Guess :

T(n)

= $4T(\frac{n}{2}) + \Theta(1)$

= $c + 4 * c + 16T(\frac{n}{4})$

= 1 * c + 4 * c + 16 * c + ……

= c + $\frac{4 * C * (4^{log_2 n} - 1)}{4-1}$

= c + $\frac{4 * C * (n^{log_2 4} - 1)}{4-1}$

= c + $\frac{4}{3}$ **$* c * (n^2 - 1)$**

$\Theta(n^2)$

Proof :

Let $\Theta(1) = c$ (a constant)

1.

Assume $T(n) = O(n^2) \leqq 2 * c * n^2 - c$

For $n = 1$ $T(1) = c \leqq 2 * c * 1 - c = c$ (boundary)

If for $m < n$ the equation holds, take $m = \frac{n}{2}$

$T(n)$

$= 4T(\frac{n}{2}) + c$

$\leqq 4 * (2 * c * (\frac{n}{2})^2 - c) + c$

$= 2 * c * n^2 - 3 * c$

$\leqq 2 * c * n^2 - c$

The equation also holds on n

Therefore, we can induce that $T(n)$ is $O(n^2)$

2.

Assume $T(n) = \Omega(n^2) \geqq c * n^2$

For $n = 1$ $T(1) = c \geqq c * 1 = c$ (boundary)

If for $m < n$ the equation holds, take $m = \frac{n}{2}$

$T(n)$

$= 4T(\frac{n}{2}) + c$

$\geqq 4 * (c * (\frac{n}{2})^2) + c$

$= c * n^2 + c$

$\geqq c * n^2$

The equation also holds on n

Therefore, we can induce that $T(n)$ is $\Omega(n^2)$

Since $T(n)$ is $O(n^2)$ and $\Omega(n^2)$, we can conclude that

$T(n)$ is $\Theta(n^2)$

Answer

1. $T(n) = \Theta(n)$ false
2. $T(n) = \Theta(n + \log(n))$ false
3. $T(n) = \Theta(n^2)$ true
4. $T(n) = \Theta(n^2 + \log(n))$ false