, 0)$$

$$\langle 0, 0, 0.1075d, 0 \rangle = \langle 0, 0, 1, 0 \rangle$$

$$\frac{1}{P} = 1 \times \frac{1}{P} \times \frac{1}{P} = (0, 1)$$

c)  $\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{array} \times \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} = \begin{array}{c} (0) \\ (0) \\ (0) \\ (1) \\ (1) \\ (1) \end{array}$

0	1	2	3	
0	$\frac{5}{64}$	$\frac{1}{16}$	0	-1
1	$\frac{5}{64}$	$\frac{1}{8}$	$\frac{1}{4}$	1
2	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{3}{4}$	1
3	0	0	1	1

For  $t=2$ :

$$P(0,0) = 0$$

$$P(0,1) = \frac{1}{4} \times 1 \times 1 = \frac{1}{4}$$

$$P(1,0) = \frac{1}{2} \times 1 \times 1 + \frac{1}{4} \times 1 \times 1 \\ = \frac{3}{4}$$

$$P(1,1) = \frac{1}{4} \times 1 \times 1 + \frac{3}{4} \times 1 \times 1 \\ = 1$$

For  $t=1$

$$P(0,0) = \frac{1}{4} \times \frac{1}{4} \times 1 = \frac{1}{16}$$

$$P(0,1) = \frac{1}{2} \times \frac{1}{4} \times 1 = \frac{1}{8}$$

$$P(1,0) = \frac{1}{4} \times \frac{1}{4} \times 1 = \frac{1}{16}$$

$$P(1,1) = 0$$

for  $t = 0$

$$\begin{aligned} P(0,0) &= \frac{3}{4} \times \frac{1}{16} \times 1 + \frac{1}{4} \times \frac{1}{8} \times 1 \\ &= \frac{5}{64} \end{aligned}$$

$$\begin{aligned} P(0,1) &= \frac{1}{4} \times \frac{1}{16} + \frac{1}{2} \times \frac{1}{8} \\ &= \frac{5}{64} \end{aligned}$$

$$P(1,0) = \frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$$

$$P(1,1) = 0$$

So, using forward and backward table:

	0	1	2	3
00	0.0175d	$\frac{d}{32}$	0	0
01	0.0195d	$\frac{d}{16}$	$0.1075d$	0
10	$\frac{1}{128}d$	0	0	d
11	0	0	0	0

→

Normalizing

	0	1	2	3
00	0.92	0.33	0	0
01	0.42	0.67	1	0
10	0.16	0	0	1
11	0	0	0	0

d)

	0	1	2	3	$t = 1$
00	0.25	0.1875	0.14	0	
01	0.25	0.125	0.0625	<del>0.03125</del>	$t = 1$
10	0.25	0	0	<del>0.03125</del>	0.015625
11	0.25	0	0	0	(Ans)

For  $t = 1$

$$P(00) = 1 \times \max \left\{ 0.25 \times \frac{3}{4}, 0.25 \times \frac{1}{4} \right\}$$

$$= 0.1875$$

$$P(01) = 1 \times \max \left\{ \frac{1}{4} \times 0.25, \frac{1}{2} \times 0.25 \times \frac{1}{4} \times 0.25 \right\}$$

$$= 0.125$$

$$P(10) = 0, P(11) = 0$$

for  $t = 2$

$$P(00) = 1 \times \max \left\{ 0.1875 \times \frac{3}{4}, 0.125 \times \frac{1}{4} \right\}$$

$$= 0.14$$

$$P(01) = 1 \times \max \left\{ \frac{1}{4} \times 0.1875, \frac{1}{2} \times 0.125 \right\}$$

$$= 0.0625$$

$$P(10) = 0, \quad \text{(01020)} \quad P(11) = 0$$

for  $t = 3$

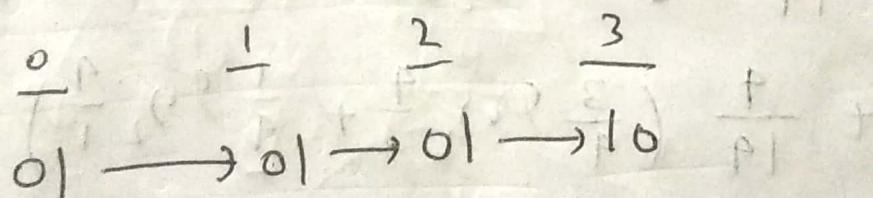
$$P(00) = 0, \quad P(01) = 0$$

$$P(10) = 1 \times \max \left\{ \frac{1}{4} \times 0.0625 \right\}$$

$$= 0.015625$$

$$P(11) = 1 \times \max \left\{ 0 \right\} = 0$$

So, most ~~likely~~ sequence,



$$\left( \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \right) \frac{1}{P(1)} = (H)$$

4. a)

1st step:

$$R(0) = \frac{9}{19} (\log_2 1) + \frac{5}{19} \left( \frac{2}{5} \log_2 \frac{5}{2} + \frac{3}{5} \log_2 \frac{5}{3} \right) + \frac{5}{19} \left( \frac{9}{5} \log_2 \frac{5}{9} + \frac{1}{5} \log_2 \frac{5}{1} \right) \\ = 0.605$$

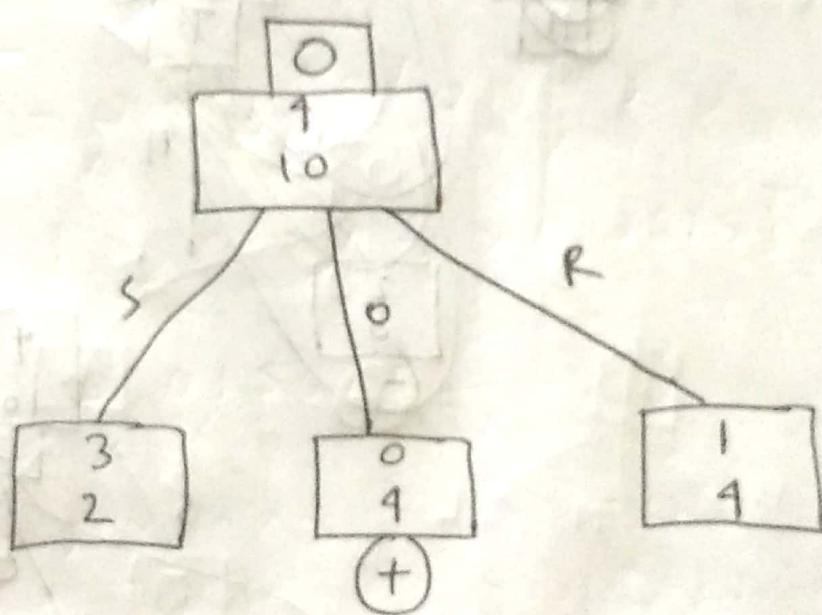
$$R(T) = \frac{9}{19} \left( \frac{2}{9} \log_2 \frac{4}{2} + \frac{2}{9} \log_2 \frac{4}{1} \right) + \frac{6}{19} \left( \frac{5}{6} \log_2 \frac{6}{5} + \frac{1}{6} \log_2 \frac{6}{1} \right) \\ + \frac{9}{19} \left( \frac{3}{4} \log_2 \frac{4}{3} + \frac{1}{4} \log_2 \frac{4}{1} \right) \\ = 0.796$$

$$R(H) = \frac{7}{19} \left( \frac{3}{7} \log_2 \frac{7}{3} + \frac{1}{7} \log_2 \frac{7}{1} \right) + \frac{7}{19} \left( \frac{6}{7} \log_2 \frac{7}{6} + \frac{1}{7} \log_2 \frac{7}{1} \right) = 0.788$$

$$R(W) = \frac{3}{19} \times \left( \frac{6}{8} \vartheta_0, \frac{8}{6} + \frac{2}{8} \vartheta_1, \frac{8}{2} \right)$$

$$= + \frac{6}{19} \times \left( \frac{4}{6} \vartheta_0, \frac{6}{4} + \frac{2}{6} \vartheta_1, \frac{6}{2} \right)$$

$$= 0.857$$

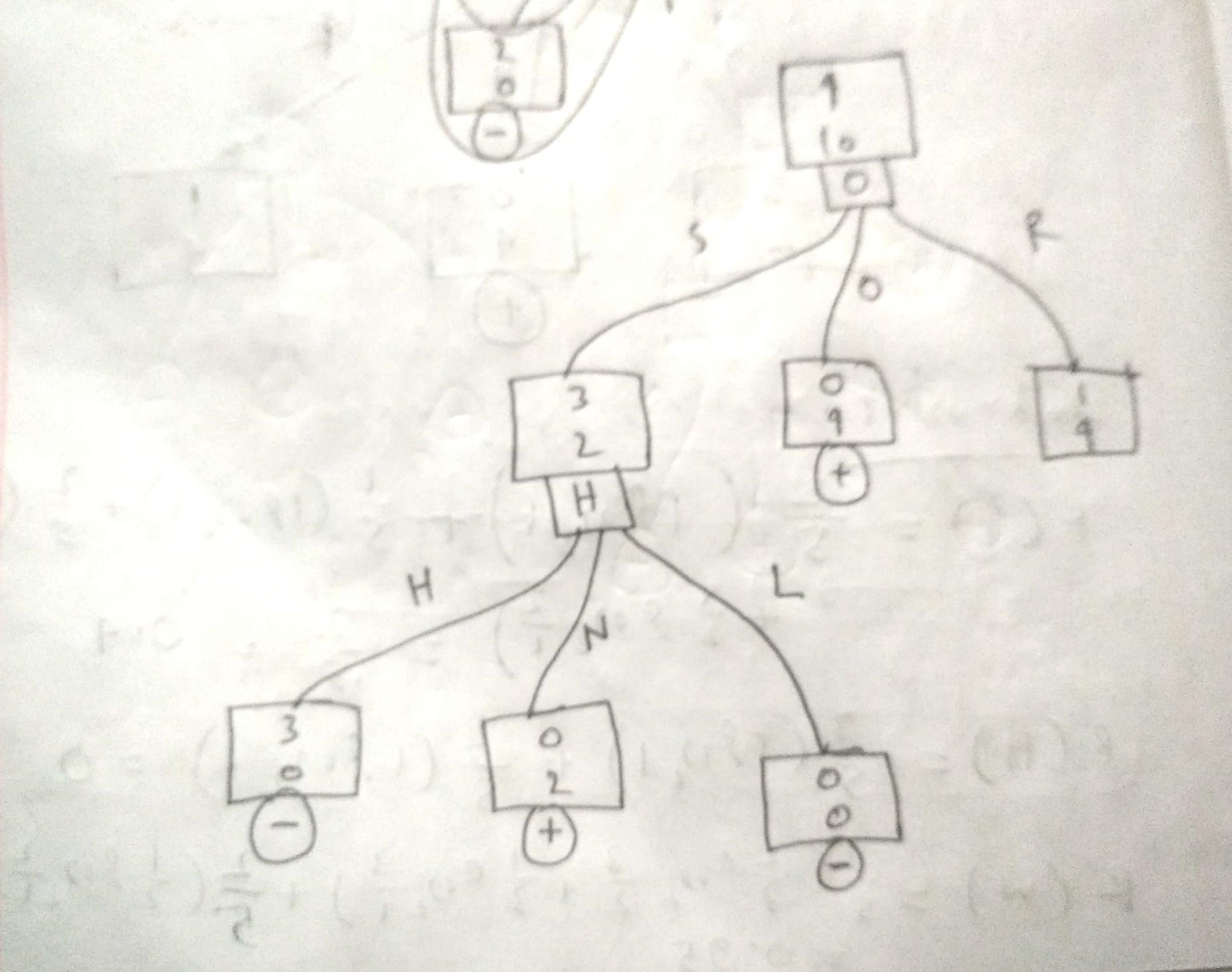


2nd step:

$$R(T) = \frac{2}{5} ((1\vartheta_0)_L 1) + \frac{1}{5} ((1\vartheta_0)_L 1) + \frac{2}{5} \left( \frac{1}{2}\vartheta_0, \frac{2}{1} \right) + \frac{1}{2} \vartheta_0, \frac{2}{1} ) = \frac{2}{5} = 0.4$$

$$R(H) = \frac{3}{5} ((1\vartheta_0)_L 1) + \frac{2}{5} ((1\vartheta_0)_L 1) = 0$$

$$R(W) = \frac{3}{5} \left( \frac{1}{3} \vartheta_0, \frac{3}{2} + \frac{1}{3} \vartheta_0, \frac{3}{1} \right) + \frac{2}{5} \left( \frac{1}{2} \vartheta_0, \frac{2}{1} + \frac{1}{2} \vartheta_0, \frac{2}{1} \right) = 0.95$$



3rd step:

$$R(T) = \cancel{0.6} \cdot \frac{3}{5} (\frac{1}{2} \delta_{0,1})$$

$$+ \frac{2}{5} \left( \frac{1}{2} \delta_{0,2} + \frac{1}{2} \delta_{0,2} \right)$$

$$= 0.4$$

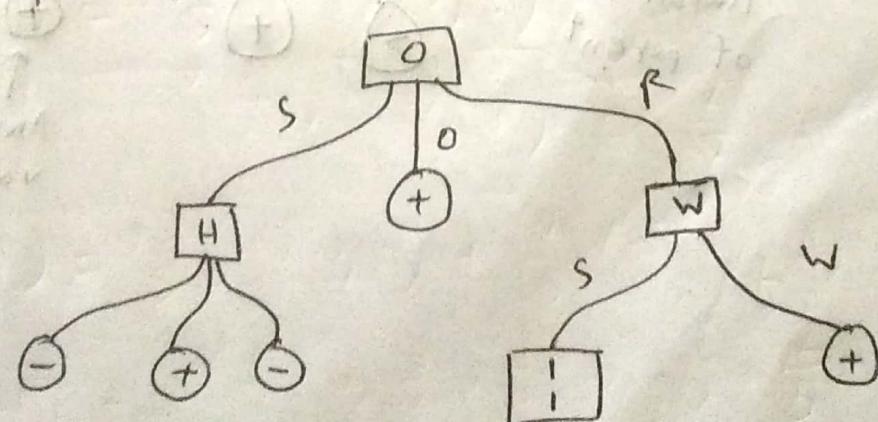
$$R(H) = \frac{2}{5} (\delta_{0,1}) + \frac{3}{5} \left( \frac{2}{3} \delta_{0,2} + \frac{1}{3} \delta_{0,2} \right)$$

$$= 0.55$$

$$R(W) = \frac{3}{5} (\delta_{0,1}) + \frac{2}{5} \left( \frac{1}{2} \delta_{0,2} + \frac{1}{2} \delta_{0,2} \right)$$

$$= 0.4$$

As, T has missing value, choosing W.



4th step:

$$R(T) = 0 \quad \text{and} \quad R(H) = 0$$

So, any one can be used.

Final tree

