Class Test 4 **Date: February 8, 2023**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

Department of Computer Science and Engineering

L-3/T-II CSE 301: Mathematical Analysis for Computer Science

Time: 30 minutes **Marks: 20**

Student Name: ______________________________ Student ID: ____________

1. Evaluate the following sum: (10)

$$\sum_{k=1}^{n} \frac{2k+3}{(2k-1)(2k+1)}$$

You are allowed to include harmonic numbers in the closed form.

Solution: $\sum_{k=1}^{n} \frac{2k+3}{(2k-1)(2k+1)} = \sum_{k=1}^{n}(\frac{2}{2k-1} - \frac{1}{2k+1}) = \sum_{k=1}^{n} \frac{1}{2k-1} + \sum_{k=1}^{n}(\frac{1}{2k-1} - \frac{1}{2k+1})$

Now, $\sum_{k=1}^{n} \frac{1}{2k-1} = 1 + \frac{1}{3} + \cdots + \frac{1}{2n-1} = (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2n-1} + \frac{1}{2n}) - (\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2n})$

$= (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{2n-1} + \frac{1}{2n}) - \frac{1}{2}(1 + \frac{1}{2} + \cdots + \frac{1}{n}) = H_{2n} - \frac{1}{2}H_n$

And, $\sum_{k=1}^{n}(\frac{1}{2k-1} - \frac{1}{2k+1}) = (1 - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{5}) + \cdots + (\frac{1}{2n-3} - \frac{1}{2n-1})$

$= 1 - \frac{1}{2n-1}$ (The internal values all cancel out, this is called telescoping)

So, the sum is $H_{2n} - \frac{1}{2}H_n + 1 - \frac{1}{2n-1}$. [Answer]

2. Evaluate the following sum: (10)

$$\sum_{k=1}^{n} ke^{-k}$$

Answer: Let $S_n = \sum_{k=1}^{n} ke^{-k} = \sum_{1 \le k \le n} ke^{-k}$

Using perturbation, $S_{n+1} = S_n + (n+1)e^{-(n+1)} = e^{-1} + \sum_{2 \le k \le n+1} ke^{-k}$

$= e^{-1} + \sum_{2 \le k+1 \le n+1} (k+1)e^{-(k+1)} = e^{-1} + \sum_{1 \le k \le n} ke^{-(k+1)} + \sum_{1 \le k \le n} e^{-(k+1)} = e^{-1} + e^{-1}S_n + \frac{e^{-2}(1-e^{-n})}{1-e^{-1}}$

$\Rightarrow (1-e^{-1})S_n = e^{-1} + \frac{e^{-2}(1-e^{-n})}{1-e^{-1}} - (n+1)e^{-(n+1)}$

$\Rightarrow S_n = (1-e^{-1})^{-1}(e^{-1} + \frac{e^{-2}(1-e^{-n})}{1-e^{-1}} - (n+1)e^{-(n+1)})$ [Answer]