Class Test 1 **Date: June 21, 2023**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

Department of Computer Science and Engineering

L-3/T-II CSE 301: Mathematical Analysis for Computer Science

Time: 30 minutes **Marks: 20**

Student Name: _____ Student No: _____

1. We are already acquainted with the Josephus problem. We are now presented with a new variation of the problem. As usual, we begin with a group of n individuals numbered from 1 to n arranged in a circular formation and eliminate every second remaining person until only one individual remains. However, in this variant, we perform the eliminations in reverse order. For instance, when $n = 10$, the sequence of eliminations is $9, 7, 5, 3, 1, 8, 4, 10, 2$, resulting in the survival of individual 6.

   Now, answer the following questions:

   (a) Formulate the recurrence relations for determining the survivor's number in this new variation of the Josephus problem. Remember to specify the base case too. (7)

   (b) Show that the survivor's number can be expressed as $n - 2l$, where $n = 2^m + l$ and $0 \leq l < 2^m$. (3)

2. We have previously studied the formulation for the minimum number of moves required in the Tower of Hanoi problem. Now, we are presented with a new variation where the tower comprises $2n$ disks having n distinct sizes and exactly two disks of each size. As usual, we can only move one disk at a time and cannot place a larger disk onto a smaller one at any time. For this particular variation, we have devised the following steps to solve the problem:

   1. We 'miraculously' transfer the top $2 \times (n-1)$ disks to the intermediary peg.
   2. Next, we move the two largest disks to the destination peg.
   3. Finally, we transfer the $2 \times (n-1)$ disks, once again 'miraculously', from the intermediary peg to the destination peg.

   Now, answer the following questions:

   (a) Formulate a recurrence relation to determine the number of moves required based on the aforementioned steps. Remember to specify the base case too. (3)

   (b) Show that the number of moves required for the given steps is equal to $2^{n+1} - 2$. (3)

   (c) Prove that this solution preserves the original order of the top $2 \times (n-1)$ disks while reversing the order of the two largest disks. (4)