

B03902062 資工二 董文捷

Guess :

$$T(n)$$

$$= 4T\left(\frac{n}{2}\right) + \Theta(1)$$

$$= c + 4 * c + 16T\left(\frac{n}{4}\right)$$

$$= 1 * c + 4 * c + 16 * c + \dots\dots$$

$$= c + \frac{4 * C * (4^{\log_2 n} - 1)}{4 - 1}$$

$$= c + \frac{4 * C * (n^{\log_2 4} - 1)}{4 - 1}$$

$$= c + \frac{4}{3} * c * (n^2 - 1)$$

$$\Theta(n^2)$$

Proof :

Let $\theta(1) = c$ (a constant)

1.

Assume $T(n) = O(n^2) \leq 2 * c * n^2 - c$

For $n = 1$ $T(1) = c \leq 2 * c * 1 - c = c$ (boundary)

If for $m < n$ the equation holds, take $m = \frac{n}{2}$

$T(n)$

$$= 4T\left(\frac{n}{2}\right) + c$$

$$\leq 4 * (2 * c * \left(\frac{n}{2}\right)^2 - c) + c$$

$$= 2 * c * n^2 - 3 * c$$

$$\leq 2 * c * n^2 - c$$

The equation also holds on n

Therefore, we can induce that $T(n)$ is $O(n^2)$

2.

Assume $T(n) = \Omega(n^2) \geq c * n^2$

For $n = 1$ $T(1) = c \geq c * 1 = c$ (boundary)

If for $m < n$ the equation holds, take $m = \frac{n}{2}$

$T(n)$

$$= 4T\left(\frac{n}{2}\right) + c$$

$$\geq 4 * \left(c * \left(\frac{n}{2}\right)^2 \right) + c$$

$$= c * n^2 + c$$

$$\geq c * n^2$$

The equation also holds on n

Therefore, we can induce that $T(n)$ is $\Omega(n^2)$

Since $T(n)$ is $O(n^2)$ and $\Omega(n^2)$, we can conclude that

$T(n)$ is $\Theta(n^2)$

Answer

1. $T(n) = \Theta(n)$ **false**

2. $T(n) = \Theta(n + \log(n))$ **false**

3. $T(n) = \Theta(n^2)$ **true**

4. $T(n) = \Theta(n^2 + \log(n))$ **false**